SLAC-PUB-2697
February 1981
(T)

COSMIC STRINGS IN UNIFIED GAUGE THEORIES*

Allen E. Everett†
Stanford Linear Accelerator Center
Stanford University, Stanford, CA 94305

ABSTRACT

Some spontaneously broken gauge theories can give rise to stringlike vacuum structures (vortices). It has been pointed out by Vilenkin that in grand unified theories these can be sufficiently massive to have cosmological implications, e.g., in explaining the formation of galaxies. The circumstances in which such structures occur are examined. They do not occur in the simplest grand unified theories, but can occur in some more elaborate models which have been proposed. The cross section for the scattering of elementary particles by strings is estimated. This is used to evaluate the effect of collisions on the dynamics of a collapsing circular string, with particular attention to the question of whether energy dissipation by collision can reduce the rate of formation of black holes by collapsed strings, which may be unacceptably large in models where strings occur. It is found that the effect of collisions is not important in the case of grand unified strings, although it can be important for lighter strings.

Submitted to Physical Review D

---

* Work supported by the Department of Energy, contract DE-AC03-76SF00515.

† Permanent address: Department of Physics, Tufts University, Medford, Mass. 02155.

Spontaneously broken gauge theories are presently of great interest, both in the context of unified theories of the electromagnetic and weak interactions, and, more speculatively, in connection with so-called grand unified theories (GUTs) incorporating color SU(3), which is presumed to be the underlying symmetry of the strong interactions. Spontaneously broken symmetries can be restored at temperatures, T, greater than some critical temperature, T_c,[1] where, in a standard "big bang" cosmology, T_c will be exceeded in the very earliest stages of the universe. A phase transition will then occur, as the universe cools below T_c, in which a multiplet of scalar Higgs fields develops a vacuum expectation value (VEV) $\langle\varphi\rangle = \eta$. Such phase transitions can result in the development of various kinds of vacuum structures, having the forms of regions of normal vacuum where $\langle\varphi\rangle = 0$. These structures result from the fact that, immediately following the phase transition, the direction of η in the abstract space in which the gauge group operates is expected to be different at different points in ordinary space; such differences will arise from considerations of causality if for no other reason. In order to minimize its energy, the vacuum will evolve toward a situation in which it is not spatially dependent. However, the structure of the gauge group may be such that all spatial dependence cannot be eliminated without leaving regions of normal vacuum. These regions may take one of three possible forms, depending on the topology of the gauge group.[2] One may have a "domain wall", separating regions in which η has different directions. Secondly, there may be stringlike regions of normal vacuum, sometimes called vortices but henceforth referred to as strings.[3] Finally, there may be localized regions corresponding to 't Hooft-Polyakov monopolies.[4] Domain walls appear likely to produce an unobserved anisotropy in the 3° blackbody radiation[2,5]; hence theories leading to their formation are probably unacceptable. Monopoles have, of course, been extensively discussed, recently in connection with the fact that at least the most straightforward forms of GUTs may predict a production of heavy monopoles in the early universe too large to be compatible with observation.

In the present paper we shall be concerned with vacuum strings, which have been less extensively studied, but which also appear to have potentially significant cosmological implications. Strings will occur if the manifold M of degenerate vacuum states which exist following spontaneous symmetry breaking is not simply connected, i.e., if the first homotopy group $\pi_1(M)$ of the homotopy classes of maps of the circle into M is nontrivial.[2] The possible cosmological significance of strings has been studied by Vilenkin.[7] He shows that in models having grand unified strings the collapse of closed strings may well lead to a significant density of black holes; the corresponding density in the case of electroweak strings is found to be negligible. (The linear mass density of strings is approximately proportional to the temperature at which they are formed.[2] Consequently, strings associated with the breaking of grand unification, which presumably occurs at a mass scale $\approx 10^{15}$ GeV,[8] are much more massive than those which might be associated with the breaking of, say, the Weinberg-Salam[9] electroweak symmetry at a mass scale $\approx$ 100 GeV. Throughout the paper, unless otherwise specified, the units used have $\hbar = c = k = 1$, where k is Boltzman's constant.) In the case of GUTs, the simplest estimates of black hole formation, as given in Ref. 7, lead to a nonthermal spectrum for the cosmic background radiation as a result of radiation emitted in the evaporation of mini-black holes; thus if these estimates are correct, GUTs leading to string formation would be excluded. In Ref. 7 the dissipation of the energy of the closed loops as a result of the viscosity of the surrounding medium, i.e., as a result of particle scattering from the collapsing string, is neglected. (The effect of oscillations of the collapsing string, which may also be important, is also neglected there.) If a sufficiently high percentage of the strings' mass is dissipated by friction during the collapse, black hole formation could be avoided. In the present work this question is investigated by examining the dynamics of the collapse of a circular relativistic vacuum loop, ignoring gravity but taking into account friction with the surrounding medium. In order to do this, the cross section for the scattering of particles from the collapsing string is estimated; this is also relevant to a discussion of the importance of oscillations. If excessive black hole formation can be avoided, then theories with strings may in fact be preferred, since heavy closed strings could serve as seeds for the production of the density fluctuations which are required for galaxy formation.[10,11]

The outline of the present work is as follows. Section I is devoted to a discussion of the conditions under which vacuum strings arise in a gauge theory. We shall see that strings do not occur in the simplest physically interesting theories, but may occur in more elaborate models. In the following section the cross section for scattering of particles by a vacuum string is estimated, and in the final section the dissipation of energy by friction in the collapse of a closed circular loop of vacuum strings is considered, and the question of black hole formation discussed.

I. CONDITIONS FOR VACUUM STRING FORMATION IN SPONTANEOUSLY BROKEN GAUGE THEORIES

The types of gauge theories in which strings can occur are strongly limited by the simple observation that if the symmetry group G is simply connected, and if G acts transitively on the manifold M of degenerate vacuum states, then M is simply connected, and strings do not occur when G is spontaneously broken. This is almost trivially demonstrated. Since G is a symmetry group, if $\sigma \in M$, then $g\sigma$ and σ are degenerate and $g\sigma \in M$ for all $g \in G$. The statement that G acts trasitively on M means that if $\sigma \in M$, then any $\sigma_i \in M$ can be written as $\sigma_i = g_i\sigma$ for some $g_i \in G$. Hence, corresponding to a closed curve $\sigma(\theta) \in M$ there will be a closed curve $\alpha(\theta)$ in the parameter space of G, where $\sigma(\theta) = g[\alpha(\theta)]\sigma$. Since by hypothesis G is simply connected, one can find a sequence of closed curves $\alpha(\theta,t)$ such that $\alpha(\theta,1) = \alpha(\theta)$ and $\alpha(\theta,0) = \alpha(g_o)$, where g_o is the identity element of G. There will be a corresponding sequence of closed curves $\sigma(\theta,t) = g[\alpha(\theta,t)]\sigma$ through which the curve $\sigma(\theta)$ in M can be contracted to the point σ, and thus M is simply connected.

