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EVIDENCE FOR LARGE-x CORRECTIONS IN QCD*

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ABSTRACT

We show there is evidence that a method of summing important logarithmic corrections which are significant in the large-x region leads to a superior description of both neutrino and electron deep-inelastic scattering data (analyzed using the evolution equations). Next-to-leading-order calculations can imitate the impact of this summation method, but at high x it appears that there are higher-order and higher-twist corrections which separate those approaches.

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Considerable effort has been devoted in the last few years to obtain clean and quantitative predictions in Quantum Chromodynamics (QCD) for various physical processes.[1] Among these, deep-inelastic processes could, in principle, represent an excellent laboratory for such tests. Although QCD predicts an observable logarithmic violation of scaling for the structure functions, there are difficulties which have prevented a completely satisfactory comparison between theory and experiment. Resonance production, elastic and diquark scattering, and more general higher-twist effects are among these.[2] Even when considering only the leading-twist contributions in the operator-product expansion (OPE) there are contributions of higher order in α_s the running coupling constant. And, as stated recently,[3-9] the summation of certain logarithmic terms (in moments they are characterized as $\alpha_s \ell n^2 n$) leads to large corrections in the $x \to 1$ region ($x = Q^2/2p\cdot q$).

In this paper, we use the evolution equations[10] to analyze the impact of both second-order and $x \to 1$ corrections to the deep-inelastic structure functions, and we consider the relationship between them. We make direct comparisons between theory and experiment for both electron and neutrino deep-inelastic scattering, using Stanford Linear Accelerator Center - Massachusetts Institute of Technology (SLAC-MIT)[11] and CERN-Dortmund-Heidelberg-Saclay (CDHS)[12] collaboration data. We find that there are dramatic indications in the data to support theoretical expectations.

Brodsky and Lepage[3] have observed that large nonleading contributions to the structure functions, which arise in the $x \to 1$ region due to the gluon radiation corrections of the theory[13], are related to kine-

matical constraints. Analyzing deep-inelastic scattering, they observed that the use of the correct kinematic constraints plus the correct argument of α_s in evolution equations introduces an additional term. For the moments of the structure functions, this reproduces the sum of the large $\alpha_s \log^2 n$ corrections (which come from the $x \to 1$ region). These corrections are the same ones as computed using the operator-product expansion[8,9] to order α_s. Similar arguments about the resummation of gluon bremsstrahlung effects (including also the Drell-Yan case) have been developed by Parisi[4] and by Curci and Greco.[5] Amati, Bassetto, Ciafaloni, Marchesini and Veneziano[6] by a careful analysis of the strong-ordering contributions to the invariant charge have proposed a modified evolution equation which, due to the rescaled argument of the running coupling constant, resums[14] the large corrections. The proof that such a resummation occurs has been given only at the leading infrared singularity level. The kinematics-dependent scale in the coupling constant is given by the upper limit on the emitted-gluon invariant mass. The modified leading-order evolution equation for the structure functions is, in the non-singlet (NS) case, then:

$$Q^2 \frac{\partial}{\partial Q^2} F^{NS}(x,Q^2) = \int_x^1 dz\, F^{NS}\left(\frac{x}{z},Q^2\right)\left[\frac{\alpha_s\left(Q^2(1-z)/z\right)}{2\pi} P(z)\right]_+ \qquad (1)$$

where $F^{NS} = (F_2^{ep} - F_2^{en})$ or $xF_3^{\nu N}$. The + notation indicates the regularization procedure (see Ref. 10). This equation sums the large logarithms which arise when the real emission of gluons cannot compensate for the large, opposite effects of the virtual contributions.

The perturbative approach of Eq. (1) breaks down for $(1-z) < Q_c^2/Q^2$ (where Q_c^2 is a mass of order 1 GeV^2 such that $\alpha_s(Q_c^2)/2\pi < 1$).

