SLAC-PUB-2677
PEP-NOTE-341
February 1981
(A)

WIGGLERS - THE NEWEST PROFESSION?*

J. E. Spencer
Stanford Linear Accelerator Center
Stanford University, Stanford, CA 94305

Introduction

Wiggler systems have been used in storage rings within the last year to increase the intensity of synchrotron radiation available for experiments as well as to increase the reaction rates in high energy physics experiments.[1] Multiperiod wigglers or undulators have also been used recently to make quasi-monochromatic photon beams as well as amplify existing photon beams such as in the free electron laser.[2] If one defines a wiggler to be any system of transverse, periodic electromagnetic fields, then recent results on photon production via charged particle channeling in crystals[3] also fall within this sphere. Of course, any periodic modulation of a charge or magnetic moment (e.g., by a laser) could produce coherent radiation or, conversely, passage through a periodic aperture (e.g., a metal bellows). This discussion is limited to a typical, active, macroscopic device and how it provides some unique advantages which are practical to achieve in storage rings. As implied, the subject divides into two basic parts - one related to the radiation from the wiggler and the other related to machine physics applications, e.g., tailoring the phase space of the particle beam, modifying its damping rates or possibly optimizing a ring for production of radiation. Neither area is exhausted nor hopefully the reader, since our goal is only to present enough information to allow one to make reasonable estimates of some important effects.[4]

Trajectories such as shown in Fig. 1 can be produced with magnetic fields that are transverse to the beam direction. The planar trajectory (Fig. 1a) results from a series of alternating polarity, dipole magnets and the axial trajectory (Fig. 1b) from a bifilar, helically wound air core magnet. The maximum angle the trajectory makes with the longitudinal axis is α. Free electron laser work has tended to use the helix to obtain small wiggler wavelengths λ (appellations such as FELIX), whereas applications of wigglers for production of synchrotron radiation or for improved accelerator or storage ring performance have tended to use planar wigglers. Since these systems are intrinsically quite similar except for the obvious difference of circular and linear polarization, only planar wigglers will be discussed. They are more easily inserted into existing synchrotrons and storage rings and also provide more capabilities. In both cases, the minimum practical wavelength presently achievable in this context is $\lambda_w \gtrsim 1$ cm.

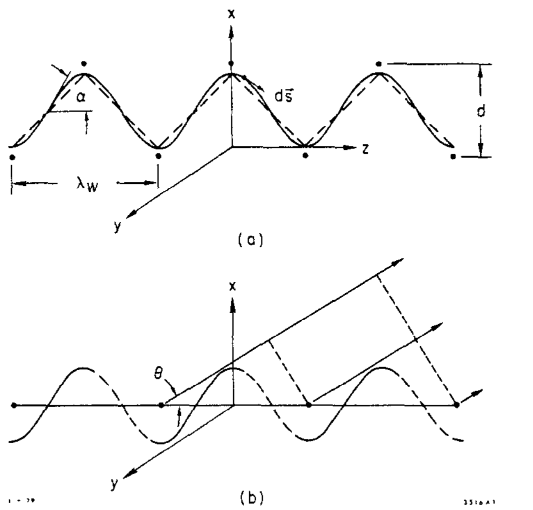

Fig. 1. Charged particle's trajectory through (a) sinusoidal planar wiggler and (b) helical wiggler. If the lines of dots along the z-axis are understood as lattice sites in a monatomic cubic crystal with plane spacing d, the planar trajectory (a) might represent positive charged particle channeling and the axial trajectory (b) that for negative particles, but greatly exaggerated because the resulting wavelength λ_w is much greater than d.

High and Low Field Wigglers

Figures 2 and 3 show a design for an efficient high-field wiggler that produces a large range of fields with minimal perturbation to the closed orbit of a storage ring. From the standpoint of the radiation that is produced, multiperiod wigglers can have two extremes of operation that depend on the field strength and the wiggler wavelength. At "high" fields, they produce radiation with a smooth spectral distribution similar to normal synchrotron radiation from the uniform

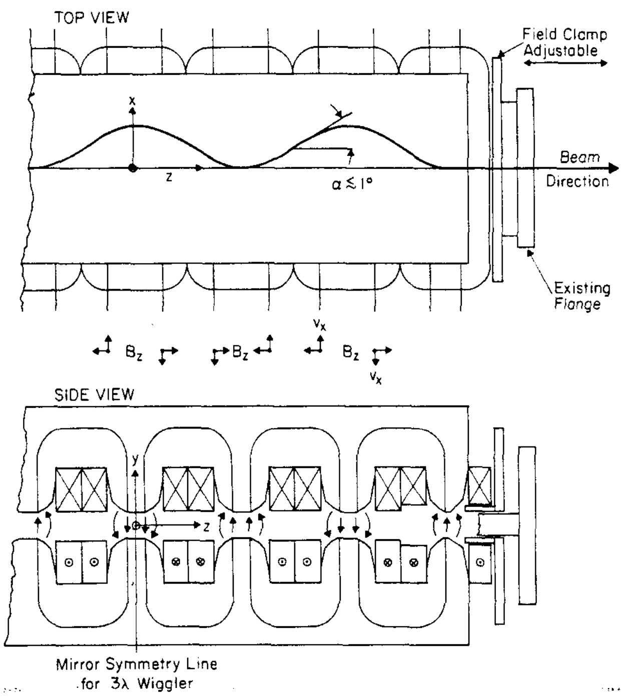

Fig. 2. Schematic layout of a three-wavelength (3λ) planar wiggler at the Stanford Linear Accelerator Center. Exaggerated trajectory is shown to illustrate edge focusing for particles out of the median plane ($y \equiv 0$) of the device.