The most commonly discussed GUTs are based on the groups SU(5) or Spin(10),[12] where we denote by Spin(10) the simply connected universal covering group of SO(10). The choice of SU(5) or Spin(10), rather than the multiply connected groups SU(5)/5 or SO(10) to which they are, respectively, locally isomorphic is dictated by the fact that the fermions in each model are assigned to representations (the fundamental 5 and the spinor 16, respectively) which are multiple valued in terms of SU(5)/5 and SO(10), so that the simply connected covering group must be chosen in order that the representation matrices be single valued and well defined. In particular, if we have a spatially dependent field $\psi(\theta)$ defined along a circular path in coordinate space by

$$\psi(\theta) = g^{-1}(\theta)\,\psi\,(0)g(\theta) \tag{1}$$

the continuity of ψ and hence the finiteness of kinetic energy terms involving ψ in the Hamiltonian will be guaranteed by the condition $g(2\pi) = 1$ only if the representation is single valued. The two most widely discussed GUTs thus satisfy the criterion that the gauge group G is simply connected; clearly this will also be true of a wide class of other gauge theories, e.g., any theory based on SU(N) or Spin(N) in which any of the basic fields are assigned to multiple valued representations of SU(N) or SO(N).

Before we conclude that such theories do not lead to string formation, a brief discussion of the question of transitivity is perhaps in order. In discussing this it is important to note that the set of possible vacuum states in a spontaneously broken gauge theory is rather different from the usual set of energy eigenstates in the presence of a symmetry group G in quantum mechanics in that the states do not belong to some irreducible representation of G. If the set of vacuum states did provide an irreducible representation of G, then in fact G would not in general act on them transitively. (This is easy to see, e.g., in the case of SU(N). The only irreducible representations of SU(N) on which SU(N) acts transitively are the fundamental representation N and its conjugate N^*. However, since the Clebsch-Gordon decomposition of $N \times N^*$ contains only the adjoint representation and the singlet, if the Higgs particles belong to anything other than the adjoint representation, the Wigner-Eckart theorem would require that the vacuum states belong to some representation other than N in order for the Higgs fields to have nonvanishing vacuum expectation values.)

The difference in the case of a spontaneously broken gauge theory can be thought of as being due to the fact that G represents a symmetry of the classical theory, before quantization, rather than of the quantized theory, and hence does not determine the structure of the Hilbert space of quantum states. This is most easily seen in the unitary gauge.[13] Let there be a multiplet of Higgs fields φ_i. In an SU(N) or SO(N) theory, e.g., in order to minimize the potential the VEVs $\langle\varphi_i\rangle$ must satisfy a condition of the form

$$\sum_i \langle\varphi_i\rangle^2 = \eta^2 \tag{2}$$

together with possible additional conditions if there are other gauge invariant combinations of $\langle\varphi_i\rangle$ of fourth order or less in the Higgs fields. Let $\langle\varphi_i\rangle = \eta_i$ be a field configuration satisfying Eq. (2). One can now impose the gauge condition of the unitarity gauge, namely[13]

$$\vec{\varphi}\cdot M^a\vec{\eta} = 0, \text{ all } a \tag{3}$$

where $\vec{\varphi}$ and $\vec{\eta}$ are vectors whose components are φ_i and η_i, and the M^a are the matrices of the generators of G in the representation to which the φ_i belong. We now define a set of unit vectors n_j in the space of the M^a to be an orthonormal basis in the subspace orthogonal to all $M^a\vec{\eta}$; in particular, we can choose $n_1 = \vec{\eta} / \sqrt{\eta}$. Then the nonzero fields are the fields $\vec{\varphi}\cdot n_j$. One now quantizes the theory, taking the fields $\vec{\varphi}\cdot n_j$ as the scalar fields in the theory. The vacuum state of the system is then characterized at the point $\vec{r}$ (up to renormalization effects) by the relations

$$\langle\vec{\varphi}\cdot n_1(\vec{r})\rangle = \eta, \quad \langle\vec{\varphi}\cdot n_j(\vec{r})\rangle = 0, \; j \neq 1 \tag{4}$$

where, if the initial conditions are such as to correspond to the presence of a string (or monopole or domain wall) the direction of $\vec{\eta}$ and hence the n_j will vary with position. The set of fields $\vec{\varphi}\cdot n_j$, being a subset of the φ_i, do not provide a representation of the full gauge group G; if they did, the representation provided by the φ_i would be reducible. They do, however, provide a representation for the subgroup H whose generators H^α correspond to the M^α for which $M^\alpha\vec{\eta} = 0$. This is easily seen. A general element of H can, with appropriate choice of basis, be written as $h = \exp(iH^\alpha\theta) = O(H^\alpha)$ for some θ. Then, from the transformation properties of the φ_i,

$$h^{-1}\vec{\varphi}\cdot n_j h = O(M^\alpha)_{ik}(n_j)_k\varphi_i = \vec{\varphi}\cdot O(M^\alpha)n_j \quad . \tag{5}$$

However, $M^\alpha\vec{\eta}\cdot O(M^\alpha)n_j = O^{-1}(M^\alpha)M^a O(M^\alpha)O^{-1}(M^\alpha)\vec{\eta}\cdot n_j = \sum_b c_b M^b\vec{\eta}\cdot n_j = 0$, where the c_b are constants, since the M^a transform among themselves under the operations of the group G. Hence $O(M^\alpha)n_j$ is just a linear combination of the n_k, and the $\vec{\varphi}\cdot n_j$ transform among themselves under the operations of H. In particular, since $M^\alpha\vec{\eta} = 0$, $\vec{\varphi}\cdot n_1$ is a scalar under the group H. Thus, as might have been expected, the Hilbert space is a representation space for the unbroken symmetry group H rather than the full gauge group. Moreover, since $\vec{\phi}\cdot n_1$ is a scalar under H, there are no selection rules which prevent it from having a nonzero VEV.

The manifold of equivalent choices of vacuum state is determined by the manifold of vectors $\vec{\eta}_g$ which are equivalent to $\vec{\eta}$ as choices for the set of $\langle\varphi_i\rangle$. However, strictly speaking, states having different values of $\vec{\eta}_g(x)$ lie in different Hilbert spaces. Classically, the set $\vec{\eta}_g$ would be the set of all vectors whose components satisfy condition (2); this is the set of all vectors obtained from $\vec{\eta}$ by operations of the invariance group G_V of the potential, where G_V may be larger than G. In the quantized theory, however, with radiative corrections taken into account, the vectors $\vec{\eta}$ and $\vec{\eta}_g$ will describe systems with the same energy only if they are obtained from one another by a gauge transformation, i.e., if $\vec{\eta}_g = M(g)\vec{\eta}$, where M(g) is the matrix in the representation of the φ_i of some element $g \in G$, and hence G does indeed act transitively on the manifold M of possible vacuum states.