The structure function becomes of the quark form-factor type and shows a Sudakov-type damping.[1]

Considering the new argument of α_s in Eq. (1), we see that as x becomes large and Q^2 is held fixed, the running coupling constant becomes large. In order to investigate this problem we introduced the following definition[15] of α_s:

$$\alpha_s\left(\frac{Q^2(1-z)}{z}\right) = \frac{4\pi}{\beta_0}\,\frac{1}{\log(V^2/\Lambda^2)} \tag{2}$$

where we define[16] $V^2 \equiv Q^2(1-z)/z + m^2$, with m a free mass parameter. Such a definition, which leaves α_s unchanged at large values of the argument, makes it possible to make a limited estimate of the sensitivity of calculations to this region.[17] Use of this definition (with the parameter m^2) is equivalent to the inclusion of higher-twist terms since:

$$\frac{1}{\ell n\,\dfrac{Q^2(1-z)/z+m^2}{\Lambda^2}} \approx \frac{1}{\ell n\,\dfrac{Q^2(1-z)/z}{\Lambda^2}}\left(1 - \frac{zm^2}{Q^2(1-z)\,\ell n\,\dfrac{Q^2(1-z)/z}{\Lambda^2}}\right) . \tag{3}$$

When we (later) make use of the second-order-in-$\alpha_s(Q^2)$ evolution equations, we follow the work of Curci, Furmanski and Petronzio[18] and write (using the $\overline{\text{MS}}$ renormalization scheme[9]):

$$Q^2\,\frac{\partial}{\partial Q^2}\,F^{NS}(x,Q^2) = \int_x^1 dz\; F^{NS}\left(\frac{x}{z},Q^2\right)\left[\frac{\alpha_s(Q^2)}{2\pi}\,P^{(1)}(z) + \frac{\alpha_s^2(Q^2)}{(2\pi)^2}\,P^{(2)}(z)\right]_+ \tag{4}$$

where $P^{(1)}$ and $P^{(2)}$ are defined in Ref. 18.

Now turning to our results: What is the impact of using the variable $V^2 \left(\equiv Q^2(1-z)/z + m^2 \right)$ instead of Q^2 in Eq. (1)? Clearly one expects to find a lower value[19] of Λ. For CDHS ν data, Λ decreases from $\Lambda \approx 0.33$ GeV for Q^2 evolution to $\Lambda \approx 0.20$ GeV for V^2 evolution. For SLAC data, $\Lambda = 0.66$ GeV decreases to 0.43 GeV.

More important is the result that the data are better described by V^2 evolution than by Q^2 evolution (first order). We find that the χ^2 for fitting data with V^2 evolution are noticeably better: $\chi^2 = 90.7$ versus 93.3 for Q^2 evolution (76 dof) for neutrino data and $\chi^2 = 62.8$ versus 67.8 (72 dof) for electron data.

This can be understood, in part, by noting that whereas the first-order Q^2-evolution equation implies a fixed, constant value of Λ, the use of this Q^2-evolution to extract Λ from the data results in a Λ which is dependent of x (see Fig. 1). Such a contradiction is not found when V^2 evolution is used to extract Λ from the data; here we find Λ consistent[20] within errors with being independent of x (see Fig. 1). Because V^2 evolution leads to a constant Λ, the resulting global (all x) fit to the data is superior to that from the leading-order Q^2 approach.

How do results using the evolution equation calculated to second-order in $\alpha_s(Q^2)$ compare with those from leading-order Q^2 evolution? The impact on the overall value of Λ is very small (e.g., $\Lambda^{(1)} \approx 0.33$ GeV and $\Lambda^{(2)} \approx 0.35$ GeV). One notes, however, that the χ^2 for second-order fits to the data are somewhat better than for first order. The cause of this improved fit is that (as for V^2 evolution) use of second-order evolution, Eq. (4), to extract Λ from data can give a Λ relatively independent of x. That it is possible for second-order evolution to imitate the effects of V^2 evolution is easy to see since Eq. (4) can be

rewritten as[21]:

$$Q^2 \frac{\partial}{\partial Q^2} F^{NS}(x,Q^2) = \int_x^1 dz F^{NS}\left(\frac{x}{z},Q^2\right)\left[\alpha_s\left(Q^2 f(z)\right)P^{(1)}(z) + \mathcal{O}(\alpha_s^3)\right]_+ . \quad (5)$$