*Work supported by the Department of Energy, contract DE-AC03-76SF00515. This report is an extended version of an article prepared for McGraw-Hill's 1980 Yearbook of Science and Technology.

(Invited paper presented at the 7th International Conference on Magnet Technology, Karlsruhe, West Germany, March 30 - April 3, 1981.)

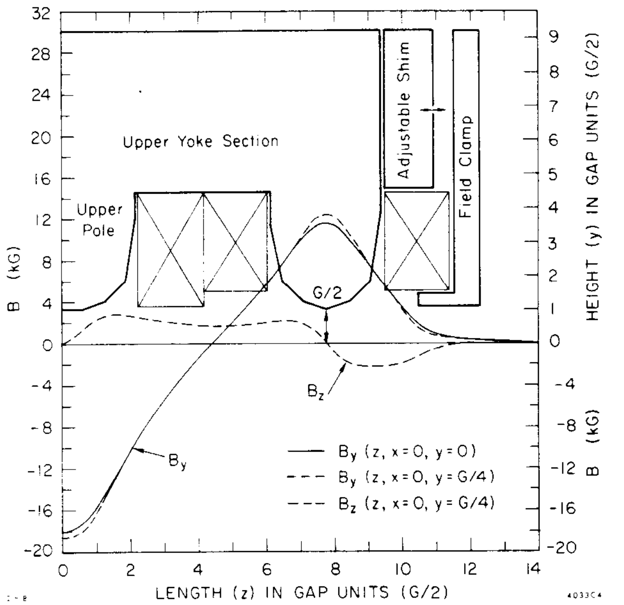

Fig. 3. Detailed layout of the end section of the wiggler of Fig. 2, showing the predicted field distributions.

field of dipole magnets. However, at "low" fields, coherent emission of synchrotron radiation from all wiggles is possible. In both regimes, the radiation falls within a cone of half-angle $\theta \sim 1/\gamma$ about the radiating particle's velocity (more precisely, θ is the rms polar emission angle). Here, $\gamma = 1/\sqrt{1-\beta^2}$ and $\beta = v/c$, where v is the particle velocity and c is the speed of light; the energy of particles of rest-mass m_0 is $\gamma m_0 c^2$.

The synchrotron radiation seen by an observer from a wiggle or dipole element will have a time duration τ given by Eq. (1),

$$\tau \simeq \frac{1}{2\gamma^2} (\rho/c\gamma) \qquad (1)$$

where ρ is the bend radius in a dipole field of strength B. The associated frequency spectrum is expected to extend up to $\omega \sim 1/\tau$. If 2α is the total angular excursion of a characteristic central trajectory through half a period or wiggle (Figs. 1 and 2), then synchrotron radiation will pass from view if the value of $\alpha\gamma$, given by Eq. (2), is much greater than 1.

$$\alpha\gamma = \frac{1}{2} (q/\beta m_0 c) \int B ds \simeq \frac{1}{2} (q/m_0 c) B_1 \lambda_w / \pi \qquad (2)$$

Here, q is the charge of the particle, the integral is taken over one-half wavelength, and B_1 is the peak amplitude of an assumed magnetic field of the form $B_y(x,o,z) = B_1 \cos(2\pi z/\lambda_w)$. All quantities in this paper are in SI (mksA) units, unless explicitly specified. Equation (2) is independent of the parent energy γ and depends only on the field and wavelength for any particle of charge q and rest-mass m_0.

The two extremes of operation of any wiggler are then given by inequalities (3).

$\alpha\gamma \gg 1$ standard wiggler [normal synchrotron radiation (SR)]

(3)

$\alpha\gamma \ll 1$ undulator or interference wiggler [undulator radiation (UR)]

The undulator produces lower energy, more directed or collimated beams of radiation which are quite comparable to lasers in many characteristics, as discussed in the next few sections. For electrons in a sinusoidal undulator with field amplitude B_1, Eq. (2) in numerical form gives Eq. (4).

$$\alpha\gamma = 0.934\, B_1(\text{T})\, \lambda_w\, (\text{cm}) \qquad (4)$$

For $B\lambda$ much greater than 1.0 T · cm = 10 kG · cm one has a conventional synchrotron radiation source, and for $B\lambda$ much less than 1.0 T · cm one expects rather monochromatic radiation if the wiggler has a sufficient number of periods. As $\alpha\gamma \to 0$, the source brightness goes quadratically to zero. The energy per unit solid angle is maximized for $\alpha\gamma \lesssim 1$. Even though the particle slope stays within the forward light-cone, we will inquire furhter into the conditions for interference since this is the key to most of the important properties of UR.

