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RENORMALIZATION GROUP FOR MANY PARTICLE EXCLUSIVE AND EXCLUSIVE SEMI-INCLUSIVE PROCESSES*

Subhash Gupta
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

In this paper it is shown that in QCD a new set of high energy processes are calculable. Firstly, many meson exclusive processes are shown to be controllable. Secondly, a new type of exclusive semi-inclusive meson process (for example $e^+ + e^- \to \Pi_{cone}$ + anything, where Π is unaccompanied in phase space) is shown to be controllable by renormalization group approach. Possible experimental confrontation is discussed.

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I. Introduction

Precise quantitative test of QCD is very desirable. A lot of recent work has focussed itself in making rigorous prediction from QCD which do not depend on detailed uncalculable bound state dynamics. Most of such predictions are based on the proof of factorizability of soft dynamics from the short distance process and then using Callan Symanzik[1] equations to control the evolution of the process as the large momentum increases.

So far noteworthy progress has been made in rigorously predicting inclusive, semi-inclusive and exclusive processes of certain types. In the inclusive processes deep inelastic lepton scattering,[2] total e^+e^- cross section, inclusive annihilation of heavy quarks and level widths of an onium state below the continuum are noteworthy.[3,4] All such process though in quantitative agreement with experiment, are not conclusive. The distinctive feature of QCD, the scaling violations which scale like $\log Q^2$ are difficult to distinguish from a small power. Also, as all the details of final states are ignored, the other distinctive features of QCD get averaged over. It is thus useful to study semi-inclusive and exclusive channels. Most of the semi-inclusive processes so far studied have the feature of isolating a few particles all having large invariants with each other, and "anything". The unidentified particles in anything in fact involve a lot of particles which are travelling parallel to the particles identified. The simplest and a crude way to think about such semi-inclusive processes is to think of the cross section of production of partons in the directions of the final hadron desired, convoluted with the fragmentation of these partons into the desired hadrons. Again, because of large averaging over final states such processes have had great disadvantage in becoming a good quantitative test.

In the exclusive process where every particle is identified such averaging problems do not exist, but they are usually too small a cross section that as quantitative tests they have not as yet been decisive,[5-9] although the power law behavior is consistent with QCD.

Theoretically, in non-Abelian gauge theories, no connection exists between exclusive processes, semi-inclusive and inclusive processes. In this paper we present more processes that can be controlled rigorously in non-Abelian theories. The first set of processes is in fact nothing but just a generalization of the exclusive processes to many more particles in final state. The second set of processes is a unusual admixture of exclusive, semi-inclusive processes. In these it is required that the identified hard particle does not have any hard parallel moving particles.

These processes though rigorously controllable are down by powers of Q^2 as compared with inclusive processes, but may provide important tests as numerically they are much larger than the exclusive processes, but still have the information of exclusive processes. These processes also have a very distinctive signature and that makes us more hopeful that they might be experimentally accessible. Of course, the higher order α correction to these processes would not be necessarily small.

The organization of the paper is as follows:

II. For the sake of notation $e^+e^- \to 2\Pi$ is reviewed.

III. The proof of factorization for $e^+e^- \to N\Pi$ is given.

IV. The proof of factorization for $e^+e^- = \Pi + \text{anything}$ where in a cone of angle $\sim a$ around Π there are no hard particles, is given; and predictions for the process are made. The predictions for the case when is a heavy quark state, will be treated somewhere else. In that case everything is calculable.

V. Other processes where we expect such factorization to work are pointed out, and finally we discuss the nice feature and possible drawbacks of the processes that we study here.

II. $e^+e^- \to 2\Pi$

We begin by considering the exclusive process of two pion production. Our treatment closely resembles that of Duncan and Mueller.[6] We repeat the essential details of the pion form factor in the time like regime so as to state the notation used in the rest of the paper.

So consider the following five point function illustrated in Fig. 1 with external legs amputated

$$\tau_\mu(p_1,k_1,p_2,k_2) = \frac{1}{16}\left(\gamma_5\gamma_+\right)_{\alpha_1\beta_1}\left(\gamma_5\gamma_-\right)_{\alpha_2\beta_2}\int d^4x_1\, d^4y_1\, d^4x_2\, d^4y_2$$

$$\times \exp\left\{-i\left(k_1-p_1\right)x_1 + i\left(p_1+k_1\right)y_1 + i\left(k_2+p_2\right)y_2 - i\left(k_2-p_2\right)x_2\right\}$$

$$\times \left\langle 0\left|T\ j_\mu(0)\ \psi_{\beta_1'}(y_1)\ \psi_{\beta_2'}(y_2)\ \bar{\psi}_{\alpha_1'}(x_1)\ \bar{\psi}_{\alpha_2'}(x_2)\right|0\right\rangle$$

$$\times S^{-1}_{\alpha_1'\alpha_1}\left(p_1+k_1\right)S^{-1}_{\beta_1\beta_1'}\left(k_1-p_1\right)S^{-1}_{\beta_2\beta_2'}\left(p_2+k_2\right)S^{-1}_{\alpha_2'\alpha_2}\left(k_2-p_2\right) . \quad (1)$$

