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τ DECAYS WITH SPIN-3/2 τ AND ν_τ AND PECULIARITIES OF MASSLESS RARITA-SCHWINGER PARTICLES*

David H. Miller
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

Abstract

τ decays involving spin-3/2 τ and ν_τ are discussed. In contrast to a previous study, it is argued that a current-current interaction amplitude involving the most general V,A currents for spin-3/2 τ and ν_τ can be chosen to be consistent with existing experimental data on τ decays. An apparent discontinuity in the $M_{\nu_\tau} \to 0$ limit of V,A currents constructed from spin-3/2 τ and ν_τ is considered. In connection with this discontinuity, the general problem of the helicity states allowed to a massless particle is reviewed, and the possibility is raised that states corresponding to nonmaximal helicities of a massless spin-3/2 particle may exist.

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1. Introduction

In recent years, the existence of a charged heavy lepton, called the τ, has been established by evidence from electron-positron annihilation experiments [1]. The accumulated experimental data — production cross section in $e^+e^- \to \tau^+\tau^-$, decay branching ratios, etc. — are all consistent with the τ and an associated massless neutrino ν_τ both being spin-1/2 particles. Indeed, the evidence is consistent with ν_τ,τ being sequential leptons exactly analogous to ν_e,e and ν_μ,μ — point Dirac particles fitting into the standard SU(2) $\otimes$ U(1) model [2] with τ in a right-handed SU(2) singlet and τ and ν_τ in a left-handed doublet.

A priori, there are a number of possible alternatives to this conventional picture: para- or ortho-leptons, nonstandard multiplet assignments of τ and ν_τ, etc. [3]. In this paper, we consider the possibility that τ and ν_τ both have spin 3/2.

The possibility that τ and/or ν_τ have spin 3/2 has been raised in the past [4]. For example, the initial difficulty in observing the decay mode $\tau \to \pi\nu_\tau$ produced the suggestion that ν_τ has spin 3/2 and τ has spin 1/2: with a massless ν_τ restricted to helicities of $\pm 3/2$, $\tau \to \pi\nu_\tau$ would then be strictly forbidden by helicity conservation [5]. (Subsequent observation of $\tau \to \pi\nu_\tau$ therefore rules out this possibility.)

Tsai [6] has argued that if the τ has spin 3/2, the behavior of the cross section $\sigma(e^+e^- \to \tau^+\tau^-)$ would be inconsistent with experiment. However, Kane and Raby [7] have suggested possible subterfuges by which nature might evade Tsai's argument; they therefore hold that the possibility that τ and ν_τ both have spin 3/2 remains open.

Alles [8] claims to dispose of this possibility by showing that spin-3/2 τ and ν_τ imply τ branching ratios and an electron energy spectrum in $\tau \to \nu_\tau e^- \bar{\nu}_e$ that are inconsistent with experiment; however, as Kane and Raby have pointed out, Alles fails to consider the most general V,A current that can be constructed from spin-3/2 τ and ν_τ.

In this paper, we assume, as does Alles, that the τ decay amplitude is of the current-current form $J^\mu_{(\tau-\nu_\tau)} \cdot J_{\mu(\text{other})}$, where $J_{\mu(\text{other})}$ is the standard V,A current which has been observed in other weak-interaction processes involving e, μ, hadrons, etc. Unlike Alles, we consider the most general form for $J^\mu_{(\tau-\nu_\tau)}$ which is consistent with proper Lorentz invariance for τ and ν_τ spins of 3/2.

The nonexistance of a renormalizable field theory for fundamental point-like spin-3/2 particles might be thought to rule out consideration of spin-3/2 leptons. However, as Kane and Raby suggest, τ and ν_τ might be composite particles with spins of 3/2; then, the fundamental constituent particles which make up the spin-3/2 τ and ν_τ could have spins less than 3/2. The fundamental interaction involving these constituent particles would not then involve spin-3/2 particles and could therefore be renormalizable. Of course, even though the fundamental theory would be renormalizable, the effective low-energy form of the interaction involving the composite spin-3/2 particles would not necessarily be renormalizable. However, one would still expect that the low-energy phenomenological amplitudes involving the spin-3/2 composite particles could be expressed in a current-current form with one current involving only the spin-3/2 composite τ and ν_τ and the other current involving the other particles participating in the reaction. An analogous situation presumably occurs in the weak decay $\Delta^+ \to \Delta^{++} e^- \bar{\nu}_e$. Although the fundamental (renormalizable) interaction presumably involves spin-1/2 quarks, one expects the phenomenological amplitude to be of the form $J^{\mu}_{(\Delta^{++}-\Delta^{+})} \cdot J_{\mu(e-\nu_e)}$ where $J^{\mu}_{(\Delta^{++}-\Delta^{+})}$ is constructed of Rarita-Schwinger spinors [9] representing the two spin-3/2 particles.

Since spin-3/2 τ and ν_τ might well be composite, one must allow nonconstant form factors, analogues of a Pauli anomalous magnetic moment term, etc., in $J^{\mu}_{(\tau-\nu_\tau)}$. Just as the p-n weak current is not the simple V-A current of point particles, so one should not expect $J^{\mu}_{(\tau-\nu_\tau)}$ for composite particles to have the simplest conceivable form. (In fact, for spin-3/2 particles, it is difficult to decide which current _is_ the "simplest conceivable".)