By expanding in powers of $\ell n(Q^2/\Lambda^2)$, one finds

$$\left[P^{(1)}(z)\ell n f(z)\right]_+ \approx \left[\frac{4\pi}{\beta_0} P^{(2)}(z)\right]_+ + \left[P^{(1)}(z)\frac{\beta_1}{\beta_0^2}\ell n\ell nQ^2/\Lambda^2\right]_+ . \quad (6)$$

To see whether this approximate equivalence does in fact occur, we treated the output of the V^2-evolution equation (1) as "data", and then checked the x-dependence of Λ extracted using the second-order evolution equation (4). The results are shown in Fig. 2, where we also compare with the use of the first-order Q^2-evolution equation. Clearly the second-order equation (unlike the first-order equation) is consistent with giving a constant Λ. The fact that both methods simultaneously provide x-independent values for Λ confirms that the Q^2-evolution equation corrected to second order can to some extent handle the essential kinematic constraints as the V^2 evolution does.

How do curves from these different approaches compare? In Fig. 3a we show $\left(xF_3^{(2)} - xF_3^{(1)}\right)/xF_3^{(1)}$ (which we found looks very similar to $\left(xF_3^{(V^2)} - xF_3^{(1)}\right)/xF_3^{(1)}$). These results indicate that large differences for xF_3 in these approaches only occur for large x (as expected).[22] Figure 3b, showing the relatively small difference between the V^2-evolution and the second-order evolution results, raises an interesting question. The fact that the differences are significant only at very large x may indicate that the ability of the second-order equation to

reproduce the impact of the V^2-evolution approach fails at these x values, so that the higher-order terms (third order and higher) are increasingly important (and higher-twist corrections become increasingly important).[23] This is not unexpected since the second-order equation cannot account for terms such as $\alpha_s^2(Q^2)\log^4\left(1/(1-x)\right)$ and $xm^2/Q^2(1-x)$.

We also note that if one adjusts the V^2-evolution parameters to fit the second-order evolution curves, we find that the best fit has the parameter m^2 roughly equal to 0.9 GeV^2 (close to $m_p^2 = 0.88$). If we choose much different values (say $m^2 \approx 3$ GeV^2), the agreement is much poorer. It is not possible to determine m^2 from present data, since they are not precise enough at large x.

We believe that the analysis of the structure functions via the evolution equations has clear advantages over moment analyses. The data are found as structure functions, not as moments. The integration over x needed to obtain moments requires the extrapolation of data into unmeasured regions. In doing moment analyses we found that our results were critically sensitive to the nature of the extrapolation.

In conclusion, our results indicate that the use of the modified evolution equation (in terms of V^2) which should sum the most important logarithmic corrections at large x, leads to clearly superior descriptions of the neutrino and electron data (compared to that from leading-order Q^2 evolution). We also have shown that the second-order evolution equation (in Q^2) can imitate the impact of the V^2-evolution equation (by making Λ roughly x-independent). However, at large x the two approaches diverge, and one can hypothesize that it is higher-than-second-order-in-α_s and higher-twist-type corrections inherent in the V^2 approach which

distinguish them.

We feel that much could be learned from having more data in the large x region. Even for moderate x regions, use of the V^2-evolution equations for the analysis of data is indicated by our results.

A more detailed report of our work will appear elsewhere.

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16. Note that this definition has the same form as that of W^2, but it is not W^2 : $W^2 = Q^2(1-x)/x + m_p^2$.

17. The value of Λ from fits can vary by $\pm$ 20 MeV (for $\Lambda \approx 200$ MeV) depending on the value of m^2. Other implications of varying m^2 are discussed later.

18. G. Curci, W. Furmanski and R. Petronzio, Nucl. Phys. B175, 27 (1980); see also A. Gonzalez-Arroyo, C. Lopez and F. J. Yndurain, Nucl. Phys. B159, 512 (1979) and B153, 161 (1979).

19. One must, of course, beware not to apply inappropriate significance to the particular values of Λ here, since Λ is not well defined at leading order and since by use of the V^2 variable we modify the definition of Λ.

20. The statistics of the data are poor when single x bins are used, because low x data are mostly at low Q^2 and high x data are at high Q^2.

21. Analogous arguments were presented in Refs. 8 and 9 in relating the Λ_n-scheme to other second-order renormalization schemes.

22. It is possible to adjust parameters to minimize the differences at large x, but then the differences for $x < 0.8$ are significantly increased.

