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SUPERCONDUCTING CAVITIES AND MODULATED RF*

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Abstract

If a cavity has an infinite Q_o, 81.5% of the energy contained in a pulse incident upon the cavity is transferred into the cavity by the end of the pulse if the cavity Q_e is chosen so that the cavity time constant is 0.796 pulse width (T_a). As Q_o decreases, the energy in the cavity at the end of the pulse decreases very slowly as long as T_a is much less than the unloaded cavity time constant, T_{co}. SC cavities with very high Q_o enable us to obtain very high gradients with a low power cw source. At high gradients, however, we often do not attain the high Q_o predicted by theory. Therefore, if we are interested in attaining maximum energy in the cavity, as is the case for RF processing and diagnostics, for a given available source energy there is no point in keeping the power on for longer than 0.1 T_{co} because the energy expended after 0.1 T_{co} is wasted. Therefore, to attain high fields at moderate Q_o, pulsed operation is indicated. This note will derive the fields and energy stored and dissipated in the cavity when Q_e is optimized for a given T_a. It will show how to use this data to measure Q_o of an SC cavity as a function of field level, how to process the cavity with high RF fields, how to operate SC cavities in the pulsed mode to obtain higher efficiencies and gradients. Experimental results will also be reported.

Theory

In this paragraph I'll derive the expression for the cavity fields after it has been subjected to an incident RF pulse of amplitude P_i of duration T_a. Also, the expression for the fraction of incident energy stored in the cavity at T_a and the fraction of energy dissipated during T_a. Asymptotic expression will be derived for the special case when the unloaded cavity time constant is much greater than T_a. A cavity with the incident, emitted, reverse, and internal fields E_i, E_e, E_r and E_c respectively is shown in Fig. 1. We equate the power into the cavity, P_i, minus the power traveling away from the cavity, P_r, to the power dissipated, P_d, plus the rate of change of the instantaneous energy, U, in the cavity dU/dt, and obtain

$$E_i^2 - (E_e - E_i)^2 = P_d + dU/dt \quad .$$

Substituting the following definitions:

$$P_e = E_e^2 = \omega U/Q_e \;\; ; \;\; P_d = E_d^2 = \omega U/Q_o \;\; ; \;\; E_c^2 = \omega U \;\; ;$$

$$q = 1/\left(1 + Q_e/Q_o\right) \;\; ; \;\; T_c = 2q\left(Q_e/\omega\right) \;\; ; \;\; E_{cf} = 2q\, Q_e^{\frac{1}{2}} E_i \;\; ,$$

we obtain $T_c dE_c/dt + E_c = E_{cf}$ whose solution is:

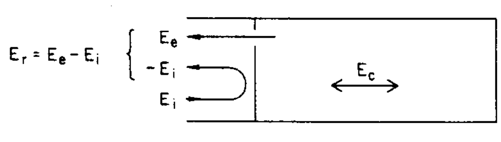

Fig. 1. Cavity with incident, emitted, reverse and internal fields, E_i, E_e, E_r and E_c respectively.

$$E_c = E_{cf} + \left(E_{ci} - E_{cf}\right)\ell^{-t/T_c} \qquad (1)$$

where E_{ci} is the cavity field at $t = 0$. Using $E_c = E_d Q_o^{\frac{1}{2}} = E_e Q_e^{\frac{1}{2}}$ we obtain identical expressions for E_d and E_e except that $E_{df} = 2q(Q_e/Q_o)^{\frac{1}{2}} E_i$, $E_{ef} = 2qE_i$, E_d and E_e, but not E_c follow abrupt changes in Q_o and Q_e respectively and therefore E_c will be used in our analysis. E_d and E_e are obtained from E_c using $E_d = E_c/Q_o^{\frac{1}{2}}$, $E_e = E_c/Q_e^{\frac{1}{2}}$. The accelerating and surface fields are proportional to $U^{\frac{1}{2}}$, hence, they are also proportional to E_c. Thus, $E_s = K_u U^{\frac{1}{2}} = k_u E_c/\omega^{\frac{1}{2}}$.