Allowing the most general $J^{\mu}_{(\tau-\nu_\tau)}$ with arbitrary form-factors, we find that Alles' conclusions ruling out spin-3/2 τ and ν_τ cannot be sustained: τ decays involving spin-3/2 τ and ν_τ can be made indistinguishable from the spin-1/2 case so long as one does not measure the τ or ν_τ spin or helicity.

Before discussing the general V,A currents for spin-3/2 τ and ν_τ and their applications to τ decay in section 4, we first discuss in the next section an apparent discontinuity in the $M_{\nu_\tau} \to 0$ limit of certain currents (and total rates) involving a spin-3/2 ν_τ. This discontinuity is related to the problem of the helicity states allowed to a massless particle. In section 3, this problem is reviewed with emphasis on two theorems due to Wigner and Weinberg. It is concluded that one may eliminate the discontinuity discussed in section 2 by allowing states corresponding to nonmaximal helicities of a massless ν_τ. In light of this possibility, in section 4 we discuss τ-decays both in the case that ν_τ is restricted to maximal helicities and in the case that all four helicities are involved.

2. An apparent discontinuity as $M_{\nu_\tau} \to 0$

In calculating weak decay amplitudes for the τ, Alles assumes the τ-ν_τ current to be

$$J^\mu_{(\tau-\nu_\tau)} = \frac{2ia}{M_\tau} \bar{u}^{\mu\beta} \left(1 - y\gamma_5\right) \overline{\bar{u}_\beta} \quad , \tag{1}$$

where $u^{\mu\beta}$ is the curl of the standard Rarita-Schwinger spinor corresponding to a particle of four-momentum k:

$$u^{\mu\beta} = k^\mu u^\beta - k^\beta u^\mu \quad . \tag{2}$$

The quantities a and y are (arbitrary) constants. With $M_{\nu_\tau} = 0$ and restricting the ν_τ to have maximal helicity ($|\lambda_{\nu_\tau}| = 3/2$), the rate for $\tau \to \nu_\tau e^- \bar{\nu}_e$ is:

$$\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right) = \frac{G_F^2 a^2 M_\tau^5}{216(2\pi)^3} \left(1 + y^2\right) \quad . \tag{3}$$

One might also attempt to calculate this rate by first calculating the rate for a massive ν_τ and then taking the limit as $M_{\nu_\tau} \to 0$. Proceeding this way, one finds a rate

$$\Gamma\left(\tau \to \nu_\tau e^- \bar{\nu}_e\right) = \frac{17}{15} \frac{G_F^2 a^2 M_\tau^5}{216(2\pi)^3} \left(1 + y^2\right) \quad . \tag{4}$$

The limit as $M_{\nu_\tau} \to 0$ appears to be discontinuous.

The occurence of a discontinuity in the zero mass limit has a precedent elsewhere. It has been known for a decade that a theory with massive gravitons does not approach the standard zero mass theory in the limit that the graviton mass goes to zero [10].

For spin-3/2 particles, the $M \to 0$ discontinuity has a straightforward origin. A Rarita-Schwinger spinor possesses both a Lorentz vector index and a Dirac spinor index. It can be conceived of as being a spin-1 field combined with a spin-1/2 field. The combination, of course, produces both total spin 1/2 and 3/2. Imposition of the standard condition

$$\gamma_\mu u^\mu = 0 \tag{5}$$

constrains u so that only the total-spin-3/2 portion remains. Writing out the helicity states of the resulting spin-3/2 field in terms of those of the spin-1 and spin-1/2 components, one finds that this condition insures that

$$\begin{aligned}
|3/2,\ 3/2\rangle &= |1,\ 1\rangle\ |1/2,\ 1/2\rangle \\
|3/2,\ 1/2\rangle &= \sqrt{2/3}\ |1,\ 0\rangle\ |1/2,\ 1/2\rangle + \sqrt{1/3}\ |1,\ 1\rangle\ |1/2,\ -1/2\rangle \\
|3/2,\ -1/2\rangle &= \sqrt{1/3}\ |1,-1\rangle\ |1/2,\ 1/2\rangle + \sqrt{2/3}\ |1,\ 0\rangle\ |1/2,\ -1/2\rangle \\
|3/2,\ -3/2\rangle &= |1-1\rangle\ |1/2-1/2\rangle \quad ,
\end{aligned} \tag{6}$$

which are nothing but the standard Clebsch-Gordan relations for combining spin 1/2 and spin 1 to form total spin 3/2.

As $M \to 0$, the longitudinal (helicity zero) vector contribution, $|1,\ 0\rangle$, to the $|3/2, \pm 1/2\rangle$ states has components which blow up; in a coordinate system where k^μ is $\begin{pmatrix} k \\ o \\ o \\ k \end{pmatrix}$ (note that the following are four-vectors not Dirac spinors),

$$|1,\ 0\rangle = \frac{1}{M} \begin{pmatrix} \sqrt{k+M^2} \\ 0 \\ 0 \\ k \end{pmatrix} \xrightarrow[M \to 0]{} \begin{pmatrix} \infty \\ 0 \\ 0 \\ \infty \end{pmatrix} \quad ,$$

where $|1,\ 0\rangle$ is normalized to unity.

