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NUMERICAL SIMULATION OF THE TRANSPORT OF INTENSE BEAMS OF HEAVY IONS IN AN ELECTROSTATIC QUADRUPOLE SYSTEM*

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Abstract

A computer program which models the behavior of an intense beam of charged particles in a periodic focusing system is described. The program solves for electrostatic fields in two dimensions, i.e., the x-y coordinates of a typical electrostatic quadrupole lens, including space-charge fields due to the presence of an array of macroparticles. Adjacent drift sections and quadrupoles are defined by "hard edges", i.e., no fringing fields. However, longitudinal changes in electrostatic potential are used to calculate applied axial fields. Particles are tracked through short segments for which Poisson's equation is solved to update the self-fields. Examples shown include the transport of an intense (space charge limited) beam in a periodic structure and the optics of a matching system between ion source and transport system. The program is equally applicable to magnetic or electrostatic focusing. In either case it accounts for space-charge image field effects which are frequently ignored in other treatments of this problem.

Introduction

Interest in using intense beams of heavy ions to implode and ignite inertial confinement fusion pellets has greatly stimulated interest in the transport of intense beams of charged particles. Several papers[1-4] at the last Particle Accelerator Conference dealt with the theoretical and numerical understanding of the stability limits to the transport of high intensity beams in vacuum, i.e., without neutralizing the space charge.

The present effort is not intended to break new ground in the understanding of these stability limits. Rather, it is an effort to provide a general purpose tool that can be used to design and study transport systems such as the matching system between an ion source and the accelerator. Since low-energy, low-charge state heavy ions respond much more effectively to electrostatic focusing than to magnetic focusing, the emphasis has been on developing a program suitable for studying electrostatic quadrupole transport systems.

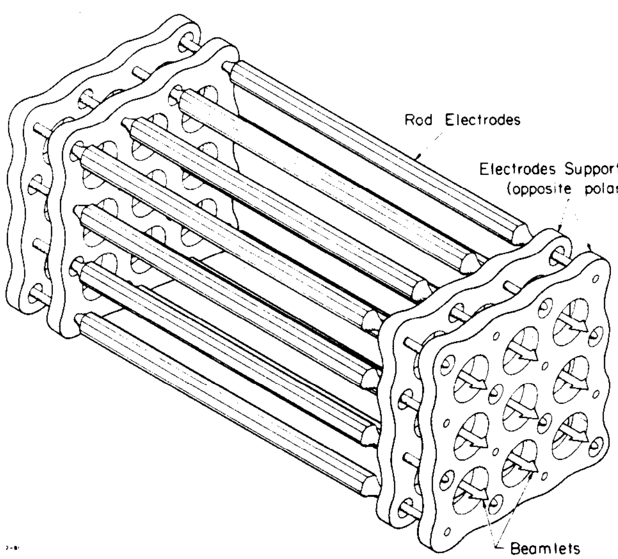
Some stability studies have been made with the program, primarily as program checks, notably the case of a matched K-V distribution in a system with $\sigma_0 = 60^\circ$ (phase advance without space charge) depressed to $\sigma = 24^\circ$ by the presence of space charge. This case was extensively studied by Laslett.[5] We found that beams behaved as expected under these conditions and were thus encouraged to experiment with beams mismatched in either phase (reduced divergence) or configuration space.

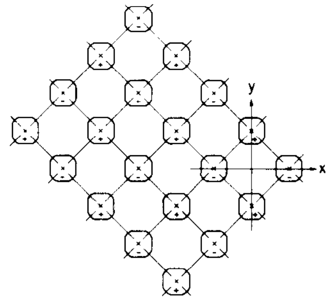
Below we describe the program and illustrate its operation with selected examples from problems related to studies presently underway at LBL to develop a linear induction accelerator as the heavy ion fusion accelerator. Because the induction linac is well matched to a very high current, short pulses, it was an early candidate as a fusion driver. The problem of transverse confinement of the beam, particularly at low velocities, has led us toward the structure, illustrated in Fig. 1, of many rods which are the elements of multiple electrostatic quadrupoles. This concept is the induction linac version of the MEQALAC structure developed by Maschke.[6] Many small beamlets, each of modest current, are accelerated together in a large bundle. The beamlets only "see" each other briefly at the acceleration gaps where the transverse space charge fields are attenuated by the plates with the multiple apertures.

