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EFFECT OF TRIPLE GLUON COUPLING IN SEMI-INCLUSIVE POLARIZED SCATTERING PROCESSES*

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ABSTRACT

We show that the gluon helicity inside a proton and a photon can be deduced from a knowledge of a special combination of cross sections of the semi-inclusive processes $e + p \to e + \pi + \pi + \ldots$ and $e^+ + e^- \to e^+ + e^- + \pi + \pi + \ldots$ Such a measurement could thus be used to check the QCD prediction that the gluon helicity increases linearly with $\ell n Q^2$.

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1. Introduction

It follows from the Altarelli-Parisi [1] equations that the gluon helicity, $\Delta G^{(1)}(t) = \int_0^1 dx \Delta G(x,Q^2)$ varies as $\ell n Q^2$, in the leading logarithm approximation. This behavior is quite unique in strong interactions, all other parton moments are either constant or decrease as some power of $\ell n Q^2$. It is true that in deep inelastic scattering on a photon target an additional $\ell n Q^2$ dependence, discovered by Witten, is also present. Here, however, this dependence of the gluon helicity on $\ell n Q^2$ is a result of the triple gluon coupling, a unique feature of non-Abelian theories. Thus, a measurement of this quantity would be very interesting, since it would help establish the relevance of such theories in understanding strong interactions. In a previous paper [2] we showed that the gluon helicity cannot be measured in inclusive reactions. In this paper we show how it can be measured in semi-inclusive reactions, at least in the parton model.

One might think that such behavior of the gluon helicity is inconsistent with the fact that the proton has definite helicity. This is not the case, however, because due to angular momentum conservation, the gluon must have a small transverse momentum. This means that orbital angular momenta are mixed in, so that there is no reason for the helicity alone to be fixed.

To measure the polarized gluon distribution functions of a proton and real photon we consider the polarized scattering processes $e^-(\uparrow) + p(\uparrow) \to e^- + \pi + \pi + \ldots$ and $e^+(\uparrow) + e^-(\uparrow) \to e^+ + e^- + \pi + \pi + \ldots$, respectively. Following ref. [3], we consider the following combination of cross sections for large transverse momentum two-pion production: $d\sigma(\pi^+\pi^-) + d\sigma(\pi^-\pi^+) - d\sigma(\pi^+\pi^+) - d\sigma(\pi^-\pi^-)$, with the two pions in different jets. As pointed out in ref. [3], only the subprocess shown in fig. 1a contributes to the above combination of cross sections. We can thus measure the polarized gluon distribution function of the corresponding target. Taking then the first moment, we obtain the gluon helicity. These two processes are discussed in sects. 3 and 4, respectively.

In sect. 2 we show how the logarithmic dependence of the gluon helicity in Q^2 follows from the Altarelli-Parisi equations and sect. 5 contains a summary.

2. Q^2-Dependence of Gluon Helicity

In the proton target case, solving the Altarelli-Parisi equation the gluon helicity is

$$\Delta G^{(1)}(t)_p = -\frac{3}{4\pi b}C_2(R)\Delta q^{(1)}(t_0)_p + \left\{\frac{3}{4\pi b}C_2(R)\Delta q^{(1)}(t_0)_p + \Delta G^{(1)}(t_0)_p\right\} \times \frac{\ln Q^2/\Lambda^2}{\ln Q_0^2/\Lambda^2} \tag{1}$$

where

$$t = \ell n \frac{Q^2}{\Lambda^2}$$

$$\Delta q^{(1)}(t)_p = \int_0^1 dx \;\; \Delta q(x,Q^2)_p$$

$$C_2(R) = \frac{N^2-1}{2N}$$

$$b = \frac{1}{12\pi}\left\{11C_2(G) - 2f\right\}$$

f - quark flavor number. (2)