The large momentum components in the above are $p_{1-} \sim k_{1-} \sim p_{2+} \sim k_{2+} \sim \mathcal{O}(Q)$ and the rest of the components are of the order of $\mathcal{O}(m/Q)$. The dominant contribution to the amplitude τ_μ as $Q^2 \to \infty$ comes only from the following regions of momenta. (A) Large momenta to the right, that is $(k_1-p_1-r_1)^2 \sim r_1^2 \sim \mathcal{O}(m^2)$, and $(r_2-p_2-k_2)^2 \sim r_2^2 \sim \mathcal{O}(Q^2)$; (B) Large momenta to the left, i.e., $(k_1 - p_1 - r_1)^2 \sim r_1^2 \sim \mathcal{O}(Q^2)$, and $(r_2 - p_2 - k_2)^2 \sim r_1^2 \sim \mathcal{O}(m^2)$; and (C) All momenta $\mathcal{O}(Q^2)$. The regime where all these momenta are order $\mathcal{O}(mQ)$ and r_3^2 is order m^2 is suppressed for spinor theories (see Fig. 2).

This allows us to oversubstract and extract the dominant contribution and obtain a factorization of soft parts and hard parts. The hard part is the five point two particle irreducible Green's function $\mathcal{T}_\mu(Q,\overset{o}{p}_1,\overset{o}{k}_1,\overset{o}{p}_2,\overset{o}{k}_2)$ which is now evaluated at the special point such that the small components of order m are set to zero and large components are the same. The soft parts are then the oversubstracted wave functions or distribution amplitudes.[5,6] Obviously for each pion there exists such an amplitude function. Therefore,

$$\mathcal{T}_\mu(p_1,k_1,p_2,k_2) \xrightarrow[4(p_1+p_2)^2\to\infty]{} \frac{1}{16}(\gamma_5\gamma_+)_{\beta_1\alpha_1}(\gamma_5\gamma_-)_{\beta_2\alpha_2}\int\frac{d^4k_1'}{(2\Pi)^4}\,\frac{d^4k_2'}{(2\Pi)^4}$$
$$\times K^{reg}_{\alpha_1\beta_1\beta_1'\alpha_1'}(p_1,k_1,k_1')(\gamma_5\gamma_-)_{\alpha_1'\beta_1'}\,\hat{\mathcal{T}}_\mu(Q,\overset{o}{p}_1,\overset{o}{k}_1,\overset{o}{p}_2,\overset{o}{k}_2)(\gamma_5\gamma_+)_{\alpha_2'\beta_2'}$$
$$\times K^{reg}_{\beta_2'\alpha_2'\alpha_2\beta_2}(p_2,k_2,k_2') \quad ; \tag{2}$$

$$K^{reg}_{\beta\alpha\alpha_1\beta_1}(p_1,k_1,k_1') \xrightarrow[(2p_1)^2-m_\Pi^2\to 0]{} \frac{i}{(2p_1)^2-m_\Pi^2+i\varepsilon}\,\chi^{reg}_{\alpha_1\beta_1}(p_1+k_1'\,;\,k_1'-p_1)$$
$$\times\,\bar{\chi}^{reg}(p_1+k_1\,;\,k_1-p_1) \quad ; \tag{3}$$

$$\chi^{reg}_{\alpha_1\beta_1}(p_1+k_1\,;\,k_1-p_1) \equiv \int d^4x\,\left\langle 0\left|N_3\left(\psi_{\alpha_1}\left(-\frac{x}{2}\right)\bar{\psi}_{\beta_1}\left(\frac{x}{2}\right)\right)\right|2p\right\rangle e^{-ik_1x} \quad . \tag{4}$$

Now defining

$$V^{n_1}_{\alpha_1\beta_1} \equiv \int \frac{d^4k_1'}{(2\Pi)^4} K^{reg}_{\alpha_1\beta_1\beta_1'\alpha_1'}\left(p_1,k_1,k_1'\right)\left(\gamma_5\gamma_-\right)_{\alpha_1'\beta_1'}\left(k_{1-}'\right)^{n_1}$$

$$= \int d^4x_1\, d^4y_1\, e^{i\left(p_1+k_1\right)x_1}\, e^{-i\left(k_1-p_1\right)y_1}$$

$$\times \left\langle 0 \middle| T\,\psi_\alpha(y_1)\,\bar\psi_\beta(x_1)\, N_3\left(\bar\psi\gamma_5\gamma_-\left(\tfrac{1}{2}\, iD_-^{n_1}\right)\psi\right)\middle| 0\right\rangle$$

$$\times\, S^{-1}_{\alpha_1\alpha}\left(k_1-p_1\right)\, S^{-1}_{\beta\beta_1}\left(p_1+k_1\right) \quad . \tag{5}$$