This of course also occurs for the electromagnetic field. There, the longitudinal contribution can be eliminated for the zero-mass photon by going from the field A^{μ} to the field strength tensor $F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$. In the case of the photon, by eliminating the longitudinal contribution ($\lambda = 0$), one leaves only maximal helicity states ($|\lambda| = 1$).

A similar use of the curl in the massless spin-3/2 case also removes the (infinite) longitudinal contribution associated with the vector index* [11]. However, unlike the electromagnetic case, the nonmaximal helicity states $\left(|\lambda_{\nu_\tau}| = \pm 1/2\right)$ for spin 3/2 involve not only a longitudinal vector piece which is eliminated by the curl, but also [as shown in eq. (6)] a portion which is transverse in the vector index and which is not eliminated by the curl.

Therefore, use of the curl formalism for massless spin-3/2 Rarita-Schwinger particles, while it will eliminate the infinite longitudinal vector contribution, will not—unlike electromagnetism—completely eliminate the states with nonmaximal helicities.

As $M \to 0$, the contributions to $\tau^- \to \nu_\tau e^- \bar{\nu}_e$ from these transverse-vector parts of the nonmaximal helicity ($|3/2, \pm 1/2\rangle$) states of ν_τ survive. As is verified by explicit calculation, it is these contributions which make the rate in eq. (4) greater than the rate which is due solely to maximal helicities $\left(|\lambda_{\nu_\tau}| = 3/2\right)$ and which is given by eq. (3).

According to the conventional wisdom of particle physics, only maximal helicities can exist for a particle the mass of which is strictly

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* In electromagnetism, A^{μ} couples only to conserved currents and the longitudinal piece will therefore not contribute to matrix elements even if one uses A^{μ} rather than $F^{\mu\nu}$. Since u^{μ} need not couple to a conserved current, in the Rarita-Schwinger case, one must employ the curl formalism to ensure finite matrix elements.

zero. Since, as $M \to 0$, nonmaximal helicities continue to contribute in the case under discussion, a discontinuity as $M \to 0$ appears unavoidable.

One cannot rule out *a priori* the possibility of such a discontinuity, but it is rather unsettling. For example, it implies that one could experimentally distinguish between the case of M_{ν_τ} finite but unbelievably small (e.g., $M_{\nu_\tau} = 10^{-1000}$ eV) and the case that M_{ν_τ} is strictly zero.

However, if it were possible for a massless particle to have a full set of helicity states rather than being restricted to maximal helicities, then it would be possible to avoid this discontinuity as $M \to 0$.

In the next section we review the general problem of the helicity states of a massless particle and conclude that one need not throw out the $\lambda = \pm 1/2$ states of ν_τ in the specific problem with which we are concerned when $M_{\nu_\tau} = 0$ and that therefore the discontinuity can be avoided.

Although our interest in the subject of the next section is motivated by the apparent discontinuity discussed in this section, our arguments in the next section rest solely on general considerations concerning massless particles. We do not claim that the goal of eliminating a discontinuity validates any of the following arguments.

3. Helicity states of a massless particle

In this section, we will review classical analyses concerning the helicity states of a massless particle and discuss their relevance to a massless spin-3/2 ν_τ.

The assumption that strictly massless particles must have only maximal helicity states rests on two theorems due to Wigner and Weinberg. Contrary to what one might expect, there are certain circumstances, including that of a massless spin-3/2 Rarita-Schwinger particle, in which these two theorems do not suffice to rule out the possibility of there being a full set of states corresponding to the full rnage of helicities for a massless particle.

A. Wigner's theorem

Wigner's theorem is the familiar statement ~~that~~ the helicity of a massless particle is invariant under the restricted Lorentz group — helicities do not mix. A single helicity forms an irreducible representation of the little group* [12].

It might be thought that the requirement that a massless particle have only maximal helicities is an immediate consequence of this theorem. For, Wigner and others have chosen to define "particle" as "an irreducible representation of the Lorentz group". With this definition, it of course follows that a massless "particle" has only a single helicity state. A massless particle with a full set of helicity states would form a reducible representation and would therefore, by definition, not constitute a "particle" but rather a set of several distinct "particles".

However, this conclusion clearly conveys no information about the nature of the physical world beyond the information contained in the statement that if a massless particle does have several helicity states they will not mix under Lorentz transformations. In particular, this conclusion does not tell us whether or not there exists in nature a

* We will take "Wigner's theorem" to refer only to this strict statement that different helicities of a massless particle may not mix — i.e., "Wigner's theorem" will not be used to refer to the restriction on the allowed helicities of a massless particle which is generally believed to be a corollary of this theorem.

full set of states corresponding to all helicities of a massless particle. It merely informs us that if such a full set of helicity states exists, and if we choose to define the word "particle" in a certain manner, then we must talk about this set of helicity states in a certain way — i.e., as several "particles" rather than as a single "particle".

