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GLUON DISTRIBUTION FROM DEEP INELASTIC GRAVITON SCATTERING*

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ABSTRACT

We discuss how polarized and unpolarized gluon distributions may be obtained in Gedanken deep inelastic graviton scattering experiments. We also discuss how the Pontryagin current $a_\xi(x)$ evades entering into the Wilson expansion for such processes.

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I. INTRODUCTION

The internal structure of a proton is often studied by doing the deep inelastic leptonic scattering experiments.[1] However, since the gluon does not carry an electromagnetic or a weak charge, it cannot interact directly with leptonic probes. To probe the gluon structure directly, we must resort to gravitational deep inelastic scattering processes. It is true that such experiments are unlikely to be carried out in the foreseeable future, but a Gedanken experiment of this kind is useful to resolve theoretical questions concerning gluon structures and distributions.[2] The present paper consists of a generalization of such an analysis of Ref. 2, to include in our present considerations all the structure functions, both from polarized and from unpolarized beams and targets.

Our original motivation for carrying out such an analysis is the following. In a previous paper,[3] we discussed how the total helicity ΔG carried by the gluons inside a nucleon target affects the first moment of the flavor-singlet part of the deep inelastic electron scattering structure function νG_1. We concluded that ΔG appears in this moment together with $\frac{1}{2}\Delta q$, the total helicity carried by the quarks inside the nucleon, in the combination $\Delta q_G \equiv \Delta q + (\alpha_s/4\pi)(2y)\Delta G$, where y is a quantity that depends on how in detail the renormalization of the operator $a_\xi(x)$ is carried out. Here a_ξ is the Pontryagin current,[4] which is an axial vector operator of the gluon fields that corresponds to ΔG when operator product expansion method is applied.

The mathematical reason for such a combination can be traced to the the fact that the <u>operator</u> $a_\xi(x)$, is not gauge invariant, and thus cannot be present in an operator product expansion. Its occurrence in deep inelastic scattering, via the gauge-invariant diagonal matrix element $\langle a_\xi(x)\rangle$, must thus be absorbed into the quark degrees of freedom. This corresponds to the physical statement that any gluon in the problem actually resides in the cloud around the quark, at least as far as this moment of νG_1 is concerned. In this way, the necessity of a separate occurrence of ΔG, or the operator $a_\xi(x)$, is thereby avoided.

One reason why this evasion mechanism is tenable is that in deep inelastic scattering by a lepton, gluons interact with photons only in order α_s, and in the same order, the quark can also acquire a gluon cloud. If we were to carry out deep inelastic graviton scattering, when gluons interact with gravitons in order $(\alpha_s)^0$, then the cloud mechanism would no longer be a possible evasion mechanism. How then is it possible to absorb the contribution of $a_\xi(x)$ to this graviton process into the quarks so that the operator $a_\xi(x)$ effectively does not occur in the expansion of the product of two stress-energy-momentum tensor operator $\theta_{\mu\nu}(x)$? In short, how does $a_\xi(x)$ evade being seen?

We shall show in Sec. IV that in this gravitational case, an evasion mechanism is not necessary because $\langle a_\xi(x)\rangle$ does not even enter in the inclusive reactions. First, $\theta_{\mu\nu}(x)$ contains one more derivative than the current operator $J_\mu(x)$, thus giving it a tendency to yield only higher than the first moments of the gluon distribution. This prevents $\langle a_\xi(x)\rangle$ from entering all but onto structure function. On the other hand, there is a structure function which is $x\Delta G(x)$ in the parton model. $\Delta G(x)$ is a polarized gluon distribution function, whose first moment corresponds to the operator $a_\xi(x)$. However, the calculation shows that the Wilson coefficient of the first moment $\int_0^1 \Delta G(x,\theta^2)dx$ is zero in the leading order, that is, the operator $a_\xi(x)$ which corresponds to the first moment does not appear in Wilson expansion of two $\theta_{\mu\nu}$ products at least in the leading order.

Section II contains kinematical discussions for deep inelastic graviton scattering on nucleon targets. Parton model results are given in Sec. III. In Sec. IV, Wilson expansion formalism is discussed. How polarized and unpolarized gluon distributions may be probed in these Gedanken experiments can be seen in the resulting formulae. How the presence of $\langle a_\xi(x)\rangle$ is avoided is also discussed there. Finally, Sec. V contains a conclusion and Appendix A contains a kinematical calculation necessary in carrying out the Wilson expansion discussions.

II. KINEMATICS

The stress-energy-momentum tensor $\theta_{\mu\nu}$ is given by[2]

$$\theta_{\mu\nu} = \theta^q_{\mu\nu} + \theta^G_{\mu\nu} \tag{1}$$

where

$$\theta^q_{\mu\nu}(x) = i\bar{q}(x)\,\gamma_\mu\,\nabla_\nu\,q(x) \tag{2}$$

and

$$\theta^G_{\mu\nu}(x) = G^a_{\mu\alpha}(x)\,G^a_{\alpha\nu}(x) - \frac{1}{4}\,g_{\mu\nu}\left(G^a_{\alpha\beta}\,G^a_{\beta\alpha}\right) \tag{3}$$

are respectively the quark and gluon components of the tensor. The operator ∇_ν operating on the quark fields $\bar{q}(x)$ and $q(x)$ is defined to be

$$\nabla_\nu = \frac{1}{2}\overleftrightarrow{\partial}_\nu - igt_a\,B^a_\nu(x) \tag{4}$$

where t_a is the usual SU(3) color matrix and $B^a_\nu(x)$ is the gluon field.

Symmetrization over μ and ν is understood in the definition of $\theta^q_{\mu\nu}$. The operator $G^a_{\mu\nu}$ is defined by

$$G^a_{\mu\nu}(x) = \partial_\mu B^a_\nu(x) - \partial_\nu B^a_\mu(x) + gC_{abc} B^b_\mu(x) B^c_\nu(x) \quad . \tag{5}$$

The stress-energy-momentum tensor is a conserved tensor,

$$\partial_\mu \theta^{\mu\nu}(x) = 0 \quad , \tag{6}$$

and it can be divided into a spin two traceless part $\theta^{(2)}_{\mu\nu}$ and a spin zero part $\theta^{(0)}_{\mu\nu}$

$$\theta_{\mu\nu}(x) = \theta^{(2)}_{\mu\nu}(x) + \theta^{(0)}_{\mu\nu}(x) \quad , \tag{7}$$

$$\theta^{(0)}_{\mu\nu}(x) = \frac{1}{3}\left(g_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2}\right) \theta^o(x) \quad . \tag{8}$$

Thus

$$\theta^{(2)\mu}_{\mu}(x) = 0 \quad , \tag{9}$$

$$\theta^\mu_\mu(x) = \theta^{(0)}(x) \quad , \tag{10}$$

and

$$\partial^\mu \theta^{(2)}_{\mu\nu}(x) = 0 \quad . \tag{11}$$

The total cross section for a graviton of momentum q to scatter on a polarized nucleon target of momentum P, mass M, and spin vector S is given by the tensor

$$W_{\mu\nu,\kappa\lambda}(S) \equiv 2\pi^2 \int e^{iqx} \langle P,S| \theta_{\mu\nu}(x)\, \theta_{\kappa\lambda}(0) |P,S\rangle\, d^4x \quad . \tag{12}$$