It is convenient to define $(k_{1-}/p_{1-}) = x_1$ and $(k_{2+}/p_{2+}) = x_2$; and then

$$\overset{\Lambda}{\mathscr{T}}_\mu\left(Q,\overset{o}{p}_1,\overset{o}{p}_2,\overset{o}{k}_1,\overset{o}{k}_2\right) = \sum_{n_1 n_2} x_1^{n_1}\, \overset{\Lambda}{\mathscr{T}}_{\mu n_1 n_2}(Q^2)\, x_2^{n_2} \quad ,$$

therefore

$$\overset{\Lambda}{\mathscr{T}}_\mu\left(Q,p_1,p_2,k_1,k_2\right) \xrightarrow[Q^2\to\infty]{} \sum_{n_1n_2} \frac{1}{4p_{1-}^{n_1}}\left(\gamma_5\gamma_+ V^{n_1}\left(p_1,k_1\right)\right)\overset{\Lambda}{\mathscr{T}}_{\mu n_1 n_2}(Q^2)$$

$$\times\, \frac{1}{4p_{2+}^{n_2}}\left(\gamma_5\gamma_- V^{n_2}\left(p_2,k_2\right)\right) \quad . \tag{6}$$

In the above the x_1 and x_2 are the momentum fraction variables. By expanding in x_1 and x_2 one has separated the soft wave function (distribution amplitudes) which comes with the oversubstracted operators inserted into the wave function. The Eq. (6) is just the statement of light cone expansion where $\overset{\Lambda}{\mathscr{T}}_{\mu n_1 n_2}$ is the usual coefficient function or the singular functions and the $\mathrm{tr}\left(\gamma_5\gamma_- V^{n_1}(p_1,k_1)\right)$ are the matrix elements of the operators.

It is now easy to obtain a Callan Symanzik equation for $\overset{\Lambda}{\mathscr{T}}_{\mu n_1 n_2}(Q^2)$ by making a soft mass insertion in $\mathscr{T}_\mu\left(Q,p_1,k_1,p_2,k_2\right)$. If such an equation exists, when one makes such an insertion in the $\mathscr{T}_\mu\left(Q,\overset{o}{p}_1,\overset{o}{k}_1,\overset{o}{p}_2,\overset{o}{k}_2\right)$ it is down by powers of 1/Q, as all momentum are of order of Q. But, if such an insertion produces terms of the same order as $\mathscr{T}_\mu\left(Q,p_1,k_1,p_2,k_2\right)$ as would happen in theories where the hard moments may go around a soft line (as in ϕ^3 in 6-dimensions) a useful C.S. equation would not exist. On inserting it in the distribution amplitudes one gets the anomalous dimensions. Hence

$$\mathscr{D}\overset{\Lambda}{\mathscr{T}}_{\mu n_1 n_2}(Q^2) = \sum_{n_1'} \gamma_{n_1 n_1'} \overset{\Lambda}{\mathscr{T}}_{\mu n_1' n_2}(Q^2) + \sum_{n_2'} \gamma_{n_2 n_2'} \overset{\Lambda}{\mathscr{T}}_{\mu n_1 n_2'}(Q^2) \quad (7)$$

where $\mathscr{D}$ is the usual Callan Symanzik operator as defined in Ref. 6. Using the above formalism and calculating the cross section for $e^+e^- \to 2\Pi$ after averaging over initial electron positron direction

$$(2\Pi)^6\, 4E_1E_2 \frac{d\sigma}{d^3p_1\, d^3p_2} = -\frac{8\Pi^2\alpha^2}{3Q^4} F_N$$

$$F_N = |\mathscr{M}_\mu|^2 (2\Pi)^4 \delta^4\left(p_1 + p_2 - Q\right)$$

$$\mathscr{M}_\mu = \langle 2\Pi \,|\, j_\mu \,|\, 0\rangle \quad . \quad (8)$$

So, we finally get to the leading anomalous dimension

$$\sigma = \frac{\Pi\alpha^2}{3Q^6} \frac{(36)^2}{N_c^2} C_2^2(f)\, f_\Pi^4\, \alpha^2_{eff}\left(Q^2\right)\left(Q^2_{ch}\right)$$

$$+ \text{ inverse fractional powers of } \log Q^2 \quad ,$$

$\left(Q^2_{ch}\right)$ = Is the charge^2 for the relevant quarks in the given pions.

III. $e^+ + e^- \to N\Pi$

After having set down the notation in the last section and learning the necessary rules for factorization and derivation of Callan Symanzik rules, now we would turn to the rather simpler related task of proving that the same can be done for $e^+ + e^- \to N\Pi$ in a certain kinematic regime. Here, we would do so for the case of pions at large angles with each other.