It thus follows that, if one begins with a simply connected gauge group G, strings will not arise in a phase transition in which G is spontaneously broken. In particular, strings will not arise at the first stage of symmetry breaking in any GUT based on SU(N) or Spin(N). More generally, let us suppose that grand unification is based, as the name implies, on a simple Lie algebra so that there is only a single coupling constant in the symmetry limit. The grand unification group then contains no U(1) factors. Then if the group G is the universal covering group of the algebra it will be simply connected and strings will not occur at the first stage in the breaking of grand unification. As noted above, the question of whether the group G is the universal covering group or a multiply connected group to which it is locally isomorphic depends on the representation content of the theory. Thus in a theory based on the algebra of SO(N), if no particles were assigned to multiple valued representations of SO(N), G would be the doubly connected group SO(N). That is to say, solutions are allowed of the form of Eq. (1) for which $g(2\pi)$ is equal to the identity element of SO(N) but not of Spin(N). (For the familiar case of the ordinary rotation group, these solutions correspond to situations in which the direction in abstract space of the vacuum state is rotated by $2n\pi$ with n odd in going around a circular path in coordinate space.) For these solutions the curve described in M when a circular path in coordinate space is traversed cannot be contracted to a point, and hence a string is present. In the case of SU(N), if one requires that only fermions belonging to the representations 1, 3, or 3^* of color SU(3) be present, then the fermions must be assigned to the fundamental representation N of SU(N), or to antisymmetrized Kronecker products of N with itself.[14] In this case, single valuedness forces one to take SU(N) as the group. In the case of SO(N), the possibility exists of assigning fermions to the fundamental representation N, which is a single valued representation of SO(N).

If strings are not formed at the first stage of symmetry breaking of the grand unified group G, they may occur at a later stage in a chain of spontaneous symmetry breaking of the form

$$G \to G' \to G'' \;...... \tag{6}$$

in which the group G', say, is not simply connected but has the form

$$G' = K \times U(1) \quad . \tag{7}$$

From the point of view of cosmological implications one is primarily interested in the case that the phase transition from G' to G'' in which the strings are produced also occurs at something like the mass scale associated with the breaking of grand unification, so that the strings are very massive. There are two different situations which can occur in the phase transition from G' to G'' which must be distinguished in their implications for string formation. Let us suppose that the original breaking of the symmetry group G at temperature T_c occurs when a multiplet of Higgs fields φ_i develop vacuum expectation values η_i which minimize the effective potential at T_c. The phase transition from G' to G'' may occur because, at the temperature T_c' at which it happens a second multiplet of Higgs fields σ_i develop vacuum expectation values δ_i, with η_i remaining largely unchanged. Let M' be the manifold of equivalent vacuum states after the symmetry has been broken to G''. Then, given one state in M', other states in M' are obtained from it only by operators which leave the η_i invariant; these are just the operators of the group G'. Thus, in this situation it is the properties of G' which determine whether string formation is possible in the second stage of symmetry breaking. In particular, if G' is not simply connected, then string formation is not necessarily forbidden; its occurrence will depend on the details of G' and G''. In this case G'' will be a subgroup of G', consisting of those transformations which leave both η_i and δ_i unchanged.

The second situation that can occur is that the phase transition at T_c' from G' to G'' arises because, due to the temperature dependence of the potential, the vacuum expectation values of the φ_i required to minimize the potential change from η_i to η_i', with G'' the group of transformations which leave η_i' unchanged. For example, it has been pointed out[15] that in an SU(5) model where the breaking of SU(5) to SU(3) × SU(2) × U(1) is associated with a Higgs multiplet transforming by the 24 dimensional adjoint representation, there may well be an intermediate phase (corresponding to G') in which the form of the vacuum expectation values of the Higgs fields result in an SU(4) × U(1)' symmetry. In this situation, given any state in M', other states in M' may be generated from it by acting with any operator in G; there is no requirement that the η_i be left unchanged, and G'' is not a subgroup of G'. Hence in this case the topology of M' is determined by G, not G', and if the grand unification group G is simply connected, string formation will not occur at the second stage of symmetry breaking, regardless of the properties of G'. Thus in the example cited there will be no strings formed in the phase transition from SU(4) × U(1)' to SU(3) × SU(2) × U(1), even though U(1) and U(1)' are different and symmetry under the multiply connected group U(1)' is broken in the transition.

In the subsequent discussion we suppose that the first of these two situations obtains, with G'' being a subgroup of G'. We suppose G' has the form K × U(1). There are then two situations of interest. In the first the symmetry breaking from G' to G'' has the form

$$K \times U(1) \to K' \tag{8}$$

where K' is a (possibly improper) subgroup of K. Here string formation obviously occurs, since, e.g., the closed curve in M' given by

$\sigma(\theta) = (1, e^{iY})\, \sigma(0)$, $0 \leqq \theta \leqq 2\pi$, clearly cannot be contracted to a point; here 1 is the identity of K and Y is the generator of U(1), normalized to $e^{2\pi iY} = 1$.

The type of symmetry breaking in Eq. (8) may be illustrated by two models which have appeared in the literature. One of these is a GUT based on the group SU(7) and embodying some flavor unification.[16] One possible sequence of symmetry breakings which can occur in this model is $SU(7) \rightarrow SU(4) \times SU(3) \times U(1)' \rightarrow SU(4) \times SU(3) \rightarrow SU(3) \times SU(2) \times U(1)$. String formation will occur in this model at the second stage of symmetry breaking, which may occur close to the grand unification mass scale. This model avoids the production of heavy monopoles,[17] but can lead to the production of massive strings. Symmetry breaking as in Eq. (8) also can occur in an SU(5) model in which electromagnetic gauge invariance is broken at intermediate energies and restored at low temperature.[18] This model also avoids overproduction of heavy monopoles, and was indeed proposed for that reason. The chain of symmetry breaking is $SU(5) \rightarrow [SU(3) \times SU(2) \times U(1)] \rightarrow SU(3) \rightarrow SU(3) \times U(1)$. The brackets around the second stage indicate that the $SU(3) \times SU(2) \times U(1)$ phase is not necessarily required in the model but could well be present. If it is, then again string formation will occur at the second stage of symmetry breaking, which may occur at a very heavy mass scale, leading to the production of massive strings. In this case, because of the fact that the U(1) symmetry is restored at low temperature, the linear mass density of the strings is approximately proportional to the temperature,[5] which must be borne in mind in analyzing the potential cosmological significance of strings in this model.