In obvious notations, the decomposition (7) induces a decomposition in W:

$$W_{\mu\nu,\kappa\lambda} = W^{(22)}_{\mu\nu,\kappa\lambda} + W^{(20)}_{\mu\nu,\kappa\lambda} + W^{(02)}_{\mu\nu,\kappa\lambda} + W^{(00)}_{\mu\nu,\kappa\lambda} \quad . \tag{13}$$

First consider the decomposition of $W^{(22)}_{\mu\nu,\kappa\lambda}$ in terms of the structure functions F_i:

$$W^{(22)}_{\mu\nu,\kappa\lambda} = \sum_{i=1}^{8} F_i \, A^{(i)}_{\mu\nu,\kappa\lambda} \equiv W^{(22)}_{\mu\nu,\kappa\lambda}(S) \quad . \tag{14}$$

In terms of the following quantities:

$$\begin{aligned}
\bar{\pi}_{\mu\nu} &\equiv \bar{P}_\mu \bar{P}_\nu - \frac{1}{3} \bar{g}_{\mu\nu} \bar{P}^2 \\
\bar{P}_\mu &\equiv P_\mu - (P\cdot q) q_\mu / q^2 \\
\bar{g}_{\mu\nu} &\equiv g_{\mu\nu} - q_\mu q_\nu / q^2 \\
\tilde{S}_\mu &\equiv P\cdot q \, S_\mu - q\cdot S \, P_\mu
\end{aligned} \tag{15}$$

each of which is orthogonal to q, and the following conventional notations,

$$\begin{aligned}
\varepsilon_{\mu\nu\alpha\beta} &\equiv [\mu\nu\alpha\beta] \; ; \qquad \varepsilon^{0123} = 1 \\
\varepsilon_{\mu\nu\alpha\beta} B^\alpha C^\beta &\equiv [\mu\nu BC] \, , \quad \text{etc.} \quad ,
\end{aligned} \tag{16}$$

the eight independent tensor in (14) can be taken to be:

$$A^{(1)}_{\mu\nu,\kappa\lambda} = \bar{\pi}_{\mu\nu} \, \bar{\pi}_{\kappa\lambda} \tag{17}$$

$$\begin{aligned}
A^{(2)}_{\mu\nu,\kappa\lambda} = {} & \bar{P}_\mu \bar{P}_\kappa \bar{g}_{\nu\lambda} + \bar{P}_\nu \bar{P}_\kappa \bar{g}_{\mu\lambda} + \bar{P}_\mu \bar{P}_\lambda \bar{g}_{\nu\kappa} + \bar{P}_\nu \bar{P}_\lambda \bar{g}_{\mu\kappa} \\
& - \frac{4}{3}\left[\bar{P}_\mu \bar{P}_\nu \bar{g}_{\lambda\kappa} + \bar{P}_\lambda \bar{P}_\kappa \bar{g}_{\mu\nu}\right] + \frac{4}{9} \bar{P}^2 \bar{g}_{\mu\nu} \bar{g}_{\kappa\lambda}
\end{aligned} \tag{18}$$

$$A'^{(3)}_{\mu\nu,\kappa\lambda} = \bar{g}_{\mu\kappa}\bar{g}_{\nu\lambda} + \bar{g}_{\mu\lambda}\bar{g}_{\kappa\nu} - \frac{2}{3}\bar{g}_{\mu\nu}\bar{g}_{\kappa\lambda} \tag{19}$$

$$\begin{aligned} A^{(4)}_{\mu\nu,\kappa\lambda} &= \bar{P}_\mu\bar{P}_\kappa[\nu\lambda qS] + \bar{P}_\nu\bar{P}_\kappa[\mu\lambda qS] + \bar{P}_\mu\bar{P}_\lambda[\nu\kappa qS] + \bar{P}_\nu\bar{P}_\lambda[\mu\kappa qS] \\ &+ \frac{2}{3}\Big\{\bar{g}_{\mu\nu}\bar{P}_\kappa[\lambda PqS] + \bar{g}_{\mu\nu}\bar{P}_\lambda[\kappa PqS] \\ &- \bar{g}_{\kappa\lambda}\bar{P}_\mu[\nu PqS] - \bar{g}_{\kappa\lambda}\bar{P}_\nu[\mu PqS]\Big\} \equiv A^{(4)}_{\mu\nu,\kappa\lambda}(S) \end{aligned} \tag{20}$$

$$\begin{aligned} A^{(5)}_{\mu\nu,\kappa\lambda} &= A^{(4)}_{\mu\nu,\kappa\lambda}(\tilde{S}) = P\cdot q\; A^{(4)}_{\mu\nu,\kappa\lambda} - q\cdot S\Big\{\bar{P}_\mu\bar{P}_\kappa[\nu\lambda qP] \\ &+ \bar{P}_\nu\bar{P}_\kappa[\mu\lambda qP] + \bar{P}_\mu\bar{P}_\lambda[\nu\kappa qP] + \bar{P}_\nu\bar{P}_\lambda[\mu\kappa qP]\Big\} \end{aligned} \tag{21}$$

$$\begin{aligned} A^{(6)}_{\mu\nu,\kappa\lambda} &= \bar{g}_{\mu\kappa}[\nu\lambda qS] + \bar{g}_{\nu\kappa}[\mu\lambda qS] + \bar{g}_{\mu\lambda}[\nu\kappa qS] + \bar{g}_{\nu\lambda}[\mu\kappa qS] \\ &\equiv A^{(6)}_{\mu\nu,\kappa\lambda}(S) \end{aligned} \tag{22}$$

$$\begin{aligned} A^{(7)}_{\mu\nu,\kappa\lambda} &= A^{(6)}_{\mu\nu,\kappa\lambda}(\tilde{S}) = P\cdot q\; A^{(6)}_{\mu\nu,\kappa\lambda} - q\cdot S\Big\{\bar{g}_{\mu\kappa}[\nu\lambda qP] \\ &+ \bar{g}_{\nu\kappa}[\mu\lambda qP] + \bar{g}_{\mu\lambda}[\nu\kappa qP] + \bar{g}_{\nu\lambda}[\mu\kappa qP]\Big\} \end{aligned} \tag{23}$$

$$A^{(8)}_{\mu\nu,\kappa\lambda} = \bar{\pi}_{\mu\nu}\Big\{\bar{P}_\kappa[\lambda PqS] + \bar{P}_\lambda[\kappa PqS]\Big\} - \bar{\pi}_{\kappa\lambda}\Big\{\bar{P}_\mu[\nu PqS] + \bar{P}_\nu[\mu PqS]\Big\}\,. \tag{24}$$