We divide both sides of any of the field equations by $|E_i|$, and as a result the fields are normalized to $P_i^{\frac{1}{2}}$ and we can replace E_i by sign E_i, that is +1 or −1. We cannot normalize to $E_i = 0$, but then the field equations hold if they have been normalized to any E_i other than zero. Define $\tau = t/T_c$. Thus,

$$E_c = 2q\, Q_e^{\frac{1}{2}}\left(\text{sign}\, E_i\right) + \left[E_{ci} - 2q\, Q_e^{\frac{1}{2}}\left(\text{sign}\, E_i\right)\right]\ell^{-\tau} . \; (2)$$

E_c and the other fields are normalized to the square root of the incident power.

The normalized energy in the cavity is $U' = U/P_i = E_c^2/\omega$.

The change in normalized energy from the beginning to the end of the pulse, U_c' is:

$$U_c' = U'(T_a) - U'(0) = \left[E_c^2(T_a) - E_c^2(0)\right]/\omega \quad .$$

The energy incident upon the cavity during the pulse is $P_i T_a$. The efficiency of energy transfer into the cavity, η_c, is:

$$\eta_c = U_c'/U_i' = \left[E_c^2(T_a) - E_c^2(0)\right]/\omega T_a \quad .$$

During T_a, the normalized energy dissipated in the cavity, U_d', is:

$$U_d' = \int_0^{T_a} E_d^2\, dt = T\left[E_{df}^2 + 2\left(E_{di} - E_{df}\right)E_{df}\left(1 - \ell^{-\tau}\right)/\tau + 0.5\left(E_{di} - E_{df}\right)^2\left(1 - \ell^{-2\tau}\right)/\tau\right] \quad .$$

If we start with an empty cavity, $E_{ci} = 0$, then $E_c = 2q\, Q_e^{\frac{1}{2}}(1 - \ell^{-\tau})$ and $\eta_c = E_c^2/\omega T_a = 2q(1 - \ell^{-\tau})^2/\tau$. The energy dissipated during charging is:

$$U_{dc}' = TE_{df}^2\left(1 - 2(1 - \ell^{-\tau})/\tau + 0.5(1 - \ell^{-2\tau})/\tau\right) = T\left(q^2 4Q_e/Q_o\right)f(\tau) \quad .$$

The energy dissipated during discharge is:

$$U_{dd}' = \int_0^{\infty} E_d(T_a)\,\ell^{-t/T_c}\, dt = q\,\eta_c\, T_a\, Q_e/Q_o \quad .$$

The total fraction of energy dissipated during a period is:

$$U_d' = \left(U_{dc}' + U_{dd}'\right)/U_i' = 4q^2 Q_e/Q_o\left(f(\tau) + \eta_c/q\right) . \quad (3)$$

U_d' also equal the fraction of average power dissipated $P_d/P_i = P_d'$ and therefore can be measured. We maximize η_c with respect to Q_e at $Q_o = \infty$, hence $q = 1$, and obtain

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$\eta_{max} = 0.815$ at $\tau = 1.26$ hence $Q_e = \omega T_a/2(1.26) = 2.5\ fT_a$. Using this value of Q_e we obtain $q = 1/(1+2.5fT_a/Q_o)$, $E_c = 3.16q(fT_a)^{1/2}(1-\ell^{-1.26q})$, $\eta_c = 1.59q^2(1-\ell^{-1.26q})^2$, $\tau = 1.26q$, $T_c = 0.8qT_a$, $E_d = 3.16q(fT_a/Q_o)^{1/2}(1-\ell^{-1.26q})$ $E_e = 2q(1-\ell^{-1.26q})$, $U'_d = 10q^2(fT_a/Q_o)[f(1.26\ ) + \eta_c(q)/q]$. If $q = 1$ then $E_c = 2.26(fT_c)^{1/2}$, $\eta_c = 0.815$, $U'_c = 0.815T_a$, $T_c = 0.796T_a$, $E_d = 2.26(fT_a/Q_o)^{1/2}$, $E_e = 1.43$, $f(\tau) = f(1.26) = 0.227$, $\eta_c = 0.815$ and $U'_d = 10.4fT_a/Q_o$.