An alternative to Eq. (8) is for the breaking of G' to have the form

$$K \times U(1) \rightarrow U(1)' \tag{9}$$

where the generator of U(1)' is a linear combination of the generator of U(1) and generators of K. (Some proper subgroup K' of K could appear on the right side of (9) as a factor in a direct product without affecting the discussion.) The conditions for string formation in this case have been discussed by Schwarz and Tyupkin.[19] Let Q and Y be the generators of U(1)' and U(1), respectively, normalized so that the smallest nonzero eigenvalues of Q and Y have absolute values equal to 1. Q will be given by an expression of the form

$$Q = aY + \sum_i b_i \tau_i \tag{10}$$

where the τ_i are the generators of K. In order for the manifold of vacuum states to be simply connected, it must be possible to deform the curve $e^{iY\theta}$, $0 \leqq \theta < 2\pi$ continuously onto the curve $e^{inQ\theta}$, $0 \leqq \theta < 2\pi$, with n an integer. Hence, strings will occur for the type of symmetry breaking of Eq. (9) provided $a \neq 1/n$ in Eq. (10).[20] Strings do not occur in the case of the spontaneous breaking of the Weinberg-Salam $SU(2) \times U(1)$ electroweak symmetry, where $Q = 3t_3 + Y/2$. Strings would occur, as shown in Ref. 19, in the $SU(3) \times U(1)$ electroweak model of Lee and Weinberg.[21]

II. SCATTERING OF A PARTICLE BY A STRING

In this section the order of magnitude of the cross section for the scattering of particles by a string will be obtained. The string thickness is of order $\varepsilon = m_H^{-1}$, the inverse Higgs mass, where $m_H = g\langle\varphi\rangle$, with g the coupling constant for the Higgs self-coupling, and $\langle\varphi\rangle$ the magnitude of the VEV of the scalar field which minimizes the potential[2]; ε may also be written as $\varepsilon = (gm_V/e)^{-1}$, where m_V is the mass acquired by the gauge bosons coupled to the generators of the symmetries broken by the spontaneous symmetry breaking, and e is the gauge coupling constant.

Considering for the moment only the gauge and Higgs fields, the Lagrangian is given by

$$L = -F_{a\alpha\beta}F_a^{\alpha\beta}/4 - D_\alpha\varphi_a D^\alpha\varphi_a/2 - g(\varphi_a\varphi_a - \eta^2)^2/4 \qquad (11)$$

where

$$F_{a\alpha\beta} = \partial_\alpha A_{a\beta} - \partial_\beta A_{a\alpha} + ef_{abc}A_{b\alpha}A_{c\beta} \qquad (12)$$

where the f_{abc} are the structure constants of the gauge group. Repeated Greek Lorentz indices are summed from 0 to 3, and Latin indices over the gauge degrees of freedom. Acting on an object φ_a belonging to an irreducible representation of the gauge group, the covariant derivative D_α is given by

$$D_\alpha\varphi_a = \partial_\alpha\varphi_a - ie\tau^j_{ab}A_{j\alpha}\varphi_b \qquad (13)$$

where τ^j is the matrix of the j-th generator of the gauge group in the representation to which the φ^b belong. Equations (11 - 13) yield the field equations

$$D^{\alpha}D_{\alpha}\varphi_a = g(\varphi_b\varphi_b - \eta^2)\varphi_a \tag{14}$$

$$D_{\alpha}F_a^{\alpha\beta} = ef_{abc}\varphi_b D^{\beta}\varphi_c \quad . \tag{15}$$

Let the VEV $\langle\varphi_a\rangle = \eta_a$. To find η_a in the case of a string, we seek a static stringlike solution of these equations which minimizes the potential, i.e., for which $\eta_a\eta_a = \eta^2$ outside the string. Take the string to lie along the z-axis, and introduce cylindrical coordinates r, θ, and z. We will choose to define the basis in the space of the φ_a in such a way that $\eta_a(\vec{r})$ varies in the 1-2 plane as $\vec{r}$ follows a circular path around the z-axis; this is, take

$$\eta_1 = \eta \cos\theta, \; \eta_2 = \eta \sin\theta, \; \eta_a = 0, \; a \neq 1,2 \tag{16}$$

in the region $r >> \varepsilon$, outside the string. Note that for all the states described by Eq. (16) to be possible vacuum states they must be connected by transformations of the gauge group. That is, the gauge group must contain the U(1) subgroup of rotations of $\vec{\varphi}$ in the 1-2 plane, where $\vec{\varphi}$ is the vector whose components are the φ_a. We will label the generators of the gauge group in such a way that the generator of this U(1) group is referred to as τ^3.

Take the vacuum expectation value of Eq. (14). A sufficient condition for a solution is that

$$D_{\alpha}\eta_{1,2} = 0, \; \alpha = 0, 1, 2, 3 \; . \tag{17}$$

The only nontrivial equation which results when Eq. (16) is substituted in (17) is

$$D_\theta \vec{\eta} = \left(\frac{1}{r}\frac{d}{d\theta} - ie\tau^j A_{j\theta}\right)\vec{\eta}(\theta) = 0 \tag{18}$$

where $\vec{\eta}$ is a column vector whose components are η_a, and the subscript θ indicates the component in the direction of increasing θ. Since $\vec{\eta}(\theta)$ makes an angle θ with the 1-axis in abstract space, τ^3, which is the generator of intinitesimal rotations in the 1-2 plane, has the effect of generating infinitesinal increases in θ; hence, acting on $\vec{\eta}$, τ^3 has the same effect as the operator $-i\frac{d}{d\theta}$, the generator of infinitesimal rotations about the z-axis. Thus Eq. (18) can be satisfied by making the coefficients of $-i\frac{d}{d\theta}$ and τ^3 equal, and choosing $A_j = 0$, $j \neq 3$. Hence Eq. (17) is satisfied if

$$\langle A_{3\theta}\rangle = 1/(er) \tag{19}$$

with all other $\langle A_{i\alpha}\rangle = 0$. Equations (16) and (19) also provide a solution of Eq. (15) for the gauge field when the VEV is taken. The right side of (15) vanishes By Eq. (17). On the left side, we assume that $\langle A_{a\alpha}\rangle$ can be treated like a classical field, as will be justified further below, so that $\langle A_{a\alpha}A_{b\beta}\rangle = \langle A_{a\alpha}\rangle\langle A_{b\beta}\rangle$. Then the nonlinear terms in $F_{a\alpha\beta}$ all vanish, since $f_{a33} = 0$ by the antisymmetry of the structure constants. Hence $F_{a0\beta} = 0$ and $F_{aij} = \varepsilon_{ijk}(\nabla\mathrm{x}\langle\vec{A}_a\rangle)_k$. Since Eq. (19) can be written as

$$\langle\vec{A}_3\rangle = \nabla\theta/e \tag{20}$$

$F_{aij} = 0$ since it is the curl of a gradient, so that the left side of Eq. (15) also vanishes. Thus outside the string we can take the VEVs of φ_a

and $A_{a\alpha}$ to be given by Eqs. (16) and (19). Within the string, of course, $\langle\varphi_a\rangle \xrightarrow[r\to 0]{} 0$, since Eq. (16) cannot be extended continuously to the origin.