Every $A^{(i)}_{\mu\nu,\kappa\lambda}$ satisfies (9) and (11), viz., it is traceless and orthogonal to q. Furthermore, as a consequence of time reversal invariance, we can check explicitly that

$$W_{\mu\nu,\kappa\lambda}(S) = W_{\kappa\lambda,\mu\nu}(-S) \quad . \tag{25}$$

The tensors $A^{(1)}$-$A^{(3)}$ are present already in unpolarized scattering, as no S is involved, but the structure functions F_4-F_8 can be measured only with polarized beam and targets. Of these four, $A^{(4)}$ and $A^{(6)}$ contain S, thus F_4 and F_6 are similar to G_1 in electron deep inelastic scattering, and can thus be obtained when the nucleon spin is polarized parallel to its direction of motion. In contrast, $A^{(5)}$, $A^{(7)}$ and $A^{(8)}$ all vanish if S is replaced by P, thus F_5, F_7 and F_8 are similar to G_2 in electron deep inelastic scattering, and can be measured only when the nucleon spin is polarized perpendicular to its direction of motion.[3]

That there are eight independent amplitudes in $W^{(22)}$ can best be seen in the helicity representation $W^{(22)}_{bs,ar}$, where b,s,a,r refer to the final graviton, final nucleon, initial graviton, and initial nucleon helicities, respectively. Because total cross section involves only the forward amplitude, helicity is conserved:

$$b - s = a - r \qquad . \tag{26}$$

Time reversal and parity invariance also demand

$$\begin{aligned} W_{bs,ar} &= W_{ar,bs} \\ W_{bs,ar} &= \eta W_{-b-s,-a-r} \end{aligned} \tag{27}$$

where η is some suitable phase factor. Using (26)-(27), the independent helicity amplitudes are (bs,ar) = $(2\tfrac{1}{2},1\tfrac{1}{2})$, $(2\tfrac{1}{2},1-\tfrac{1}{2})$, $(2-\tfrac{1}{2},2-\tfrac{1}{2})$, $(1\tfrac{1}{2},1\tfrac{1}{2})$, $(1\tfrac{1}{2},0-\tfrac{1}{2})$, $(1-\tfrac{1}{2},1-\tfrac{1}{2})$, $(0\tfrac{1}{2},0\tfrac{1}{2})$, $(0\tfrac{1}{2},-1,-\tfrac{1}{2})$, and there are eight of them.

The independent amplitudes and structure functions for $W^{(20)}$, $W^{(02)}$ and $W^{(00)}$ are:

$$W^{(20+02)}_{\mu\nu,\kappa\lambda}(S) = F_9\, A^{(9)}_{\mu\nu,\kappa\lambda} + F_{10}\, A^{(10)}_{\mu\nu,\kappa\lambda}$$

$$W^{(00)}_{\mu\nu,\kappa\lambda} = F_{11}\, A^{(11)}_{\mu\nu,\kappa\lambda} \tag{28}$$

$$A^{(9)}_{\mu\nu,\kappa\lambda} = \bar{g}_{\mu\nu}\left\{\bar{P}_\kappa[\lambda PqS] + \bar{P}_\lambda[\kappa PqS]\right\} - \bar{g}_{\kappa\lambda}\left\{\bar{P}_\mu[\nu PqS] + \bar{P}_\nu[\mu PqS]\right\}$$

$$A^{(10)}_{\mu\nu,\kappa\lambda} = \bar{\pi}_{\mu\nu}\,\bar{g}_{\kappa\lambda} + \bar{g}_{\mu\nu}\,\bar{\pi}_{\kappa\lambda} \tag{29}$$

$$A^{(11)}_{\mu\nu,\kappa\lambda} = \bar{g}_{\mu\nu}\,\bar{g}_{\kappa\lambda}$$

They correspond to the helicity amplitudes $W^{(20)}_{0\frac{1}{2},0\frac{1}{2}} = W^{(02)}_{0\frac{1}{2},0\frac{1}{2}}$, $W^{(20)}_{-1-\frac{1}{2},0\frac{1}{2}} = W^{(02)}_{0\frac{1}{2},-1,-\frac{1}{2}}$, and $W^{(00)}_{0\frac{1}{2},0\frac{1}{2}}$. The tensors $A^{(10)}$ and $A^{(11)}$ are unpolarized but the tensor $A^{(9)}$ is linear in S. Moreover, $A^{(9)}$ vanishes if S is replaced by P, so F_9 acts like G_2 in electron deep inelastic scattering. Finally, we can also check from (29) that Eq. (25) remains valid.

III. PARTON MODEL

It is straightforward to compute the structure functions F_i defined in (14) and (28). From (1)-(3), and (12), we let

$$H_i \equiv K_i\, F_i \tag{30}$$

then

$$
\begin{aligned}
H_1 &= x^4\left[q(x)+\bar{q}(x)+G(x)\right] \\
H_2 &= x^2\left[q(x)+\bar{q}(x)+4G(x)\right] \\
H_3 &= G(x) \\
H_4 &= x^3\left[\frac{1}{2}\Delta q(x)+\frac{1}{2}\Delta\bar{q}(x)+\Delta G(x)\right] \\
H_6 &= x\,\Delta G(x) \\
H_i &= 0 \qquad (i=5\,,\,7-11)
\end{aligned}
\tag{31}
$$

provided the normalization factors K_i are chosen to be

$$
\begin{aligned}
K_1 &= -\,q^2/2 \\
K_2 &= -\,8 \\
K_3 &= -\,8/q^2 \\
K_4 &= -\,2q^2/iM \\
K_6 &= -\,8/iM\ .
\end{aligned}
\tag{32}
$$

In Eq. (31),

$$
q(x) = \sum_{i=1}^{f}\left[q_{i+}(x)+q_{i-}(x)\right] \quad , \quad \bar{q}(x) = \sum_{i=1}^{f}\left[\bar{q}_{i+}(x)+\bar{q}_{i-}(x)\right] \tag{33}
$$

are the distribution functions for quarks and antiquarks with momentum fraction x, of any flavor i and carrying either + or − helicity. Similarly, G(x) is the gluon distribution function. The polarized quark, antiquark, gluon distribution functions are denoted by $\frac{1}{2}\Delta q$, $\frac{1}{2}\Delta\bar{q}$

and ΔG respectively. For example,

$$\Delta q(x) = \sum_{i=1}^{f} \left[q_{i+}(x) - q_{i-}(x) \right] \quad . \tag{34}$$

The structure function H_1 is given in Ref. 2 where it is called xS. It measures x^3 times the total momentum density carried by all the partons. Similarly, the structure function H_3 measures the gluon density and H_4 measures x^3 times the parton total helicity distribution.