Pulsed RF Processing of SC Cavities and Experimental Results

Figure 2 curve (a) is a plot of P'_d vs Q_o with $Q_e = 10^8$, (b) and (c) are plots of E_s vs Q_o with $Q_e = 10^8$ and $Q_e = Q_o$ respectively with $P_i = 1$ watt cw. Curves (d) and (e) are P'_d and E_s respectively with a 1 kW 7.0 μs pulse input and the asymptotic optimum Q_e of 50,000. Curve (f) is a plot E_s vs Q_o with Q_e optimized at each Q_o. Note that as Q_o decreases, the pulsed power disipation increases, whereas cw power dissipation decreases, and that the pulsed cavity electric field essentially remains constant whereas the cw electric field diminishes rapidly. Assume that a given field level ionizes a speck of dielectric or causes a gas burst, field emission, or a tiny fraction of the cavity surface to go normal. At that point, Q_o decreases drastically. With pulsed RF a larger fraction, but still a very small fraction, of the incident energy is diverted to be dissipated, but the field will essentially not diminish. But with a cw source as soon as Q_o decreases drastically, the field also drops drastically. Since we need high fields for RF processing we need pulsed RF. If the requisite peak power is available RF processing should be done at room temperature in order to remove the processing byproducts. If it is not available, the next best thing is to pulse RF process with the cavity superconducting. It could work because the processing byproducts are rearranged and some are removed. If possible, the coupling probe should be superconducting. If it is not, it should be at the same temperature as the SC cavity and be made of a material, such as copper, that has a high-heat conductivity. Otherwise, the probe will get much hotter than the cavity walls and outgas into the cavity. At the tight coupling which is required for pulsed RF processing, if the probe is not SC, most of the incident power is dissipated in the probe tip. The SC cavity acts as a matching network between the generator and probe tip.

The niobium TM_{010} cavity we tested has a room temperature Q_o of 5000, and a $k_u = 82$ MV/M/joule$^{1/2}$. With our presently available 30 dW, 7 μs pulse source, we could reach 7.5 MV/m at room temperature and 35.8 MV/m when superconducting. Unfortunately, we had a niobium coupling probe tip which was thermally isolated from the cavity. When we first cooled down the pressure in the cavity as measured at the top of the dewar, decreased from 10^{-9} to 10^{-8}, the cavity was cryopumping. In our first test we reached a cw $E_s = 20$ MV/m. After that, presumably because it was damaged, or because of the lowered vacuum, the cavity broke down consistently at 10 MV/m. With pulsed power we could reach 30 MV/m. At 10 MV/m, radiation was observed and the pressure measured at the top of the probe increased to 10^{-7} Torr, mostly hydrogen. Obviously, large quantities of gases desorbed from the cavity wall and the probe, the hydrogen being the tip of the iceberg. At 30 MV/m the radiation through the dewar wall plus 1/4-inch of lead was 15mR. There was no improvement or deterioration of cw Q_o and breakdown field.

We are now constructing a helium-cooled copper probe which will enable us to RF process at SC temperatures and are also preparing a high-power source that will enable us to RF process at nonsuperconducting temperatures.

$E_e(T_a)$, $E_c(T_a)$, are weak functions of Q_o and therefore, not suitable for its indication. P'_d, however, varies inversely as Q_o, hence, it is a good indication of Q_o. Measureing P'_d at different pulse height indicates how Q_o varies with field level. From a plot of P'_d vs field level one can deduce the field level at which Q_o changes. Pulsed RF can also be used to distinguish between magnetic and thermal breakdown. The rise in temperature and hence thermal breakdown is proportional to the pulse repetition rate n_{pps}. We can ascertain that thermal breakdown does not occur by decreasing the n_{pps}. Thus, if the cavity breaks down each pulse we have magnetic breakdown, and if it breaks down at low field at high repetition rate the breakdown is thermal.