Coherence/Interference Effects

One can begin by considering the light from a single electron (source) as it propagates through a periodic system such as shown in Fig. 1. For simplicity, assume it is moving parallel to the z-axis with no significant perturbation to its motion (straightline motion) and that a wave train is emitted each time it passes a lattice site. Ignore the interaction responsible for the radiation except to assume that the amplitude (and phase) at the emission points $z = \pm\lambda_w/2$ are essentially the same. When the particle is at $z = \lambda_w/2$, the separation between these two wave trains will be

$$\delta = c\lambda_w / \langle v_z \rangle - \lambda_w \approx (\lambda_w/\beta)(1-\beta) \approx \lambda_w/(2\gamma^2) , \qquad (5)$$

where $\langle v_z \rangle$ is the average particle velocity in the z-direction and by assumption $\beta_z \approx \beta \approx 1$. For the wave train produced at $\lambda_w/2$, any wavelength λ will have a phase shift,

$$\phi = 2\pi(\delta/\lambda) = 2\pi(\lambda_w/\lambda)/(2\gamma^2) \quad . \qquad (6)$$

Complete constructive interference only occurs when $\phi = 2\pi n$, with n a positive integer since in a non-dispersive medium, the wave and particle velocity can never be the same ($\delta > 0$) so we can write

$$\lambda_n = (\lambda_w/n)/(2\gamma^2) \qquad (n = 1,2,3...) \ . \qquad (7)$$

This is a necessary condition on enhanced radiation in the forward direction ($\theta = 0$). The result is only slightly more complicated for arbitrary wiggler excitations (i.e., realistic interactions) and angles of observation. In the orbital plane of the particle, there is only the σ-component of polarization and from Fig. 1b

$$\delta = (\lambda_w / \langle\beta_z\rangle)(1 - \langle\beta_z\rangle \cos\theta)$$

$$\lambda_n = \frac{(\lambda_w/n)}{\langle\beta_z\rangle} (1 - \langle\beta_z\rangle \cos\theta) \simeq \frac{(\lambda_w/n)}{2\gamma_z^2} (1 + \gamma_z^2 \theta^2) , \qquad (8)$$

where the small angle approximation is consistent with $\beta \simeq 1$. The characteristic wavelengths are therefore expected to increase with increasing magnet excitation and/or observation angle (Doppler shift) but decrease with particle energy. The decreasing photon energy with wiggler excitation causes the "line" spectrum to come together but since it also enhances the higher harmonics, this is consistent with the expected progression into a conventional SR spectrum.

The degree to which constructive interference can actually be observed for any wavelength depends on how well the phase and amplitude relation between successive wiggler periods is maintained for each electron path as well as having a large number of cycles in the wave train from each pole. From Eq. (7) we can estimate the number of cycles as $\lesssim n\,\gamma^2$. In a wiggler with N periods (2N poles) we can then define a coherence length which can be expressed in units of λ_w as

$$N_{max} = Cn(\gamma^2/2)$$
$$\gamma_{min} = \sqrt{2N/(Cn)} \quad (9)$$

where C is a constant with a value between zero and one. Solving for γ assuming $C \approx 0.5$ and $N \approx 160$ (Ref. 2) gives $E_{min} \approx 13$ MeV, whereas the coherence length for 24 MeV electrons would be $N_{max} \approx 550$.

If the SR from a single pole or half-period of the wiggler has time duration τ(frequency spread $\omega \sim 1/\tau$) and the pulses come at intervals $T(\sim\lambda_w/\langle v_z\rangle)$, then $\tau \leq T/2$. The relative intensities of the allowed harmonics, e.g., those actually excited, will be determined by the motion induced by the single pole. Thus, one wants a long-time, smooth, sinusoidal motion for an undulator, rather than a short, abrupt one like the "billiard-ball" process shown in Fig. 1a. The latter ceases to differentiate between normal modes as the accelerating impulse $[(-2v)\sin\alpha]/\Delta t$ becomes infinite $(\Delta t \to 0)$ so it is clearly best for SR. Furthermore, such motion could not produce UR even if we could find or produce such a periodic interaction with $\alpha\gamma \ll 1$.

More quantitatively, the power per unit area incident on an observer from a single pole is given by the Poynting vector $|\vec{S}_1| = \varepsilon_o c\, \vec{E}_o(\vec{R},t)^2$ and the total energy per unit solid angle and frequency interval is

$$\frac{d^2I_1}{d\omega d\Omega} = R^2\varepsilon_o c|\vec{E}_o(\omega)|^2 = \frac{R^2\varepsilon_o c}{4\pi^2}\left|\int \vec{E}_o(t')e^{-i\omega t}dt\right|^2. \quad (10)$$

If $E_j(R,t)$ is the amplitude function for the j-th pole, that for 2N poles with centers separated by $\lambda_w/2$ will be written as

$$\vec{E}_{2N}(\vec{R},t) = \sum_{j=0}^{2N-1} \vec{E}_j e^{i\phi_j} \approx \vec{E}_o(\vec{R},t)\left[\sum_j e^{i2\pi(\delta_j/\lambda)} \cdot e^{\pm i\pi j}\right]$$
$$= \vec{E}_o(\vec{R},t)\sum e^{ij\pi[(\delta/\lambda)\pm 1]} = \vec{E}_o(\vec{R},t)e^{i\frac{\pi}{\lambda}(\lambda_1\pm\lambda)(N-\frac{1}{2})} \left[\frac{\sin\pi N(\lambda_1\pm\lambda)/\lambda}{\sin\frac{\pi}{2}(\lambda_1\pm\lambda)/\lambda}\right] \quad (11)$$

where δ is given by Eq. (8) and either sign can be used. Although the coherent phase slip between successive poles is π, the acceleration also changes sign so all poles act as coherent sources. The intensity, Eq. (10), then becomes a product of the single-pole intensity and