IV. WILSON EXPANSION

We start with the tensor

$$\begin{aligned} \widetilde{W}_{\mu\nu,\kappa\lambda} &\equiv 2\pi^2 i \int d^4x \; e^{iqx} \left\langle P,S \middle| T\left(\theta_{\mu\nu}(x)\, \theta_{\kappa\lambda}(0)\right) \middle| P,S \right\rangle \\ &\equiv \sum_{i=1}^{11} \widetilde{F}_i \; A^{(i)}_{\mu\nu,\kappa\lambda} \equiv \sum_{i=1}^{11} \widetilde{H}_i \; B^{(i)}_{\mu\nu,\kappa\lambda} \quad . \end{aligned} \tag{35}$$

Whose discontinuity produces the cross sectional tensor W in (12):

$$W_{\mu\nu,\kappa\lambda} = - iD_{isc}\left(\widetilde{W}_{\mu\nu,\kappa\lambda}\right) \quad . \tag{36}$$

Making an operator product expansion of the product of two θ's, the tensor $\widetilde{W}$ can be written as

$$\widetilde{W}_{\mu\nu,\kappa\lambda} = \frac{1}{\pi} \sum_n q^{\mu_1} q^{\mu_2} \cdots q^{\mu_n} \left(\frac{2}{Q^2}\right)^n \sum_{a=q,G,\ldots} \left\langle P,S \middle| V^a_{\mu\nu,\lambda\kappa;\mu_1\ldots\mu_n} \middle| P,S \right\rangle \tag{37}$$

where $Q^2 = -q^2 > 0$.

The tensors $A^{(i)}$ and $B^{(i)}$ in (35) are related by

$$A^{(i)}_{\mu\nu,\kappa\lambda} = K_i \, B^{(i)}_{\mu\nu,\kappa\lambda} \quad . \tag{38}$$

The factors K_i for i = 1-4,6 are defined in Eq. (32). We shall now define the rest to be:

$$\begin{aligned} K_5 &= -\,q^4/iM \\ K_7 &= -\,2q^2/iM \\ K_8 &= -\,q^4/iM \\ K_9 &= K_4 \\ K_{10} &= K_2 \\ K_{11} &= K_3 \end{aligned} \tag{39}$$

In order to write down and compute V in a somewhat compact form, it is convenient to rewrite the tensors $A^{(i)}$ and $B^{(i)}$ so that their explicit dependences on q,P,S are factored out. We thus write, for i = 1,2,3,10,11,

$$A^{(i)}_{\mu\nu,\kappa\lambda} = \left(Q^2\right)^{-b_i} t^{(i)}_{\mu\nu,\kappa\lambda;\mu_1\ldots\mu_{e_i};\nu_1\ldots\nu_{f_i}} \left(\prod_{k=1}^{e_i} q^{\mu_k}\right)\left(\prod_{j=1}^{f_i} P^{\nu_j}\right) \tag{40}$$

where b_i, e_i, f_i are integers listed in Table I. For i = 4-8, we let

$$A^{(i)}_{\mu\nu,\kappa\lambda} = \left(Q^2\right)^{-b_i} t^{(i)}_{\mu\nu,\kappa\lambda;\mu_1\ldots\mu_{e_i};\nu_1\ldots\nu_{f_i},\xi} \left(\prod_{k=1}^{e_i} q^{\mu_k}\right)\left(\prod_{j=1}^{f_i} P^{\nu_j}\right) S^{\xi} \tag{41}$$

In Eq. (40) [Eq. (41)] the tensor t is of rank $4+e_i+f_i$ $(4+e_i+f_i+1)$ symmetric respectively in all the μ and all the ν indices, and is independent of P, q and S.

For later purposes, we would need to know what happens if we interchange S with one of the P's in (41). The resulting tensors A', defined by

$$A'_{\mu\nu,\kappa\lambda} = \left(Q^2\right)^{-\bar{b}} t_{\mu\nu,\kappa\lambda;\mu_1\ldots\mu_e;\nu_1\ldots\nu_f,\xi} \left(\prod_{i=1}^{e} q^{\mu_i}\right)$$

$$\times \left(\frac{1}{f}\sum_{k=1}^{f} S^{\nu_k} \prod_{\substack{j=1\\ j\neq k}}^{f} P^{\nu_j}\right) P^{\xi} \equiv K\, B'_{\mu\nu,\kappa\lambda} \tag{42}$$

are computed in Appendix A. The result is:

$$B'^{(i)}_{\mu\nu,\kappa\lambda} = 0 \qquad (i = 6,7)$$

$$B'^{(4)}_{\mu\nu,\kappa\lambda} = \frac{7}{4} B^{(4)}_{\mu\nu,\kappa\lambda} + xB^{(5)}_{\mu\nu,\kappa\lambda} + \frac{1}{4x^2} B^{(6)}_{\mu\nu,\kappa\lambda} + x^2 B^{(8)}_{\mu\nu,\kappa\lambda}$$

$$B'^{(5)}_{\mu\nu,\kappa\lambda} = -\frac{1}{3} B^{(5)}_{\mu\nu,\kappa\lambda} \tag{43}$$

$$B'^{(8)}_{\mu\nu,\kappa\lambda} = -\frac{1}{4} B^{(8)}_{\mu\nu,\kappa\lambda}$$

$$B'^{(9)}_{\mu\nu,\kappa\lambda} = -\frac{1}{2} B^{(9)}_{\mu\nu,\kappa\lambda}$$

We also need to know what happens to $A^{(i)}$ and $B^{(i)}$ if the S inside is replaced by P. Denoting the resulting tensors by $A^{11(i)}$ and $B^{11(i)}$, we have by definition

$$A^{11}_{\mu\nu,\kappa\lambda} = \left(Q^2\right)^{-b} t_{\mu\nu,\kappa\lambda;\mu_1\ldots\mu_e;\nu_1\ldots\nu_f,\xi}\left(\prod_{i=1}^{e} q^{\mu_i}\right)\left(\prod_{j=1}^{f} p^{\nu_j}\right)p^{\xi}$$

$$= K\,B^{11}_{\mu\nu,\kappa\lambda} \quad . \tag{44}$$

Then

$$B^{11(i)}_{\mu\nu,\kappa\lambda} = 0 \qquad (i = 5,7,8,9)$$

$$B^{11(4)}_{\mu\nu,\kappa\lambda} = -\frac{q^2}{2xq\cdot S}\left(B^{(4)}_{\mu\nu,\kappa\lambda} + xB^{(5)}_{\mu\nu,\kappa\lambda}\right) \tag{45}$$

$$B^{11(6)}_{\mu\nu,\kappa\lambda} = -\frac{q^2}{2xq\cdot S}\left(B^{(6)}_{\mu\nu,\kappa\lambda} + xB^{(7)}_{\mu\nu,\kappa\lambda}\right) \quad .$$