Let us now introduce additional particles into the theory, and study their scattering by the string. (A similar problem for the case of domain walls has been studied previously.[22]) For simplicity, take them to be a set of scalar particles described by the real scalar fields σ_i; the conclusions will not depend on the spin structure. Let T^a be the matrix of the ath generator of the gauge group in the representation of the σ_i, and $\vec{\sigma}$ a column vector with σ_i as components. The σ particles have an intrinsic real mass m, and also develop a mass as a result of a gauge invariant coupling to φ, with coupling constant h, of the form $h\vec{\sigma}\cdot T^a\vec{\sigma}\varphi_a$, where, for specificity, the Higgs fields φ_a are now taken to transform under the adjoint representation of the group to which the generators also belong. Consider, to begin with, a representation of the T^i in which T^1 is diagonal, so that we are considering scattering of particles with definite T^1. The Klein-Gordon equation satisfied by a component of $\vec{\sigma}$ with energy E will be, for $r > \varepsilon$,

$$(-D^iD_i + M^2(\theta) - E^2)\vec{\sigma} = 0 \tag{21}$$

where the mass-squared matrix M^2 is given by

$$M^2 = m^2 + h\eta(T^1 \cos\theta + T^2 \sin\theta) \quad . \tag{22}$$

Consider an incident particle of energy E travelling in the negative x-direction and described by the incident wave function

$$\sigma_{oi}(x) = e^{-ikx}u_i$$

where u_i is the wave function in the space of the gauge group whose jth component is $(u_i)_j = \delta_{ij}$ and

$$k = (E^2 - M_{ii}^2(0))^{1/2} = (E^2 - m^2 - h\eta T_{ii}^1)^{1/2} \quad . \tag{24}$$

Scattering will clearly take place out of the state σ_{oi}, because of the angular dependence of M_{ii}^2, because of the coupling to other σ_{oj}, $j \neq i$, due to the off-diagonal elements of T^2 which enter M^2 for $\theta \neq 0$, and because of the additional r-dependence in D_θ due to $\langle A_{3\theta}\rangle$; e.g., if $T_{ii}^1 < 0$, there will be a range of energies for which $M_{ii}^2(\pi) > E^2 > M_{ii}^2(0)$, in which case the particle is energetically forbidden from penetrating undeviated into the region of negative x while remaining in the state u_i. However, it is easy to see that the scattering is of a trivial character, and is, in fact, an artifact of the choice of gauge. Namely, define

$$\sigma_{Ri}(x) = \exp(iT^3\theta)\sigma_{oi}(x) \equiv R(\theta)\sigma_{oi}(x) \quad . \tag{25}$$

One easily finds that $\sigma_{Ri}(x)$ is a solution of Eq. (21). Namely, $R^{-1}(\theta)M^2(\theta)R(\theta) = M^2(0)$, so that $M^2(\theta)$ is a diagonal operator acting on σ_{Ri}. Moreover, $D_i\sigma_{Ri} = \partial_i\sigma_{oi}$. This is trivial except for D_θ. In the case of D_θ, the additional term $(iT^3/r)\sigma_{Ri}$ which comes from $(1/r)d/d\theta$ acting on $R(\theta)$ is just cancelled by the term $-ie\langle A_{3\theta}\rangle T^3\sigma_{Ri}$, by Eq. (19). Hence, Eq. (21) for σ_{Ri} reduces to the statement that σ_{oi} satisfies the free particle Klein-Gordon equation with $M^2 = M^2(0)$, which, of course, it does. Thus, apart from the deviation from Eq. (21) for $r < \varepsilon$, within the string, the only effect of the string is to replace σ_{oi} by σ_{Ri}, that is, to cause the particle's direction in abstract space to rotate adiabatically to follow the direction of $\vec{\eta}$.

This result can be understood by noticing that, under the local gauge transformation consisting of a rotation about the 3-axis in abstract space by the angle $\alpha(\theta) = -\theta$, the solutions found in Eqs. (16) and (19) behave as

$$\eta_1(\theta) \to \eta, \ \eta_a(\theta) \to 0, \ a \neq 1 \tag{26a}$$

$$\langle A_{3\theta} \rangle \to \langle A_3\theta \rangle + (\nabla\alpha(\theta))_\theta / e = 0 \tag{26b}$$

where the last equality in Eq. (26b) will hold except on the half plane where $\alpha(\theta)$ is discontinuous. Hence, by a suitable choice of gauge one can arrange things so that $\vec{\eta}$ has a constant direction and $\langle A_{a\alpha} \rangle = 0$ for $r > \varepsilon$ except on a singular surface, which is the analog of the Dirac string singularity in the case of a magnetic monopole. By choosing the definition of α, one can take the position of the singular surface, which is the only indication for $r > \varepsilon$ of the existence of the string in this gauge, to be anywhere desired; e.g., if α is defined to run from $-\pi$ to π, the singular surface will be $\theta = \pi$, the xz-plane with $x < 0$. Thus the singular surface can always be taken to be behind the string with respect to the incident particle, so that the only evidence of the string seen by the incident particle will be the stringlike region itself at $r < \varepsilon$.[23] The fact that $\langle A_{3\theta} \rangle$ is a pure gauge field, obtainable by gauge transformation from a field which is identically zero (almost everywhere) is, of course, consistent with the result that the $F_{a\alpha\beta}$ to which it gives rise are zero, since the statement that all $F_{a\alpha\beta} = 0$ is invariant under local as well as global gauge transformations. Also, since the effect of a local gauge transformation is to add a c-number field to the vector potential, this justifies our treatment of $A_{a\alpha}$ as a classical field as far as its VEV is concerned.

One can now treat the scattering from the string using the familiar techniques of partial wave analysis, following, e.g., the discussion in Schiff's text,[24] with the appropriate changes to go from spherical to cylindrical geometry. We write the wave function for an incident wave σ_{Ri} for $r > \varepsilon$ as

$$\sigma_{si} = \left[\sum_n A_n(\cos\delta_n J_n(kr) + \sin\delta_n N_n(kr))\cos n\theta\right] R(\theta)u_i \tag{27a}$$

$$\sim \sigma_{Ri} + \left(f(\theta)e^{ikr}/\sqrt{r}\right) R(\theta)u_i \tag{27b}$$

where N_n is the solution (Neumann function) to Bessel's equation of order n which is singular at the origin. The differential cross section per unit length of string is given, as expected, by $d\sigma/d\theta = |f(\theta)|^2$. (We have neglected possible inelastic scattering to states σ_{Rj}, $j \neq i$. This does not affect the general conclusions.) Using the standard expansion for e^{ikx} as a series in $J_n(kr)\cos n\theta$, as well as the asymptotic form of the Bessel and Neumann functions, and equating the expressions for σ_{si} on the right sides of Eqs. (27a) and (27b) in the asymptotic region, yields an expression for $f(\theta)$ in terms of the phase shifts

$$f(\theta) = (2/k\pi)^{1/2} e^{i\pi/4} \sum_n c_n e^{i\delta_n} \sin\delta_n \cos n\theta \tag{28}$$

whence, on integrating over angles, one obtains for the total cross section

$$\sigma_{tot} = (2/\pi k) \sum_n c_n^2 \sin^2\delta_n \tag{29}$$

where $c_o = 2$, $c_n = 1$, $n > 0$. The phase shifts are obtained by equating the logarithmic derivatives of the coefficient of cos $n\theta$ in the exterior solution for σ_{si} in Eq. (27a), evaluated at $r = \varepsilon$, to the corresponding quantity for the interior solution valid inside the string. Denoting the logarithmic derivatives of the interior partial waves evaluated at $r = \varepsilon$ by γ_n, one finds