Operation of SC Cavities with Pulsed RF

To enable one to chose which system of pulsed and superconducting combinations to use we determine the overall system efficiency η_{ox} which is defined as the ratio of V^2, the voltage^2 gained by a single bunch traversing the structure each T_q seconds, to the lengths of the structure, ℓ, times U_{ac}, the energy into the system each T_q seconds.

$$\eta_{ox} = V^2/U_{ac}\ell = E_a^2/(U_{ac}/\ell) \quad .$$

A general system is shown in Fig. 3. The overall efficiency, η_{ox}, is the product of efficiencies of the components comprising the system which is calculated as follows. This approach is similar to that of P. Wilson,[1] except that he considered RF efficiency only. The energy into the system $U_{ac} = U_{arf} + U_{acr} + U_{asr} = U_{arf}(1 + U_{acr}/U_{arf} + U_{asr}/U_{arf}) = U_{arf}/\eta_r$ where U_{arf} is the energy to be converted into RF; U_{acr}, U_{asr} are the ac energy into the cavity and structure refrigerators respectively. The RF energy $U_i = \eta_{arf}\eta_t U_{arf} = \eta_{arf}\eta_t\eta_r U_{ac}$ where η_t is the transmission efficiency; η_r is the refrigeration efficiency. $\eta_r = 1/(1+\eta_{arf}U'_{dc}R_{fc}+\eta_{arf}\eta_c U'_{ds}R_{fs})$. U'_{dc} U'_{ds} are fractions of incident RF energy dissipated in the storage cavity and structure, respectively; R_{fc}, R_{fs} are the respective refrigeration factors of the

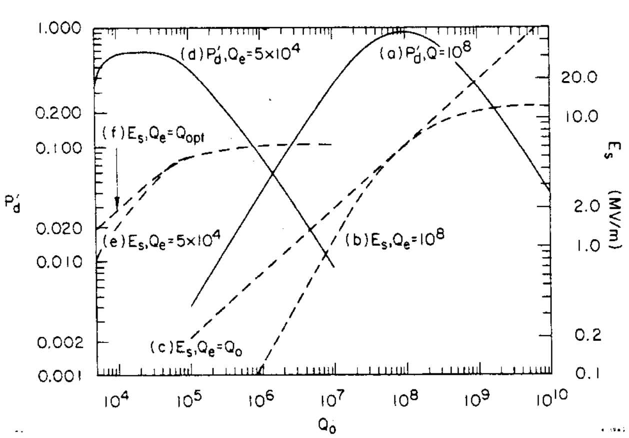

Fig. 2. Power dissipated and electric field in a superconducting cavity vs unloaded Q

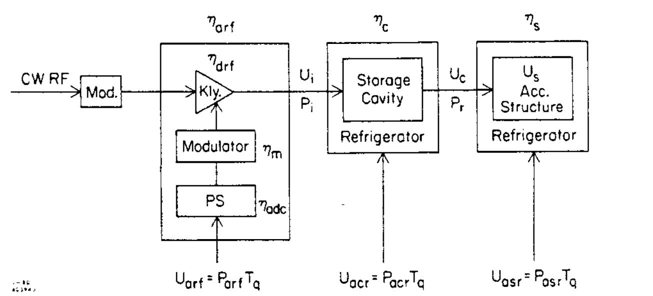

Fig. 3. AC to accelerating RF conversion system.

