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QUARK STRUCTURE OF NUCLEI *

by

R. Blankenbecler

Stanford Linear Accelerator Center

Stanford University, Stanford, California 94305

ABSTRACT

A brief review is given of selected topics involved in the relativistic quark structure of nuclei such as the infinite momentum variables, scaling variables, counting rules, forward-backward variables, thermodynamic-like limit, QCD effects, higher quark bags, confinement, and many unanswered questions.

The study of nuclei and of their intersections (as well as the nucleon-nucleon interaction) has historically proceeded from the large distance and global properties to smaller distances as the energy of accelerators has increased. In the large distance regime nucleons can be considered point-like and there is no particle production. In the intermediate regime, the finite nucleon size is important but particle production effects are small. In the small distance regime, nucleons are "large", particle production is important and the internal degrees of freedom of the nucleons and mesons (the quarks) are fully excited.

Theoretically, one would like to start from a theory of quarks and their interactions, compute the properties of their bound states, i.e., nucleons and mesons, and then predict the properties of bound states of these bound states (nuclei). This is obviously a tall order and it may be some time before this program can be carried out in quantitative terms. In the roughest qualitative terms, it does seem to work, or at least tie together quite different phenomena.

In this regard it may be of some benefit to develop models that are valid in the regimes listed above <u>and</u> which continue correctly and bridge the gaps between these regimes. Ironically, it may prove to be more difficult to develop suitable models if nature is too smooth than if there are sharp delineations between these regimes.

I will try to organize this talk into three overlapping topics: kinematics, descriptive-parametrizations, and finally dynamics. However, of course, the

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dynamical model (and we will be particularly interested in QCD here) and its associated calculational scheme will suggest convenient parametrizations and useful kinematic variables. This will also lead to problems in that any acceptable fundamental theory will be relativistically invariant and will lead to a relativistic description of bound states.

How can one treat this problem so that the connection to the nonrelativistic problem, where one has developed considerable insight and phenomenology, is obvious and can be used? I shall attempt to demonstrate that the use of the infinite momentum frame, or rather the infinite momentum variables, provides this close connection. In listening to some earlier talks and questions at this conference, it is clear that there is considerable misunderstanding about the meaning and uses of the infinite momentum frame. Excuse me for spending an extraordinary amount of time on this point, but if you take anything from this talk, please remember the clear physics of this choice of variables.

Our notation will be simple: A will denote a particle's name, A_μ its four-momentum, and A its mass. Confusion is therefore impossible! In the finite momentum frame, the general four-vector A_μ is written as:

$$A_\mu = \left(A_o;\ A_T,\ A_z\right) \quad ,$$

$$A_\mu = \left(yP + \frac{1}{4yP}\left(A^2 + A_T^2\right)\ ;\ \ A_T,\ yP - \frac{1}{4yP}\left(A^2 + A_T^2\right)\right) \quad , \tag{1}$$

where P is a parameter

$$A_\mu A_\mu = A^2 \quad ,$$

$$y = \left(A_o + A_z\right)\big/2P \quad , \tag{2}$$

and $$d^4A = dA_o\ d^3A = d^2A_T\ dA^2\ dy\big/2|y| \quad .$$

The variable y is the misnamed momentum fraction. The infinite momentum frame can be achieved by taking the limit $P \to \infty$ but this is unnecessary since all relevant quantities will, in fact, be independent of the parameter P. The rest frame is achieved by choosing P so that A_Z vanishes and by setting $A_T = 0$.

There are, at least, three general approaches to the problem of the relativistic description of bound states.[1] The first is an explicitly four-dimensional approach using Feynman rules which leads to the familiar Bethe-Salpeter type of equation. The second is the time ordered approach using old-fashioned noncovariant perturbation theory which actually is an integral over the fourth component P_O of some relative four-momentum in the first approach leaving $\vec{p}$ as the variable. The third is the "infinite momentum frame" approach which uses the parametrization illustrated for A_μ and an integration over dp^2 which leaves $\vec{p}_T$ and y as the three variables. The last two approaches can be made to yield similar final results but I prefer the latter because of its simplicity (one does not have to worry about all possible time orderings, for example). In addition, and contrary to what one would expect, the (P_T,y) variables yield a result that is very close to that from the nonrelativistic Schrodinger equation.