For $N \geq 2$, the amplitude depends on 3N-2 variables. We shall choose a reference frame where the incoming photon is at rest $q = (Q,0,0,0)$ and the other 3N-3 variables would label the magnitudes and angles of the various Π's. Thus,

$$
\begin{aligned}
p_{1\mu} &= \left(E_1, 0, 0, -p_1\right) \\
p_{2\mu} &= \left(E_2, -p_2\sin\theta_2, 0, -p_2\cos\theta_2\right) \\
p_{i\mu} &= \left(E_i, -p_i\sin\theta_i\cos\phi_i, -p_i\sin\theta_i\sin\phi_i, -p_i\cos\theta_i\right) \quad . \qquad (9)
\end{aligned}
$$

The independent variable are taken to be Q, p_i, θ_i, ϕ_i. And θ_i, ϕ_i are held fixed as $Q \to \infty$. Define

$$
\begin{aligned}
\mathcal{T}_\mu\left(Q, p_i, k_i\right) = \frac{1}{4^N} \prod_N \left(\gamma_5\gamma_-\left(\theta_i, \phi_i\right)_{\alpha_i\beta_i}\right) \int \prod_N d^4x_i\, d^4y_i \\
\times \exp\left\{-i\sum_N \left(k_i - p_i\right)x_i + i\sum_N \left(p_i + k_i\right)y_i\right\} \\
\times \left\langle 0 \middle| Tj^\mu(0) \prod_N \left(\psi_{\beta_i'}(y_i)\right) \prod_N \bar{\psi}_{\alpha_i'}(x_i) \middle| 0 \right\rangle \\
\times \prod_N S^{-1}_{\alpha_i'\alpha_i}\left(p_i + k_i\right) S^{-1}_{\beta_i\beta_i'}\left(k_i - p_i\right) \quad . \qquad (10)
\end{aligned}
$$

It is rather trivial to check that factorization works as in the previous section with almost no change. The result finally of following the oversubstraction on each leg finally gives

$$\mathscr{T}_\mu(Q,p_i,k_i)\xrightarrow[Q^2\to\infty]{}\frac{1}{4^N}\prod_N\Big(\gamma_5\gamma_-(\theta_i\phi_i)\Big)_{\alpha_i\beta_i}$$

$$\times\int\prod_N d^4k_i'\,K^{\mathrm{reg}}_{\alpha_i\beta_i\beta_i'\alpha_i'}(p_i,k_i,k_i')\Big(\gamma_5\gamma_+(\theta_i\phi_i)\Big)_{\alpha_i'\beta_i'}\;\overset{\Lambda}{\mathscr{T}}_\mu(Q,\overset{o}{p}_i,\overset{o}{k}_i)\;. \quad (11)$$

where $\gamma_-(\theta_i,\phi_i) \doteq (1/\sqrt{2})(E_i+|p_i|)$. Expanding in the usual variables

$$x_i = \frac{k_{i-}(\theta_i,\phi_i)}{p_{i-}(\theta_i,\phi_i)} \qquad ,$$

we get

$$\overset{\Lambda}{\mathscr{T}}_\mu(Q,p_i,k_i) = \sum_{n_1\ldots n_N}\prod_N\frac{1}{4p_{i-}^{n_i}(\theta_i\phi_i)}\,\mathrm{tr}\Big(\gamma_5\gamma_-(\theta_i\phi_i)V^{n_i}(p_i,k_i)$$

$$\times\,\overset{\Lambda}{\mathscr{T}}_{\mu n_1\ldots n_N}(Q^2)\Big) \qquad . \quad (12)$$

The power law of course agrees with the usual power counting arguments.[10] To show that a Callan Symanzik equation exists is altogether different. Let us recall all the regimes and difficulties that can exist.

Firstly note that the nuclear form factors do not have a Callan Symanzik equation although factorization works. This happens because of a double flow regime, which leaves certain propagators in the hard part on the mass shell, and the large momenta route around it (Fig. 3). This also happens in pion form factor in ϕ_6^3 theory. (See Ref. 6, Page 1643, last paragraph.) See Fig. 4. Also, the Callan Symanzik equation would not exist if Landshoff pinch singularities exist. This happens in Π-Π scaling at large angles (see Fig. 5). These singularities occur in Π-Π scattering when the near on-shell quarks scatter with another near on-shell quark, into on-shell quarks moving in the direction of the final pions. That is all the quarks in the diagram are near shell. Such pinch singularities spoil Callan Symanzik equation.[5,7] (See Ref. i, Page 163).

It is easy to convince one self that the pinch singularities of Π-Π scattering diagram, that could occur in $e^+ + e^- \to 4\Pi$ do not occur and are in fact down by powers. The reasons for why this occurs is easy to see. The hard momenta from the photon has to be transferred to all the fermions lines and hence there exist at least 3 propagators which are off-shell by order Q^2 at least, thus obviating this problem. This will be true in general. The double flow regimes are also suppressed by powers because of the fermion propagators near the photon line have to be always $\mathcal{O}(Q^2)$ off mass shell, and then one simply looses too much in phase space in the double flow regimes.

Let us therefore sketch the argument in general. Consider the diagrams where a set of pions cannot be separated from the rest of the pions, in the hard part, by just cutting gluon lines (see Fig. 7a), the usual arguments for 2 pion form factor work and generalize because the fermion lines near the photon are always off-shell by $\mathcal{O}(Q^2)$.

The part of the argument where a set of pions can be separated from the rest of pions by cutting only gluon lines is argued in two stages (see Fig. 7b). The pions which are produced by gluons work essentially in the way the heavy quarkonian decay into 2 pions work (Ref. 8). Firstly, one notes that if the two gluons finally create more than two pions then they must be again off-shell by $\mathcal{O}(Q^2)$. If they give rise to only two pions then once again the argument is just the one that was relevant to the four pion case dealt with before.

