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SUPERCOLOR *

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ABSTRACT

We propose new types of theories which combine supersymmetry and some new strong interaction which we generically refer to as supercolor. In some cases which we discuss, supercolor is identical with the familiar Technicolor. These theories are natural. They explain the scale of weak interactions and they do not require any unnatural adjustments. They possess naturally light scalars which give mass to ordinary quarks and leptons.

Naturalness imposes strong constraints on the U(1) gauge structure of the theory. These constraints appear <u>not</u> to be satisfied by the electroweak hypercharge. If this is true then, the symmetry of the world at energies above ~1 TeV cannot be standard $SU_{3C} \times SU_{2L} \times U(1)_Y$ with only ordinary families.

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1. Introduction

There are two fundamental dimensional parameters in particle physics. These are the Fermi constant $G_F \simeq 10^{-5}$ GeV^{-2} and Newton's constant $G_N \simeq 10^{-38}$ GeV^{-2}. Associated with the huge ratio of these numbers there are two problems of "naturalness". The first problem is to explain why their ratio is so huge. In the standard scalar models, this ratio is an input. Both G_F and G_N are introduced by hand in the theory. The second problem of naturalness is the great sensitivity of low energy physics to minute changes in the short distance bare quantities of the theory [1,2].

Technicolor [2] was introduced to solve these problems. However, Technicolor alone could not account for the current algebra masses of quarks and leptons. Extended Technicolor [3] was introduced to solve this difficulty. In extended Technicolor theories there are no elementary scalar fields. All light scalars are composites of new degrees of freedom. Unfortunately, these theories seem to suffer from phenomenological problems with flavor changing neutral currents [4]. No elegant Technicolored analog to the ingenious GIM mechanism has been found.

In this paper, we propose a new set of natural theories which replace Extended Technicolor. They are theories with elementary scalars. The scalars are light because they are protected by supersymmetry [5] which is a good symmetry down to ~ TeV. The fermions acquire their mass via Yukawa couplings to the light scalars. The scale of the supersymmetry breaking is determined by the scale at which a new strong interaction becomes strong. We call this interaction Supercolor. Supercolor may be different from Technicolor, since it is not necessarily involved in the breaking of $SU(2)_L \otimes U(1)_Y$. The characteristic scale of Supercolor can be as high as ~ 10 TeV.

2. Supersymmetry

In this section we recall the obvious virtues of supersymmetric theories [5,6]. In the standard model of electroweak interactions, the mass of the $W^{\pm}$ and Z^{o} boson is proportional to the expectation value of a scalar field ϕ, i.e., $M_W \sim \langle\phi\rangle$. Since ϕ is not protected by chiral or gauge symmetries from receiving a large mass, the natural value for $\langle\phi\rangle$ is of order 10^{19} GeV . The way to remedy this difficulty is to introduce supersymmetry. The scalars are then protected from obtaining mass by the chiral symmetries of their fermionic partners. Since phenomenologically there are no scalar partners to ordinary quarks and leptons, supersymmetry must be spontaneously broken at a scale of the order of the weak interactions. Thus, m_ϕ will be of order $G_F^{-\frac{1}{2}}$. Quarks and leptons receive mass in this scheme via standard Yukawa couplings. This scenario as outlined still suffers from the first problem of naturalness since it does not provide an explanation for the weak interaction scale. The value of the weak interaction scale is put in by hand as a fundamental parameter in the Lagrangian. Naively these theories do not seem to suffer from the second problem of naturalness because scalars are protected from quadratically divergent mass corrections. However, as shown in the next section, these types of models may in fact suffer from the second problem of naturalness (in some important cases).

3. Supersymmetric U(1) problem

The purpose of this section is to point out the existence of a very important class of super symmetric theories which are unnatural in the sense that they do not protect scalars from receiving huge linearly divergent masses. To be specific we will prove the following statement:

"Supersymmetric theories are unnatural if: (1) they contain a U(1) gauge multiplet [5,6]: $V \equiv (A_\mu, \chi; D)$, and (2) the symmetry $V \to -V$ is broken by dimension-four operators (we shall call the operation $V \to -V$ "parity")."

To prove this statement we begin by introducing left-handed chiral multiplets [5,6] $S_i \equiv (\phi_i, \psi_i; F_i)$ with U(1) charges e_i; i = 1, 2, ... We assume that the U(1) is anomaly free, i.e.,

$$\sum_i e_i^3 = 0 \tag{3.1}$$

Consider now the following terms in the Lagrangian that contribute to the scalar potential*

$$\mathscr{L} \supset \frac{1}{2} D^2 + D \sum_i e_i \phi_i^\dagger \phi_i + \xi D \tag{3.2}$$

We assume that there are chiral symmetries which forbid explicit scalar masses. In addition, we have omitted from the scalar potential eq. (3.2) any terms that are proportional to Yukawa couplings. Equation (3.2) contains all the gauge contributions to the scalar potential. Inclusion of Yukawa couplings will <u>not</u> change any of our conclusions on quadratic mass divergences since the gauge and Yukawa couplings are totally unrelated.

Another convenient expression for the scalar potential eq. (3.2) is obtained by integrating out the auxilary D-field:

$$\mathscr{L} \supset -\frac{1}{2} \left\{ \sum_i e_i \phi_i^\dagger \phi_i + \xi \right\}^2 \tag{3.3}$$

Equation (3.3) shows that scalars have masses squared proportional to ξ.

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* Note that D is the Fayet-Illiopoulos D term [7].

Thus, if ξ is quadratically divergent then so are scalar masses squared. Note that eq. (3.3) also shows that in an anomaly free theory (eq. (3.1)), where there are both positive and negative charges e_i, a nonvanishing ξ always implies that the gauge symmetry is spontaneously broken whereas the supersymmetry is not broken.

If the theory is symmetric under the "parity" operation $D \to -D$, then ξ vanished identically to all orders. If the "parity" operation is softly broken, then ξ can be at most logarithmically divergent. If however the "parity" operation is broken by dimension-four operators, then ξ will be quadratically divergent.