$$\tan \delta_n = (kJ_n'(k\varepsilon) - \gamma_n J_n(k\varepsilon))/(kN_n'(k\varepsilon) - \gamma_n N_n(k\varepsilon)) \tag{30}$$

where the prime denotes the derivative of the function with respect to its argument. Our primary concern is with the case of a thin string, $k\varepsilon << 1$. In this limit J_n and N_n, for $n > 0$, go as $(k\varepsilon)^n$ and $(k\varepsilon)^{-n}$, respectively, so that the J_n' and N_n' terms dominate and one gets $\tan \delta \sim (k\varepsilon)^{2n}$ for $n > 0$, leading to a contribution of order $\varepsilon(k\varepsilon)^{4n-1}$ to the total cross section from the partial waves with $n > 0$; thus, as expected, one gets negligible contributions equal to the string thickness multiplied by powers of the thickness divided by the wave length. However, for $n = 0$, the situation is somewhat different, reflecting the difference between cylindrical and spherical geometry. For $k\varepsilon \to 0$, $J_o \sim$ constant, and $N_o \sim \log k\varepsilon$. Now the J_o and N_o terms dominate in Eq. (30), and one obtains $\tan \delta_o \sim 1/\log k\varepsilon$, so that the $n = 0$ phase shift only vanishes logarithmically as $k\varepsilon \to 0$. One thus finds, for $k\varepsilon << 1$, that the total cross section is given by

$$\sigma_{tot} = (\pi^2/k)/(\log^2 k\varepsilon) \quad . \tag{31}$$

Thus the magnitude of the total cross section in the case of a thin string is controlled not by the thickness of the string, but by the wavelength of the incident particle, albeit divided by a logarithmic factor which can become numerically important if $k\varepsilon$ is sufficiently small.

We have derived this result for the case of a scalar field. However, from the derivation, it will hold for any field whose components obey the Klein-Gordon equation. Equation (31) will fail only in the event that γ_o becomes very close to $J_o'(k\varepsilon)/J_o(k\varepsilon)$. It may be worth observing that there is one case where this happens, namely, in the scattering of electromagnetic radiation in the special case that it is polarized with the electric field perpendicular to the string axis. Since the boundary condition requires the continuity of the tangential component of $\vec{E}$, which equals $E \cos \theta$, the scattered wave which must be added to the incident wave to fulfill the boundary condition has no $n = 0$ piece, and the lowest nonzero phase shift is $n = 1$. However, if the electric field is polarized along the string direction, $E_{tan} = E$, and $\delta_o \neq 0$.

III. DYNAMICS OF A COLLAPSING CIRCULAR STRING

Let us now consider the problem of the collapse of a closed loop of string, using Eq. (31) in estimating the effect of energy dissipation as a result of collisions. We will consider the case of a circular loop, thus neglecting the possible effect of oscillations in the case of noncircular loops; energy dissipation by gravitational radiation will also be neglected. Vilenkin's work[7,10] indicates that both oscillations and gravitational radiation are potentially important, and must also be considered in a complete treatment of the problem, but that will not be attempted here.

Closed loops of string may form in the initial phase transition which gives rise to the strings, or they may be formed later when, as the strings move, two of them cross and a "change of partners" occurs. The strings will have a mass per unit length given by[2]

$$\mu \approx \eta^2 \quad . \tag{32}$$

Consider a closed loop which, when formed, is a circle of radius R, and in particular, a small portion of the loop subtending an angle δ which remains constant as the string collapses; δ will be used as a symbol for the small element of string as well as for the value of the angle. The overall rest frame of the loop and the instantaneous rest frame of δ will be designated by S and S', respectively; S is assumed to be also the rest frame of matter, i.e., the frame in which the average particle velocity is zero.

To begin with, consider the free collapse of the loop, neglecting the frictional force due to collisions with the surrounding particles. Let v designate the inward radial speed of δ when the loop has collapsed to a radius r (all three-vector components will refer to the inward radial direction). Since energy is conserved in S, $E = \mu R\delta$, while $E' = \mu r\delta$ and $p' = 0$; hence $\mu R\delta = \gamma\mu r\delta$, or

$$\gamma = (1 - v^2)^{-1/2} = R/r \quad . \tag{33}$$

Since $p = \mu\delta Rv = \mu\delta R(1 - r^2/R^2)^{1/2}$, differentiating yields for the 4-vector force $T_\mu = dp_\mu/d\tau$ due to the tension

$$T_r = \gamma dp/dt = \mu\delta \tag{34a}$$

while

$$T_o = \gamma dE/dt = 0 \quad . \tag{34b}$$

It is interesting to look at the result of transforming these equations to S'. One finds $T'_r = dp'/dt' = \gamma\mu\delta$. Initially, when $\gamma = 1$, so that S' and S coincide and the string appears circular in S', this is just what is expected for a string whose tension is given by the mass per unit length μ. The additional factor of γ arises from the fact that $T'_r = 2T' \sin\delta'/2$, where T' is the string tension in S'. The arc of string appears Lorentz contracted in the radial direction in S; i.e., the arc is less bowed in S than in S'. Thus

$$\delta = \delta'/\gamma \tag{35}$$

and hence the result for T'_r corresponds to a string tension equal to μ for arbitrary values of γ. This value for the tension is also consistent with the result $T'_o = dE'/dt' = -\gamma v\mu\delta = \mu d(r\delta)/dt'$; since the transverse arc length is Lorentz invariant and thus given by $r\delta$ in either frame, dE'/dt' is just the rate in S' at which work is done by the tension μ as the arc length decreases.

To include the effect of collisions, write

$$dp_\mu/dt = T_\mu/\gamma + (dp_\mu/dt)_{coll} = t_\mu + f_\mu \quad . \tag{36}$$

The quantities t_μ and f_μ are, of course, not 4-vectors; their spatial components $t_r = T_r/\gamma$ and f_r represents the forces acting on δ in S due to the string tension and to collisions, respectively. The collision force f_μ is given by

$$f_\mu = nv\sigma r\delta \, \overline{\Delta p}_\mu \tag{37}$$

where σ is the total string cross section/unit length, n is the number of particles/unit volume, and $\overline{\Delta p}_\mu$ represents the average transfer of p_μ to the string per collision. Take the string to be infinitely massive, and the surrounding gas of particles relativistic. Then, working in the frame S', one has $\overline{\Delta E}' = 0$ and $\overline{\Delta p}' = -2\bar{p}'$, where $\bar{p}'$ is the average momentum in S'. Since $\bar{p} = 0$, $\bar{p}' = \gamma\bar{E} = \gamma T$ at temperature T. By Lorentz transformation,

