SLAC-PUB-2712
March 1981
(T/E)

HIGHER ORDER QUANTUM-CHROMODYNAMIC CORRECTIONS TO THE THIRD STRUCTURE FUNCTION OF THE PHOTON *

Ken Sasaki †
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

The moments of the photon's third structure function W_3^γ are considered in quantum chromodynamics up to the next-to-the-leading order. The one-loop and two-loop anomalous dimensions of the relevant operators are calculated. Using these results, the part of the higher-order corrections is evaluated.

Submitted to Physical Review D

---

*Work supported in part by the Department of Energy, Contract DE-AC03-76SF00515, and in part by the Nishina Memorial Foundation.
†On leave of absence from Department of Physics, Yokohama National University, Japan.

In contrast to the photon structure functions W_1^γ and W_2^γ in the deep-inelastic photon-photon scattering, the third structure function W_3^γ in the leading order is not affected by strong interaction effects and agrees with the result calculated in the simple parton model. This fact has been found independently by several authors.[1-5] The origin of this interesting result is that W_3^γ is related to the s-channel helicity-flip forward amplitude of the photon-photon scattering; i.e., W_3^γ requires helicity flip both at the target and the high-Q^2 virtual photon.

The authors of Refs. 1-4 have arrived at the above conclusion in the framework of perturbative QCD and/or the leading logarithm approximation, observing that quarks acquire the transverse momentum cut-off at the photon target in the case of W_3^γ. This observation corresponds to the absence of twist-2 quark operators contributing to W_3^γ in the framework of the operator-product-expansion (OPE) and renormalization group (RG) method. Delduc et al., in their first paper of Ref. 4, and the present author have derived the same conclusion using the OPE and RG method.

In this short note I shall put an addendum to the results of Ref. 5 and calculate the one-loop and two-loop anomalous dimensions relevant to the new gluon operator, which is typical of the photon structure function W_3^γ. The expressions of these anomalous dimensions turn out to be very simple. Using these results, I shall evaluate part of the next-to-leading-order corrections of asymptotic freedom to the structure function W_3^γ. In what follows I shall adopt the notation of Ref. 5 unless stated otherwise below, and shall refer to equations there, denoting by V, followed by the same numbers as in Ref. 5.

First I comment that the expression (V.3.11) for the moments of W_3^γ is incomplete.[6] Another term which is the order of $1/(\ell n Q^2/\Lambda^2)$ should be added. This comes from the next-to-the-leading (i.e., two-loop) corrections to the photon coefficient function $C_{3,n}^\gamma(1,\bar{g}^2,\alpha)$ (V.3.8). It is expanded in powers of $\bar{g}^2$ as follows:

$$C_{3,n}^\gamma\left(1,\bar{g}^2,\alpha\right) = \frac{e^4}{16\pi^2}\,\delta_\gamma\left[B_{3,\gamma}^{0,n} + \frac{\bar{g}^2}{16\pi^2}\,B_{3,\gamma}^{1,n} + \ldots\right] \tag{1}$$

Then adding the contribution of the second term in the parenthesis of Eq. (1), I obtain the quantum chromodynamic (QCD) prediction for the moments of W_3^γ as

$$\int_0^1 dx\, x^{n-1}\, W_3^\gamma = \frac{1}{2}\alpha^2\left\{a_{3,n} + \frac{\bar{g}^2}{16\pi^2}\left(b_{3,n} + c_{3,n}\right) + O\left[\left(\frac{\bar{g}^2}{g^2}\right)^{1+\left(\gamma_{3,GG}^{0,n}/2\beta_0\right)}\right]\right\} \tag{2}$$

where

$$a_{3,n} = \delta_\gamma\, B_{3,\gamma}^{0,n} \tag{3}$$

$$b_{3,n} = \frac{\delta_\psi\, K_{3,G}^{1,n}\, B_{3,G}^{n}}{\gamma_{3,GG}^{0,n}} \tag{4}$$

$$c_{3,n} = \delta_\gamma\, B_{3,\gamma}^{1,n}\quad , \tag{5}$$

and $\gamma_{3,GG}^{0,n}$ is the one-loop anomalous dimension for the gluon operator U_G^n (see Eq. (6) below), $K_{3,G}^{1,n}$ is the two-loop anomalous dimension which arises from the mixing between U_G^n and the photon operator U_γ^n, and $B_{3,G}^n$ is the one-loop correction to the coefficient function $C_{3,n}^G(1,\bar{g}^2,\alpha)$ (V.3.8.).