Now we are ready to write down V of Eq. (37). To avoid too long a formula, we shall divide V into the sum of unpolarized and polarized parts. Then we may write

$$V^{a}_{\mu\nu,\kappa\lambda;\mu_1\ldots\mu_n}(\text{unpolarized}) = \sum_{i=1-3,10-11}\left(K_i\right)^{-1}\left(Q^2\right)^{-b_i}\left(\frac{Q^2}{2}\right)^{e_i}$$

$$\times\, t^{(i)}_{\mu\nu,\kappa\lambda;\mu_1\ldots\mu_{e_i};\nu_1\ldots\nu_{f_i}}\ \frac{1}{2}\tilde{C}^{i}_{a}\ O^{a}_{\mu_{e_i+1}\ldots\mu_n;\nu_1\ldots\nu_{f_i}} \tag{46}$$

$$V^{a}_{\mu\nu,\kappa\lambda;\mu_1\ldots\mu_n}(\text{polarized}) = \sum_{i=4-9}\left(K_i\right)^{-1}\left(Q^2\right)^{-b_i}\left(\frac{Q^2}{2}\right)^{e_i}$$

$$\times\, t^{(i)}_{\mu\nu,\kappa\lambda;\mu_1\ldots\mu_{e_i};\nu_1\ldots\nu_{f_i},\xi}$$

$$\times\,\frac{1}{4M}\left\{\tilde{C}^{i}_{a}\ R^{aS}_{\xi\mu_{e_i+1}\ldots\mu_n;\nu_1\ldots\nu_{f_i}} + \tilde{D}^{i}_{a}\ R^{aA}_{\xi\mu_{e_i+1}\ldots\mu_n;\nu_1\ldots\nu_{f_i}}\right\} \tag{47}$$

where $\tilde{C}$ and $\tilde{D}$ are the Wilson coefficients. The operator O^a is a tensor operator symmetric and traceless in all its indices. Its nucleon matrix element has the form

$$\left\langle P,S\middle|O^a_{\alpha_1\ldots\alpha_\ell}\middle|P,S\right\rangle = 2F^a_\ell\left(\prod_{i=1}^{\ell} P_{\alpha_k} - \text{traces}\right) \quad . \tag{48}$$

The operators R^{aS} and R^{aA} are pseudotensor operators; the former is symmetric and traceless in all its indices, and the latter is anti-symmetric in the first two indices and is symmetric and traceless in the rest of the indices. Their nucleon matrix elements may be written as

$$\left\langle P,S\middle|R^{aS}_{\alpha_1\ldots\alpha_\ell}\middle|P,S\right\rangle = 4M\,\Delta F^a_\ell\left(\sum_{i=1}^{\ell} S_{\alpha_i}\prod_{j\neq i} P_{\alpha_j} - \text{traces}\right) \tag{49}$$

$$\left\langle P,S\middle|R^{aA}_{\alpha_1\ldots\alpha_\ell}\middle|P,S\right\rangle = 4M\,\delta F^a_\ell\left(S_{\alpha_1}P_{\alpha_2} - S_{\alpha_2}P_{\alpha_1}\right)\left(\prod_{j=3}^{\ell} P_{\alpha_j} - \text{traces}\right) . \tag{50}$$

To proceed with the calculation, let us ignore the trace terms in (48)-(50) because they give rise to corrections down by powers of Q^2. Then using (37), (38), (40), (46) and (48), we get

$$\tilde{W}_{\mu\nu,\kappa\lambda}(\text{unpolarized}) = \frac{1}{\pi}\sum_n\sum_a\sum_{i=1-3,10-11}\tilde{C}^i_{an}\,F^a_{\ell ni}\left(\frac{1}{x}\right)^{m_{ni}} B^{(i)}_{\mu\nu,\kappa\lambda} \tag{51}$$

where the values of

$$\ell_{ni} = n - e_i + f_i \tag{52}$$
$$m_{ni} = n - e_i$$

are given in Table I. Thus

$$\tilde{H}_i = \frac{1}{\pi}\sum_n\sum_a\left(\frac{1}{x}\right)^{m_{ni}} F^a_{\ell ni}\, \tilde{C}^i_{an} \qquad (i = 1\text{-}3, 10\text{-}11) \quad . \tag{53}$$

Similarly,

$$\tilde{W}_{\mu\nu,\kappa\lambda}(\text{polarized}) = \frac{1}{\pi}\sum_n\sum_a\sum_{i=4-9}\left(\frac{1}{x}\right)^{m_{ni}}$$

$$\times\left\{\tilde{C}^i_{an}\,\Delta F^a_{\ell ni}\left[B^{(i)}_{\mu\nu,\kappa\lambda} + f_i B'^{(i)}_{\mu\nu,\kappa\lambda} + m_{ni}(P\cdot q)^{-1}(S\cdot q)B^{11(i)}_{\mu\nu,\kappa\lambda}\right]\right.$$

$$\left. + \tilde{D}^i_{an}\,\delta F^a_{\ell ni}\left[B^{(i)}_{\mu\nu,\kappa\lambda} - \delta_i B'^{(i)}_{\mu\nu,\kappa\lambda} - (1-\delta_i)\frac{S\cdot q}{P\cdot q}B^{11(i)}_{\mu\nu,\kappa\lambda}\right]\right\} \tag{54}$$

where $\delta_i = 0$, if $f_i = 0$, and $\delta_i = 1$, if $f_i \neq 0$. Using (43), (45) and Table I, we get

$$\tilde{H}_4 = \frac{1}{\pi}\sum_n\sum_a\left(\frac{1}{x}\right)^{n-5}\left[\left(n-\frac{1}{2}\right)\tilde{C}^4_{an}\,\Delta F^a_{n-3} - \frac{3}{4}\tilde{D}^4_{an}\,\delta F^a_{n-3}\right] \tag{55}$$

$$\tilde{H}_5 = \frac{1}{\pi}\sum_n\sum_a\left(\frac{1}{x}\right)^{n-6}\left[2\tilde{C}^4_{an}\,\Delta F^a_{n-3} - \tilde{D}^4_{an}\,\delta F^a_{n-3} + \frac{4}{3}\tilde{D}^5_{an}\,\delta F^a_{n-3}\right] \tag{56}$$

$$\tilde{H}_6 = \frac{1}{\pi}\sum_n\sum_a\left(\frac{1}{x}\right)^{n-3}\left[\frac{1}{2}\tilde{C}^4_{an}\,\Delta F^a_{n-3} + (n-2)\tilde{C}^6_{an}\,\Delta F^a_{n-3} - \tilde{D}^4_{an}\,\delta F^a_{n-3}\right] \tag{57}$$

$$\tilde{H}_7 = \frac{1}{\pi}\sum_n\sum_a\left(\frac{1}{x}\right)^{n-4}\left[(n-3)\tilde{C}^6_{an}\,\Delta F^a_{n-3} + \tilde{C}^7_{an}\,\Delta F^a_{n-3}\right.$$

$$\left. - \tilde{D}^6_{an}\,\delta F^a_{n-3} + \tilde{D}^7_{an}\,\delta F^a_{n-3}\right] \tag{58}$$

$$\tilde{H}_8 = \frac{1}{\pi}\sum_n\sum_a\left(\frac{1}{x}\right)^{n-7}\left[2\tilde{C}^4_{an}\,\Delta F^a_{n-3} - \tilde{D}^4_{an}\,\delta F^a_{n-3} + \frac{5}{4}\tilde{D}^8_{an}\,\delta F^a_{n-3}\right] \tag{59}$$

$$\tilde{H}_9 = \frac{1}{\pi}\sum_n\sum_a\left(\frac{1}{x}\right)^{n-5}\frac{3}{2}\tilde{D}^9_{an}\,\delta F^a_{n-3} \quad . \tag{60}$$