$$\overline{\Delta E} = -2\gamma v\bar{p}' \tag{38a}$$

$$\overline{\Delta p} = -2\gamma\bar{p}' \quad . \tag{38b}$$

The foregoing discussion is based on the assumption that (in the absence of gravitational radiation which we are neglecting) the total energy of the string is conserved except for energy losses due to collisions. Thus we are neglecting the possibility of particle production during the collapse of the string. This is presumably valid as long as the radius of curvature of the string is large enough that different parts of the string do not interact with one another; in this case the string can be validly approximated as locally straight on the length scale appropriate to elementary particle interactions, and a straight string will not decay into elementary particles. Since the vacuum is invariant under the unbroken symmetries of the gauge group, the string is therefore neutral with respect to the charges which generate the unbroken symmetries; these are the charges which couple to the gauge fields which remain massless and give rise to long-range interactions after the spontaneous symmetry breaking. Hence, the forces by which the parts of the string can interact with one another have a range of order m_V^{-1}. Thus if the angle δ' corresponding to a segment of loop of length m_V^{-1} in its rest frame differs appreciably from 0, such a segment will not be locally straight and one expects elementary particle radiation to become important. From Eqs. (33) and (35), $\delta' \gtrsim 1$ for $r\delta = m_V^{-1}$ when

$$r = (Rm_V)^{1/2} m_V^{-1} \equiv r_0 \quad . \tag{39}$$

If r_0 is greater than the Schwarzchild radius, the present discussion certainly becomes invalid, and one may expect the string energy to be converted into elementary particle radiation before black hole formation occurs. Hence the present argument is valid only for the case $GM = G2\pi R\mu > r_0$, or

$$R \gtrsim m_V^{-1}/G^2\mu^2 \quad . \tag{40}$$

If energy loss due to friction is significant in the collapse, then Eq. (40) must be modified by replacing M, the total mass of the loop, by the fraction of the initial energy which is not dissipated in collisions. As long as Eq. (40) is satisfied, then the collapsing string will form a black hole before elementary particle radiation becomes important.

The conditions expressed by Eqs. (39) and (40) may also be thought of in terms of the proper acceleration of an element of the loop. The magnitude of the proper acceleration a' of the element δ is given by $a' = T_r'/\mu r\delta = \gamma/r \approx R/r^2$ from Eq. (33). Hence $r = r_0$ corresponds to $a' = m_V$, so that $r < r_0$ corresponds to values of a' which are appreciable on the mass scale m_V.[25]

The strings are first produced at temperature $T_c \approx \eta \approx m_V/e$. In the standard cosmology, with the early universe treated as an expanding relativistic gas, time and temperature are related by

$$t = C/T^2 \tag{41}$$

where the constant $C = [45/32\pi^3 GN]^{1/2}$ in our units,[26] and N is approximately the total number of different particle species present in the gas. Taking $N \approx 100$, which is a typical order of magnitude in GUTs, gives $C \approx 10^{30}$ in our units. The largest possible value of R is $R = t$, namely a radius equal to the horizon length. Taking $m_V \approx 10^{15}$ GeV $\approx 10^{28}$, one finds $t \approx 10^{-28} \approx 10^{-39}$ sec as the time of first formation of grand unified strings. From Eq. (40) one has $R/t = RT^2/C > (T/T_c)^2(Cm_V G^2\mu^2)^{-1}$; numerically this gives $R/t > 10^{15}T^2/T_c^2$. Thus loops formed in the initial phase transition at $T = T_c$ fail to satisfy the condition (40). Hence in the grand unified case we shall only be concerned with loops formed at temperatures $T < T_0 = (T_c/10^{15})^{1/2}$, i.e., at $t > 10^{-24}$ sec, at which time the largest possible loops, with $R \approx t$, may produce black holes before losing their energy by elementary particle radiation. At later times loops with $R > (T/T_0)^2 t$ will collapse to black holes if friction may be neglected. The formation of closed loops at $T < T_c$ may come about, as already mentioned, as a result of collisions between strings in which a "change of partners" occurs. It may also come about as the result of the formation of closed loops in the initial phase transition with radii greater than the horizon length at that time. Vilenkin has shown[7] that such loops undergo conformal expansion, with the expansion of the universe, up to the time $t \approx R$ at which the entire loop is included within the horizon, at which point they begin to collapse. Hence, for the present discussion, such loops may be thought of as produced at $t = R$, the time at which collapse starts. Since all loops produced at $T \ll T_c$ are expected[7] to have $R \approx t$, once black hole formation has become possible at all one expects, assuming frictional effects are not important, that almost all loops which are produced will be large enough to collapse to black holes.

A similar calculation for the case of electroweak strings, with $m_V \approx 100$ GeV, gives $t \approx 10^{-13}$ sec as the time of the initial phase transition. However the condition (40) requires $(T_c/T)^2 \gtrsim 10^{54}$, which, from Eq. (41), implies $t \gtrsim 10^{41}$ secs, far greater than the age of the universe. Hence electroweak strings will always decay by elementary particle radiation before black hole formation occurs.

From the foregoing, it is clear that the discussion will be concerned with temperatures $T \ll 1/\varepsilon$, and thus, in evaluating Eq. (37), we shall be concerned with the case of long wavelength scattering. Hence, apart from logarithmic factors, σ will be of order $(\bar{p}')^{-1}$. Combining Eqs. (31), (37), and (38), one obtains

$$f_r = -2nvr\gamma\delta\left(\pi^2/\log^2(\gamma T\varepsilon)\right) \tag{42}$$

and

$$f_o = vf_r \quad . \tag{43}$$

Suppose the loop undergoes a collapse to a final radius r between times t_i and t_f. The energy dissipated during this collapse as a result of collisions is $W(r) = -\int_{t_i}^{t_f} f_o dt$. As a result of Eq. (43), this can be written in the usual way as the line integral of the force, so

$$W(r) = \int_R^r f_r dr' \quad . \tag{44}$$

(It is perhaps worth noting that in the case of the tension the result, analogous to (44), that would normally hold in special relativity, namely that $\int t_o dt = \int t_r dr$, is not correct. This is because the mass of δ changes during the motion, and hence there are additional terms in $dE/dt = d(\gamma m)/dt$ and in dp/dt, involving dm/dt, which spoil the usual relation between force and the rate of change of energy.) The equation of motion, Eq. (36), is quite complicated, primarily because of the highly nonlinear form of the velocity dependent frictional force in Eq. (42), even when we omit, as we shall, the final parenthetical factor, which is slowly varying because of the logarithmic dependence and numerically sufficiently close to 1 so as not to affect estimates in an important way. The frictional force can be written in the usual form $f_r = -\alpha v$, but here α depends on r and v. In addition, α has an explicit dependence on the independent variable t through the density n. In a relativistic gas the energy density $n_E \approx NT^4$;[27] combining this with Eq. (41) gives for the number density

$$n \approx NC^{3/2}/t^{3/2} \quad . \tag{45}$$

Fortunately, it turns out that an exact solution is not needed; it is possible to find an upper bound to W(r) which shows that, in cases of interest, the effect of collisions is not important. To do this, note that, since the effect of f_r is to slow down the collapse of the ring, an upper bound for γ is obtained by using the result of Eq. (33) for the collapse of the loop in the absence of collisions. Since $v < 1$ and $n \lesssim NC^{3/2}/t_o^{3/2}$, where t_o is the time at which the loop began to collapse, we have from Eqs. (42), with the factor in parentheses neglected, and (44), that W(0), the total energy dissipated by collision, satisfies