Let us now give an explicit example for which "parity" is broken by dimension-four operators. The example consists of a U(1) gauge multiplet together with N left-handed chiral multiplets with charges $e_i = g$ $(i = 1, \ldots, N)$ and a left-handed chiral multiplet with charge $e_{N+1} = -gN^{\frac{1}{3}}$.

The Lagrangian of this model is:

$$\mathscr{L} = -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2} i\chi^\dagger \not{\partial}\chi - \frac{1}{2} i \sum_{i=1}^{N+1} \psi_i^\dagger \not{D} \psi_i - \sum_{i=1}^{N+1} |D_\mu \phi_i|^2$$
$$+ i\sqrt{2}\, g \left[\sum_{i=1}^{N} \psi_i \chi \phi_i^* - N^{\frac{1}{3}} \psi_{N+1} \chi \phi_{N+1}^* - \text{h.c.} \right] + \frac{1}{2} D^2$$
$$+ gD \left(\sum_{i=1}^{N} \phi_i^\dagger \phi_i - N^{\frac{1}{3}} \phi_{N+1}^\dagger \phi_{N+1} \right) + \xi D \quad . \qquad (3.4)$$

Note that the symmetry $D \to -D$ is broken by the hard operator

$$gD \left(\sum_{i=1}^{N} \phi_i^\dagger \phi_i - N^{\frac{1}{3}} \phi_{N+1}^\dagger \phi_{N+1} \right) \qquad (3.5)$$

because for any scalar ϕ of a given charge there is no corresponding scalar of opposite charge.

Let us now compute the one-loop corrections to ξ. They are given by the graphs of fig. 1. From these we see that the correction $\delta\xi$ to ξ is

$$\delta\xi \sim \Lambda^2 \sum_{i=1}^{N+1} e_i \tag{3.6}$$

where Λ is the cutoff. As a consequence scalars in this model obtain quadratically divergent masses squared.

From now on we define a U(1) to be <u>safe</u> if it admits a "parity" operation $V \to -V$ which is <u>not</u> broken by dimension-four operators.

It is important to notice that in the standard model with the usual families the electroweak hypercharge Y is <u>not</u> a safe U(1). To see this note that the hypercharge assignments of a family are

$$\begin{aligned} &Y\left(u_L\right) = Y\left(d_L\right) = \frac{1}{3} \\ &Y\left(\bar{d}_L\right) = \frac{2}{3} \; ; \quad Y\left(\bar{u}_L\right) = -\frac{4}{3} \\ &Y\left(\nu_L\right) = y\left(e_L\right) = -1; \quad Y\left(\bar{e}_L\right) = +2 \end{aligned} \tag{3.7}$$

Thus, for each supersymmetric scalar partner of ordinary fermions, there is no corresponding scalar with opposite hypercharge. Note that in the example given in sect. 3, the U(1) gauge generator was not traceless; i.e., $\sum_i^{N+1} e_i \neq 0$. Since hypercharge is traceless, the one-loop corrections to ξ_y vanish. In fact, the two-loop contributions to ξ of purely gauge interactions also vanishes if the U(1) is anomally free. However, without any symmetry preventing a D_y term for hypercharge, we expect a quadratically divergent ξ_y to be generated in higher loops. We are however cautioned by the knowledge that miraculous unexplained cancellations have been known to occur in supersymmetric theories.* We thus remark that if no such cancellations occur, then the result is very important. It changes our traditional point-of-view which was to have the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ symmetry with only standard families up to some extremely high energy $\Lambda >> \text{TeV}$.

What are possible solutions to this difficulty? Two possibilities suggest themselves:

a) There is no gauged U(1) for energies greater than $\Lambda(U_1) \gtrsim 1$ TeV. Note that this implies the existence of problematic light momopoles [8].

b) There are only safe U(1)'s for energies greater than $\Lambda(U_1) \gtrsim 1$ TeV. There are two ways to implement (b):

i) Hypercharge itself is a safe U(1) due to the existence of three heavy right-handed families. These families are expected to be in the 100 GeV range since their masses carry $\Delta I_L = \frac{1}{2}$.

ii) The gauge symmetry above $\Lambda(U_1) \gtrsim$ TeV is different than $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ and contains only new safe U(1)'s.

In the next section we elaborate on the above possibilities.

4) Uniscale and Biscale Supersymmetric Scenarios

In the previous section we introduced the energy scale $\Lambda(U_1)$ above which the theory changes. So far, the only thing that we know about $\Lambda(U_1)$ is that it cannot be very much higher than the electroweak scale or else the scalars would get masses much larger than $G_F^{-\frac{1}{2}}$. The value of

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* We thank Dr. M. Roĉek for this observation.

$\Lambda(U_1)$ differentiates between various Supersymmetric scenarios. Two possibilities suggest themselves:

$$\text{I.}\quad \text{(Uniscale scenario)}\ \Lambda\left(U_1\right) \sim G_F^{-\frac{1}{2}}$$

$$\text{II.}\quad \text{(Biscale secnario)}\ \Lambda\left(U_1\right) >> G_F^{-\frac{1}{2}}\ .$$

Under scenario I there are two inequivalent possibilities. The first possibility is that the gauge group changes above $\Lambda(U_1)$. In this case, there will be new gauge bosons with masses of order M_W. These bosons are potential hazards for such models since they typically mediate rare processes at rates comparable to those of ordinary weak interactions. The second possibility is that the gauge structure remains unchanged at $\Lambda(U_1)$, but three new heavy right-handed generations appear. An example of such a model is the grand unified theory with the gauge group O(18) and one chiral multiplet in the spinor representation* [9]. If O(18) breaks down to $SP(4)_{TECHNICOLOR} \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ at the grand scale and Technicolor becomes strong at ~ 1 TeV, then there exists three light ordinary generations. In addition there is a quintet of left-handed Techni-generations and two quartets of right-handed Techni-generations. Thus standard hypercharge in such a model is safe. Technicolor in this scenario breaks $SU(2)_L \otimes U(1)_Y$ at $\Lambda(U_1) \sim G_F^{-\frac{1}{2}}$. In both cases of the uniscale scenario, supersymmetry is assumed to break at $\Lambda(U_1)$. Clearly, much more work is required to see if a uniscale scenario can be viable.