Also note all other soft cancellations are automatically guaranteed to occur as in the two pion case, (Ref. 6, Sec. III and also last of Ref. 3, Secs. IIC, IIIC) as all relevant invariants between any given two pions are order Q^2. This can be best seen by rotating the reference frame to let the pion whose soft interactions with others are under observation to fly in the -Z direction and boost if further in the -Z direction, so as to have all the rest of the particles only have -- components large. In this case it is clearly seen to be the same (as far as soft objects are concerned) as the two pion case.

Hence, using this we verify the existence of a Callan Symanzik equation. We get

$$\mathscr{D}\hat{\mathscr{T}}_{\mu n_1 \ldots n_N}(Q^2) = \sum_k \sum_{n_k'} \gamma_{n_k n_k'} \hat{\mathscr{T}}_{\mu n_1 \ldots n_k' \ldots n_N}(Q^2) \tag{13}$$

where $\gamma_{n_i n_i'}$ are the same as in the pion form factor in time like regime.

IV. $\underline{e^+ + e^- \to \Pi_{\text{cone}} + \text{anything}}$

In this section we would like to show that factorization works not only in semi-inclusive Π as discussed earlier,[3] but also in this regime which we refer to as the exclusive-inclusive regime. Let us therefore start by stating the kinematic regime. Let the pion have a momentum 2p, the photon Q. We work in a frame where

$$q = (Q,0,0,0)$$

$$p = (p,0,0,-p) = \left(p_{1+} \sim \mathscr{O}(Q)\ , p_{1-} \sim \mathscr{O}\left(\frac{m^2}{Q}\right),\ p_\perp = 0\right)$$

Define $(4p\cdot q)/Q^2 = x$.

We demand that in a cone of finite angle around the +Z direction there are no particles that carry energy $\gtrsim \mu$ where μ is the QCD scale. That is all hard particles in 'anything' satisfy the requirement that $x_- \geq x_+ a$, where 'a' is order 1. So in the center-of-mass frame we are looking for a typical event that looks as shown in Fig. 8.

The relevant cut — amplitude is, as shown in Fig. 9,

$$W\left(Q,p_1,k_1,\ell_1\right) = \frac{1}{16}\left(\gamma_5\gamma_-\right)_{\beta_1\alpha_1}\left(\gamma_5\gamma_-\right)_{\beta_2\alpha_2} \int d^4x_1\, d^4x_2\, d^4y_1\, d^4y_2\, d^4x$$

$$\times\ e^{iqx}\, e^{i\left(p_1+k_1\right)x_1}\, e^{-i\left(k_1-p_1\right)y_1}\, e^{i\left(p_1+\ell_1\right)x_2}\, e^{-i\left(\ell_1-p_1\right)y_2}$$

$$\times \left\langle \bar{T}\left(\psi_{\alpha_2}(x_2)\, \bar{\psi}_{\beta_2}(y_2)\, j_\mu(x)\right) T\left(\psi_{\alpha_1}(x_1)\, \bar{\psi}_{\beta_1}(y_1)\, j^\mu(0)\right)\right\rangle_{trun} . \qquad (14)$$

Now, in this process it is clear that any line that runs across a cut must have a large p_{1-} component unless all the components are zero. Using this observation and the fact that in the pion form factor double momentum flow regimes are not possible, we shall derive the factorization and Callan Symanzik equation for these processes.

Define τ to the set of six point completely amputated connected diagrams and σ, λ, σ', λ' four point connected diagrams amputated on the right but not left as shown in Fig. 10.

It is easy to see that all the propagators in τ are the hard propagators in the dominant regime and the hard momentum can reach up to a certain point in the pion legs denoted here by σ and σ'. λ and λ' are the diagrams with soft momentum only. It is easy to see that the large logs build up in the pion legs. The usual twist two large legs that one might get from the loop integrations like k_1 in Fig. 11 are in fact not large at all. They are in fact seen to be log a and as a is chosen $\mathcal{O}(1)$ these are in fact constants that do not scale with Q^2 and are therefore $\mathcal{O}(\alpha_s)$ corrections.

The usual problems of structure functions at $x \to 1$ are also not present as we have restricted ourselves to x fixed and not near 1. Also by restricting ourselves to be away from x near 1 we eliminate the problem of large logs of the exclusive pion by the d^4k integration in Fig. 12. This is so as the fermions are moving at large angles with respect to each other the k^2 integral is not allowed to go to μ^2. Once, one has convinced oneself of the topology of large momentum flows and seeing that all propagators in τ are hard, it is trivial to derive factorization and the final result is

$$W(Q,p_1,k_1,\ell_1) \xrightarrow[Q^2 \to \infty]{} \frac{1}{16}(\gamma_5\gamma_-)_{\beta_1\alpha_1}(\gamma_5\gamma_-)_{\beta_2\alpha_2}\int d^4k_1'\, d^4\ell_1'$$

$$\times\ K^{reg}_{\alpha_1\beta_1\beta_1'\alpha_1'}(p_1,k_1,k_1')(\gamma_5\gamma_+)_{\alpha_1'\beta_1'}\ \hat{W}(Q,\mathring{p}_1,\mathring{k}_1,\mathring{\ell}_1)$$

$$\times\ (\gamma_5\gamma_+)_{\alpha_2'\beta_2'}\ \bar{K}^{reg}_{\beta_2'\alpha_2'\alpha_2\beta_2}(p_1,\ell_1,\ell_1') \qquad . \qquad (15)$$