In Eqs. (53)-(60), n must be taken to be $\geq e_i$. Thus $m_{ni} \geq 0$ and in these equations the powers of $1/x$ are always non-negative. Since the rank of R^{aS} is $\ell_{ni} + 1 = m_{ni} + f_i + 1 \geq f_i + 1$, the axial vector gluon operator

$$a_\xi(x) = R^{aS}_\xi(x) = -\epsilon_{\xi\alpha\beta\gamma}\left[G^{\alpha\beta}_a(x)\, B^\gamma_a(x) - \frac{g}{3} C_{abc}\, B^\alpha_a(x)\, B^\beta_b(x)\, B^\gamma_c(x)\right] \tag{61}$$

mentioned in the Introduction may enter only when $m_{ni} = 0$ and $f_i = 0$. This happens only when $i = 6$ and $n = 3$. Whether this operator enters effectively is therefore a question whether the Wilson coefficient C^6_{an} for $a = G$ and $n = 3$ may be taken to be zero or not. From (53)-(60), this coefficient affects only H_6, thus it is this structure coefficient that we should study for a resolution of the question of evasion of $a_\xi(x)$ mentioned in the Introduction.

The explicit forms of the operators O^a, R^{aS} and R^{aA} are well known[5] but are not relevant for our purpose here. We shall only need to know that they are normalized in such a way that F^a, ΔF^G, δF^G are unity for all ℓ when the nucleon states in (48)-(50) are replaced by free quark (for $a = q$) and gluon states (for $a = G$). Similarly, we shall take ΔF^q and δF^q to be ½ when the nucleon is replaced by a free quark.

With this normalization, we may now calculate the Wilson coefficient in the parton model approximation. To do that we make use of the result of Sec. III and the connection between $\tilde{W}$ and W in Eq. (36).

Crossing symmetry for $\tilde{W}_{\mu\nu,\kappa\lambda}$ demands that

$$\tilde{W}_{\mu\nu,\kappa\lambda}(q,P) = \tilde{W}_{\kappa\lambda,\mu\nu}(-q,P) \quad . \tag{62}$$

Combined with the time-reversal-invariance relation (25), we see that $\tilde{H}_i$, as a function of $x = Q^2/2P\cdot q$, is symmetric in x for i = 1-4,6,8-11 and is antisymmetric for i = 5,7.

We start with the parton relation for H_1 given in Eq. (31). If we examine the calculation leading to Eq. (31), we can see that strictly speaking, the expression for H_1 should be written as

$$H_1 = x\int \xi^4 \;\delta(\xi-x)\left[q(\xi)+\bar{q}(\xi)+G(\xi)\right]\frac{d\xi}{\xi} \quad . \tag{63}$$

From (36) and the crossing symmetry, we conclude that

$$\begin{aligned}\tilde{H}_1 &= \frac{x}{2\pi}\int \xi^4\left[\frac{1}{x-\xi}+\frac{1}{x+\xi}\right]\left[q(\xi)+\bar{q}(\xi)+G(\xi)\right]\frac{d\xi}{\xi} \\ &= \frac{1}{\pi}\sum_{\substack{m=0\\ m\ \text{even}}}^{\infty}\left(\frac{1}{x}\right)^m\int_0^1 \xi^{m+3}\left[q(\xi)+\bar{q}(\xi)+G(\xi)\right]d\xi \quad .\end{aligned} \tag{64}$$

We may now compare this with (53). From Table I for i = 1, we see that $m_{n1} = n-8 \equiv m$ and $\ell_{n1} = m+4$. Thus in the parton approximation

$$\begin{aligned} 2F^a_{m+4}\;\tilde{C}^1_{q,m+8} &= \int_0^1 \xi^{m+4}\left[q(\xi)+\bar{q}(\xi)\right]d\xi \\ 2F^G_{m+4}\;\tilde{C}^1_{G,m+8} &= \int_0^1 \xi^{m+3}\;G(\xi)\;d\xi \end{aligned} \tag{65}$$

In particular, if we choose the target to be a free quark, instead of a nucleon, then $q(\xi) = \delta(\xi-1)$, and also $F^q_{m+4} = 1$ by the normalization we agreed on. Thus in the parton approximation

$$\begin{aligned} \tilde{C}^1_{q,m+8} &= 1 \qquad\qquad (m\ \text{even}) \\ \tilde{C}^1_{q,m+8} &= 0 \qquad\qquad (m\ \text{odd}) \end{aligned} \quad . \tag{66}$$

Since the Wilson coefficient is independent of the target, we conclude that for a nucleon target in the parton approximation

$$F^q_{m+4} = \int_0^1 \xi^{m+3}\left[q(\xi) + \bar{q}(\xi)\right] d\xi \quad , \quad (m \text{ even}) \quad . \tag{67}$$

Similarly

$$\begin{aligned} \tilde{C}^1_{G,m+8} &= 1 \qquad (m \text{ even}) \\ \tilde{C}^1_{G,m+8} &= 0 \qquad (m \text{ odd}) \end{aligned} \tag{68}$$

and

$$F^G_{m+4} = \int_0^1 \xi^{m+3}\, G(\xi)\, d\xi \quad , \quad (m \text{ even}) \quad . \tag{69}$$

Thus we may interpret F^a_ℓ and measuring the $\ell + 1$ moment of parton distribution of type a.[2]

The calculation for other structure function H_i $(i > 1)$ is similar. All x^k factors in (31) are to be interpreted as $x\xi^{k-1}$. We obtain thus in the parton approximation

$$\tilde{H}_2 = \frac{1}{\pi} \sum_{\substack{m=0 \\ m \text{ even}}}^{\infty} \left(\frac{1}{x}\right)^m \int_0^1 \xi^{m+1}\left[q(\xi) + \bar{q}(\xi) + 4G(\xi)\right] d\xi \tag{70}$$

$$\tilde{H}_3 = \frac{1}{\pi} \sum_{\substack{m=0 \\ m \text{ even}}}^{\infty} \left(\frac{1}{x}\right)^m \int_0^1 \xi^{m-1}\, G(\xi)\, d\xi \tag{71}$$

$$\tilde{H}_4 = \frac{1}{\pi} \sum_{\substack{m=0 \\ m \text{ even}}}^{\infty} \left(\frac{1}{x}\right)^m \int_0^1 \xi^{m+2}\left[\frac{1}{2}\Delta q(\xi) + \frac{1}{2}\Delta\bar{q}(\xi) + \Delta G(\xi)\right] d\xi \tag{72}$$

$$\tilde{H}_6 = \frac{1}{\pi} \sum_{\substack{m=0 \\ m \text{ even}}}^{\infty} \left(\frac{1}{x}\right)^m \int_0^1 \xi^m \, \Delta G(\xi) \, d\xi \quad . \tag{73}$$

On comparing with (53), (55)-(60), we may obtain as in the H_1 case, expression for the Wilson coefficients, F^a, ΔF^a and δF^a, in the parton approximation. Again we may interpret ΔF^a, δF^a as the moments of the longitudinally and transversely polarized parton distributions for type a partons. Since the anomalous dimensions for the operators O^a, R^{aS}, R^{aA} are known,[5] we may also easily obtain the Q^2-variation of these distribution functions in the leading log approximation. The result is the same as those obtained from leptonic deep inelastic scattering and we shall not repeat them here.