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FIGURE CAPTIONS

Fig. 1. The values of Λ extracted from SLAC-MIT[11] and CDHS[12] data for $(F_2^p - F_2^n)$ and xF_3 using first-order Q^2 evolution and using V^2 evolution with the data in large x bins.

Fig. 2. The values of Λ extracted using first-order and using second-order Q^2 evolution from "theoretical data" created from the output of the V^2-evolution equation. The input value of Λ was 0.2 GeV. Below $x = 0.2$, there is little sensitivity to the value of Λ.

Fig. 3. The fractional differences between (a) second-order and first-order-in-$\alpha_s(Q^2)$ structure functions and (b) the V^2-evolved and the second-order structure functions obtained by use of the corresponding evolution equations. The same x parametrizations were used in all cases. In (b) the particular Q^2 value at which the high-x difference is (approximately) zero is not significant; it varies with the choice of Q_0^2 (the starting point of evolution).

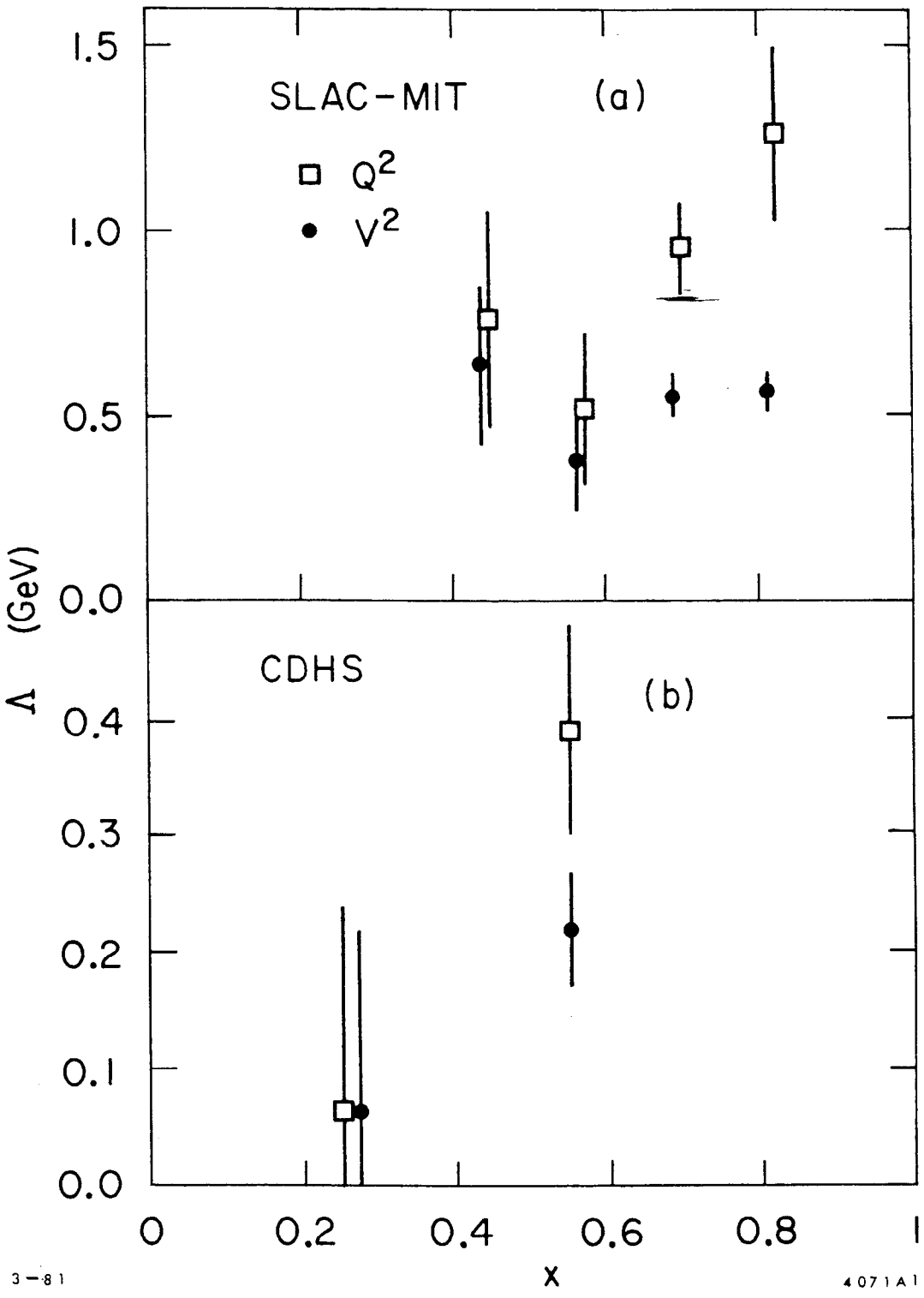
SLAC-MIT
(a)
Q^2
ν^2
CDHS
(b)
Λ (GeV)
1.5
1.0
0.5
0.0
0.4
0.3
0.2
0.1
0.0
0
0.2
0.4
0.6
0.8
1
x
3-81
4071A1