In scenario II, there are necessarily two breaking scales $\Lambda(U_1)$ and $G_F^{-\frac{1}{2}}$. At $\Lambda(U_1)$ a gauge symmetry G is broken down to the standard

* Note that the naive scheme as just outlined cannot work since $SP(4)_{TC}$, with the given states, is unfortunately not asymptotically free.

$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. Supersymmetry in general may or may not be broken at $\Lambda(U_1)$. We shall however, only discuss the case where the supersymmetry is broken at $\Lambda(U_1)$. This is because as we shall show in the next section, if the breaking at $\Lambda(U_1)$ is dynamical, then supersymmetry is always broken.

As a result of the breaking of the supersymmetry at $\Lambda(U_1)$ the standard Higgs fields will obtain finite radiatively induced mass terms denoted by μ_h^2. As we shall show later on μ_h^2 will be at most of order

$$\mu_h^2 \sim \pm\, \alpha_1\, g_1^2\, \Lambda^2(U_1) \tag{4.1}$$

The sign of μ_h^2 appears to be model dependent and Higgs dependent. If the sign of μ_h^2 is negative, then we have the exciting possibility that the electroweak breaking scale $G_F^{-1/2}$ is radiatively induced. If the sign of μ_h^2 for all Higgs' is positive then the second scale of symmetry breaking would have to be introduced as an explicit scale in the Lagrangian.

Next we turn to a simple toy model to illustrate some of the ideas discussed in scenario II.

BISCALE TOY

We introduce an example of a biscale model. It includes the gauge interactions

$$SU(4)_{PS} \otimes SU(2)_L \otimes T_{3R} \tag{4.2}$$

where $SU(4)_{PS}$ is the Pati-Salam group, T_{3R} is the third component of right-handed isospin and $SU(2)_L$ is the standard left-handed weak isospin. Weak hypercharge in this model is a linear combination of T_{3R} and the

15th component of $SU(4)_{PS}$, i.e.,

$$Y = T_{3R} + \sqrt{\frac{2}{3}}\, P_{15} \tag{4.3}$$

where

$$P_{15} = \frac{1}{\sqrt{24}} \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -3 \end{pmatrix}_{PS}$$

The model is a toy since we shall omit any strong Supercolor group. As a result, the first breaking scale $\Lambda(U_1)$ is put in by hand; i.e., it arises from vacuum expectation values of explicit scalars and their associated auxilary fields.

At $\Lambda(U_1)$ we suppose $SU(4)_{PS} \otimes T_{3R}$ breaks down to $SU(3)_{color} \otimes Y$ leaving $SU(2)_L$ intact. Supersymmetry also breaks at this scale. We are then interested in calculating quadratic mass corrections to the Higgs potential which arise as a result of this breaking. We shall show that in this example, the Higgs mass squared vanishes to order $\alpha_R g_R^2$ (one loop) and thus obtains mass only to order $\alpha_R \alpha_P g_R^2$ (two loops).

We consider the following supermultiplets, transforming under $SU(4)_{PS} \otimes SU(2)_L \otimes T_{3R}$,

$$\begin{array}{ll} H^{\pm} & (1,\ 2,\ \pm\tfrac{1}{2}) \\ S & (10,\ 1,\ 1) \\ \bar{S} & (\overline{10},\ 1,\ -1) \\ N_1 & (1,\ 1,\ 0) \\ N_2 & (15,\ 1,\ 0) \end{array} \tag{4.4}$$

We use the superfield formalism, where the above fields are all left-handed chiral superfields [5,6]. For example, S is given by

$$S = \frac{1}{\sqrt{2}}\,\phi + (\theta\psi) + (\theta\theta)\,\frac{1}{\sqrt{2}}\,F \qquad .$$

θ_α is a two-component left-handed Grassman variable, $(\theta\psi) \equiv \theta_\alpha\psi_\beta\varepsilon^{\alpha\beta}$, $\phi = (1/\sqrt{2})(\phi_1 - i\phi_2)$ is a complex scalar, $F = (1/\sqrt{2})(F_1 + iF_2)$ is a complex auxilary field and ψ is a two-component Weyl spinor. The fields ϕ,ψ,F all transform in the (10,1,1) representation of $SU(4)_{PS} \otimes SU(2)_L \otimes T_{3R}$.

$H^\pm$ are the standard Higgs doublets in a two-Higgs model. S and $\bar{S}$ are introduced in order to break $SU(4)_{PS} \otimes T_{3R}$ down to $SU(3)_c \otimes Y$. They mimic the supercolor condensates. N_1 and N_2 are necessary to construct an effective potential whose minimum breaks both supersymmetry and $SU(4)_{PS} \otimes T_{3R}$ [6]. Finally, we shall ignore the ordinary quarks and leptons which transform as $(4,2,0) \oplus (\bar{4},1,\pm\tfrac{1}{2})$ for each generation.