Next I present the results of calculation of $\gamma_{3,GG}^{0,n}$ and $K_{3,G}^{1,n}$. The twist-2 gluon operator U_G^n which contributes to W_3^γ is

$$U_G^{[\mu_1,\alpha]\mu_2\cdots\mu_{n-1}[\mu_n,\beta]} = \frac{1}{4}\, i^{n-2}\, S'\, G_A^{\mu_1\alpha} D^{\mu_2}\cdots D^{\mu_{n-1}} G^{\mu_n\beta} - \text{trace terms} \tag{6}$$

where $G_A^{\mu\nu}$ is the gluon field tensor, and S' denotes complete symmetrization over $\mu_1,\mu_2,\ldots,\mu_n$. The twist-2 photon operator U_γ^n for W_3^γ is the analog of the gluon operator U_G^n with the gluon field tensor $G_A^{\mu\nu}$ replaced by the electromagnetic field tensor $F^{\mu\nu}$.

The calculation of the one-loop anomalous dimension $\gamma_{3,GG}^{0,n}$ for the operator U_G^n is very similar to the case of gluon operators contributing to the nucleon structure functions. The symmetrization and removal of the trace terms can be done by multiplying the operator by the tensor

$$\Delta^{\mu_1}\Delta^{\mu_2}\ldots\Delta^{\mu_n}$$

where Δ^μ is an arbitrary vector subject to the constraint $\Delta^2 = 0$. The diagrams contributing to $\gamma_{3,GG}^{0,n}$ are shown in Fig. 1. The result is

$$\gamma_{3,GG}^{0,n} = 2C_G\left[\frac{1}{3} + \sum_{j=2}^{4} \frac{4}{j}\right] + \frac{8}{3}\ T(R) \quad . \tag{7}$$

Where $C_G = 3$ and $T(R) = f/2$ with f being the number of quark flavors.

The calculation of the two-loop mixing anomalous dimension $K_{3,G}^{1,n}$ is much involved. I perform the calculation using dimensional regularization and the minimal subtraction scheme.[7]

In the renormalization of the operator U_G^n (i.e., the calculation of its matrix elements in higher orders) it mixes not only with the operator U_γ^n but also may mix with other operators which are in general not gauge invariant.[8] The zero-loop matrix element of the operator U^n_γ between photon states with momentum p, multiplied by the tensor $\Delta^{\mu_1}\Delta^{\mu_2}\ldots\Delta^{\mu_n}$ with $\Delta^2 = 0$, has the following form:

$$\left\langle\gamma(P),\mu\left|U^n_\gamma\right|\gamma(p),\nu\right\rangle_0 \Delta^{\mu_1}\ldots\Delta^{\mu_n} = \frac{1}{4}(p\cdot\Delta)^{n-2}\left\{\left[g^{\mu\alpha}g^{\nu\beta} + g^{\mu\beta}g^{\nu\alpha}\right](p\cdot\Delta)^2 - \left[\left(g^{\mu\alpha}\Delta^\nu + g^{\nu\alpha}\Delta^\mu\right)p^\beta + \left(g^{\mu\beta}\Delta^\nu + g^{\nu\beta}\Delta^\mu\right)p^\alpha\right](p\cdot\Delta) + 2\,p^\alpha p^\beta\,\Delta^\mu\Delta^\nu\right\}, \quad (8)$$

where μ and ν are the Lorentz indices of the external photon. Following the argument of Ref. 8 it can be shown that the matrix element of the operator U^n_G between photon states

$$\left\langle\gamma(p),\mu\left|U^n_G\right|\gamma(p),\nu\right\rangle\ \Delta^{\mu_1}\ldots\Delta^{\mu_n} \quad (9)$$

has a counterterm whose $[g^{\mu\alpha}g^{\nu\beta} + g^{\mu\beta}g^{\nu\alpha}]$ coefficient is obtained from the matrix element of U^n_γ only. Therefore the first step in the calculation is to find the projection operator which projects out the coefficient of $[g^{\mu\alpha}g^{\nu\beta} + g^{\mu\beta}g^{\nu\alpha}]$ and reduces tensor forms into scalars.