The Altarelli-Parisi equations satisfied by the polarized quark, gluon and photon distribution functions inside a photon are, respectively,

$$\frac{d}{dt}\Delta q^i(x,t) = \frac{1}{2\pi}\int_x^1 \frac{dy}{y}\left\{\alpha(t)\Delta q^i(y,t)\Delta P_{qq}\left(\frac{x}{y}\right) + \alpha(t)\Delta G(y,t)\Delta P_{qG}\left(\frac{x}{y}\right)\right.$$

$$\left. + \alpha_\gamma \Delta q^i(y,t)\Delta P^\gamma_{q_i q_i}\left(\frac{x}{y}\right) + \alpha_\gamma \Delta\Gamma(y,t)\Delta P_{q_i\gamma}\left(\frac{x}{y}\right)\right\}$$

(3)

$$\frac{d}{dt}\Delta G(x,t) = \frac{\alpha(t)}{2\pi}\int_x^1 \frac{dy}{y}\left\{\Delta q_s(y,t)\Delta P_{Gq}\left(\frac{x}{y}\right) + \Delta G(y,t)\Delta P_{GG}\left(\frac{x}{y}\right)\right\}$$

$$\frac{d}{dt}\Delta\Gamma(x,t) = \frac{1}{2\pi}\int_x^1 \frac{dy}{y}\left\{\alpha_\gamma \sum_i \Delta q_i(y,t)\Delta P_{\gamma q_i}\left(\frac{x}{y}\right) + \alpha_\gamma \Delta\Gamma(y,t)\Delta P_{\gamma\gamma}\left(\frac{x}{y}\right)\right\}$$

where

$$\Delta P^{\gamma}_{q_i q_i}(z) = e_i^2 \left\{ \frac{1+z}{(1-z)_+} + \frac{3}{2}\,\delta(1-z) \right\} \; ; \; \Delta P_{qG}(z) = \frac{1}{2}\left[z^2 - (1-z)^2\right]$$

and

$$\Delta P_{q_i\gamma}(z) = 3e_i^2\left[z^2 - (1-z^2)\right] \; ; \; \Delta P_{Gq_i} = \Delta P_{Gq} = C_2(R)\,\frac{1-(1-z)^2}{z}$$

$$\Delta P_{\gamma q_i}(z) = e_i^2\,\Delta P_{Gq_i}(z)\,/\,C_2(R) \; ; \qquad (4)$$

$$\Delta P_{\gamma\gamma}(z) = -\,2\left[\sum_{i=1}^{f} e_i^2 + \frac{1}{3}\,N_e\right]\delta(1-z) \equiv -\,C\delta(1-z)$$

$$\Delta q_s(x,t) = \sum_{i}^{f}\left\{\Delta q_i(x,t) + \Delta\bar{q}_i(x,t)\right\} \quad .$$

N_e is the lepton number, and N is the number of colors. Δq_i is $O(\alpha_\gamma)$ and $\Delta\Gamma(x,t) = \delta(x-1) + O(\alpha_\gamma)$ (the photon helicity is assumed to be +1). Keeping only the terms of $O(\alpha_\gamma)$, the above equations become

$$\frac{d}{dt}\,\Delta q^i(t) = \frac{\alpha_\gamma}{2\pi}\,\Delta P_{q_i\gamma}(x) + \frac{\alpha(t)}{2\pi}\int_x^1 \frac{dy}{y}\left\{\Delta q^i(y,t)\,\Delta P_{q_i q_i}\left(\frac{x}{y}\right) + \Delta G(y,t)\,\Delta P_{qG}\left(\frac{x}{y}\right)\right\} \qquad (5)$$

$$\frac{d}{dt}\,\Gamma(x,t) = -\,\frac{\alpha_\gamma}{2\pi}\,\Delta\Gamma(x,t)C \quad .$$

The equation for ΔG remains unchanged.

Using Eqs. (3), we find the following solutions for the first moments of Δq_i and ΔG, in the case of photon target.

$$\Delta q^{(1)}(t) = \text{const.}$$

$$\Delta G^{(1)}(t) = -\frac{3}{4\pi b} C_2(R)\Delta q^{(1)}(t_0) + \left\{\frac{3}{4\pi b} C_2(R)\Delta q^{(1)}(t_0) + \Delta G^{(1)}(t_0)\right\} \frac{\ell n(Q^2/\Lambda^2)}{\ell n(Q_0^2/\Lambda^2)} \quad , \tag{6}$$

and for the polarized photon distribution function

$$\Delta\Gamma(x,t) = \Delta\Gamma(x,t_0)\, e^{-\alpha_\gamma tC/2\pi} \quad . \tag{7}$$