The Lagrangian density for the model is as follows:

$$\begin{aligned}\mathscr{L} = \mathscr{L}_0 &+ \Big[S^\dagger \exp\left\{g_P V_P - 2g_R V_R\right\} S + \bar{S}\exp\left\{-g_P V_P + 2g_R V_R\right\}\bar{S}^\dagger \\ &+ N_2^\dagger \exp\left\{g_P V_P\right\} N_2 + N_1^\dagger N_1 + H^{+\dagger}\exp\left\{g_2 V_2 - g_R V_R\right\} H^+ \\ &+ H^{-\dagger}\exp\left\{g_2 V_2 + g_R V_R\right\} H^-\Big]_{D\,TERM} \\ &+ \Big[2h_P\,\bar{S}T_P\,SN_2^P + 4h_R\,\bar{S}SN_1 + \Lambda N_1\Big]_{F\,TERM} \qquad (4.5)\end{aligned}$$

where $\mathscr{L}_0$ is the Lagrangian density for the gauge multiplets. The constants, g_P, g_R, g_2 are the dimensionless couplings for the gauge interactions $SU(4)_{PS}$, T_{3R} and $SU(2)_L$, respectively. The constants h_P and h_R are arbitrary dimensionless constants and Λ is a parameter with dimensions of mass squared. In units of m_{PLANCK} we have $\Lambda \sim 10^{-28}$. Nevertheless, the model is natural in the second sense; i.e., low-energy physics does not sensitively depend on minute adjustments of the bare parameters. Note that T_{3R} is a "safe" U(1). This is guaranteed by the discrete symmetry

$$S \leftrightarrow \bar{S}^{\dagger}, \quad H^{+} \leftrightarrow H^{-}, \quad V_P \leftrightarrow -V_P, \quad V_2 \leftrightarrow V_2$$

$$V_R \leftrightarrow -V_R, \quad N_1 \leftrightarrow N_1^{\dagger}, \quad N_2 \leftrightarrow N_2^{\dagger} \tag{4.6}$$

Also no additional terms can be generated via radiative corrections. For example, a term like $H^{+}H^{-}N_1$ is forbidden by the discrete symmetry $H^{+} \leftrightarrow -H^{+}$ with all other fields unchanged.

The Lagrangian density must then be expanded in terms of the component fields. The minimum of the scalar potential must be found and perturbation theory is defined by small fluctuations about the minimum. The component fields are defined by the following expressions:

Matter multiplets:

$$\begin{aligned} S &: \left(\phi_{ab}, \psi_{ab}, F_{ab}\right) \\ \bar{S} &: \left(\bar{\phi}^{ab}, \bar{\psi}^{ab}, \bar{F}^{ab}\right) \\ H^{\pm} &: \left(h_i^{\pm}, \psi_{hi}^{\pm}, f_{hi}^{\pm}\right) \\ N_2 &: \left[\frac{1}{\sqrt{2}}\left(a_2 - ib_2\right)_P, n_{2P}, \frac{1}{\sqrt{2}}\left(f_2 + ig_2\right)_P\right] \\ N_1 &: \left[\frac{1}{\sqrt{2}}\left(a_1 - ib_1\right), n_1, \frac{1}{\sqrt{2}}\left(f_1 + ig_1\right)\right] \end{aligned} \tag{4.7}$$

Gauge multiplets:

$$\begin{aligned} V_P &: \left(V_P^{\mu}, \lambda_P, D_P\right) \\ \vec{V}_2 &: \left(\vec{V}_2^{\mu}, \vec{\lambda}_2, \vec{D}_2\right) \\ V_R &: \left(V_R^{\mu}, \lambda_R, D_R\right) \end{aligned} \tag{4.8}$$

where (a,b = 1,2,3,4) ϵ $SU(4)_{PS}$, (i = 1,2) ϵ $SU(2)_L$ and (P = 1, ..., 15) labels the adjoint representation of $SU(4)_{PS}$. Upon studying the scalar potential, we find that:*

$$\left\langle \phi_{44} \right\rangle = \left\langle \bar{\phi}^{44} \right\rangle \equiv F \neq 0 \tag{4.9}$$

minimizes the potential with

$$2F^2 = -\frac{h_R \ \Lambda}{\left(\frac{3}{2} h_P^2 + h_R^2\right)} \tag{4.10}$$

In addition, one linear combination of the auxilary fields obtains a nonvanishing vacuum expectation value; i.e.,

$$\langle \tilde{f} \rangle = \left\langle \cos\alpha f_1 + \sin\alpha f_2^{15} \right\rangle = -\cos\alpha\Lambda \tag{4.11}$$

where

$$\tan\alpha = \sqrt{\frac{2}{3}} \frac{h_R}{h_P} \quad .$$

Thus both supersymmetry and gauge symmetry are broken at the scale F. The remaining gauge symmetry is just $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ where Y is defined in Eq. (4.3).

We now want to calculate quadratic corrections to the Higgs mass. The relevant graphs are in fig. 2. They are obtained from the following terms in the Lagrangian:

$$\begin{aligned} \delta\mathscr{L}_{HIGGS} = {} & i\sqrt{2}\,\frac{g_R}{2}\left(\psi_{h^+}\, h^{+*}\ \lambda_R - h.c.\right) - i\sqrt{2}\,\frac{g_R}{2}\left(\psi_{h^-}\ h^{-*}\ \lambda_R - h.c.\right) \\ & - i\,\frac{g_R}{2}\left(h^{+*} \overleftrightarrow{\partial}_\mu h^+ - h^{-*} \overleftrightarrow{\partial}_\mu h^-\right)V_R^\mu + \frac{g_R^2}{4}\left(h^{+*}h^+ + h^{-*}h^-\right)\left(V_{\mu R}\right)^2 \\ & - \frac{1}{8}\ g_R^2\left(h^{+*}h^+ - h^{-*}h^- + 2\phi^*_{ab}\phi_{ab} - 2\bar{\phi}^{*ab}\bar{\phi}^{ab} + \begin{matrix}\text{other scalars} \\ \text{coupled to } D_R\end{matrix}\right)^2 \end{aligned} \tag{4.12}$$

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* Note that in this case $F = \Lambda(U_1)$.

After diagonalizing the mass matrix at the tree level, we make the following observation. The Higgs bosons do not receive any mass at the tree level. This is a result of the discrete symmetry $\phi_{ab} \leftrightarrow \bar{\phi}^{*ab}$ which is preserved by the vacuum. We note that the Higgs boson would receive mass at the tree level, if the term $H^+H^-N_1$ were present.