$$W(0) < 2NC^{3/2}R^2\delta/t_o^{3/2} \quad . \tag{46}$$

The condition that the fractional loss of energy due to collisions be small is that $W(0) << \mu R\delta$, which is just the condition $t_o^{3/2} >> 2NC^{3/2}R/\mu$. This restriction clearly becomes more stringent as R is increased. Since the maximum value of R is t_o, the fraction of energy lost due to collisions will be small for all loops formed at time t_o if

$$t_o >> 4N^2C^3/\mu^2 \quad . \tag{47}$$

In the grand unified case, this becomes $t_0 >> 5\times 10^{-35}$ sec. Since only those closed loops formed at $t > 10^{-24}$ sec satisfy inequality (40), the energy lost due to collisions during the collapse of a circular loop is negligible for all cases of interest, that is, for all cases in which (40) is satisfied. Thus collisional energy loss will not prevent the loop collapsing to a black hole before elementary particle production can become important.

For electroweak strings we have already seen that the inequality (40) is never satisfied, so that elementary particle radiation can always be expected to prevent black hole formation. We may also note that frictional effects, as well, are important in the electroweak case. From Eq. (47) one obtains $t_0 > 10^{19}$ secs in the case of electroweak strings. That is, only in the case of loops formed at $t > 10^{19}$ secs, which exceeds the present age of the universe by at least an order of magnitude, could one be sure that the fraction of energy dissipated by collision during collapse is small.

Finally we can examine models of the type in Ref. 16, where the U(1) symmetry whose spontaneous breakdown is responsible for string formation is restored at low temperature. As mentioned earlier, in this case μ is temperature dependent, vanishing at the temperature T_c' at which the symmetry is restored; if $T_c' << T_c$, then the VEV's of the Higgs fields are just proportional to the temperature,[5] and thus $\mu = T^2 = C/t$. To study the importance of collisions in such a model one simply has to replace μ by C/t_o in Eq. (45). The sense of the inequality is then reversed, and one finds $t_o < 1/4N^2C \approx 10^{-45}$ sec $< t_c$. Hence, in such a model one expects that the effect of collisions will always be important, and collapse to black holes will not occur. Black hole production is, in any event, not a problem in such models; because of the decrease of μ with t the observational consequences of black hole production in such models would be characteristic of models with a mass scale much less than the grand unification scale, and hence not important cosmologically. On the other hand, for the same reason closed loops in such models formed near the recombination time will be too light to serve as seeds in the production of galaxies.

Let us sum up the conclusions briefly. Frictional effects on the collapse of closed loops in standard GUTs are negligible, and will not serve to suppress the possible overproduction of black holes discussed in Ref. 7. It is quite possible that black hole production is suppressed for other reasons, in particular oscillations and gravitational radiation. It also could be that the estimates of the rate of formation of closed loops in Ref. 7 are too large; as pointed out there, this especially might be true if the likelihood of a change of partners occurring in a collision between two strings is very small, and this question requires further study. If black hole production cannot be suppressed by mechanisms other than friction, then GUTs in which string formation occurs are experimentally excluded. On the other hand, if such suppression is possible, then GUTs with strings may offer an attractive explanation of galaxy formation. Frictional effects are important in the collapse of closed strings with values of μ smaller than that corresponding to the grand unification mass scale. This applies both to electroweak strings, and to grand unified strings in models in which a U(1) symmetry is restored at low temperatures, leading to a value of μ which decreases with time. There do not appear to be potential arguments against GUTs of this type based on the fact that they lead to string formation. On the other hand, strings in these models are not capable of serving as seeds for the production of galaxies.

ACKNOWLEDGEMENTS

It is a pleasure to express my thanks to Professor Sidney Drell for his hospitality in the SLAC theory group where most of this work was carried out, and to Professor Alex Vilenkin for many useful conversations throughout the course of the work.

REFERENCES

1. D. Kirzhnits and A. Linde, Phys. Lett. 42B, 471 (1972); S. Weinberg, Phys. Rev. D 9, 3357 (1974); L. Dolan and R. Jackiw, Phys. Rev. D 9, 3320 (1974).
2. T. Kibble, J. Phys. A 9, 1387 (1976).
3. H. Nielsen and P. Olesen, Nucl. Phys. B61, 45 (1973).
4. G. 't Hooft, Nucl. Phys. B79, 276 (1974); A. Polyakov, Pis'ma Zh. Eksp. Teor. Fiz. 20, 430 (1974) [JETP Lett. 20, 194 (1974)].
5. A. Vilenkin, Phys. Rev. D 23, 852 (1981).
6. Ya. B. Zel'dovich, I. Yu. Kobzarev, and L. B. Okun, Zh. Eksp. Teor. Fiz. 67, 3 (1974) [Sov. Phys. JETP 40, 1 (1975)].
7. A. Vilenkin, Phys. Rev. (to be published).
8. H. Georgi, H. Quinn, and S. Weinberg, Phys. Rev. Lett. 33, 451 (1974).
9. S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in Elementary Particle Physics, ed. by N. Svarthold (Almquist and Wiksels, Stockholm, 1968) p. 367.
10. A. Vilenkin, Private communication, and to be published.
11. Ya. B. Zel'dovich, Mon. Not. Ast. Soc. 192, 663 (1980).
12. H. Georgi and S. Glashow, Phys. Rev. Lett. 32, 438 (1974); M. Chanowitz, J. Ellis, and M. Gaillard, Nucl. Phys. B128, 506 (1977).
13. S. Weinberg, Phys. Rev. D 7, 1068 (1973).
14. M. Gell-Mann, P. Ramond, and R. Slansky, Rev. Mod. Phys. 50, 721 (1978).
15. A. Guth and S. Tye, Phys. Rev. Lett. 44, 631 (1980).

16. J. Kim, Phys. Rev. Lett. 45, 1916 (1980).

17. J. Preskill, Phys. Rev. Lett. 43, 1365 (1979).

18. P. Langacker and S.-Y. Pi, Phys. Rev. Lett. 45, 1 (1980).

19. A. Schwarz and Yu. S. Tyupkin, Phys. Lett. 90B, 135 (1980).

20. Note that only the requirement $a \neq 1$ is specifically mentioned in Ref. 19.

21. B. Lee and S. Weinberg, Phys. Rev. Lett. 38, 1237 (1977).

22. A. Everett, Phys. Rev. D 10, 3161 (1974).

23. I am grateful to Prof. A. Vilenkin for pointing this out to me.

24. L. I. Schiff, Quantum Mechanics (McGraw-Hill, New York, 1968) p. 118.

25. I am grateful to Professor S. Coleman for a comment pointing out the relevance of the proper acceleration in formulating the inequality (40).

26. See, e.g., S. Weinberg, Gravitation and Cosmology (Wiley, New York, 1972), p. 538.

27. See, e.g., Ref. 25, p. 509.