3. Measurement of the Polarized Gluon Distribution Function (Proton Target)

In this section we discuss how $\Delta G_p(x,t)$ can be measured in the proton case. Fontannaz et al. [3] have suggested that $\Delta G_p(x,t)$ can be measured in polarized photon production of two pions in large transverse momentum. We look at the process $e + p \rightarrow e + \pi + \pi + \ldots$, where the initial electron and target proton are polarized, and the pions have large transverse momenta. Since $D_g^{\pi^-} = D_g^{\pi^+}$, only the process shown in fig. 1a contributes to the following combination of cross sections:

$$d\sigma \equiv d\sigma(\pi^+\pi^-) + d\sigma(\pi^-\pi^+) - d\sigma(\pi^+\pi^+) - d\sigma(\pi^-\pi^-) \quad .$$

The parton model [4] gives

$$d\sigma(e\uparrow p\uparrow) - d\sigma(e\downarrow p\uparrow) = \frac{d^3k_1 d^3k_2 d^3k'}{E_1E_2E'} \int_0^1 dx_a \int_0^1 dx_b \int_0^1 dx_c \, \frac{1}{x_b^2 x_c^2} \Delta G_p(x_a, Q^2)$$

$$\times \sum_i e_i^2 \left\{ D_{q_i}^{\pi^+}(x_b, Q^2) - D_{q_i}^{\pi^-}(x_b, Q^2) \right\} \left\{ D_{q_i}^{\pi^+}(x_c, Q^2) - D_{q_i}^{\pi^-}(x_c, Q^2) \right\}$$

$$\times \delta^{(4)}(p + p_g - p_q - p_{\bar{q}}) \, \frac{2}{\pi} (s - q^2) \frac{d\sigma}{\frac{d^3k'}{E'} dt} \tag{6}$$

where the arrows denote the polarization of the corresponding particles k_1, k_2 are the pion momenta, E_1 and E_2 the pion energies, k' the momentum of the final electron, E' the energy of the final electron, $D_{q_i}^{\pi}(x_b, Q^2)$ is the fragmentation function of the i-th quark, p_g is the gluon momentum, and p_q, $p_{\bar{q}}$ are the momenta of the quark and antiquark (see fig. 1a). Also

$$p_g = x_a p \,;\, k_1 = x_b p_q \,;\, k_2 = x_c p_{\bar{q}} \,;\, s = (p_g + q)^2 \,;\, u = (p_g - p_q)^2 \,;\, t = (p_g - p_{\bar{q}})^2 . \tag{7}$$

Finally, p is the momentum of the initial proton and q the momentum of the virtual photon, and $d\sigma / (d^3k'/E')dt$ is the differential cross section for the subprocess e(polarized) + g(polarized) $\rightarrow q + \bar{q}$ (see fig. 1a); it is given by

$$(s - q^2) \frac{d\sigma}{\frac{d^3k'}{E'} dt} = \frac{\alpha_\gamma^2 \alpha_s(Q^2)}{\pi q^2 \, t u \, \omega_g E_e |\vec{v}_g - \vec{v}|} \Big\{ (s + t) k \cdot p_{\bar{q}} + (s + u) k \cdot p_q - \frac{1}{4} (t^2 + u^2 + 2sq^2) \Big\} \tag{8}$$

where ω_g and E_e are the gluon and initial electron energies, respectively, $\vec{v}_g$ and $\vec{v}$ their respective velocities (they are antiparallel, thus $|\vec{v}_g - \vec{v}_e| = 2$), k is the initial electron momentum, α_γ the fine structure constant, and α_s the running coupling constant. Following Feynman [5] we take

$$Q^2 = \frac{2tus}{t^2 + u^2 + s^2} \quad . \tag{9}$$

Using the δ-function, we carry out the x_a, x_b, x_c and azimuthal angle of the final electron integrations, to obtain