To believe that this line of argument reveals information about the allowed particle states which can exist in the real world is therefore to confuse physics with semantics. Obviously, the existence or nonexistence of certain states in nature does not depend on how one chooses to define the word "particle".

Wigner's definition of "particle" is of course convenient for some purposes, but it may prove rather inconvenient for other purposes. For example, when one is taking the massless limit of a finite-mass theory as in the previous section, it is natural to define "particle" in the strictly massless case to be the set of massless states, if it exists, which corresponds to the limit of the finite-mass states. With this definition of "particle" for the massless case, the question of whether or not a particle can possess a full set of massless helicity states can be settled not by definition but only by investigation: do nonmaximal helicity states decouple when $M = 0$?, will a full set of massless helicity states mix and violate Wigner's theorem?, etc.

We shall employ this definition, which differs from Wigner's, and which is more convenient for our purposes, throughout this paper.

It should now be clear that the conclusion that one must throw out nonmaximal helicities of a massless spin-3/2 ν_T because massless particles must have maximal helicities and that, therefore, the $M \to 0$

discontinuity discussed in sect. 2 is unavoidable is, in fact, an invalid conclusion resulting from a misunderstanding involving Wigner's definition of "particle".

Unfortunately, Wigner's definition seems somewhat prone to this sort of misunderstanding. For example, suppose an experimenter discovers a very light particle, so light that he is unable to determine whether its mass is strictly zero or is an extremely small but finite number. The experimenter might decide, wrongly of course, that if he can observe a full set of helicity states for the new particle, he will have proven that its mass must not be zero. Similarly, if an experimenter is confident that a particle is strictly massless, he may falsely conclude that it would be fruitless to investigate whether states corresponding to nonmaximal helicities exist.

Thus, even where it is useful, Wigner's definition can be rather misleading and should be handled with care.*

Wigner's theorem does not then trivially rule out the possibility that states corresponding to nonmaximal helicities of a massless particle may exist. However, if such states exist, Wigner's theorem does require that they not mix under Lorentz transformations.

If the curl formalism for spin-3/2 massless particles were not used, different helicities would mix under restricted Lorentz transformations, violating Wigner's theorem. However, with employment of the curl formalism, mixing of helicities does not occur and the situation is in fact in accord with Wigner's theorem.

* We apologize to the reader who is quite immune to misuse of Wigner's definition and who views the preceding discussion as overemphasizing a trivial and obvious point. However, a majority of the established particle theorists with whom we discussed the result of sect. 2 did misapply Wigner's definition to this specific problem with which we are concerned; hence, we thought it necessary to discuss this matter in some detail.

The same situation arises for a massless vector particle. If one does not use the curl formalism, different helicities mix in violation of Wigner's theorem. As in the Rarita-Schwinger case, the curl formalism ensures that different helicities do not mix.

However, in the massless vector case, the curl also eliminates the nonmaximal helicity state. This does not occur, as we've emphasized, in the massless Rarita-Schwinger case.

In the Rarita-Schwinger case, the curl formalism allows nonmaximal helicities to exist without violating Wigner's theorem. We conclude that for spin-3/2, Wigner's theorem is consistent with there being a full range of helicities for a massless spin-3/2 particle.

B. Weinberg's theorem

Weinberg's theorem [13] explicitly specifies which helicity states can exist for a massless particle in a given representation of the Lorentz group. First define

$$\vec{A} = \frac{\vec{J} + i\vec{K}}{2} \quad \text{and} \quad \vec{B} = \frac{\vec{J} - i\vec{K}}{2} \quad , \tag{7}$$

where $\vec{J}$ and $\vec{K}$ are the usual generators of rotations and boosts, respectively. Since $\vec{A}$ and $\vec{B}$ commute, and since each generates an SU(2) algebra, any representation of the Lorentz group can be specified in terms of its representation content with respect to $\vec{A}$ and $\vec{B}$ and can be labelled accordingly: (A,B). A Dirac spinor corresponds to (1/2, 0) + (0, 1/2). A four-vector behaves as (1/2, 1/2). A Rarita-Schwinger spinor, which combines a vector and a Dirac index, corresponds to

$$(1/2, 1/2) \otimes [(1/2, 0) + (0, 1/2)] = (1, 1/2) + (0, 1/2) + (1/2, 1) + (1/2, 0).$$

Parts of these last representations are eliminated by the standard constraint [eq. (5)].

Weinberg's theorem is the statement that a massless particle in the representation (A,B) can only have a single helicity:

$$\lambda = B - A \tag{8}$$

At first glance, it therefore appears to vindicate the common belief that a massless particle cannot have a full range of helicities. However, if one applies Weinberg's criterion to some specific examples, one finds that, in fact, it is not at all in agreement with the usual belief that massless particles have only maximal helicities.

For example, for a spin-1 particle field described by a four-vector (e.g., electromagnetism with the photon field A_μ), (A,B) = (1/2, 1/2), so that Weinberg's criterion implies $\lambda = 1/2 - 1/2 = 0$. Thus, Weinberg's theorem requires that a massless vector field can only have a longitudinal component, that it can only have nonmaximal helicity!