The appropriate projection operator is

$$\begin{aligned} R_{\mu\nu\alpha\beta} &= \frac{4}{\ell(\ell-2)} P_{\mu\nu}P_{\alpha\beta} \\ &- \frac{4}{\ell(\ell-3)}\Bigg\{g_{\mu\nu}P_{\alpha\beta} + P_{\mu\nu}g_{\alpha\beta} - g_{\mu\nu}g_{\alpha\beta} + \frac{g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha}}{2} \\ &- \frac{1}{2}g_{\nu\beta}P_{\mu\alpha} - \frac{1}{2}g_{\nu\alpha}P_{\mu\beta} - \frac{1}{2}g_{\mu\alpha}P_{\nu\beta} - \frac{1}{2}g_{\mu\beta}P_{\nu\alpha} \\ &- \frac{p_\mu p_\nu \Delta_\alpha\Delta_\beta}{(p\cdot\Delta)^2} - \frac{\Delta_\mu\Delta_\nu p_\alpha p_\beta}{(p\cdot\Delta)^2} + \frac{\left[p_\mu\Delta_\nu + \Delta_\mu p_\nu\right]\left[p_\alpha\Delta_\beta + \Delta_\alpha p_\beta\right]}{2(p\cdot\Delta)^2}\Bigg\}, \end{aligned} \quad (10)$$

where ℓ is the dimension of space-time, and

$$P_{\mu\nu} = g_{\mu\nu} - \frac{p_\mu\Delta_\nu + \Delta_\mu p_\nu}{p\cdot\Delta} + p^2\frac{\Delta_\mu\Delta_\nu}{(p\cdot\Delta)^2}\quad. \quad (11)$$

Since $\Delta^2 = 0$, the projection operator satisfies the following relations:

$$\frac{1}{4}\left[g^{\mu\alpha}g^{\nu\beta} + g^{\mu\beta}g^{\nu\alpha}\right] R_{\mu\nu\alpha\beta} = 1$$

$$g^{\mu\nu}R_{\mu\nu\alpha\beta} = g^{\alpha\beta}R_{\mu\nu\alpha\beta} = 0$$

$$p^{\mu}R_{\mu\nu\alpha\beta} = p^{\nu}R_{\mu\nu\alpha\beta} = p^{\alpha}R_{\mu\nu\alpha\beta} = p^{\beta}R_{\mu\nu\alpha\beta} = 0$$

$$\Delta^{\mu}R_{\mu\nu\alpha\beta} = \Delta^{\nu}R_{\mu\nu\alpha\beta} = \Delta^{\alpha}R_{\mu\nu\alpha\beta} = \Delta^{\beta}R_{\mu\nu\alpha\beta} = 0 \qquad (12)$$

The two-loop diagrams contributing to $K_{3,G}^{1,n}$ are shown in Fig. 2. It is known that the box diagrams with one-quark-loop for the gluon-photon scattering give no divergence when they are combined together. Therefore, the diagrams of Fig. 2 when combined give a simple pole only at $\varepsilon = 0$ in $\ell = 4 - \varepsilon$ dimensions, and do not have a double pole. The anomalous dimension $K_{3,G}^{1,n}$ is obtained by taking the coefficient of simple pole part (i.e., the $1/\varepsilon$ coefficient) of these diagrams. After the straightforward calculation, the diagram (a) of Fig. 2 gives no contribution to $K_{3,G}^{1,n}$. All the contribution comes from the diagram (b), and I obtain

$$K_{3,G}^{1,n} = - C_F \frac{8}{(n-1)(n+2)} \; 3f \left\langle e^2 \right\rangle \qquad (13)$$

with $C_F = 4/3$.