$$\frac{d\sigma(e\uparrow p\uparrow) - d\sigma(e\downarrow p\uparrow)}{d^2k_{1\perp}dy_1 d^2k_{2\perp}dy_2 dk'^2_\perp dy_e} = \frac{1}{E}\left|k_{1\perp}e^{y_1}\left(\frac{1}{x_c}k^2_{2\perp} + \frac{1}{x_b}\vec{k}_{1\perp}\cdot\vec{k}_{2\perp}\right)\right.$$

$$\left. - k_{2\perp}e^{y_2}\left(\frac{1}{x_b}k^2_{1\perp} + \frac{1}{x_c}\vec{k}_{1\perp}\cdot\vec{k}_{2\perp}\right)\right|^{-1}$$

$$\times \Delta G_p(x_a,Q^2)\sum_i e_i^2\left\{D^{\pi^+}_{q_i}(x_b,Q^2) - D^{\pi^-}_{q_i}(x_b,Q^2)\right\}\left\{D^{\pi^+}_{q_i}(x_c,Q^2) - D^{\pi^-}_{q_i}(x_c,Q^2)\right\}$$

$$\times \frac{1}{\pi}(s - q^2)\,\frac{d\sigma}{\frac{d^3k'}{E'}\,dt} \quad , \tag{10}$$

where E is the proton energy.

In the CM frame of the electron and proton

$$x_a = \frac{1}{2E}\left\{q_{\|} - q_0 + (q_0 + q_{\|})e^{-2y_1} + \frac{e^{-y_2} - e^{y_2 - 2y_1}}{1 + e^{2(y_2 - y_1)} - 2e^{y_2 - y_1}\cos\phi}\right.$$

$$\left.\left[(q_0 + q_{\|})e^{-y_1}(e^{y_2 - y_1} - \cos\phi) \pm \sqrt{B}\right]\right\}$$

$$\frac{1}{x_c} k_{2\perp} = \left\{1 + e^{2(y_2 - y_1)} - 2e^{y_2 - y_1}\cos\phi\right\}^{-1}\left\{(q_0 + q_{\|})e^{-y_1}(e^{y_2 - y_1} - \cos\phi) \pm \sqrt{B}\right\}$$

$$\frac{1}{x_b} k_{1\perp} = \left\{1 + e^{2(y_2 - y_1)} - 2e^{y_2 - y_1}\cos\phi\right\}^{-1}\left\{(q_0 + q_{\|})e^{-y_1}(1 - e^{y_2 - y_1}\cos\phi)\right.$$

$$\left.\mp e^{y_2 - y_1}\sqrt{B}\right\}$$

$$B = k_{\perp}'^2\left\{(1 - e^{y_2 - y_1})^2 + 4e^{y_2 - y_1}\sin^2\frac{\phi}{2}\right\} - (q_0 + q_{\|})^2 e^{-2y_1}\sin^2\phi \quad . \quad (11)$$

Here ϕ is the angle between $\vec{k}_{1\perp}$ and $\vec{k}_{2\perp}$, $q_{\|}$ is the longitudinal momentum and q_0 the energy of the virtual photon, y_1, y_2 and y_e are the rapidities of the two pions and the electron, respectively. The kinematic conditions are

$$B \geq 0$$

$$1 \geq x_a, x_b, x_c \geq 0.$$

If $\vec{k}_{\perp}' \cdot \vec{k}_{2\perp} < 0$ the upper sign in Eqs. (11) is taken, whereas if $\vec{k}_{\perp}' \cdot \vec{k}_{2\perp} > 0$ the lower sign is taken. In the case $\phi = 180^{\circ}$, Eqs. (11)

simplify to

$$x_a = \frac{1}{2E}\left\{q_{\parallel} - q_0 + (q_0 + q_{\parallel})e^{-y_1 - y_2} \pm \frac{k'_{\perp}}{e^{y_1} + e^{y_2}}\left(e^{y_1 - y_2} - e^{y_2 - y_1}\right)\right\}$$

$$\frac{1}{x_c} k_{2\perp} = \frac{1}{e^{y_1} + e^{y_2}}\left(q_0 + q_{\parallel} \pm k'_{\perp} e^{y_1}\right)$$

$$\frac{1}{x_b} k_{1\perp} = \frac{1}{e^{y_1} + e^{y_2}}\left(q_0 + q_{\parallel} \pm k'_{\perp} e^{y_2}\right) \quad ,$$

where if $\vec{k}'_{\perp}$ is parallel to $\vec{k}_{2\perp}$ the lower sign holds, whereas if $\vec{k}'_{\perp}$ is antiparallel to $\vec{k}_{2\perp}$ the upper sign holds.