Similarly, Weinberg's theorem demands that in the massless Rarita-Schwinger case $|\lambda_{\nu_T}| = 1/2$. Again, maximal helicities are forbidden. Thus, while Weinberg's theorem does seem to prevent a massless particle from having a full set of helicities, the helicity states allowed by Weinberg's theorem are not, in general, the maximal helicities. On the contrary, the theorem forbids maximal helicities for both vector and Rarita-Schwinger fields.

There is, of course, a loophole in these results.* The helicity states allowed by eq. (8) (in both the vector and Rarita-Schwinger cases) are precisely those states which have infinite components when M = 0,

* Weinberg was, of course, aware that a loophole existed, although he was concerned with a somewhat different context than we.

with the standard normalization. If one normalizes the Rarita-Schwinger spinors (or the spin-1 vector representation) so that these components are finite, the other helicity states will indeed vanish as required by Weinberg's theorem.

If, however, one chooses the standard normalization in which these components are infinite and then employs the curl formalism to eliminate the infinite contributions, one escapes Weinberg's theorem. Equation (8) was derived as a necessary condition to ensure that different helicities do not mix, but the curl formalism guarantees this even if $\lambda \neq B - A$. Thus, Weinberg's theorem does not restrict the allowed helicities of a massless Rarita-Schwinger particle if the curl formalism is employed.

C. Is "total spin" meaningful for massless particles? *

We have concluded that neither Wigner's theorem nor Weinberg's theorem requires one to throw out the $\lambda = \pm 1/2$ states which appear in the Rarita-Schwinger formalism for a massless spin-3/2 particle. We have pointed out that whether one views these states as being a separate particle or merely as different states of the particle which has $\lambda = \pm 3/2$ is a matter of convenience. Since all four helicity states of a massless spin-3/2 ν_τ correspond to the $M \to 0$ limit of a single finite-mass particle, it is convenient to refer to the four helicity states as comprising the same particle.

However, it is standard practice to identify the spin of a massless particle as $|\lambda|$. Standard practice would thus assign the $\lambda = \pm 1/2$ states a spin of 1/2 and the $\lambda = \pm 3/2$ states a spin of 3/2, which

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* The discussion in this subsection is in response to queries raised by L. Wolfenstein.

conforms nicely with Wigner's definition which defines these states as being separate particles.

If we view all four helicities as comprising one particle, however, we would assign them all a spin of 3/2.

For a massive particle, spin is a physically measurable quantity. If the same were true for a massless particle, one could (in principle) measure the total spin of the $\lambda = \pm 1/2$ states and prove either Wigner's definition or our own to be wrong: either the $\lambda = \pm 1/2$ states would have the same spin as the $\lambda = \pm 3/2$ states, or they would not.

In fact, total spin is apparently not a meaningful quantity for a massless particle. Obviously, one cannot go to the rest frame to measure $S_x^2 + S_y^2 + S_z^2$. The Pauli-Lubanski vector, $\Gamma^\mu = \varepsilon^{\mu\nu\rho\sigma} P_\nu M_{\rho\sigma}$, has magnitude $M^2|S|(|S|+1)$ which uniquely determines the spin $|S|$ — unless $M = 0$.

For a massive particle the transformation properties under boosts and rotations of a state of helicity λ depend not only on λ but also on $|S|$, and this allows one in principle to physically measure $|S|$. However, Wigner's theorem proves that for a massless particle the transformation properties depend only on λ and cannot therefore determine $|S|$.

The standard approach to coupling angular momenta of several particles requires knowledge of each particle's spin. Which Clebsch-Gordan table one uses depends on the magnitude of the spins of the particles one is considering. One expects this to carry over to the massless case; i.e., depending on whether one assigns a spin of 3/2 or 1/2 to our $\lambda = \pm 1/2$ states one expects to use a different set of Clebsch-Gordan coefficients to combine these states with other particles to form some composite angular momentum state.

This is indeed so. If Clebsch-Gordan coefficients are—in principle—physically measurable, the assignment of total spin to a massles particle would not be arbitrary. However, Clebsch-Gordan coefficients specify a particle's component of spin along a definite fixed direction, generally not the direction of the particle's motion. For a massive particle, the component of spin along a fixed direction can be physically measured by bringing the particle to rest. For a massless particle this cannot be done and the Clebsch-Gordan coefficients therefore cannot be measured physically.

For massless particles, the only physical approach to specifying the spin state is to give the helicity. If one couples the angular momenta of several particles in the helicity basis (a generalized Jacob-Wick approach), it can be proven that the helicity coefficients analogous to Clebsch-Gordan coefficients do not depend on the spin of any of the particles—whether the particles are massive or massless. (This result therefore does not depend on Wigner's theorem).

Since only the helicity basis is physically meaningful for massless particles, the combining of angular momenta and the dependence of Clebsch-Gordan coefficients on the magnitude of the spin does not therefore allow one to give a physical meaning to the spin of a massless particle.

None of the obvious approaches to physically measuring the spin of a massless particle works. Indeed, Wigner's theorem probably rules out any such approach.