The Higgs boson can, in principle, receive mass to one-loop level. Such a correction is <u>a priori</u> proportional to $\alpha_R \bar{g}_R^2 F^2$, where $F \equiv \langle \phi_{44} \rangle$. The factor of α_R comes because the Higgs only couples to symmetry breaking effects through $\delta\mathscr{L}_{Higgs}$ (eq. (4.12)) in one-loop order. The factor g_R^2 comes from mixing of V_R and λ_R in the gauge-multiplet of T_{3R} with V_{15}^μ and λ_{15} in the gauge multiplet of P_{15} (eq. (4.3). If we were to ignore this mixing, then as far as the Higgs is concerned supersymmetry would be effectively unbroken at the one-loop level, and μ_h^2 would be zero. We shall in fact find that $\mu_h^2 = 0$ even to one-loop order. This result follows directly from the fact that the following states transform as degenerate supermultiplets at the tree level. Consider the six supermultiplets:

$$\begin{pmatrix} \phi_{44} \\ \\ \psi_{44} \end{pmatrix} \begin{pmatrix} \bar{\phi}^{44} \\ \\ \bar{\psi}^{44} \end{pmatrix} \begin{pmatrix} V_R^\mu \\ \\ \lambda_R \end{pmatrix} \begin{pmatrix} V_{15}^\mu \\ \\ \lambda_{15} \end{pmatrix} \begin{pmatrix} a_1 + ib_1 \\ \\ n_1 \end{pmatrix} \begin{pmatrix} a_2^{15} + ib_2^{15} \\ \\ n_2^{15} \end{pmatrix} . \quad (4.13)$$

After diagonalizing the mass matrix, we obtain the following massless and massive multiplets:

$$\begin{pmatrix} B^\mu \\ \\ \lambda_y \end{pmatrix} = \cos\beta \begin{pmatrix} V_R^\mu \\ \\ \lambda_R \end{pmatrix} + \sin\beta \begin{pmatrix} V_{15}^\mu \\ \\ \lambda_{15} \end{pmatrix}$$

$$\tan\beta = \sqrt{\frac{2}{3}}\ \frac{g_R}{g_P} \qquad (4.14)$$

$$M_B = m_{\lambda_y} = 0$$

B_μ is the gauge boson coupled to weak hypercharge.

$$\begin{pmatrix} B_\perp^\mu \\ \lambda_\perp \end{pmatrix} = \cos\beta \begin{pmatrix} V_{15}^\mu \\ \lambda_{15} \end{pmatrix} - \sin\beta \begin{pmatrix} V_R^\mu \\ \lambda_R \end{pmatrix}$$

$$\tilde{\psi}_{44} = \frac{1}{\sqrt{2}} \left(\psi_{44} - \bar{\psi}^{44}\right) \tag{4.15}$$

$$\tilde{B} = \frac{1}{2} \left(\phi_{44} + \phi_{44}^* - \bar{\phi}^{44} - \bar{\phi}^{44*}\right)$$

$$M_{B_\perp}^2 = m_{\lambda_\perp}^2 = \mu_{\tilde{B}}^2 = 4F^2\left(g_R^2 + \frac{3}{2} g_P^2\right) \quad .$$

$\lambda_\perp$ and $\tilde{\psi}_{44}$ form a massive Dirac fermion. The state $\tilde{B}' = (1/2i)\,(\phi_{44} - \phi_{44}^* - \bar{\phi}^{44} + \bar{\phi}^{44*})$ is the Goldstone boson associated with broken T_{3R} and is eaten by $B_\perp^\mu$.

$$\begin{pmatrix} \tilde{a}' \\ \tilde{b}' \\ n_y \end{pmatrix} = \cos\alpha \begin{pmatrix} a_1 \\ b_1 \\ n_R \end{pmatrix} + \sin\alpha \begin{pmatrix} a_2^{15} \\ b_2^{15} \\ n_2^{15} \end{pmatrix} \tag{4.16}$$

$$\mu_{\tilde{a}'} = \mu_{\tilde{b}'} = m_{n_y} = 0$$

n_y is the Goldstone fermion.

$$\tilde{A} = \frac{1}{2}\left(\phi_{44} + \phi_{44}^* + \bar{\phi}^{44} + \bar{\phi}^{44*}\right); \quad \tilde{A}' = \frac{1}{2i}\left(\phi_{44} - \phi_{44}^* + \bar{\phi}^{44} - \bar{\phi}^{44*}\right)$$

$$\begin{pmatrix} \tilde{a} \\ \tilde{b} \\ n_\perp \end{pmatrix} = \cos\alpha \begin{pmatrix} a_2^{15} \\ b_2^{15} \\ n_2^{15} \end{pmatrix} - \sin\alpha \begin{pmatrix} a_1 \\ b_1 \\ n_1 \end{pmatrix} \quad ; \quad \tilde{\tilde{\psi}}^{44} = \frac{1}{\sqrt{2}}\left(\psi_{44} + \bar{\psi}^{44}\right) \tag{4.17}$$

$$\mu_{\tilde{A}}^2 = \mu_{\tilde{A}'}^2 = \mu_{\tilde{a}}^2 = \mu_{\tilde{b}}^2 = m_{n_\perp}^2 = 4F^2\left(h_R^2 + \frac{3}{2} h_P^2\right)$$

$n_\perp$ and $\tilde{\tilde{\psi}}^{44}$ form a massive Dirac fermion.

As a result of this degeneracy at the tree level and the unbroken discrete symmetry $\phi \to \bar{\phi}^*$ we obtain

$$\mu^2_{h^+} = \mu^2_{h^-} = 0 \tag{4.18}$$

to one-loop order. This is strictly a one-loop result. The reason for this is that at the tree level the relevant particles (see multiplets in eqs. (4.13)- (4.17)) formed degenerate supermultiplets. The one-loop contributions are going to split this degeneracy. In fact, the amount of splitting will be related to the couplings of the Goldstino (see fig. 3). Once the degeneracy of the supermultiplets is lifted, the Higgs will obtain a mass of the order of

$$\mu^2_h \sim \pm\, \alpha_R \alpha_P g^2_R F^2 \tag{4.19}$$

If $\mu^2_{h^+}$ and/or $\mu^2_{h^-}$ is negative, then the scale of weak interactions is radiatively induced and does not have to be introduced by hand! In this case

$$G_F^{-1/2} \sim \sqrt{\alpha_R \alpha_P}\ F \tag{4.20}$$

which implies that

$$F \equiv \Lambda\left(U_1\right) \sim 10 \text{ TeV}$$

Note that the order in which the Higgs mass is induced is model dependent.