storage cavity and structure refrigerators. Thus $U_i = P_i T_a = P_{ac} T_q \eta_r \eta_{arf}$ and $\eta_{pps} = \eta_r \eta_{arf} P_{ac} / P_i T_a = \eta_r \eta_{arf} P_{act} / N_s P_i T_a$ where P_{act} is the total ac power available and N_s is the number of section, P_i is the power into each section. $U_c = \eta_c U_i$, the energy available at structure input, is converted into accelerating energy, $U_s = V^2/\omega X \ell = \eta_s U_c$. $X(\equiv R/Q)$ is the shunt reactance/unit length, the ratio of $E^2(z)$ to dU_s/dz at point z, η_s is the structure efficiency. Thus, $V^2/\ell = \eta_s \omega X U_c = \eta_r \eta_{arf} \eta_t \eta_c \eta_s \omega X U_{ac}$ and $\eta_{ox} = \eta_r \eta_{arf} \eta_c \eta_s \omega X \equiv \eta \omega X$. The expression for η_s for an SW structure is the same as for η_c which was derived in the first section. If the structure is TW then energy density and hence the electric field is not uniformly distributed over its length because the structure has attenuation and also because U_c is not necessarily uniformly distributed over time, i.e., $P_r(t) = dU_c/dt$ is not constant. Therefore, to obtain V/ℓ that determines η_s we must integrate the field along the structure the instant the charged particle traverses it. Thus: $V/\ell = (1/\ell) \int_0^\ell E(z)dz$; $E^2(z) = (\omega X) U(z)$; $U(z) = P(z)/v_g = P(t) \ell^{-2\alpha z}/v_g$; $t = t_o - z/v_g$. If P(t) is constant and equals P, then for a constant impedance structure $V^2 = \omega X P \ell T_s (1 - \ell^{-A_s})^2 / A_s^2$ and $\eta_s = (T_s/T_a)(1 - \ell^{-A_s})^2/A_s^2$; for a constant gradient structure $V^2 = \omega X P \ell T_s (1 - \ell^{-2A_s})/2A_s$ and $\eta_s = (T_s/T_a) \times (1 - \ell^{-2A_s})/2A_s$ where A_s, T_s are the structure attenuation and filling time respectively. The gradient is $E_a = (\eta_{ox} U_{ac}/\ell)^{\frac{1}{2}} = \eta_t \eta_c \eta_s P_i T_a / \ell$. Consider SLAC: $\eta_r = \eta_c = 1$, $\eta_{arf} = .303$; $\eta_t = .931$, $\omega X = 73.2$, $A_s = .57$, $T_s = .82$ μs, $P_{ac} = 114$ kW, $T_a = 2.5$ μs, $P_i = 38.5$ MW, $\ell = 12$ m. Thus $\eta_s = .196$, $\eta = .055$, $\eta_{pps} = 360$ and $E_a = 10.3$. To make or not to make a given change depends on the new η and on the gradient multiplication factor, $M = E_a/10.3$. The following are the changed efficiencies, P_i, and T_a and the resulting η_{pps}, η and M due to installation of several systems: 1) SLED storage cavity $Q_o = 10^5$: $\eta_c = .746$, $\eta_s = .511$, $\eta = .110$, $M = 1.40$. Note that even with 100% efficient pulse compression $M = \sqrt{2.5/.82} = 1.75$; 2) SLED and change T_a to 5 μs: $\eta_c = .601$, $\eta_s = .517$, $\eta = .088$, $\eta_{pps} = 180$, $M = 1.78$. 3) Lead plate cavities and cool them to 4.2°K, $R_f = 350$, $I_f = 3.6 \times 10^3$: $\eta_r = .997$, $\eta_c = .734$, $\eta_s = .525$, $\eta = .109$, $\eta_{pps} = 179$, $M = 1.99$. Refrigerator power = 320 W/station. 4) No storage cavities. Replace 38.5 MW, 2.5 μs klystrons with 100 MW, 1 μs klystrons: $\eta_{arf} = .212$, $\eta_s = .489$, $\eta = .097$, $\eta_{pps} = 243$, $M = 1.62$. 5) Change 38.5 MW klystron pulse width to 3.2 μs, plate accelerator section with lead and cool to 4.2°K, and connect four sections in tandem: $\eta_s = .999$, $\eta_r = .937$, $\eta = .265$, $\eta_{pps} = 264$, $M = 2.56$. Refrigerator power = 7 kW/station.

Lead plating or changing the structure material to miobium is no simple task. Nor can we be sure that the superconducting structure can sustain the 27.4 MV/m gradient but the Q_o required to achieve this gradient, 4×10^7, is two orders of magnitude less than 10^9 required with a cw source and an SW structure.