We conclude that neither Wigner's theorem nor Weinberg's theorem, nor considerations of the total spin of a massless particle, constrains the helicity states allowed for a massless Rarita-Schwinger field. It appears that when constructing a theory one can, if one chooses, assume that a massless Rarita-Schwinger field has all four helicity states: both $\lambda_{\nu_\tau} = \pm 3/2$ and $\lambda_{\nu_\tau} = \pm 1/2$. (Of course, whether nature in fact chooses to conform to such a theory is a question to be settled by experiment.) If one does choose to allow all four helicities when M = 0, the $M \to 0$ discontinuity discussed in sect. 2 disappears.*

4. V,A τ,ν_τ currents and τ decay

In sect. 2 we showed that if we start with a theory with a massive neutrino and let $M_{\nu_\tau} \to 0$, all four helicities of the ν_τ continue to contribute; none totally decouples. Furthermore, we argued in sect. 2 that, contrary to what one might expect, even when M_{ν_τ} is strictly zero one can, if one wishes, allow all four helicity states to exist.

Given these considerations, we will present results in this section based on the assumption that all four helicity states for ν_τ are present for $M_{\nu_\tau} = 0$. Of course, it is not necessary for all four helicity states to exist in the strictly massless case — it is possible to have only maximal helicity states present. We will therefore also discuss the results in this case.

More bizarre possibilities exist in the strictly massless case: e.g., one could have $\lambda_{\nu_\tau} = 3/2,\ 1/2,\ -1/2$ states existing but $\lambda_{\nu_\tau} = -3/2$ not existing. We will not discuss such possibilities.

For spin-3/2 τ and ν_τ, with arbitrary masses, there are in general seven independent pairs of V,A currents which can be formed from the τ

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* Weinberg and Witten have recently shown that a massless spin-3/2 particle cannot have a conserved Lorentz-covariant vector current or a conserved Lorentz-covariant stress-energy tensor [14]. As they point out, there are known theories which lack a Lorentz-covariant conserved vector current or conserved stress-energy tensor but which are nonetheless acceptable theories.

Of course, one can avoid Weinberg's and Witten's theorem entirely by simply giving ν_τ an arbitrarily tiny yet nonzero mass. Obviously, all four ν_τ helicities would then automatically exist.

and ν_τ Rarita-Schwinger spinors:

$$A_V^\lambda + aA_A^\lambda = \bar{u}^{\lambda\beta}(\nu_\tau)(1+a\gamma_5)\, u_\beta(\tau)$$

$$B_V^\lambda + bB_A^\lambda = \frac{(p^\lambda + k^\lambda)}{M_\tau^2}\, \bar{u}^{\alpha\beta}(\nu_\tau)p_\alpha(1+b\gamma_5)\, u_\beta(\tau)$$

$$C_V^\lambda + cC_A^\lambda = \frac{(p^\lambda - k^\lambda)}{M_\tau^2}\, \bar{u}^{\alpha\beta}(\nu_\tau)p_\alpha(1+c\gamma_5)\, u_\beta(\tau)$$

$$D_V^\lambda + dD_A^\lambda = \bar{u}^{\alpha\beta}(\nu_\tau)\, \frac{p_\alpha}{M_\tau}\, (1+d\gamma_5)\, \gamma^\lambda\, u_\beta(\tau) \tag{9}$$

$$E_V^\lambda + eE_A^\lambda = \frac{M_{\nu_\tau} M_\tau}{p\cdot k}\, \bar{u}^\beta(\nu_\tau)(1+e\gamma_5)\, u_\beta(\tau)(p^\lambda + k^\lambda)$$

$$F_V^\lambda + fF_A^\lambda = \frac{M_{\nu_\tau} M_\tau}{p\cdot k}\, \bar{u}^\beta(\nu_\tau)(1+f\gamma_5)\, u_\beta(\tau)(p^\lambda - k^\lambda)$$

$$G_V^\lambda + gG_A^\lambda = M_{\nu_\tau}\, \bar{u}^\beta(\nu_\tau)\, (1+g\gamma_5)\, \gamma^\lambda\, u_\beta(\tau) \quad .$$

Here k^λ, p^λ are the four-momentum of ν_τ, τ respectively. Other currents can be written in terms of these seven; e.g., by the Gordon decomposition, $(q = k - p)$

$$i\, \frac{p_\alpha}{M_\tau}\, \bar{u}^{\alpha\beta}\, q_\nu\, \sigma^{\lambda\nu}\, U_\beta = M_\tau\, D_V^\lambda - M_\tau\, B_V^\lambda \quad . \tag{10}$$

The currents E^λ, F^λ and G^λ involve the parts of the $\lambda_{\nu_\tau} = 1/2$ helicity states which satisfy Weinberg's criterion and the components of which become infinite as $M_{\nu_\tau} \to 0$. However, E^λ, F^λ, G^λ are constructed so as to approach a finite limit as $M_{\nu_\tau} \to 0$ even though the components of the spinors comprising E^λ, F^λ, G^λ blow up as $M_{\nu_\tau} \to 0$.

When ν_τ is strictly massless, E^λ, F^λ, G^λ are of the indeterminate form $0 \times \infty$ (assuming the standard normalization for u^β) and are hence undefined. For thie reason, we will refrain from using these currents in our analysis.