$$\frac{d^2I_{2N}}{d\omega d\Omega} = \left[\frac{\sin\pi N(\omega\pm\omega_1)/\omega_1}{\sin\frac{\pi}{2}(\omega\pm\omega_1)/\omega_1}\right]^2 \frac{d^2I_1}{d\omega d\Omega} \quad (12)$$

an interference function resulting from the 2N poles. The intensity is maximum when $\omega/\omega_1 = 2m - 1$, i.e., only odd harmonics are seen unless the detector is sensitive to only one sign of the field, in which case all harmonics result. Notice that SR, like the radiation from a single pole, excites all harmonics. The normalized intensity at the odd harmonics is $(2N)^2$ and the full width at their base is $\Delta\omega/\omega_n = 2/(nN)$ with the implicit assumption that $N < N_{max}$ and the energy $\gamma > \gamma_{min}$. With Eq. (9), the minimum intrinsic line broadening achievable with an undulator then becomes

$$\frac{2}{nN} \geq \left(\frac{\Delta\omega}{\omega_n}\right)_{min} \geq \frac{4/C}{(n\gamma)^2}, \quad (13)$$

which can still, in principle, be made comparable to a laser with high enough energy and periodicity but is not unlimited. However, if one could interpose a phase shifter every period or at least every N_{max} periods, the coherence length could be increased until inhomogeneities could not be contained. Of course, the imposition of a dielectric would affect the particle beam and irritate the containment considerably. For instance, as N increases indefinitely, the beam energy goes to zero due to the radiative loss so that energy boosters would also be required at some point.

So far we have only dealt with a single electron whereas a beam will generally broaden the spectrum beyond the intrinsic width implied by Eq. (13). We will discuss so-called inhomogeneous broadening later but first consider briefly the possibility of coherence between particles. Assume the beam to be monoenergetic, broad and parallel in both transverse directions and the wiggler to be short compared to the ring circumference $N\lambda_w \ll L$, so that all electrons in a transverse slice through the beam will be in phase over the length of the wiggler - ignoring for the moment any resultant inhomogeneities. If the bunch length were short compared to λ_n, one could then expect further enhancement proportional to the square of the number of electrons N_e^2 rather than the incoherent sum, N_e. The electron coherence length or minimum longitudinal separation between coherent electrons is just $C_e\lambda_n/4$. Although small, coherence over even a small fraction of the bunch length $(C_e\lambda_n/10\sigma_z)$ could be significant compared to $(2N)^2$ depending on whether

$$\mathscr{E} \equiv C_e\lambda_n N_e/(10\sigma_z) = C_e(\lambda_w/n)N_e/(20\sigma_z\gamma^2) \gtrless 1, \quad (14)$$

where σ_z is the rms bunch length which we assume to be gaussian. With enough electrons in a short enough pulse one can expect enhancements which should decrease with harmonic number. For instance, the SLAC Linear Collider is supposed to have $N_e = 5\times 10^{10}$/bunch and $\sigma_z = 1$ mm or an enhancement factor of roughly 3000 over the incoherent sum for $\lambda_n = 1$ Å and $C_e = 0.5$. This would be equivalent to a wiggler with N = 27 (or 54-poles). The result is similar for $\lambda_n = 10\mu$ and the parameters of Ref. 2, so that it should be observable. However, there is still the question of inhomogeneous broadening, e.g., since we made the beam broad in both transverse dimensions to reduce the angular spread and thereby energy spread [via Eq. (8) which also applies to the particle's angle] we must ask about the consequences of this. We show later that this leads us to define a coherence volume in terms of σ_x,σ_y and σ_z because the world and its things such as magnetic fields are inevitably 3-dimensional.

Energy Spectrum

In the "weak" field limit, a sinusoidal wiggler has $\langle\beta_z\rangle = \beta\sqrt{1-\alpha^2/2}$ so that peaks are expected at light wavelengths given by Eq. (15)

$$\lambda_n \simeq \frac{(\lambda_w/n)}{2\gamma^2}\left[1 + \frac{1}{2}(\alpha\gamma)^2 + (\gamma\theta)^2\right]. \quad (15)$$

For the n-th harmonic and laboratory observation angle θ, the critical or line energy, ε_1, of UR in the forward direction for $\alpha\gamma \ll 1$ is independent of field strength and given by Eq. (16),

$$\varepsilon_1(\text{keV}) = 0.950\ E(\text{GeV})^2/\lambda_w(\text{cm}) \quad (16)$$

where E is the energy of the particle beam. In this limit, the particle motion approximates an oscillating dipole in a frame moving along the wiggler axis with

In the vertical direction, the wiggler exerts edge focusing for particles out of the median plane ($y \equiv 0$) of the device (Fig. 2). In other words, it is impossible to achieve a purely transverse field over the particle envelop and the resulting small discrepancy tends to focus the particles. The longitudinal magnetic field B_z for a particle above the median plane and the transverse horizontal velocity v_x, indiated by arrows and an exaggerated trajectory in Fig. 2a, are such that the particle experiences a force towards this plane. Thus, the wiggler acts like a vertically focusing quadrupole lens(without, however, the corresponding defocusing in the horizontal plane) so that the beam should again be made vertically small in the wiggler to minimize vertical tune changes quite independently of the constraint Eq. (24) for undulators. Whereas the tune change is most relevant for standard wigglers, there is still another reason to have a vertical waist in both types of wigglers - i.e., to get the highest possible fields and shortest possible wavelengths.