Thus using the forms of the fragmentation functions given in ref. [5], a measurement of the cross section in Eq. (10) allows $\Delta G_p(x_a, Q^2)$ to be determined. The integral

$$\int_0^1 dx \; \Delta G_p(x, Q^2)$$

will then yield the gluon helicity inside a proton.

4. Measurement of Polarized Gluon Distribution Function (Photon Target)

Just as in the proton case, we can use the process $e^+ + e^- \rightarrow e^+ + e^- + \pi + \pi + X$ to measure $\Delta G_\gamma(x, Q^2)$ inside a real photon, when the initial e^+ and e^- are polarized. Since $D_g^{\pi^+}(x, Q^2) = D_g^{\pi^-}(x, Q^2)$ only the diagrams in fig. 3 contribute to the following combination of cross sections:

$$\Delta = \left\{ d\sigma(\pi^+\pi^-) + d\sigma(\pi^-\pi^+) - d\sigma(\pi^+\pi^+) - d\sigma(\pi^-\pi^-) \right\}_{e^-\uparrow e^+\uparrow \, - \, e^-\downarrow e^+\uparrow} \quad . \tag{12}$$

This cross section can be written as

$$\Delta = \frac{1}{(2\pi)^{12}} \frac{d^3p_1}{2E_1} \frac{d^3p_2}{2E_2} \frac{d^3k'}{2E'_e} \frac{d^3k'_1}{2E'_{e^+}} \frac{1}{4EE_1|\vec{v}_e - v_{e^+}|} \frac{(4\pi\alpha)^2}{p^4} \tau^{[A]}_{\mu\nu} (k_1, k'_1) W^{\mu\nu}, \tag{13}$$

where E_e and E_{e^+} are the energies of the initial electron and positron, respectively, E'_e and E'_{e^+} the energies of the final electron and positron, respectively, k and k' the momenta of the initial and final electron and k_1 and k'_1 the initial and final momenta of the positron, respectively. p_1, p_2 are the pion momenta and [A] denotes the antisymmetric part. The tensor of the lower vertex is given by

$$\tau^{[A]}_{\mu\nu} = 2im_e \varepsilon_{\mu\nu\lambda\eta} p^\lambda s^\eta \tag{14}$$

where $p = k_1 - k'_1$, $p^2 \simeq 0$ and s_η is the polarization vector of the initial positron. Ignoring the lepton mass, we have

$$m_e s_\eta = \lambda k_{1\eta} \ , \ (\lambda = \pm \frac{1}{2}) \quad . \tag{15}$$

The tensor $W_{\mu\nu}$ is given by

$$W_{\mu\nu} = \sum_{a,b=\pm 1} (-1)^{a+b} \varepsilon_\nu(a) \varepsilon^*_\mu(b) W_{ab} \quad , \tag{16}$$

where ε_μ is the photon polarization vector. Since $p^2 = 0$, only transverse photons contribute to $W_{\mu\nu}$.

The tensor W_{ab} is in turn given by

$$W_{ab} = \tau^{[A]}_{\alpha\beta}(k,k') \sum_{x} \langle\gamma_a|J^{\beta}(0)|\pi_1\pi_2 X\rangle\langle X\pi_2\pi_1|J^{\alpha}(0)|\gamma_b\rangle$$

$$\times (2\pi)^4 \, \delta^{(4)}(p+q-p_1-p_2-p_X) \ . \qquad (17)$$

Using the formulas [6]

$$\varepsilon^{*}_{\mu}(a)\varepsilon_{\nu}(a) = \frac{1}{2}\left\{R_{\mu\nu} - \frac{i}{p\cdot q}\, a\varepsilon_{\mu\nu\lambda\eta}p^{\lambda}q^{\eta}\right\} , \ (q_{\mu} = k_{\mu} - k'_{\mu})$$

$$R_{\mu\nu} = -\, g_{\mu\nu} + \frac{1}{(p\cdot q)^2}\left\{p\cdot q(p_{\mu}q_{\nu} + p_{\nu}q_{\mu}) - q^2 p_{\mu}p_{\nu}\right\} \quad , \qquad (18)$$

and Eqs. (14) and (16), we obtain

$$\tau^{[A]}_{\mu\nu}(k_1,k'_1)W^{\mu\nu} = -\frac{p^2}{p\cdot q}\,(k_1+k'_1)\cdot q(W_{++} - W_{--}) \quad . \qquad (19)$$