Whether it pays to replace a cw system driving a SW superconducting structure with a pulsed system depends on their relative efficiencies. They are:

$$\eta_{cw} = \left(\eta_{arfcw} Q_o/\omega T_q\right) / \left(1 + \eta_{arfcw} R_f\right)$$

$$\eta_{pulsed}\left(2.5\ fT_a << Q_o\right) = \eta_{arfp} .815 / \left(1 + 10.4 \eta_{arfp} R_f f T_a / Q_o\right)$$

The pulsed system has tighter coupling and therefore is advantageous for loading higher order modes.

Greater pulse compression ratios are obtained if we Q_e switch, charge slowly, high Q_e and discharge fast low Q_e. In Fig. 4, curves η_1 and E_{a1} are plots of η and E_a vs η_{pps} for a Jungle-Jim (J-J), constant impedance structure, $A_s = 0.12$, $\omega X = 120$ teraohn/m-s, and a TE_{023} mode spherical storage cavity which has a $Q_o = 240{,}000$. The RF efficiency, $\eta_c \eta_s$ was derived by P. Wilson.[1] To obtain higher gradient the pulse width is increased. The time between pulses is increased by the same factor to maintain the average power constant. The efficiency decreases with higher gradients because the pulse width becomes a larger fraction of the unloaded storage cavity time constant and η approaches $Q_o/\omega T_a$. A solution is to increase the cavity Q_o by making it superconducting. But then it is advantageous to use another system,[2] RASP (radio frequency storage pulser) where only RF and no electrostatic storage is used and therefore, 70% efficient cw klystrons, rather than 50% efficient pulse klystrons, can be used. Also, η_c approaches 1.0 instead of 0.815 and U_c is more evenly distributed. Curves η_2, η_3 and E_{a2}, E_{a3} are plots of η and E_a vs η_{pps} for a RASP system with a SLAC and J-J structure respectively. The storage cavity is the same as for curves η_1 and E_{a1}, except that it is superconducting at 1.85°K.

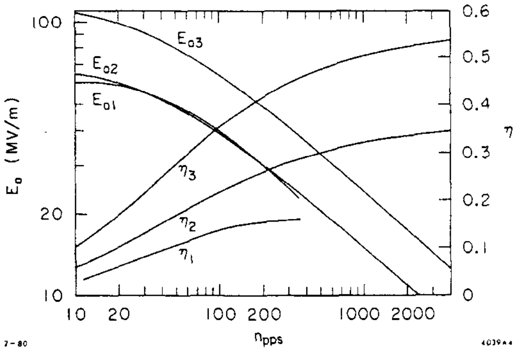

Fig. 4. System efficiency η and gradient E_a vs pulse repetition rate η_{pps} of systems using storage cavities with Q_e switching.
1) Pulsed klystron and J-J structure.
2) RASP and SLAC structure.
3) RASP and J-J structure.

There remains the formidable task to make an efficient Q_e switch. It has to have low driving power and has to turn on and off in nanoseconds. During charging the coupling network Q_o must be greater than the cavity Q_o; hence, the switch must function in an SC environment. During discharge, a 1 dB loss is tolerable, but it and the power generated by the switch have to be removed at a nonsuperconducting temperature, otherwise the dissipation is multiplied by the refrigeration factor. A promising switch is a laser-actuated, cesium-coated gallium arsenide cathode which is capable of delivering at least 180A/cm² and can be turned off in subnanoseconds with small amounts of laser power. RASP is not only more efficient but also more versatile and agile. One can have a train of pulses instead of a single pulse and can trade more easily gradient for repetition rate. It requires lower peak power because the energy into the structure is more evenly distributed than room temperature storage cavities with pulsed RF.

Conclusion

It was shown that using modulated RF with superconducting cavities and accelerating structures results in higher efficiencies, particularly at low improvement factors and it is useful for processing and diagnosing superconducting cavities.

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