We will derive the effective quad strength and tune change for an undulator since these expressions generally give results within 25% or better of detailed calculations[7] or experiments.[1] With midplane symmetry and no orbit errors

$$B_z = y \frac{\partial B_z}{\partial y} + 0(3) \simeq y \frac{\partial B_y}{\partial z} \quad . \tag{30}$$

Assuming the perturbation over the length of the wiggler does not perturb the position appreciably

$$\delta y' = 4\ N(e/p) \int_0^{\lambda_w/4} x' B_z ds = \frac{B_1 \lambda_w}{2} (e/p) \frac{N}{\rho} y \tag{31}$$

$$\delta y' = 0.045\ N\lambda_w(\mathrm{m}) \left(\frac{B_1(\mathrm{T})}{E(\mathrm{GeV})} \right)^2 y \equiv k_y(\mathrm{m}^{-1})\ y\ (\mathrm{m}) \tag{31}$$

where k_y is the effective, vertical quadrupole strength. Similarly, the tune change is

$$\Delta\nu_y \approx \frac{k_y \beta_y}{4\pi} \tag{32}$$

where β_y is the average, vertical betatron amplitude in the wiggler. A measurement of the tune change together with a calculation of k_y then measures β_y at the wiggler.

Particle Beam Phase Space

Although the ring optics experience little perturbation from the wiggler, the beam phase space may not, due to shifts in the balance between quantum excitation and damping. Local control of beam energy loss is the basis for a number of applications involving increases or decreases in transverse beam size (or emittance) and/or energy spread.

Since the distribution of energies in a stored beam is a balance between damping (I_2, I_4) and excitation (I_3, I_5), one only requires the relative increase in damping from the wiggler to be greater than the increase in excitation. It is easily shown that this occurs when the wiggler field B_w is less than the bend strength B of the ring, or equivalently, when the bending radius of the wiggler ρ_w is greater than that of the ring ρ_B. That is, an undulator can generally be expected to improve the energy spread and a standard wiggler to worsen it. Similarly, placing a wiggler at $\eta = \eta' = 0$, such as near the interaction region (IR) in Fig. 4, allows a significant decrease in beam emittance, whereas putting a wiggler near the symmetry point (SP) in Fig. 4 produces the opposite effect.[8] Within limits it is also possible to maintain a constant ratio of emittance change to energy spread change as well as other effects. These statements follow from a consideration of the following equations which we will not justify here.

$$\left(\frac{\sigma_E}{\sigma_E^o}\right)^2 = \frac{(1+\delta_3)}{(1+\delta_2)} \xrightarrow{\delta_2 \ll 1} 1 - \delta_2 + \delta_3 - \delta_2\,\delta_3$$

$$\left(\frac{\varepsilon_x}{\varepsilon_x^o}\right) = \frac{(1+\delta_5)}{(1+\delta_2)} \xrightarrow{\delta_2 \ll 1} 1 - \delta_2 + \delta_5 - \delta_2\,\delta_5 \tag{33}$$

where we designate $\delta_i \equiv \delta I_i/I_i$ with δI_i due only to the wiggler and the spreads σ_E^o and ε_x^o are without the wiggler, etc. Since we seldom want to increase σ_E, these expressions add another constraint to Eqs. (29)

$$\int_w |B|^3\ ds = \text{minimum}. \tag{34}$$

Experimental Possibilities

While there has been considerable interest and activity lately in free electron laser work with its potential benefits for applied research with high power, tunable "laser" beams, there has been very little consideration of higher energy work such as x-ray holography, gamma ray lasers (grasers) and applications of wigglers for basic research with high energy, monochromatic, polarized photon beams. The coherence length of wigglers and the coherence volume of electron beams should be explored experimentally. Ignoring the potential use of wigglers for such secondary possibilities as luminosity enhancement, beam polarization control or particle identification, we note that Compton backscattering of laser beams already provides useful yields of nearly monoenergetic, polarized photons with $w_{max} \simeq 4\ w_L\gamma^2$ at $\theta = 0$ with $w_L \lesssim 2.5$ eV from lasers whereas a wiggler with $\lambda_w = 1$ cm associated with the 50 GeV SLC beam would allow w_w to be 2.5 MeV with characteristics comparable to the laser. This would give a 50% higher photon energy (~ 50 GeV) and probe much smaller distances in the particle rest frame. In a setup such as the SLC using a pulsed superconducting wiggler, one might expect $\lambda_w \lesssim 1$ mm or variable energy photon beams within reach of 100 MeV in an arrangement that would also allow particle-photon scattering, particle-photon scattering and photon-photon scattering. Since one expects the cross sections for such nonlinear processes in quantum electrodynamics to increase with photon energy, the possibility of looking for additional quantum numbers for the photon exists from high energy accelerator development like the SLC. Perhaps the simplest and most straightforward possibility is to take advantage of the large energy variability of the monochromatic photon beam from wigglers to search for the low energy axion or Higglet via γ-γ annihilation and scattering since this particle is expected to have a mass in the vicinity of 1 MeV and should be observable as a resonance as a function of incident photon energy. Thus, the use of wigglers could allow simultaneous operation of multiple experimental areas in the SLC.

Final Comments

In this discussion of wigglers, we have constrained accelerators and storage rings from the standpoint of the undulator by specifying limits on their betatron amplitudes, emittances, etc. The problem of achieving optics which can provide both a storage ring for high energy physics and an efficient, optimal SR source wavelength through the wiggler.