According to Eq. (17), $W_{++} - W_{--}$ is the cross section of polarized electron scattering on a polarized photon. Therefore, we can calculate it in the parton model.

Defining

$$\Delta_{\gamma}(\pi_1\pi_2) = \frac{d\sigma(e\uparrow\gamma\downarrow \to e\pi_1\pi_2 X) - d\sigma(e\uparrow\gamma\downarrow \to e\pi_1\pi_2 X)}{\frac{d^3k'}{E'_e}\ \frac{d^3p_1}{E_1}\ \frac{d^3p_2}{E_2}} \quad , \qquad (20)$$

we have

$$\Delta_{\gamma}(\pi_1\pi_2) = \frac{1}{2^3(2\pi)^9}\ \frac{1}{4E_eE_{\gamma}|\vec{v}_{\gamma} - \vec{v}_e|}\,(W_{++} - W_{--}) \quad , \qquad (21)$$

where E_e and E_γ are the energies of the initial electron and real photon, respectively, and $\vec{v}_\gamma$ and $\vec{v}_e$ their velocities. The combination of cross sections of interest is then given by

$$\Delta_\gamma(\pi^+\pi^-) + \Delta_\gamma(\pi^-\pi^+) - \Delta_\gamma(\pi^+\pi^+) - \Delta_\gamma(\pi^-\pi^-)$$

$$= \int_0^1 dx_a \int_0^1 dx_b \int_0^1 dx_c \frac{1}{x_b^2 x_c^2} \Delta G_\gamma(x_a,Q^2) \sum_i e_i^2 \left\{ D_{q_i}^{\pi^+}(x_b,Q^2) - D_{q_i}^{\pi^-}(x_b,Q^2) \right\}$$

$$\times \left\{ D_{q_i}^{\pi^+}(x_c,Q^2) - D_{q_i}^{\pi^-}(x_c,Q^2) \right\} \delta^{(4)}(p + p_g - p_q - p_{\bar{q}}) \frac{2}{\pi} (s - q^2) \frac{d\sigma}{\frac{d^3k'}{E_e'} dt} , \tag{22}$$

$\Delta G_\gamma(x_a,Q^2)$ is the polarized gluon distribution function inside a real photon, All quantities are the same as in sect. 3. Since $p^2 \simeq 0$, $\vec{v}_\gamma$ is parallel to $\vec{v}_e$ in the rest frame of the beam. As before we carry out the δ-function integration to obtain

$$\frac{\Delta}{d^2p_{1\perp} dy_1 d^2p_{2\perp} dy_2 dk_1' dk_\perp'^2 dy_e'} = 2\alpha_\gamma N_2 \frac{1}{E_1^2 + E_1'^2} \frac{1}{p\cdot q} (k_1 + k_1')\cdot q$$

$$\times \left| p_{1\perp} e^{y_1} \left(\frac{1}{x_c} p_{2\perp}^2 + \frac{1}{x_b} \vec{p}_{1\perp}\cdot\vec{p}_{2\perp} \right) - p_{2\perp} e^{y_2} \left(\frac{1}{x_b} p_{1\perp}^2 + \frac{1}{x_c} \vec{p}_{1\perp}\cdot\vec{p}_{2\perp} \right) \right|^{-1}$$

$$\times \Delta G_\gamma(x_a, Q^2) \sum_i e_i^2 \left\{ D_{q_i}^{\pi^+}(x_b,Q^2) - D_{q_i}^{\pi^-}(x_b,Q^2) \right\} \left\{ D_{q_i}^{\pi^+}(x_c,Q^2) - D_{q_i}^{\pi^-}(x_c,Q^2) \right\}$$