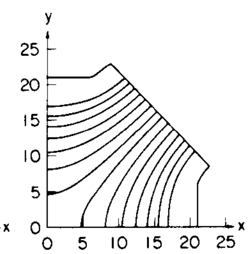
A cross section of the multiple beam structure is shown in Fig. 2. Figure 3 shows the equipotential lines within an electrostatic quadrupole for a single quadrant of this structure. The beam is transported, with quadrant symmetry, in the area nearest the origin. The flattened rod faces provide a nearly linear field to all four adjacent quadrants.

Program Description

The program is a modified version of the electron







Figs. 1-3. An example of a multiple quadrupole array.
Fig. 1. Structure Fig. 2. End Cross Section Fig. 3. Symmetry Quadrant

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trajectory program developed by W. B. Herrmannsfeldt.[7] The original program solved by numerical integration the relativistic equations of motion (Lorenze force equation) to define the particle trajectories along an ion gun axis. Charge deposited along macroparticle orbits defined a charge density that was used to solve Poisson's equation for a potential distribution which included effects from both applied electrode voltages and beam space charge. This solution was then used to recalculate the orbits. This process of orbit tracking, charge deposition, and potential solution was iterated until convergence was obtained thus giving a self-consistent solution.

The modified program used here can be understood in terms of Fig. 4. For the original program the z-axis

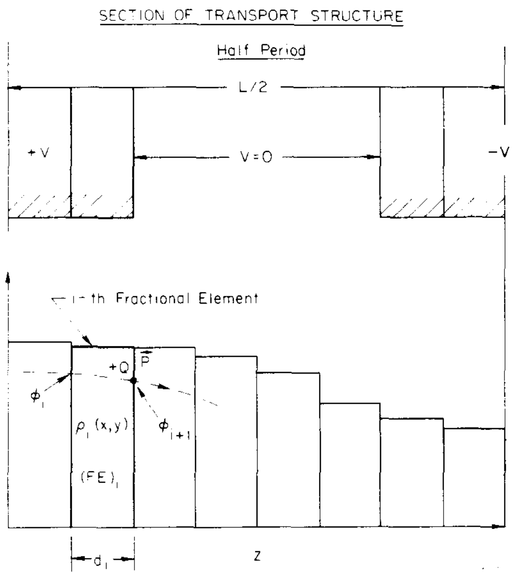

Fig. 4. One-half period of a transport structure.

would be the given axis; Poisson's equation would be solved in the r, z plane after particles were tracked from the entrance to the exit of the structure. The new version solves Poisson's equation in the transverse x,y plane as particles are tracked through the structure, their principal motion being along the z-axis.

The transport structure (Fig. 4 shows a half-period) is subdivided in the longitudinal (z) direction into short sections called fractional elements (FE). An example of an FE is shown in Fig. 5. The transverse

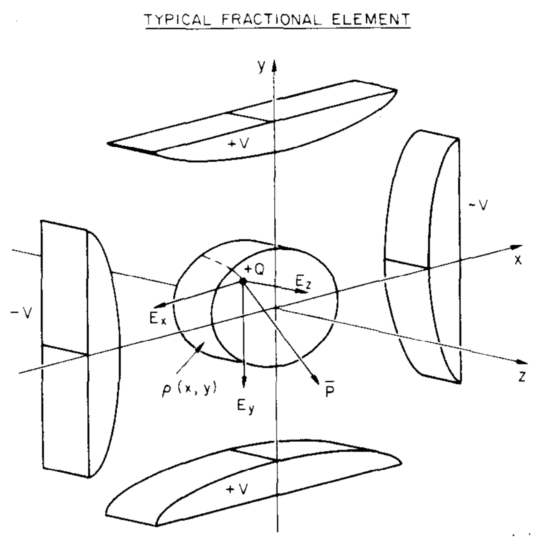

Fig. 5. Single fractional element.

projection of the electrode within an FE is shown in Fig. 3. The initial condition of a collection of particles at the entrance to the FE is used to define the beam charge density in the transverse x,y plane. A 2-dimensional Poisson's equation including space charge effects is solved to obtain the potential $\phi(x,y)$ used throughout the FE. Macroparticles are then moved through the FE along trajectories found by numerically integrating the relativistic Lorenze force equation. A 3-dimensional electric field is used; the x,y components are the gradient of the potential $\phi(x,y)$. The z component is obtained by assuming that the potential drop between adjacent FE's is experienced by a macroparticle over the length of the FE through which it is being tracked. For a hard-edge approximation this is equivalent to a linear potential drop, or rise, in the gaps over an edge width of one FE. This process is repeated for each FE in the system; charge distributions being updated and Poisson's equation being solved at the entrance to each FE.