Irradiance and Radiance

In some cases such as extended x-ray absorption fine structure (EXAFS) or topography experiments, the transverse source size is of concern, that is, the irradiance ($\text{photon/mm}^2\text{/s}$). This can be improved by increasing the number of periods N of the wiggler, proper selection of its location in the storage ring, or by varying its field or wavelength. In other experiments such as crystallography or photoemission, it is the radiance or brightness ($\text{photons/mm}^2\text{/sr/s}$) that synchrotron radiation users want optimized. In this case one should choose a location in the ring lattice where the wiggler causes the least increase in beam emittance or even decreases this quantity (see below). Such decisions are based on calculation of synchrotron radiation integrals[5] around the ring. Undulators provide high brightness and spectral brightness ($dN/dt/dA/d\Omega d\omega$) with low background but require control of the parent beam energy. Assuming all conditions of the previous section are satisfied, we can integrate the spectral brightness over the source cross section to get the intensity of the fundamental per unit relative energy as

$$\frac{d^3N_1(\theta=0)}{dt\,\frac{d\omega}{\omega}\,d\Omega} = (2N)^2\,\frac{N_e}{\ell/c}\left(\alpha_e\gamma^2\right)\frac{(\alpha\gamma)^2/2}{(1+(\alpha\gamma)^2/2)^2} \qquad (25)$$

where N_e is the number of electrons in a bunch of length ℓ and α_e is the fine structure constant. This assumes no coherence between electrons and is maximum for $\alpha\gamma = \sqrt{2}$. Substituting this value and Eqs. (13) and (21) and integrating one finds $N_1/N_e = \alpha_e$. This is the same result as for photons with ϵ_c from an SR source but with a much more collimated beam, i.e., one comparable to the particle beam that produced it [see Eq. (22)].

Power

The instantaneous power P radiated by a charge (electron) is given by Lienard's expression in Eq. (26),

$$P = \frac{2}{3}\,(r_e c/E_o)(\dot{p}_\parallel^2 + \gamma^2\dot{p}_\perp^2) \quad , \qquad (26)$$

where $\dot{p}_\perp = \gamma m_o \dot{v}_\perp$ is the transverse acceleration and $\dot{p}_\parallel = \dot{E}/\beta c$, r_e is the classical electron radius ($q^2/4\pi\epsilon_o m_o c^2$) and E_o is the particle's rest energy, $m_o c^2$. For equal forces, there will be γ^2 more power radiated for a transverse acceleration than for a longitudinal acceleration. For circular motion in the uniform field B of a ring dipole magnet, $\dot{p}_\perp = qc\beta B$ and the power is given by Eq. (27).

$$P = \frac{8}{3}\pi\epsilon_o r_e^2 c^3 \beta^2\gamma^2 B^2 \qquad (27)$$

The energy lost per turn per charge to synchrotron radiation in a storage ring is then given by Eqs. (28),

$$U = \oint P ds/\beta c = \frac{8}{3}\pi\epsilon_o r_e^2 c^2 \beta\gamma^2 \oint B^2 ds \;\propto\; \gamma^4 I_2$$

$$U \simeq \frac{8}{3}\pi\epsilon_o r_e^2 c^2 \beta\gamma^2 \Sigma B_i^2 \ell_i = \frac{4\pi}{3}\,r_e E_o \beta^3\gamma^4/\rho \qquad (28)$$

where ℓ_i is the effective length of dipole i having mean central field value B_i. I_2 is an important synchrotron integral proportional to the total damping rate which scales inversely as the ring bend radius ρ in the absence of wigglers. Similarly, to maximize the synchrotron radiation power from a wiggler, B should be made as large as practicable but alternating in sign so that Eqs. (28) hold

$$\int_W B\,ds = 0 \qquad\qquad \int_W B^2 ds = \text{maximum.} \qquad (29)$$

Here the integrals are taken only over the wiggler.

Damping

Adding a wiggler into a ring always increases the energy loss per turn U (that is, the change in I_2 in Eqs. (28) resulting from the wiggler is greater than 0) so one generally expects improving particle injection rates with wiggler excitation, since injection is limited by Liouville's theorem to the order of the damping rate which is proportional to I_2. When the wiggler is at a location in the ring where $\eta = 0$ (Fig. 4), the

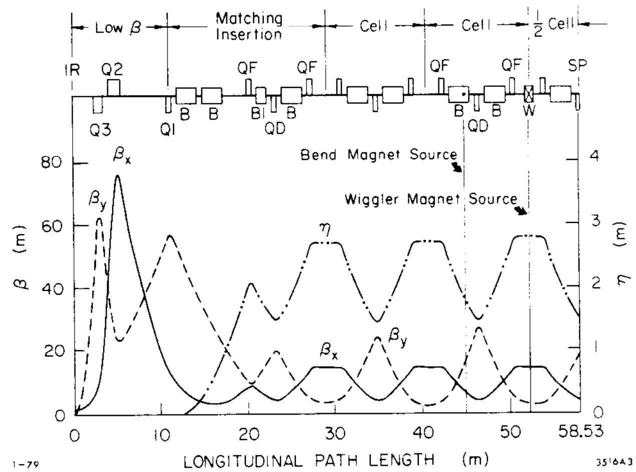

Fig. 4. Half of a SPEAR superperiod showing the machine betatron functions (β_x,β_y') and the dispersion η between the interaction region (IR) used for high energy physics and the symmetry point (SP) about which the superperiod is mirror symmetric. The magnetic elements that are tuned to obtain a particular configuration (β_x,β_y,η) are shown at the top with the designations Q (quadrupole), B (bends), and W (wiggler).