5. Supercolor

In the previous sections we outlined the advantages of supersymmetric theories and discussed some constraints in order that these theories satisfy the second criterion of naturalness. Namely, the low energy world in these theories is insensitive to minute changes of the high energy bare quantities of the theory. These theories however do not satisfy the first criterion of naturalness. That is, dimensional quantities much smaller than the fundamental cutoff have to be introduced by hand. In order to solve this problem, we introduce into the preceding scenario a new strong interaction with new fermions carrying this strong charge. These new fermions and their interactions shall replace the states S, $\bar{S}$, N_1 and N_2 (eq. (4.4)) introduced previously in order to break both supersymmetry and the gauge symmetry. The resulting theory does not explicitly contain any dimensional parameters and the huge ratio of the Planck scale to the weak interaction scale is naturally explained by the logarithmic variation of the dimensionless coupling for the new strong interaction.

In this section we want to describe how the new strong interaction can in principle break both supersymmetry and the gauge symmetry.

In a uniscale scenario (sect. 4) the new strong interaction is identical with the usual Technicolor forces. In a biscale scenario (sect. 4) however, the first scale of symmetry breaking denoted by $\Lambda(U_1) >> G_F^{-\frac{1}{2}}$ does not involve the breaking of the standard $SU(2)_L \otimes U(1)_Y$ weak forces. As a result we shall refer to this new strong force, which is responsible for the breaking at $\Lambda(U_1)$ as Supercolor.

We now wish to demonstrate how supersymmetry is broken by condensates which are bilinear in superfermions belonging to scalar multiplets. (Superfermions are the fermions that carry Supercolor in the biscale scenario or Technicolor in the uniscale scenario). Consider two scalar multiplets S_1 and S_2 given by

$$S_1 = \frac{1}{\sqrt{2}} \phi_1 + \psi_1\theta + \frac{1}{\sqrt{2}} F_1\theta\theta$$

$$S_2 = \frac{1}{\sqrt{2}} \phi_2 + \psi_2\theta + \frac{1}{\sqrt{2}} F_2\theta\theta \quad . \tag{5.1}$$

Assume now that when Supercolor becomes strong at a scale Λ_s, ψ_1 and ψ_2 condense.

$$\langle \psi_1\psi_2 \rangle \sim \Lambda_S^3 \neq 0 \quad . \tag{5.2}$$

It is easy to see that this condensate breaks the supersymmetry <u>if</u> the equation of motion for the auxiliary fields is:

$$F_1 = F_2 = 0 \quad . \tag{5.3}$$

To see this, note that by multiplying S_1S_2 we obtain a new scalar multiplet:

$$S_1 \cdot S_2 = \frac{1}{2}\phi_1\phi_2 + \frac{1}{\sqrt{2}}\left(\phi_1\psi_2 + \phi_2\psi_1\right)\theta + \frac{1}{2}\left(\phi_1F_2 + \phi_2F_1 + \psi_1\psi_2\right)\theta\theta \tag{5.4}$$

Thus under a supersymmetry transformation parametrized by α we have that:

$$\delta \frac{1}{\sqrt{2}}\left(\phi_1\psi_2 + \phi_2\psi_1\right) = \frac{1}{2} \not{\partial}\left(\phi_1\phi_2\right)\alpha + \frac{1}{2}\left(\phi_1F_2 + \phi_2F_1 + \psi_1\psi_2\right)\alpha \tag{5.5}$$

Therefore, if $F_1 = F_2 = 0$ and $\langle\psi_1\psi_2\rangle \neq 0$, we have

$$\left\langle \frac{1}{\sqrt{2}} \delta\left(\phi_1\psi_2 + \phi_2\psi_1\right)\right\rangle = \frac{1}{2}\left\langle\psi_1\psi_2\right\rangle\alpha \neq 0 \tag{5.6}$$

and supersymmetry is spontaneously broken. We can then identify $(1/\sqrt{2})\ (\phi_1\psi_2 + \phi_2\psi_1)$ as the Goldstone fermion associated with the breaking of supersymmetry.

So far we have shown that when the fermions contained in the two scalar multiplets S_1 and S_2 condense, supersymmetry is broken if $F_1 = F_2 = 0$. Under what conditions is $F_1 = F_2 = 0$ satisfied? This question is easily answered. Notice that the kinetic energy terms of the multiplet S_1 and S_2 give rise to terms in the Lagrangian which have the form

$$\mathscr{L} \supset \frac{1}{2} F_1^* F_1 + \frac{1}{2} F_2^* F_2 \quad . \tag{5.7}$$

In order to ensure that $F_1 = F_2 = 0$, it is sufficient to require that no other terms containing F_1 or F_2 appear in the Lagrangian. This is easily done. Terms of the form

$$\left(S_1 \cdot S_2\right)_F \; ; \quad \left(S_1 \cdot S_1\right)_F \; ; \quad \left(S_2 \cdot S_2\right)_F \tag{5.8}$$

are forbidden by chiral and/or gauge symmetries. Such chiral symmetries are present in out theories since masses are <u>not</u> put in by hand. Similarly, terms of the form

$$S_1^2 \cdot S_2 \; ; \quad S_1 \cdot S_2^2 \; ; \quad S_1^3 \; ; \quad S_2^3 \tag{5.9}$$

are forbidden by gauge symmetries in the realistic examples of interest.[§] The terms of eqs. (5.8) and (5.9) are the only ones allowed by renormalizability. Thus indeed the equations of motion $F_1 = F_2 = 0$ are easy to satisfy. They simply follow from chiral and gauge symmetries.