Fig. 1

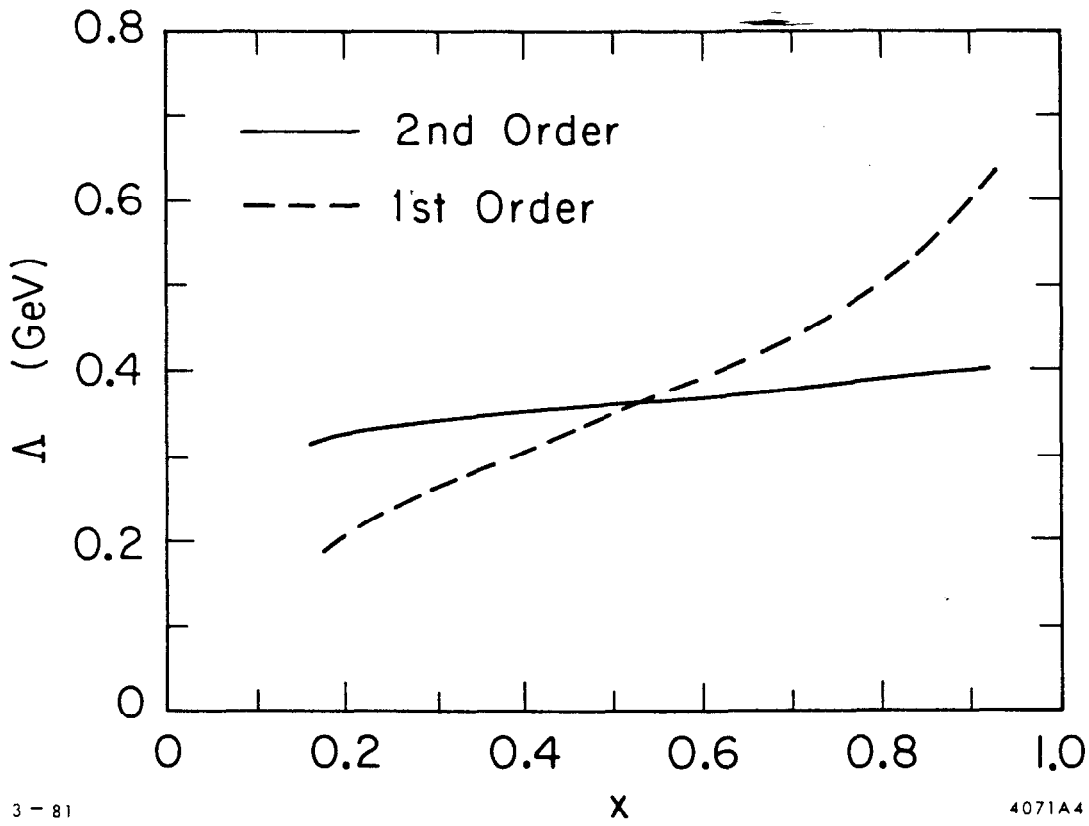
0.8
0.6
0.4
0.2
0
Λ (GeV)
2nd Order
1st Order
0
0.2
0.4
0.6
0.8
1.0
X
3 – 81
4071A4