$$\times (s - q^2) \frac{d\sigma}{\frac{d^3k'}{E_e'} dt} \tag{23}$$

where $N_2 = \frac{\alpha}{\pi} \frac{E_1^2 + E_1'^2}{E_1^2} \ell n \frac{E_1}{m_e}$ comes from the equivalent photon approximation [7]. All other quantities are the same as in sect. 3. Thus a measurement of the cross section in Eq. (23) and knowledge of the fragmentation functions, allows $\Delta G_\gamma(x_a,Q^2)$ to be determined. The integral

$$\int_0^1 dx \, \Delta G_\gamma(x,Q^2)$$

will then yield the gluon helicity inside a real photon.

There is another subprocess contributing to the cross section of the inclusive process $e^+ + e^- \to e^+ + e^- + \pi + \pi + \ldots$ This is shown in fig. 3, where the lower photon line propagates directly to the $q\bar{q}$ vertex. Its contribution is given by Eq. (23) with $\Delta G_\gamma(x_a,Q^2)$ replaced by $\Delta\Gamma(x_a,Q^2)$ and where $d\sigma/(d^3k'/E')dt$ is now the cross section for $e + \gamma \to e + q + g$ and is given by Eq. (8) with the gluon subscript replaced by γ everywhere and $\alpha_s(Q^2)$ replaced by α_γ. Since, however, $\Delta\Gamma(x_a,Q^2) = \delta(x_a - 1)$, if we do not consider the point $x_a = 1$, this term does not contribute.

5. Summary

We have shown that if a special combination of cross sections for the polarized semi-inclusive processes

$$e + p \to e + \pi + \pi + \ldots$$
$$e^+ + e^- \to e^+ + e^- + \pi + \pi + \ldots$$

were experimentally measured, then the polarized gluon distribution functions of a proton and a photon could be deduced from Eqs. (10) and

(23). The first moments of these distributions will then give the gluon helicity in the proton and in the photon, and thus its logarithmic dependence on Q^2 predicted by QCD could be checked.

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Figure Captions

Fig. 1. The two diagrams contributing to the processes

$$e(\uparrow) + p(\uparrow) \rightarrow \bar{e} + \pi + \pi + \ldots$$

$$e^+(\uparrow) + e(\uparrow) \rightarrow e^+ + e^- + \pi + \pi$$

to order α_s.

Fig. 2. The two general two-photon diagrams for the process

$$e^+ + e^- \rightarrow e^+ + e^- + \ldots$$

Fig. 3. The two subdiagrams contributing to the two-photon process to order α_s.