Expanding in the usual variables $x_1' = (k_{1+}/p_{1+})$ and $x_2' = (\ell_1'/p_{1+})$, and deriving the usual C.S. equation we get

$$\mathcal{D}\hat{W}_{n_1n_2}(Q^2) = \sum_{n_1'}\gamma_{n_1n_1'}\hat{W}_{n_1'n_2}(Q^2) + \sum_{n_2'}\gamma_{n_2n_2'}\hat{W}_{n_1n_2'}(Q^2) \quad . \quad (16)$$

The cross section is

$$(2\Pi)^3\, 2E_\Pi \frac{d\sigma}{d^3p_\Pi} = -\frac{8\Pi^2\alpha^2}{3Q^4} W(Q^2,a,x) \qquad .$$

The dominant contribution to the W in the leading approximation come from the graphs shown in Fig. 13. The cross section is for the leading term

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \sin^2\theta}{8Q^6}\, 16 f_\Pi^2\, C_2^2(f) \left(Q_{ch}\right)^2 g_{st}^4(Q^2)\, f(x,a)$$

$$+ \text{ terms down by fractional powers of logs} \quad .$$

$$f(x,a) = \frac{1}{16\Pi^3} \frac{4\sqrt{2}}{\left(1-\frac{x}{2}\right)^2} \left\{\left(x^2+4x-4\right)\log\frac{1}{c} + \frac{2(1-x)}{x^2}\left(1+\frac{x}{2}\right)^2 \frac{\left(1-c^2\right)}{c} - x\left(1-\frac{x}{2}\right)\left(\frac{1-c}{1+c}\right)\right\} \qquad (17)$$

where $c = a(1-x)$ defines the cone restriction.

V. Other Process and Discussion

It is rather obvious that the factorization of $e^+ + e^- \to \Pi_{cone}$ + anything of the last section did not very crucially depend on the fact that the particle Π came from a γ^*. The factorization and Callan Symanzik equations are therefore expected to go through in all cases where such a defined final state is produced in any given initial state reaction. Notably in $p+p \to \Pi_{cone}$ + anything also we expect the factorization to work. So also in deep inelastic electron scattering. $p+p \to \Pi_{cone}$ + anything process may be rather interesting, as the leading particle epxerimental cut, might pick out these contributions selectively and hence could be the reason for $(\gamma/\Pi^o) \sim p_\perp^2$ at fixed x_T as these contributions are in fact higher twist we expect $(d\sigma/dp_\perp)|^{\Pi} \sim (1/p_\perp^6)$ giving this result.[11]

The problem in making a rigorous prediction is to eliminate all the cut-vertices that would occur in the initial state and hence a clean prediction seems a little difficult. Therefore, we have not tried to make explicit calculations.

It is easy to combine these Π's i.e., particles (mesons) that come out singly with the particles that come in jets. The factorization in that case is just a combination of the previous cut-vertex for each hard particle in the jet, and two operators for each meson.

The cross section for $e^+ + e^- \to \Pi_{cone} +$ anything is small but has a very distinctive signature and hence it may be possible to observe it. The signature of these events are one hard pion recoiling against two "jets" with a calculable normalization in terms of f_Π makes these processes extremely attractive, and we emphasize that it can be rigorously controlled. When the meson is a heavy quark state, much more may be said about the process and it is under investigation.

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FIGURE CAPTIONS

1. $\tau_\mu(p_1,k_1,p_2,k_2)$.

2. Dominant momentum flows in $\tau_\mu(p_1,k_1,p_2,k_2)$.

3. Nuclear form factor double flow diagrams.

4. Pion form factor ϕ_6^3 double flow regime.

5. Landshoff pinch diagrams in Π-Π scattering.

6. Possible Landshoff regime in $e^+ + e^- \to 4\Pi$.

7. (a) A diagram where a set of pions cannot be separated from the rest by cutting only gluon lines.

   (b) A diagram where a set of pions can be separated from the rest by cutting only gluon lines.

8. A typical exclusive semi-inclusive event in $e^+ + e^- \to \Pi_{\text{cone}} + \text{anything}$.

9. $W(Q,p_1,k_1,\ell_1)$.