damping rate in all degrees of freedom is increased; if $\eta \neq 0$ at the wiggler, it is possible to repartition the damping rates, that is, to increase or decrease some rates relative to what they would have been without the wiggler. The first practical application of such an effect allowed an alternating gradient synchrotron to store a beam.[6]

Optics

With the condition Eq. (29) and the additional constraint that the wiggler be mirror symmetric about its midpoint (Fig. 2), there will be no horizontal deflection or displacement of the beam on passing through as long as the fractional energy loss through the wiggler is small. This implies that the wiggler is the optical equivalent of a drift space in the dispersion plan, that is, the horizontal optics of the ring in first order are independent of wiggler excitation. Of course, the increased path length resulting from the wiggles may have to be compensated by rf changes, but this is easily done.

the average longitudinal velocity $\langle\beta_z\rangle$ so only the fundamental has significant intensity. This can also be thought of as Thomson scattering of virtual photons with relativistically contracted wavelength from the wiggler field. With increasing values of $\alpha\gamma$, higher harmonics increase in importance eventually resulting in the synchrotron radiation spectrum.

The "strong" field limit is of most current interest because conventional synchrotron radiation beams can be subsequently monochromatized with low background up to fairly high photon energies with intensities that are adequate for most experiments. Similarly, accelerator physics applications such as damping or beam blow-up generally require considerable radiative power loss. In this regime, the "endpoint" or critical energy ε_c of synchrotron radiation is usually defined as in Eq. (17), where $\hbar$ is Planck's constant divided by 2π.

$$\varepsilon_c(\text{keV}) = \hbar\left(\frac{3}{2}\;\frac{c\gamma^3}{\rho}\right) = 2.218\ E(\text{GeV})^3/\rho(\text{m})$$

$$= 0.665\ B(\text{T})\ E(\text{GeV})^2 \quad . \qquad (17)$$

Notice the multiplying factor of 3/2 compared to Eq. (1). With this definition, the photon flux varies slowly up to ε_c but drops sharply thereafter with half the radiated power emitted in photons above this energy but only ~ 1/50 or 2% above $\lambda_c/5$ which is often taken as the limit of useable flux for experiments.

For a variety of technical and economic reasons, the dipole field in a storage ring is usually restricted to $B \lesssim 10 - 12$ kG (1.0 - 1.2 T) whereas wiggler fields of 20 kG (2.0 T) or more are practical using conventional iron core magnets and fields of 50 kG (5 T) or more can be obtained with superconducting magnets. Thus, standard wigglers can shift the spectrum of low energy machines (E < 1 GeV) into the hard x-ray region (ε_c > 1 keV). The signal to background in the experimental areas will generally be lower, the operating costs will be lower and the initial capital equipment costs will be lower than for a higher energy machine. To achieve the same maximum photon energy with an undulator would require machine energies of nearly 5 GeV. At SPEAR, the ring bending radius is 12.7 m, so that Eq. (17) implies that the ratio of the endpoint energy of the wiggler ε_c^W to that of the ring ε_c^B is given by Eq. (18).

$$\varepsilon_c^W/\varepsilon_c^B = 0.38\ B_1(\text{kG})/E(\text{GeV}) \qquad (18)$$

or nearly a 13-fold increase with a 50-kG wiggler at 1.5 GeV. This is equivalent to operating SPEAR at its upper limit of 3.5 GeV.

Line Broadening

The requirement that N >> 1 is not sufficient to ensure quasimonochromatic UR ($\Delta\lambda/\lambda$ << 1) because of various inhomogeneous effects such as finite beam phase space in the wiggler or nonuniformities in the wiggler itself. The inhomogeneous broadening of the radiation from a single electron can be estimated with Eqs. (4) and (15) assuming uncorrelated variations in $\lambda_n(\theta,\lambda_w,B_1,E)$

$$\Delta\lambda_n = \left(\delta\theta\,\frac{\partial}{\partial\theta} + \delta\lambda_w\,\frac{\partial}{\partial\lambda_w} + \delta B_1\,\frac{\partial}{\partial B_1} + \delta\gamma\,\frac{\partial}{\partial\gamma}\right)\lambda_n$$

$$\lambda_n \simeq \frac{(\lambda_w/n)}{2\gamma^2}\left[1 + a(\lambda_w B_1)^2 + (\gamma\theta)^2\right] \equiv \frac{(\lambda_w/n)}{2\gamma^2}\,D \qquad (19)$$

with a = 0.436 and $\lambda_w B_1 \lesssim 1$ in appropriate units of cm and Tesla. The conditions for UR can then be written

$$\frac{\Delta\lambda_n}{\lambda_n} = \frac{2}{D}\left\{\gamma^2\theta^2\left(\frac{\delta\theta}{\theta}\right) + \left[\frac{D}{2} + a(\lambda_w B_1)^2\right]\left(\frac{\delta\lambda_w}{\lambda_w}\right)\right.$$

$$\left. + a(\lambda_w B_1)^2\left(\frac{\delta B_1}{B_1}\right) + \left[(\gamma\theta)^2 - D\right]\left(\frac{\delta\gamma}{\gamma}\right)\right\} \quad . \qquad (20)$$