If $M_{\nu_\tau} = 0$ and if one is restricted to maximal helicities (but <u>not</u> if all four ν_τ helicities are allowed), then

$$A_V^\lambda = D_V^\lambda \quad , \qquad A_A^\lambda = D_A^\lambda \quad .$$

Therefore, in the maximal-helicity case there are only three pairs of independent currents: $A_{V,A}^\lambda$, $B_{V,A}^\lambda$ and $C_{V,A}^\lambda$. If all four helicity states of a massless ν_τ are allowed $D_{V,A}^\lambda$ must be included as a fourth pair of independent currents.

The general $\tau - \nu_\tau$ current, $J_{(\tau - \nu_\tau)}$ can be constructed as a linear combination of these independent currents.

Alles assumes that

$$J^\lambda_{\left(\tau - \nu_\tau\right)} = K\left(A_V^\lambda + aA_A^\lambda\right) ,$$

with K an arbitrary constant and shows that the ratio (assuming only maximal helicities for ν_τ)

$$\Gamma\left(\tau \to \pi\nu_\tau\right) / \Gamma\left(\tau \to \nu_\tau e^- \bar{\nu}_e\right)$$

is 2.25 times the standard-model value. He concludes that the disagreement of this prediction with experiment definitely excludes the hypothesis that τ and ν_τ both have spin 3/2.

In fact, if one allows a more general form for $J^{\lambda}_{(\tau-\nu_\tau)}$, Alles' conclusion is false; for, let

$$J^{\lambda}_{(\tau-\nu_\tau)} = K\,\frac{p_\alpha}{M_\tau}\left[\bar{u}^{\alpha\beta}\,\gamma^{\lambda}\,u_\beta(\tau) + \kappa\, i\,\bar{u}^{\alpha\beta}\,\frac{q_\nu}{M_\tau}\,\sigma^{\lambda\nu}\,u_\beta\right]$$

$$= K\left[(1+\kappa)\,D^{\lambda}_{V} - \kappa B^{\lambda}_{V}\right] \quad . \qquad (11)$$

Since the current $\langle\pi|J^{\lambda}|0\rangle$ is proportional to q^{λ}, the Pauli term, $q_\nu\sigma^{\lambda\nu}$, does not contribute to $\Gamma(\tau\to\pi\nu_\tau)$ at all and $\Gamma(\tau\to\pi\nu_\tau)$ is independent of κ. In particular, $\Gamma(\tau\to\pi\nu_\tau)$ is finite as $\kappa\to\infty$. Since the Pauli term gives a nonzero contribution to $\Gamma(\tau\to\nu_\tau e\bar{\nu}_e)$, $\Gamma(\tau\to\nu_\tau e\bar{\nu}_e)$ will go to ∞ as $\kappa\to\infty$. Therefore, $\Gamma(\tau\to\pi\nu_\tau)/\Gamma(\tau\to\nu_\tau e^-\bar{\nu}_e)$ goes to 0 as $\kappa\to\infty$.

Since $\Gamma(\tau\to\pi\nu_\tau)/\Gamma(\tau\to\nu_e e^-\bar{\nu}_e)$ is 2.25 when $\kappa = 0$ and 0 when $\kappa=\infty$, and since it is a continuous function of κ, it follows that there exists a κ corresponding to any value of this ratio between 0 and 2.25. Since both the experimental value and the standard-model theoretical value for this ratio lie between 0 and 2.25, there does exist, contrary to Alles, a $J^{\mu}_{(\tau-\nu_\tau)}$ involving spin-3/2 τ and ν_τ which produces the desired value of $\Gamma(\tau\to\pi\nu\tau)/\Gamma(\tau\to\nu_\tau e\bar{\nu}_e)$ with ν_τ restricted to maximal helicities.

This reasoning applies also when all four ν_τ helicities are allowed.*

The fact that $\Gamma(\tau\to\nu_\tau\pi)/\Gamma(\tau\to\nu_\tau e\bar{\nu}_e)$ can be adjusted so as to agree with experiment leaves open the possibility that other branching ratios, the e^- energy spectrum, etc., might not be similarly adjustable.

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* When all four helicities are involved and $\kappa = 0$, $\Gamma(\tau\to\pi\nu_\tau)/\Gamma(\tau\to\nu_\tau e^-\bar{\nu}_e)$ is 75/34 ($\cong$ 2.21) rather than 2.25 of the standard-model value. Otherwise, the reasoning is unchanged.

In fact, if one is willing to allow arbitrary form factors, there exists $J^{\lambda}_{(\tau-\nu_\tau)}$ such that the "lepton trace"

$$L^{\lambda\rho} = \sum_{\text{spins}} J^{\lambda}_{(\tau-\nu_\tau)} \left(J^{\rho}_{(\tau-\nu_\tau)}\right)^*$$

is identical to that for the spin-1/2 case. Such a $J^{\lambda}_{(\tau-\nu_\tau)}$ with the same $L^{\lambda\rho}$ as in the standard model will clearly reproduce the standard-model branching ratios and, in the τ rest frame, the standard-model energy spectra for unpolarized τ and undetected ν_τ helicity.