Comparing Eq. (73) with Eq. (57), we obtain

$$\int_0^1 \xi^n \Delta G(\xi) d\xi = \sum_a \left\{ \frac{1}{2} \tilde{C}^4_{an+3} \, \Delta F^a_n + (n+1) \, \tilde{C}^6_{an+3} \, \Delta F^a_n - \tilde{D}^4_{an+3} \, \delta F^a_n \right\} \quad , \quad n = \text{even} \quad . \tag{74}$$

In the leading logarithm approximation the quark's operators do not contribute, and in n = 0 case the operator R^{aA} does not contribute. Only the operator a_ξ contributes. The calculation (zeroth order in running coupling constant) shows that

$$\frac{1}{2} \tilde{C}^4_{G3} + \tilde{C}^6_{G3} = 0 \quad . \tag{75}$$

Therefore, the operator a_ξ does not exist in Wilson expansion of the product of two stress-energy-momentum tensors at least in the leading order. In order to obtain the first moment ($n = 0$) we have to use analytic continuation in n complex plane.

V. CONCLUSION

We have studied in this paper the kinematics, the parton model, and the Wilson expansion for deep inelastic gravitational scattering on a nucleon target. Both the polarized and the unpolarized cases have been studied. There are altogether five unpolarized and six polarized structure functions. Of these, eight are associated with spin 2 gravitons; spin 0 gravitons, say, coming from the tract anomaly, enter into the remaining three.

In the parton approximation, Eq. (31), H_1 can be considered as generalization of the flavor singlet part of W_1 and W_2 in leptonic deep inelastic scattering, and H_4 is the generalization of G_1 in the same. In either structure function gluons enter in the graviton case but not in the leptonic case. But like the corresponding structure functions in the leptonic case, H_1 is related to the sum of distributions of all those partons that interact with the probe, and H_4 is related to the sum of helicity distributions of all those partons that interact with the probe.

We have discussed the kinematics and the formalism of Wilson expansion, but we have not carried out explicitly any hitherto uncalculated QCD corrections. We have also discussed in Sec. IV how the axial vector gluon operator $a_\xi(x)$ is evaded in the present process.

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APPENDIX A

We compute in this Appendix $A'^{(i)}$ defined in (42), which by definition is obtained from $A^{(i)}$ by interchanging S and one of the P's in Eq. (41).

To do so, it is convenient to derive some kinematical formulas first. If we let

$$\bar{S}_\mu \equiv S_\mu - \frac{S\cdot q}{q^2} q_\mu = \frac{1}{q^2} a_{\mu\nu\alpha\beta} S^\nu q^\alpha q^\beta \tag{A.1}$$

then from (15)

$$\tilde{S}_\mu = P\cdot q\, \bar{S}_\mu - q\cdot S\, \bar{P}_\mu \quad . \tag{A.2}$$

Furthermore, if we let

$$\begin{aligned} \hat{S}_\mu &= \bar{S}_\mu - \frac{\bar{S}\cdot\bar{P}}{\bar{P}^2} \bar{P}_\mu \\ k_\mu &= [\mu P q S] = [\mu \bar{P} q \hat{S}] \end{aligned} \tag{A.3}$$

then q, $\bar{P}$, S, k are mutually orthogonal. Thus

$$\begin{aligned} \bar{P}^2 q^2 \hat{S}_\mu &= [\mu P q k] \\ \bar{P}^2 [\nu\lambda q S] &= \bar{P}_\lambda k_\nu - \bar{P}_\nu k_\lambda \quad . \end{aligned} \tag{A.4}$$

We are now ready to compute A'. If we replace S in the following vectors by P, then

$$\tilde{S}_\mu \to 0 \ , \quad \bar{S}_\mu \to \bar{P}_\mu \ , \quad \bar{P}_\mu \to \bar{P}_\mu \ , \quad \bar{g}_{\mu\nu} \to \bar{g}_{\mu\nu} \quad . \tag{A.5}$$

If we replace P in the following vectors by S, then

$$\tilde{S}_\mu \to 0 \ , \quad \bar{S}_\mu \to \bar{S}_\mu \ , \quad \bar{P}_\mu \to \bar{S}_\mu \ , \quad \bar{g}_{\mu\nu} \to \bar{g}_{\mu\nu} \quad . \tag{A.6}$$

Using these rules, we get the following from Eqs. (20)-(24)

$$A'^{(4)}_{\mu\nu,\kappa\lambda} = \frac{1}{2}\left(\bar{S}_\mu\bar{P}_\kappa + \bar{S}_\kappa\bar{P}_\mu\right)[\nu\lambda qP] + (\mu\leftrightarrow\nu) + (\kappa\leftrightarrow\lambda) + (\mu\kappa)\leftrightarrow(\nu\lambda)$$

$$- \frac{1}{3}\left[\bar{g}_{\mu\nu}\bar{P}_\kappa[\lambda PqS] + \bar{g}_{\mu\nu}\bar{P}_\lambda[\kappa PqS] - (\mu\nu)\leftrightarrow(\kappa\lambda)\right] \tag{A.7}$$

$$A'^{(5)}_{\mu\nu,\kappa\lambda} = -\frac{1}{3} A^{(5)}_{\mu\nu,\kappa\lambda} \tag{A.8}$$

$$A'^{(6)}_{\mu\nu,\kappa\lambda} = 0 \tag{A.9}$$

$$A'^{(7)}_{\mu\nu,\kappa\lambda} = - A^{(7)}_{\mu\nu,\kappa\lambda} \tag{A.10}$$

$$A'^{(8)}_{\mu\nu,\kappa\lambda} = -\frac{1}{4} A^{(8)}_{\mu\nu,\kappa\lambda} \tag{A.11}$$

$$A'^{(9)}_{\mu\nu,\kappa\lambda} = -\frac{1}{2} A^{(9)}_{\mu\nu,\kappa\lambda} \quad . \tag{A.12}$$

As for $A'^{(4)}$ we need the following considerations: From (A.3) and (A.4),

$$\bar{S}_\mu = \frac{1}{\bar{P}^2 q^2}[\mu\bar{P}qk] + \frac{\bar{S}\cdot\bar{P}}{\bar{P}^2}\bar{P}_\mu \quad . \tag{A.13}$$