This expression is independent of n and since the intrinsic line width is 2/(nN) when $N < N_{max}$, it follows that each of these terms should be smaller, e.g.,

$$\frac{1}{n\gamma}\sqrt{\frac{D}{C}} \lesssim \theta_{max}\gamma \lesssim \sqrt{\frac{D}{2nN}} \quad . \qquad (21)$$

The coherent effect of the wiggler has already been limited by Eq. (3) to $\alpha\gamma \lesssim 1$. Equation (21) is an independent and more stringent constraint on <u>both</u> the angular aperture of the detector and the angular divergence of the individual electron in the beam. Since it is preferable not to physically collimate the photons, this dictates the detector's distance from the source or alternatively, a preference toward lower energies $\gtrsim \gamma_{min}$ as well as an emphasis on the importance of coherence between electrons rather than wiggler poles, i.e., shorter bunch lengths. Similarly, since the angular spread in the beam is related to its size through the emittance $\varepsilon_{x,y} = \sigma_{x,y}^2/\beta_{x,y}$, one has

$$\frac{\varepsilon_{x,y}}{\sigma_{x,y}} \lesssim \frac{1}{2\gamma}\sqrt{\frac{D}{nN}} \quad . \qquad (22)$$

Because $1 \lesssim D \lesssim 5/2$ and the emittance is hard to decrease, one generally wants the beam size to be comparatively large at the wiggler, i.e., wigglers should be put at locations with large amplitude functions (β) even though this creates a problem with another term in Eq. (20). Notice that the emittance can be reduced by decreasing the beam energy, thereby increasing the constraining value, θ_{max}.

Suppose we have a perfect sinusoidal wiggler with a field $B_y(x,o,z) = B_1 \cos(2\pi z/\lambda_w)$. Without worrying about the hyperbolic sine or cosine dependence of the field off the median plane we know we can write the Taylor expansion for any field with median plane symmetry as in Eq. (23).

$$B_y(x,y,z) = \left(1 + \frac{y^2}{2}\,\frac{\partial^2}{\partial y^2}\right)B_y(x,o,z) + 0(4)$$

$$\delta B_y(x,y,z) \cong -\frac{y^2}{2}\,\frac{\partial^2}{\partial z^2}\,B_y(x,o,z) = 1/2\left(\frac{2\pi y}{\lambda_w}\right)^2 B_y(x,o,z) \qquad (23)$$

Thus, $\Delta\lambda/\lambda$ varies over the beam envelop as the square of the normalized distance from the median plane (in units of $k_w^{-1} = \lambda_w/2\pi$). The condition here is then

$$\left(\frac{2\pi\sigma_y}{\lambda_w}\right) \lesssim \sqrt{\frac{D/(D-1)}{nN}} \quad , \qquad (24)$$

which becomes important for short enough wiggler wavelengths and many periods.

Similarly, one has that $(\delta\lambda_w/\lambda_w) \lesssim 1/nN$ for the mechanical tolerance and $\sigma_\varepsilon/E = \delta\gamma/\gamma \lesssim 1/(2nN)$ when the angular spread is appropriately small, i.e., neither the beam energy spread nor the energy lost to radiation, should exceed 1/(nN) without compensation. Such compensation is available through either the second or third terms in Eq. (20), i.e., via a variable field or should be reconsidered with a goal of achieving better compatibility. Clearly, accelerators with higher energies, shorter pulse lengths and smaller energy spreads and emittances are called for and continue to offer new and exciting possibilities.

Acknowledgements

The author would like to thank Professor Matt Sands for suggestions on the content of the work, S. Brodsky and H. Quinn on the high energy physics possibilities and finally Margie Barnes and the SLAC publications group for its delivery.

References

1. M. Berndt, W. Brunk, R. Cronin, D. Jensen, A. King, J. Spencer, T. Taylor, and H. Winick, "Initial Operation of SSRL Wiggler in SPEAR", Proceedings of the 1979 Particle Accelerator Conference, IEEE Trans. Nucl. Sci. 26, 3812-3815 (1979). SLAC-PUB.
2. L. R. Elias, W. M. Fairbank, J. M. J. Madey, H. A. Schwettan and T. I. Smith, Phys. Rev. Lett. 36, 717-719 (1976); See also: D. A. G. Deacon, L. R. Elias, J. M. J. Madey, G. J. Ramian, H. A. Schwettman, and T. I. Smith, "First Operation of a Free Electron Laser", Phys. Rev. Lett. 38, 892-894 (1977).
3. R. L. Swent, R. H. Pantell, M. J. Algard, B. L. Berman, S. D. Bloom, and S. Datz, Phys. Rev. Lett. 43, 1783 (1979).
4. Those interested in more details and a complete set of references are referred to: J. E. Spencer and H. Winick, Synchrotron Radiation Research, "Wiggler Systems as Sources of Electromagnetic Radiation", ed. by H. Winick and S. Doniach (1980); See also: Wiggler Magnets, ed. by H. Winick and T. Knight, SSRP Report 77/05 (1977); and Frascati Wiggler Meeting Proceedings, ed. by A. Luccio, A. Reale and S. Stipcich (1978).
5. M. Sands, The Physics of Electron Storage Rings. An Introduction, ed. by B. Touschek, Proceedings of the International School of Physics "Enrico Fermi", Course XLVI (Academic Press, New York, 1971); Also see: R. Helm, M. J. Lee, P. L. Morton and M. Sands, Evaluation of Synchrotron Radiation Integrals, Proceedings of the 1973 Particle Accelerator Conference, IEEE Trans. Nucl. Sci. 20, 900-903 (1973).