In the maximal helicity case, an appropriate $J^{\lambda}_{(\tau-\nu_\tau)}$ is

$$J^{\lambda}_{(\tau-\nu_\tau)} = \frac{M_\tau}{k\cdot p}\left(A^{\lambda}_{V} + A^{\lambda}_{A}\right) - \left(\frac{1}{2} - \frac{1}{2\sqrt{3}}\right)\frac{M^3_\tau}{(k\cdot p)^2}\left(B^{\lambda}_{V} - C^{\lambda}_{V} + B^{\lambda}_{A} - C^{\lambda}_{A}\right) \quad . \tag{12}$$

When all four ν_τ helicities are allowed

$$J^{\lambda}_{(\tau-\nu_\tau)} = \frac{M_\tau}{k\cdot p}\left(D^{\lambda}_{V} + D^{\lambda}_{A}\right) - \left(\frac{1}{2} + \frac{1}{2\sqrt{5}}\right)\frac{M^3_\tau}{(k\cdot p)^2}\left(B^{\lambda}_{V} - C^{\lambda}_{V}\right) \quad . \tag{13}$$

For unpolarized τ and undetected ν_τ helicity, these currents will reproduce the branching ratios, energy spectra, angular distributions, etc., of the unpolarized standard-model spin-1/2 case.

The price one pays for achieving this mimicry of the standard model is the need to use some rather unaesthetic form factors. (Contrary to appearances, $J^{\lambda}_{(\tau-\nu_\tau)}$ does not, of course, blow up as $k^{\lambda} \to 0$.) However, as we argued in sect. 1, the fact that spin-3/2 τ and ν_τ if they exist are probably composite combined with the uncertainty as to what is the "simplest" current for spin-3/2 τ and ν_τ compels one to accept the probability of nonconstant form factors. Unless one has a specific theory concerning these form factors, one cannot rule out the possibility that $J^{\lambda}_{(\tau - \nu_\tau)}$ is of the form given by eq. (12) or (13).

It is of course impossible in general for a polarized spin-3/2 τ to reproduce the angular distributions produced by a spin-1/2 τ. Therefore, if one can produce fully polarized τ's, one could determine the τ spin, the results of this section notwithstanding.

The restriction of this section that the τ be totally unpolarized is somewhat more severe than one might suppose. For example, spin-3/2 $\tau^+\tau^-$ produced in e^+e^- annihilation would not, in general, be unpolarized—e.g., the helicities $\lambda = \pm\ 3/2$ and $\lambda = \pm 1/2$ might not be equally populated. This situation might produce not only angular correlations differing from the standard model but also energy spectra in the lab frame which differ from the standard-model spectra; for, if there is any correlation between the direction of the τ spin and the direction of the boost from the rest frame to the lab frame, then the energy spectra in the laboratory frame depend not only on the rest-frame spectra but also on the rest-frame angular distributions. The existence of such a correlation is equivalent to there being a differential population of the various τ helicity states (i.e., to there not being equal numbers of τ's of various helicities). Therefore, even if one takes $J^{\lambda}_{(\tau - \nu_\tau)}$ to be given by eq. (12) or (13), if the τ helicities are differentially populated in e^+e^- annihilation the lab-frame energy spectra (integrated over angles) will not necessarily agree with the standard model predictions even though the <u>rest-frame</u> spectra (integrated over angles) <u>will</u> agree with the standard-model predictions.

Of course, it is *a priori* possible that the four helicities of spin-3/2 τ's produced in e^+e^- annihilation could be equally populated in which case lab-frame spectra would agree with the standard-model predictions (as is the case experimentally for the e^- energy spectrum). However, even if the different helicity states are equally populated, there must at least be a correlation between the τ^+ and τ^- helicities for spin-3/2 τ. This would probably produce correlations between τ^+ and τ^- energy spectra and angular distributions which differ from the standard model.

Therefore, detailed consideration of the $\tau - \tau$ electromagnetic current for spin-3/2 τ [15] would probably reveal either energy spectra or correlations between τ^+ and τ^- angular distributions or energy spectra which differ from the standard-model predictions and which might thus enable one to distinguish experimentally between τ spin of 1/2 and 3/2.

5. Conclusion

We have considered the possibility that τ and ν_τ both have spin 3/2. We have found that, contrary to the usual assumption, it is apparently not necessary for a massless spin-3/2 ν_τ to be restricted to maximal helicity. For unmeasured ν_τ helicity and unpolarized τ, it is possible for spin-3/2 τ and ν_τ to precisely mimic the standard-model decay rates and energy distributions of a spin-1/2 τ and ν_τ. Only in situations where one has some information about the τ polarization, as in the correlations that must exist for the τ^+,τ^- helicities in $e^+e^- \to \tau^+\tau^-$, might it be possible to rule out the possibility that τ and ν_τ both have spin 3/2. Although we share the general prejudice against spin-3/2 τ

and ν_τ as unaesthetic and lacking in the simplicity of the standard model, we must conclude that existing theoretical and experimental analysis is not sufficient to rule out the hypothesis that both τ and ν_τ have spin 3/2.

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