The result (13) has turned out to be a very simple expression. It decreases as $1/n^2$ for large n. It is interesting to compare $K_{3,G}^{1,n}$ with the two-loop gluon-photon mixing anomalous dimension $K_G^{1,n}$ for the photon structure function F_2^{γ}.[9] It has the following form:[9,10]

$$K_G^{1,n} = -C_F \left\{ 8 + 16 \frac{2n^6 + 4n^5 + n^4 - 10n^3 - 5n^2 - 4n - 4}{(n-1)\ n^3 (n+1)^3\ (n+2)} \right\} 3f \langle e^2 \rangle \tag{14}$$

In the large n limit $K_G^{1,n}$ becomes constant.

Now I evaluate $a_{3,n}$ and $b_{3,n}$ in Eq. (2) using the results of $\gamma_{3,GG}^{0,n}$ and $K_{3,G}^{1,n}$. The one-loop corrections to the coefficient functions $B_{3,\gamma}^{0,n}$ and $B_{3,G}^{n}$ have been given in Ref. 5. They are

$$B_{3,\gamma}^{0,n} = - \frac{4}{n+2} \tag{15}$$

$$B_{3,G}^{n} = - \frac{f}{2} \frac{4}{n+2} \tag{16}$$

The numerical values for $a_{3,n}$ and $b_{3,n}$ are given in Table I. The parameter $b_{3,n}$ decreases very rapidly as $1/(n^3 \ln n)$ with increasing n. Considering that the effective coupling constant $\bar{g}^2/4\pi$ varies roughly[12] between 0.3 and 0.2 while Q^2 changes from 5 GeV^2 to 50 GeV^2, one of the higher-order correction terms, $(\bar{g}^2/16\pi^2)b_{3,n}$, in Eq. (2) is negligible as compared with the leading term $a_{3,n}$.

Another higher-order correction term $(\bar{g}^2/16\pi^2)c_{3,n}$ in Eq. (2) has not been evaluated. In order to do that, the two-loop correction $B_{3,\gamma}^{1,n}$ in Eq. (1) should be calculated. Some of the diagrams contributing to $B_{3,\gamma}^{1,n}$ are shown in Fig. 3. Because of the spin structure of W_3^γ, the one-loop correction $B_{3,\gamma}^{0,n}$ does not have in its expression (15) a term proportional to the sum

$$\sum_{j=1}^{n} \frac{1}{j} \tag{17}$$

It is probable that the two-loop correction $B_{3,\gamma}^{1,n}$ may also evade having a term like Eq. (17) and hence may not behave as $\ln n$ for large n. The structure function W_3^γ for large x values is governed by the large n behavior of the moments. The calculations of $B_{3,\gamma}^{1,n}$, therefore, is very interesting in order to study how W_3^γ behaves as $x \to 1$.

Acknowledgments

I would like to thank Sid Drell for warm hospitality extended to me at SLAC. I have benefited from discussion with S. J. Brodsky and with J. Kodaira. I wish to thank S. Parke for advice and help in some of the calculations and for his personality which has made my stay at SLAC enjoyable. Finally, I acknowledge the financial support of the Nishina Memorial Foundation. This work was also supported by the Department of Energy under contract number DE-AC03-76SF00515. Some calculations have been done using the MIT computer MACSYMA.

REFERENCES

1. W. R. Frazer and G. Rossi, Phys. Rev. D21, 2710 (1980).
2. C. Peterson, T. F. Walsh and P. M. Zerwas, Nucl. Phys. B174, 424 (1980).
3. A. C. Irving and D. B. Newland, Zeit. Phys. C6, 27 (1980).
4. F. Delduc, M. Gourdin and E. G. Oudrhiri-Safiani, Nucl. Phys. B174, 147 (1980); ibid., B174, 157 (1980).
5. K. Sasaki, Phys. Rev. D22, 2143 (1980).
6. The author thanks S. J. Brodsky for pointing out that another important term has been missing in Eq. (3.11) of Ref. 5.
7. G. 't Hooft, Nucl. Phys. B61, 455 (1973).
8. H. Kluberg-Stern and J. B. Zuber, Phys. Rev. D12, 3159 (1975).
9. W. A. Bardeen and A. J. Buras, Phys. Rev. D20, 166 (1979).
10. A. Gonzalez-Arroyo and C. Lopez, Nucl. Phys. B166, 429 (1980).
11. If Λ is chosen to be 0.3 GeV, the effective coupling constant $\bar{g}^2(Q^2)/4\pi$, calculated up to the two-loop approximation, varies between 0.25 and 0.17 in the three flavor case (between 0.28 and 0.19 in the four flavor case) while Q^2 changes from 5 GeV^2 to 50 GeV^2.