The macroparticle beam used to establish a charge distribution is initially defined by drawing a statistical sample from a given distribution. Runs presented here used a K-V distribution of specified width in transverse phase space (x,x', y,y'). If desired, an initial momentum spread $\delta P_z/P_z$ along the longitudinal coordinate can be specified.

Charge deposition is done by locating the macroparticle in the x,y plane mesh and distributing charge to each of the four neighboring mesh points.

Beam behavior is defined by a vector of macroparticles that contains, for example, the position and velocities of the sample. Marker particles can be tracked. Histograms showing projections in transverse phase and configuration space are calculated. Also, moments are calculated along with the unnormalized emittance in both the x and y space.

The resolution of the grid used to solve Poisson's equation, i.e., the mesh size, influences the accuracy of the solution. The number of macroparticles must be sufficient to insure that the space charge is well defined. The integration step size must be adjusted to give sufficient accuracy to the macroparticle orbits, and the FE cannot be too long. These quantities, mesh size, sample size, integration step size, and FE length all combine to determine the computing time needed per machine period. Sample sizes of 500 macroparticles yielding about 5 - 10 particles per cell, integration steps of ~ 70 per machine period, and 16 FE per machine period have yielded reasonable runs.

Extensions

By solving a 3-dimensional Laplace equation with boundary conditions that include the applied voltages, it is possible to furnish analytic expressions for the applied fields.[5] By using the program to include the effect of space charge and solving Poisson's equations with a constant, e.g., zero, potential on the boundaries and superposing the analytically defined fields, orbits of macroparticles can be tracked in fields for which only the space charge effects are calculated in a 2-dimensional manner. This would allow the accurate inclusion of specified fringe fields.

We also note that in principle, multiple beams (see Fig. 2) can be handled by the program. Whether this is practical depends on the questions asked of the program results and the particular geometry at hand.

Examples

In Fig. 6 are plotted relative emittances as a function of machine structure periods. One matched and two unmatched beams are taken through 17 full FODO structure periods. In one case the emittance (divergence) is two times less and in the other ten times less than the matched value. The matched beam shows little emittance growth, whereas the unmatched beams show growth toward the matched value.

Typical structure parameters used for the matched and mismatched beams of 0.5 MeV C_s^+ ions were: focusing strength $K = 27.252\ m^{-2}$, quadrupole apertures $A_q = 3.4$ cm and voltage $V_Q = 15.752$ kV, half period length $L = 30$ cm, and packing fraction $\eta = 50\%$. The matched beamlet parameters were: current per beamlet $I = 30.38$ mA, unnormalized emittance $\varepsilon_0 = 17.596$cm-mrad, with beam size of $x = 20.53$ mm and $y = 12.79$ mm.

Figure 7 shows a matching system that takes a beam from an ion gun to an electrostatic quadrupole periodic focusing system. It was designed[8] using only the envelope equation for the K-V distribution to account for space-charge effects. The results from this program are in substantial agreement with these envelope calculations, giving some assurances that the two approaches give consistent results.

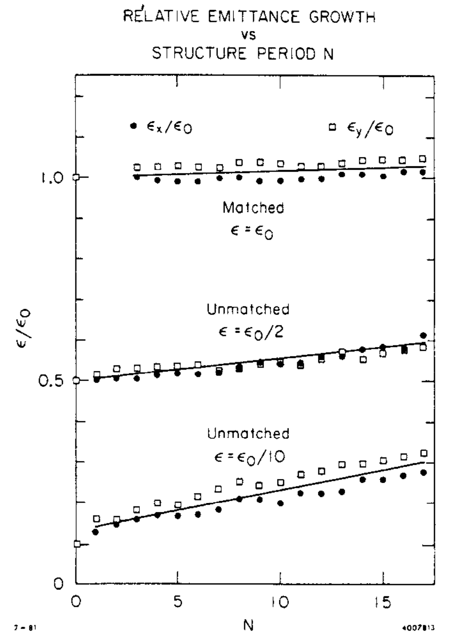

Fig. 6. Relative emittance growth of matched and unmatched beams.