As a specific example for applying these ideas consider an $SO(N)_{SC} \otimes SU(4)_{PS} \otimes SU(2)_L \otimes T_{3R}$ group with the following chiral

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§ Our argument is exact in the limit in which the Yukawa couplings g_Y of superfermions to ordinary fermions ($g_Y\, \psi_1\, \psi\, \phi_2$, ψ = ordinary fermion) vanish. Turning on a small Yukawa coupling will <u>not</u> change the results.

multiplets:

$$S_{a\alpha} = \left(N,\ 4,\ 1,\ \tfrac{1}{2}\right)$$

$$S'^{a}_{\beta} = \left(N,\ \bar{4},\ 1,\ -\tfrac{1}{2}\right) \qquad (5.10)$$

$$H^{\pm} = \left(1,\ 1,\ 2,\ \pm\tfrac{1}{2}\right)$$

where a = 1, 2, 3, 4 is an $SU(4)_{PS}$ index and $\alpha,\beta = 1, \ldots N$ are SO(N) indices.

When the Supercolor $SO(N)_{sc}$ forces become strong at the scale Λ_s we assume that the following condensates form:*

$$\left\langle \left(S_{4\alpha} S_{4\beta} \delta^{\alpha\beta}\right)_F \right\rangle = \Lambda_S^3$$

$$\left\langle \left(S'^{4}_{\alpha} S'^{4}_{\beta} \delta^{\alpha\beta}\right)_F \right\rangle = \Lambda_S^3 \qquad (5.11)$$

$$\left\langle \left(S_{c\alpha} S'^{c}_{\beta} \delta^{\alpha\beta}\right)_F \right\rangle = \Lambda_S^3$$

where c = 1,2,3 is the $SU(3)_{color}$ index. These condensates break both supersymmetry and the gauge symmetry $SU(4)_{PS} \otimes T_{3R}$ down to $SU(3)_c \otimes U(1)_Y$. They replace the scalar multiplets S_{ab}, $\bar{S}_{ab}$, N_1 and N_2 of the example of sect. 4. Since supersymmetry gets broken at Λ_S, the Higgs fields $h^{\pm}$ can obtain masses squared proportional to Λ_S^2. In the present dynamical example these masses are of order

$$\mu_h^2 \sim \alpha_R\, g_R^2\, \Lambda_S^2 \qquad (5.12)$$

The Higgs does not receive mass at the tree level as a result of the discrete symmetry $S_{a\alpha} \to S'^{a\dagger}_{\alpha}$ which we assume is not spontaneously

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* The breaking pattern we have just discussed does not satisfy the criteria of subgroup alignment as discussed by M. Peskin and J. Preskill [10].

broken. It does receive a mass squared to one loop from the graphs of fig. 4. This mass squared is of order

$$\mu_h^2 \sim \pm\, \alpha_R\, m^2 \tag{5.13}$$

where $m^2 \sim g_R^2 \Lambda_S^2$. The various factors in eq. (5.13) are simply understood as follows: the factor of α_R arises because we are computing to one loop and T_{3R} is the only common interaction of the Higgs supermultiplet and the supersymmetry breaking supermultiplets S and S'. μ_h^2 is proportional to Λ_S^2 because Λ_S is the scale of supersymmetry breaking as well as the scale below which an unsafe U(1) emerges. The reason why $m^2 \sim g_R^2 \Lambda_S^2$ instead of $m^2 \sim g_P^2 \Lambda_S^2$ is associated with the fact that all of the mass squared mixings of the $U(1)_{T_{3R}}$ gauge boson are proportional to g_R (see figs. 4). Again if $\mu_h^2 < 0$ then the weak interaction scale would arise by radiative corrections from the scale Λ_S. In this case we would have

$$G_F^{-\frac{1}{2}} \sim \sqrt{\alpha}\, \Lambda_S \tag{5.14}$$

or $\Lambda_S \sim 3$ TeV. This implies that the color triplet Pati-Salam generators would have masses of order of 3 TeV. Phenomenological constraints from $K \to \mu e$ exclude such light Pati-Salam generators [11]. Thus, if the above estimates are correct, this scenario with $\mu_h^2 < 0$ is excluded and thus the scale of weak interactions has to come from a second round of Technicolored dynamical symmetry breaking. In other models where the symmetry above Λ_S does not include $SU(4)_{PS}$ it is of course still possible to have the scenario with $\mu_h^2 < 0$ and thus a radiatively induced scale of weak interactions.

6. Closing remarks

In this paper we outlined several new scenaria for constructing natural theories with elementary scalars. Scalars were protected from receiving ultraheavy masses by having a supersymmetry down to some energy scale $\Lambda \sim .3$-10 TeV. The first consequence of these scenaria was that the symmetry of the world above Λ cannot be $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ with standard families.

The most appealing scenaria of all that we have presented are the uniscale scenario with right-handed generations at ~ 100 GeV and the biscale scenario with radiatively induces scale of weak interactions. We proposed scenaria combining supersymmetry with a new strong force (Supercolor), which would be able to explain the magnitude of the scale of low energy physics where supersymmetry and weak interactions are broken.

Our discussion has been very general and no models have been offered. Therefore, we are not able to address several important issues. These issues include: (1) understanding why the usual particles remain light whereas their supersymmetric partners become heavy or unobservable, and (2) understanding whether rare processes (i.e., $K_L - K_S$ mass difference) are indeed supressed in spite of the several new degrees of freedom that can potentially be dangerous.

Upon completion of this work, we were informed that E. Witten has proven the following theorem concerning the supersymmetric U(1) problem. If the U(1) interaction is unified at some grand scale Λ_{GUM} in a non-Abelian gauge group, then the associated D term $\xi \propto \Lambda_{GUM}^2$ does not arise. In view of this result, the simplest scenario consistent with our philosophy is to have a grand unified theory G_{GUT} of all ordinary particles, together with a Technicolored group G_{TC}, such that the theory at the grand scale is $G_{GUT} \otimes G_{TC} \otimes$ SUPERSYMMETRY. A possible breaking scheme would involve the breaking of G_{GUT} at $\Lambda_{GUM} \sim 10^{15}$ GeV to $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes G_{TC} \otimes$ SUPERSYMMETRY. Then G_{TC} becomes strong at $\Lambda_{TC} \sim$ TeV and forms T.C. condensates $\langle\bar{Q}Q\rangle$ which break supersymmetry as discussed in sect. 5. Finally, if the Higgs field couples directly to $\bar{Q}Q$ it will obtain a vacuum expectation value $\langle h\rangle$ which in turn gives mass to ordinary fermions via standard Yukawa couplings.