10. The large momentum and soft momentum subdivision.

11. The twist 2 large logarithms in the Π form factor.

12. The exclusive pion large logarithms.

13. The leading diagrams for $e^+ + e^- \to \Pi_{\text{cone}} + \text{anything}$.

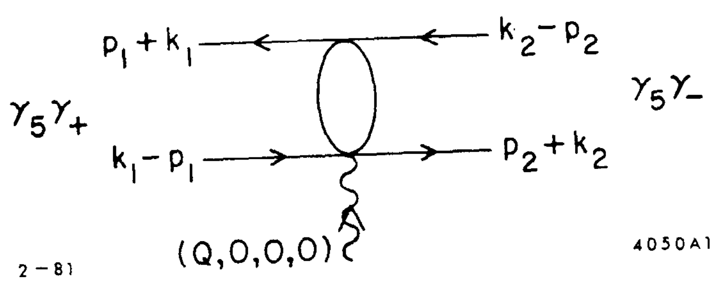

$p_1 + k_1$
$k_2 - p_2$
$\gamma_5\gamma_+$
$\gamma_5\gamma_-$
$k_1 - p_1$
$p_2 + k_2$
$(Q,0,0,0)$
2 − 81
4050A1

Fig. 1

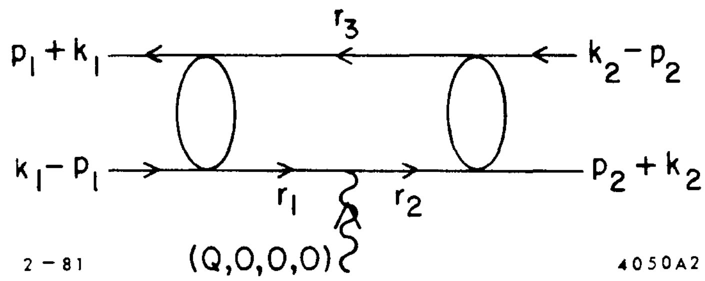


Fig. 2

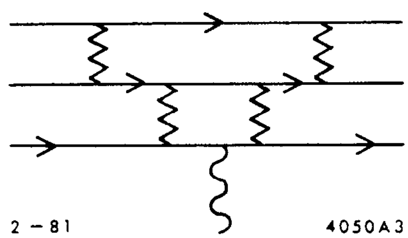
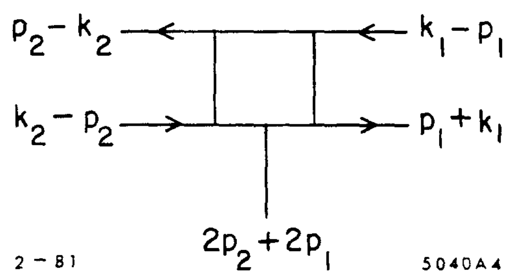


Fig. 3



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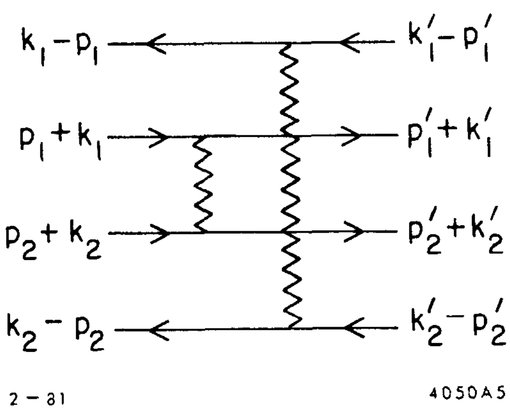

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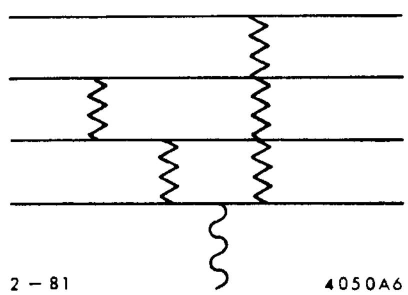

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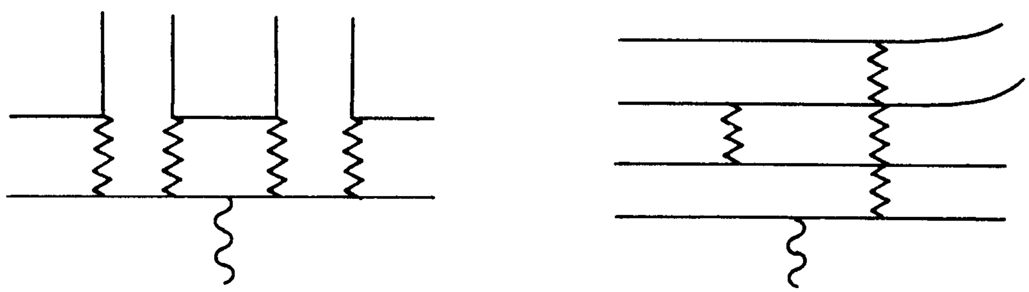