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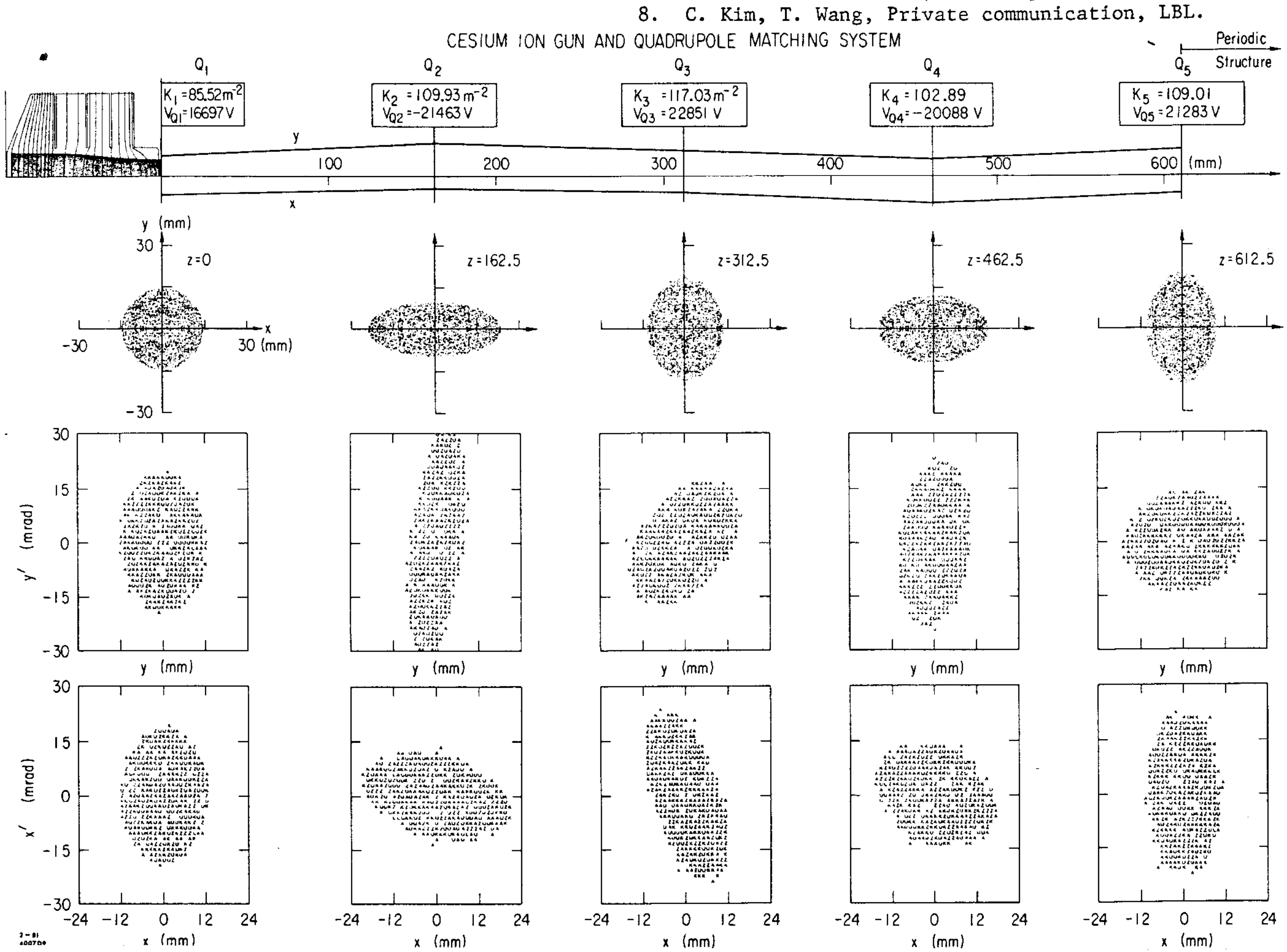

Fig. 7. Matching section between ion gun and periodic structure.