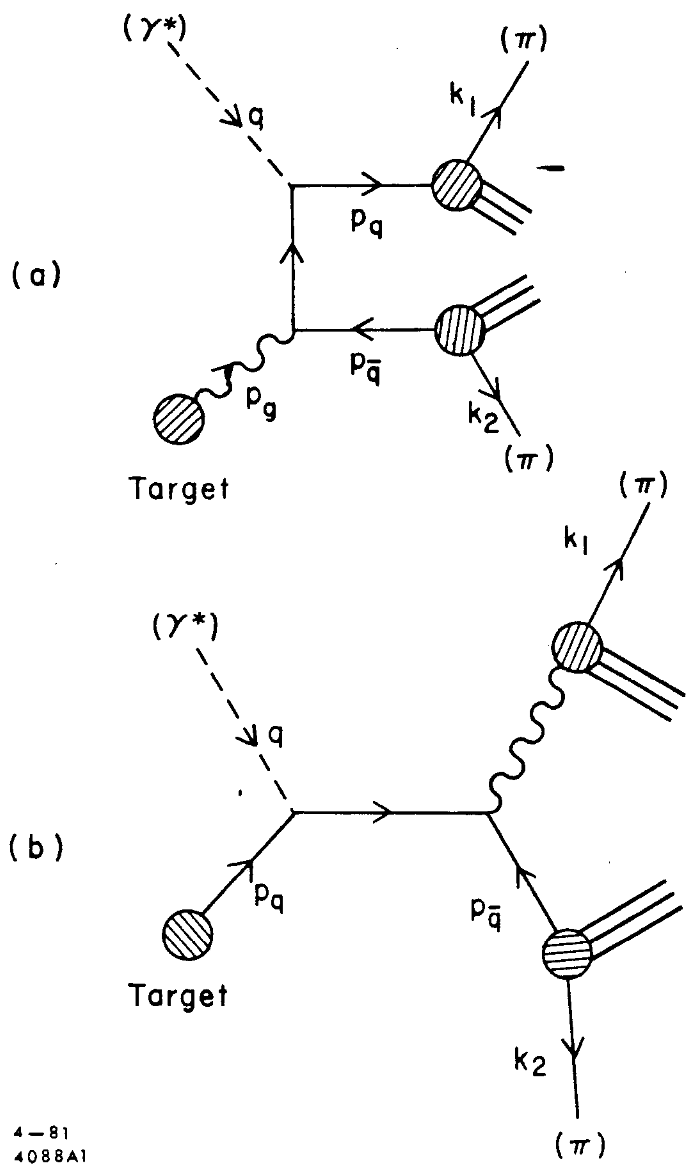

Fig. 1

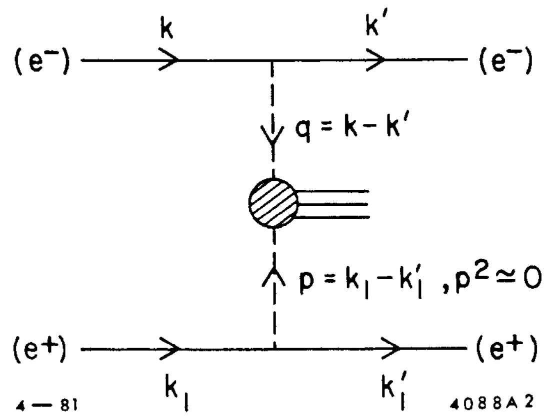

Fig. 2

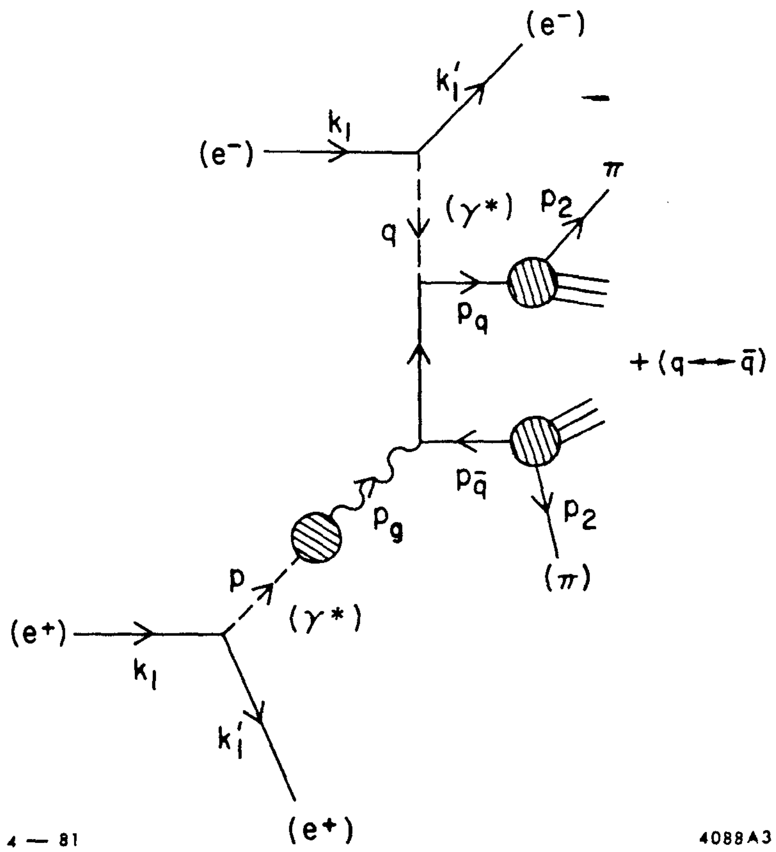

Fig. 3