Moreover

$$[\mu\bar{P}qk][\nu\lambda qP] = q^2\left[\left(\bar{g}_{\mu\nu}\bar{P}^2 - \bar{P}_\mu\bar{P}_\nu\right)k_\lambda - \left(\bar{g}_{\mu\lambda}\bar{P}^2 - \bar{P}_\mu\bar{P}_\lambda\right)k_\nu\right] \quad . \tag{A.14}$$

Thus

$$\frac{1}{2}\left(\bar{S}_\mu\bar{P}_k+\bar{S}_k\bar{P}_\mu\right)[\nu\lambda qP] = \frac{\bar{S}\cdot\bar{P}}{\bar{P}^2}\bar{P}_\mu\bar{P}_\kappa[\nu\lambda qP]$$

$$+\frac{1}{2}\left(\bar{g}_{\mu\nu}-\frac{\bar{P}_\mu\bar{P}_\nu}{\bar{P}^2}\right)\left(\bar{P}_\kappa k_\lambda\right)+\frac{1}{2}\left(\bar{g}_{\kappa\nu}-\frac{\bar{P}_\nu\bar{P}_\kappa}{\bar{P}^2}\right)\left(\bar{P}_\mu k_\lambda\right)$$

$$-\frac{1}{2}\left(\bar{g}_{\mu\lambda}-\frac{\bar{P}_\mu\bar{P}_\lambda}{\bar{P}^2}\right)\left(\bar{P}_\kappa k_\nu\right)-\frac{1}{2}\left(\bar{g}_{\kappa\lambda}-\frac{\bar{P}_\kappa\bar{P}_\lambda}{\bar{P}^2}\right)\left(\bar{P}_\mu k_\nu\right) \quad . \qquad \text{(A.15)}$$

After adding to (A.15) terms corresponding to $(\mu\leftrightarrow\nu)$, $(\kappa\leftrightarrow\lambda)$, and $(\mu\kappa)\leftrightarrow(\nu\lambda)$, (A.15) becomes

$$\frac{1}{2}\left(\bar{S}_\mu\bar{P}_\kappa+\bar{S}_\kappa\bar{P}_\mu\right)[\nu\lambda qP] + (\mu\leftrightarrow\nu) + (\kappa\leftrightarrow\lambda) + (\mu\kappa)\leftrightarrow(\nu\lambda)$$

$$= \frac{\bar{S}\cdot\bar{P}}{\bar{P}^2}\left\{\bar{P}_\mu\bar{P}_\kappa[\nu\lambda qP] + (\mu\leftrightarrow\nu) + (\kappa\leftrightarrow\lambda) + (\mu\kappa)\leftrightarrow(\nu\lambda)\right\}$$

$$+\left\{\frac{1}{2}\left(\bar{g}_{\mu\lambda}-\frac{\bar{P}_\mu\bar{P}_\lambda}{\bar{P}^2}\right)\left(\bar{P}_\nu k_\kappa-\bar{P}_\kappa k_\nu\right) + (\mu\leftrightarrow\nu) + (\kappa\leftrightarrow\lambda) + (\mu\nu)\leftrightarrow(\nu\lambda)\right\}$$

$$+\left(\bar{g}_{\mu\nu}-\frac{\bar{P}_\mu\bar{P}_\nu}{\bar{P}^2}\right)\left(\bar{P}_\kappa k_\lambda+\bar{P}_\lambda k_\kappa\right)-\left(\bar{g}_{\kappa\lambda}-\frac{\bar{P}_\kappa\bar{P}_\lambda}{\bar{P}^2}\right)\left(\bar{P}_\mu k_\nu+\bar{P}_\nu k_\mu\right) \quad . \qquad \text{(A.16)}$$

Substituting into (A.7) finally yields the result

$$A'^{(4)}_{\mu\nu,\kappa\lambda} = -\frac{1}{2\bar{P}^2}A^{(8)}_{\mu\nu,\kappa\lambda} - \frac{\bar{P}^2}{2}A^{(6)}_{\mu\nu,\kappa\lambda} + \frac{3}{4}A^{(4)}_{\mu\nu,\kappa\lambda}$$

$$+\frac{\bar{S}\cdot\bar{P}}{\bar{P}^2}\frac{1}{q\cdot S}\left[P\cdot q\, A^{(4)}_{\mu\nu,\kappa\lambda} - A^{(5)}_{\mu\nu,\kappa\lambda}\right]$$

$$= -\frac{1}{2\bar{P}^2}A^{(8)}_{\mu\nu,\kappa\lambda} - \frac{\bar{P}^2}{2}A^{(6)}_{\mu\nu,\kappa\lambda} + \left[\frac{3}{4} - \frac{(P\cdot q)^2}{\bar{P}^2q^2}\right]A^{(4)}_{\mu\nu,\kappa\lambda}$$

$$+\frac{P\cdot q}{\bar{P}^2q^2}A^{(5)}_{\mu\nu,\kappa\lambda} \quad . \qquad \text{(A.17)}$$

In the Bjorken limit, we may take

$$\bar{P}^2 = -q^2/4x^2 \tag{A.18}$$

where

$$x = -q^2/2P\cdot q \quad . \tag{A.19}$$

Thus we may write (A.17) as

$$A'^{(4)}_{\mu\nu,\kappa\lambda} = \frac{2x^2}{q^2} A^{(8)}_{\mu\nu,\kappa\lambda} + \frac{q^2}{8x^2} A^{(6)}_{\mu\nu,\kappa\lambda} + \frac{7}{4} A^{(4)}_{\mu\nu,\kappa\lambda} + \frac{2x}{q^2} A^{(5)}_{\mu\nu,\kappa\lambda} \quad . \tag{A.20}$$

Using (28)-(39) we get

$$B'^{(4)}_{\mu\nu,\kappa\lambda} = \frac{7}{4} B^{(4)}_{\mu\nu,\kappa\lambda} + xB^{(5)}_{\mu\nu,\kappa\lambda} + \frac{1}{4x^2} B^{(6)}_{\mu\nu,\kappa\lambda} + x^2 B^{(8)}_{\mu\nu,\kappa\lambda} \tag{A.21}$$

which is the form quoted in Eq. (43) in the text.

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TABLE I

Table of integers used in the definition of the tensor $A^{(i)}$ in Eqs. (38) and (39) and of the integers in Eq. (52).

i	b_i	e_i	f_i	$\bar{n}_i$	m_{ni}
1	4	8	4	n–4	n–8
2	3	6	2	n–4	n–6
3	2	4	0	n–4	n–4
4	2	5	2	n–3	n–5
5	2	6	3	n–3	n–6
6	1	3	0	n–3	n–3
7	1	4	1	n–3	n–4
8	3	7	4	n–3	n–7
9	2	5	2	n–3	n–5
10	3	6	2	n–4	n–6
11	2	4	0	n–4	n–4