Fig. 2

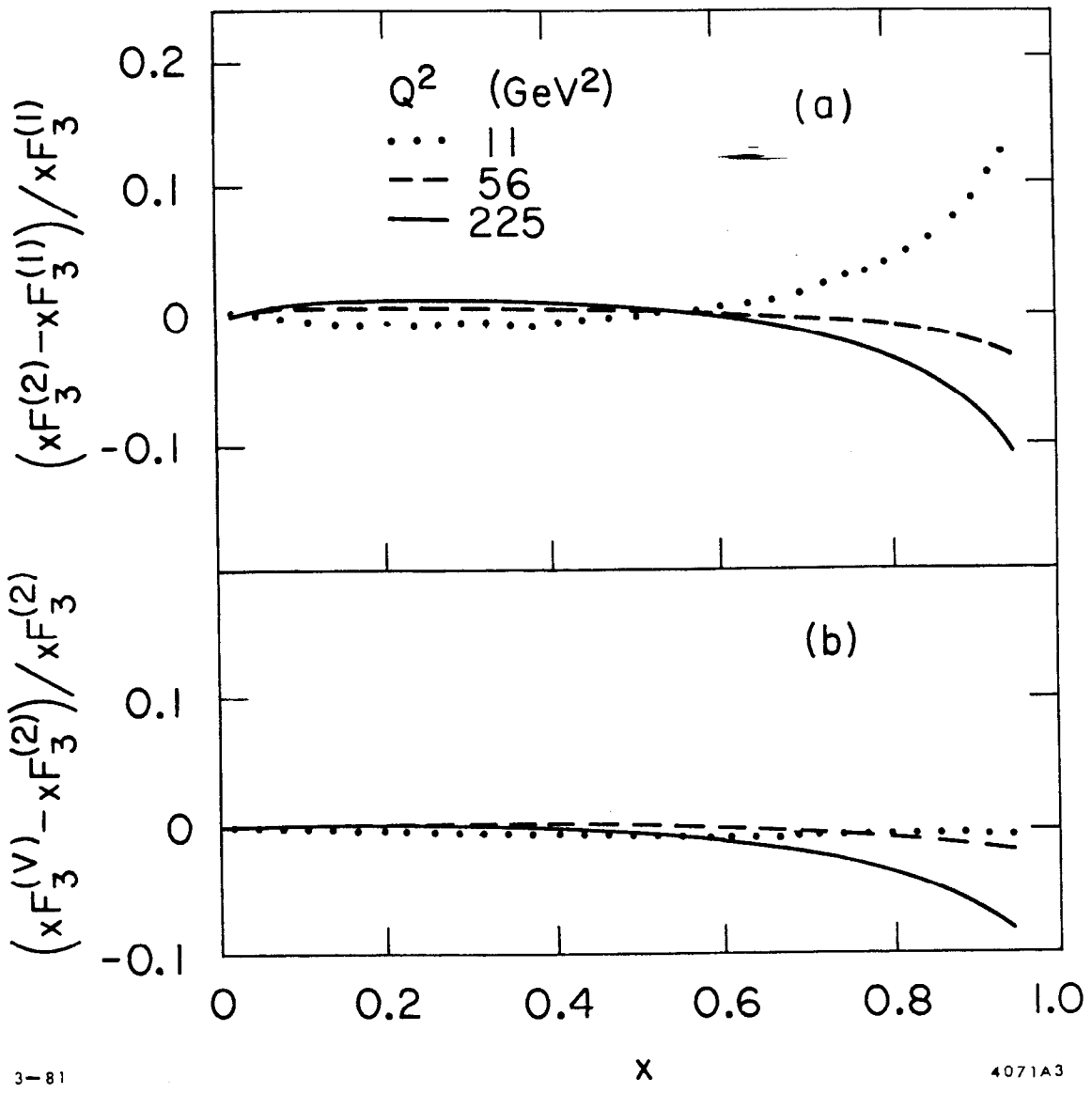
Q^2 (GeV^2)
11
56
225
(a)
(b)
$\left(xF_3^{(2)} - xF_3^{(11)}\right)/xF_3^{(11)}$
$\left(xF_3^{(V)} - xF_3^{(2)}\right)/xF_3^{(2)}$
0.2
0.1
0
-0.1
0
0.2
0.4
0.6
0.8
1.0
x
3-81
4071A3

Fig. 3