Finally, we have learned that M. Dine, W. Fischler and M. Srednicki are working on ideas similar to those discussed in this paper and that a manuscript is now in preparation.

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Figure captions

Fig. 1. One loop contribution to $\delta\xi$ is quadratically divergent in the model of sect. 3.

Fig. 2. The graphs contributing to the one loop calculation of μ_h^2 for the biscale toy.

Fig. 3. One loop coupling of the Goldstino η_y to the gauge multiplet $\lambda_\perp, B_\mu^\perp$.

Fig. 4. Lowest order contributions to μ_h^2 in a dynamical scheme. The states Ψ, Φ and Ψ', Φ' are the fermion and scalar elements of the matter multiplets S and S', respectively.

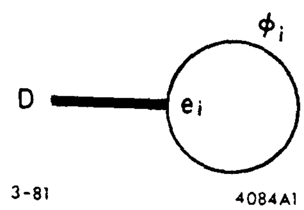


Fig. 1

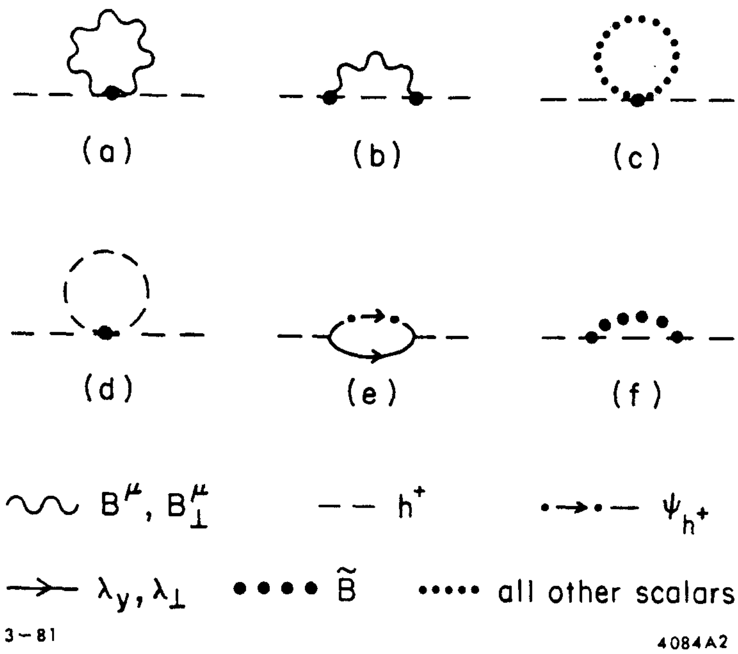
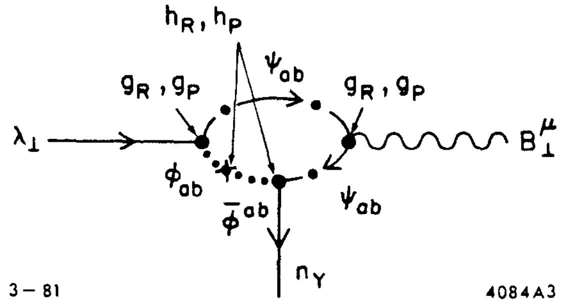

(a)
(b)
(c)
(d)
(e)
(f)
B^μ, $B_\perp^\mu$
h^+
ψ_{h^+}
λ_y, $\lambda_\perp$
$\tilde{B}$
all other scalars
3-81
4084A2

Fig. 2



h_R, h_P
g_R, g_P
ψ_{ab}
g_R, g_P
$\lambda_\perp$
$B_\perp^\mu$
ϕ_{ab}
$\bar{\phi}^{ab}$
ψ_{ab}
n_Y
3-81
4084A3

Fig. 3

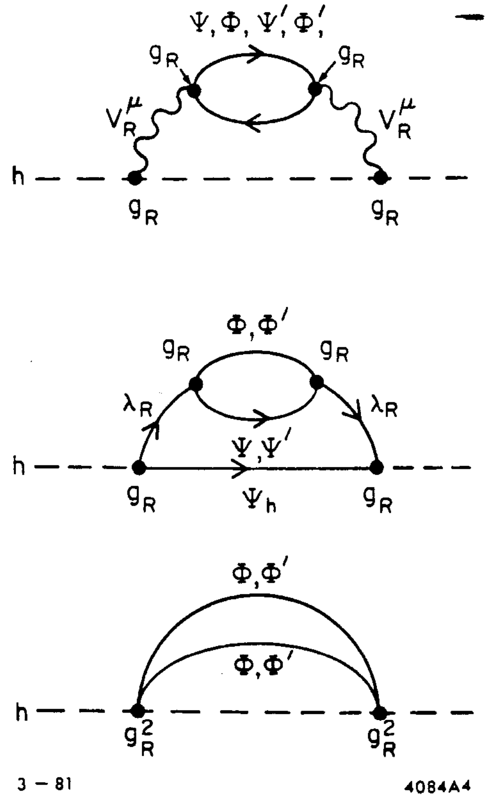

Ψ, Φ, Ψ′, Φ′
g_R
g_R
V_R^μ
V_R^μ
h
g_R
g_R
Φ,Φ′
g_R
g_R
λ_R
λ_R
Ψ,Ψ′
h
g_R
Ψ_h
g_R
Φ,Φ′
Φ,Φ′
h
g_R^2
g_R^2
3 − 81
4084A4

Fig. 4