TABLE I

Numerical values of the parameters $a_{3,n}$ and $b_{3,n}$ for f=3 and f=4.

n	$a_{3,n}$		$b_{3,n}$	
	f=3	f=4	f=3	f=4
2	-0.667	-1.259	9.86×10^{-2}	2.55×10^{-1}
4	-0.444	-0,840	8.24×10^{-3}	2.20×10^{-2}
6	-0.333	-0.630	2.18×10^{-3}	5.88×10^{-3}
8	-0.267	-0.504	8.60×10^{-4}	2.33×10^{-3}
10	-0.222	-0.420	4.20×10^{-4}	1.14×10^{-3}

FIGURE CAPTIONS

Fig. 1. Diagrams contributing to $\gamma_{3,GG}^{0,n}$.

Fig. 2. Diagrams contributing to $K_{3,G}^{1,n}$. Solid lines, curly lines and wavy lines represent quark, gluon and photon, respectively.

Fig. 3. Some of the diagrams contributing to $B_{3,\gamma}^{1,n}$.

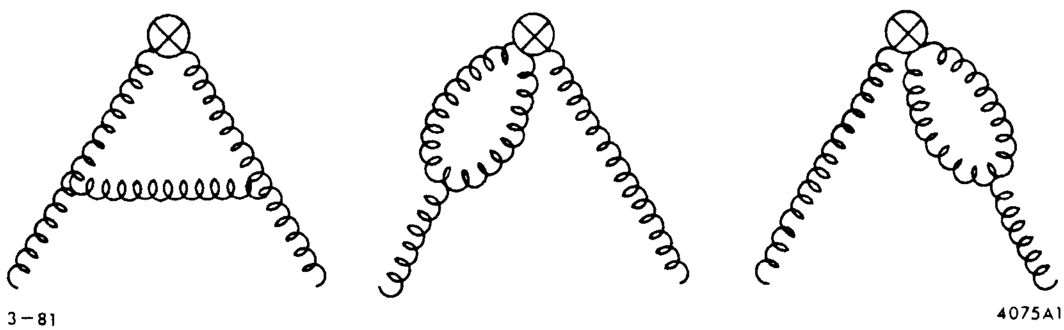


Fig. 1

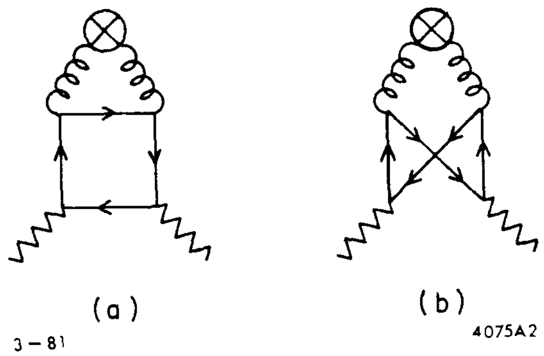
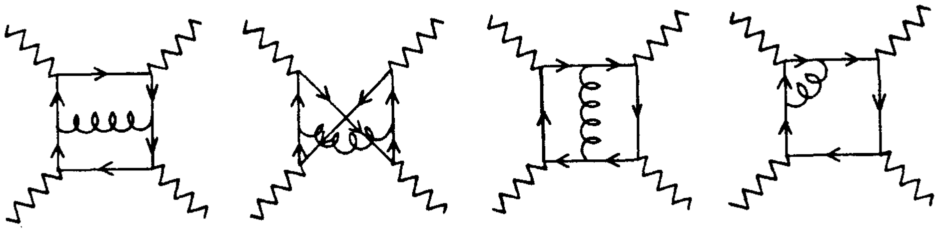


Fig. 2



+ Other Diagrams

3-81

4075A3

Fig. 3