2 - 81 (a) (b) 4050A7

Fig. 7

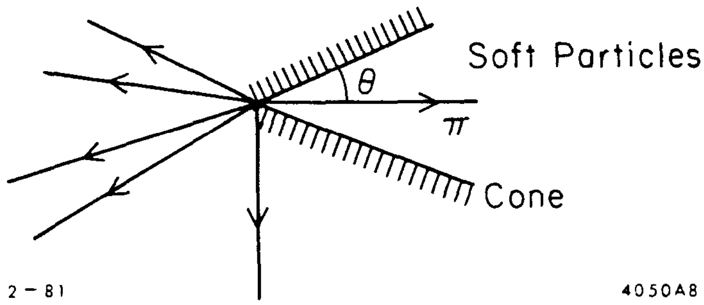
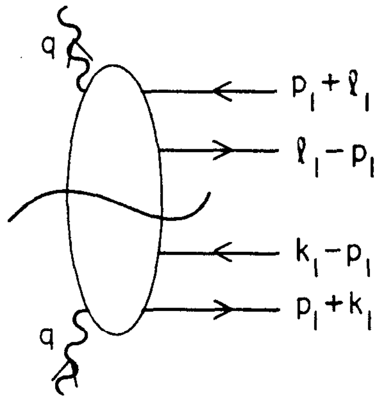


Fig. 8



2 – 81

4050A9

Fig. 9

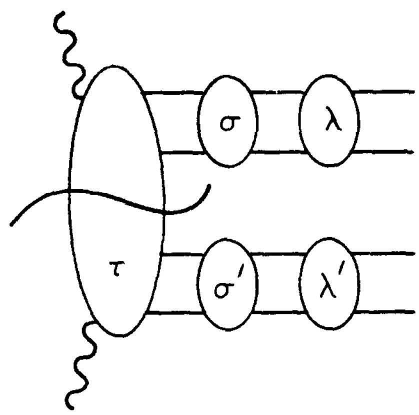


Fig. 10

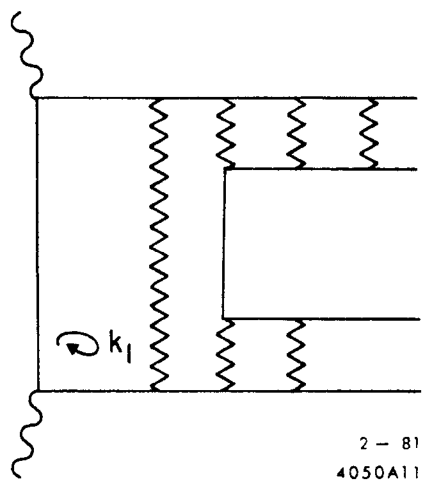


Fig. 11

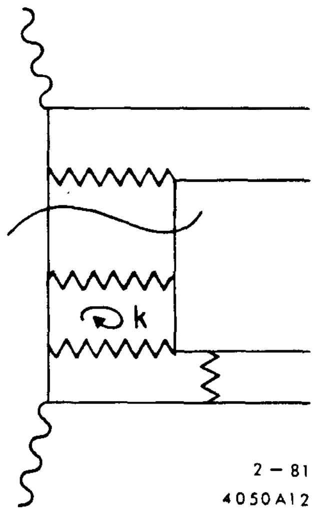


Fig. 12

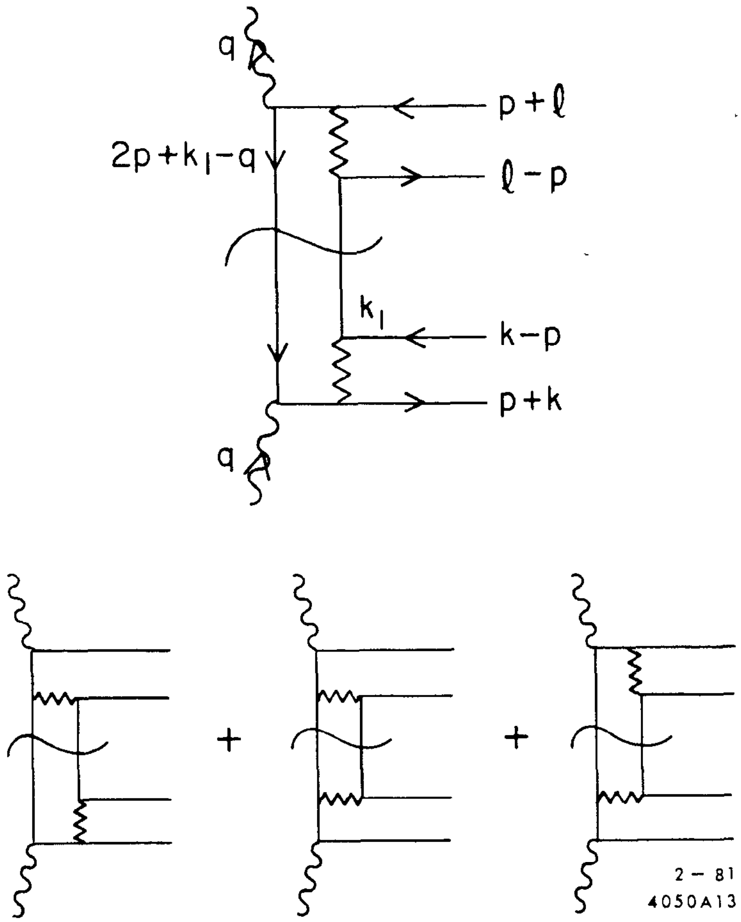


Fig. 13