To illustrate this point consider the vertex function for $B \to C + b$, where first b and then C is off-shell (this vertices could be used in the computations the processes shown in Fig. 1, for example). We will choose our frame by writing B_μ in the form of Eq. (1) with $y = 1$, and $B_T = 0$. For C on shell, we choose $y = x$, C_T and then compute the off-shell quantity b from momentum conservation. The relevant propagator for the equal mass case, $b^2 = C^2$, is

$$\left(b^2 - b_\mu b_\mu\right)^{-1} = \left[\frac{\left(C_T^2 + C^2\right)}{x(1-x)} - B^2\right]^{-1}$$

For the case of C off-shell, choose b_μ of the form of Eq. (1) with $y = 1 - x$, $b_T = -C_T$, and then

$$\left(C^2 - C_\mu C_\mu\right)^{-1} = \left[\frac{\left(C_T^2 + C^2\right)}{(1-x)} - B^2\right]^{-1} \quad .$$

These denominators differ only by a factor of x. To show that the first is closely related to the familiar Schrodinger energy denominator $H_0 - E \cong -E + \vec{k}^2/m$, simply write $B = 2C + E$, $x = \frac{1}{2}(1 + k_z/C)$, and one finds

$$b^2 - b_\mu b_\mu = 4C\left[-EC + k_T^2 + k_z^2\right] \quad ,$$

as expected in the nonrelativistic limit.

After a short calculation, one finds that it is possible to introduce probability functions for finding particle C in state B with momentum fraction x and transverse momentum C_T by [2]

$$G_{C/B}\left(x, C_T\right) = \frac{1}{2(2\pi)^3} \frac{x}{1-x} \left|\psi\left(x, C_T\right)\right|^2 \quad , \tag{3}$$

where ψ is a truncated Bethe-Salpeter amplitude. One needs a detailed dynamical model to be able to compute ψ for all x and C_T but it will be shown that the $x \to 1$ and the $C_T \to \infty$ behaviors are a simple function of the short-range nature of the force between the constituents. The inclusive distribution of detected particle C will in general be of the form [2]

$$\frac{d\sigma}{dC_T dx} \propto G\left(x, C_T\right) + \ldots \quad .$$

Let us now examine "scaling", the search for scaling variables, their uses, and a few cautions. There are many scaling variables that have been found to be useful. A few of them are discussed in Ref. 3. Here, I would just like to briefly discuss one that follows from our previous discussion of the infinite-momentum frame variables. For an excellent review of certain applications of this approach, I refer you to the articles by Chemtob.[4]

If absorption and final state interactions can be neglected (or rather, if they do not drastically change the longitudinal momentum distribution— they certainly will spread the transverse momentum distribution) then the inclusive

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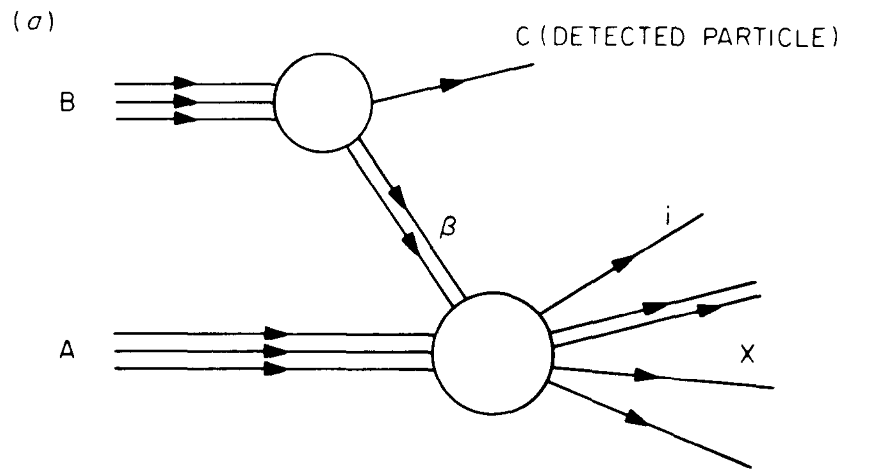

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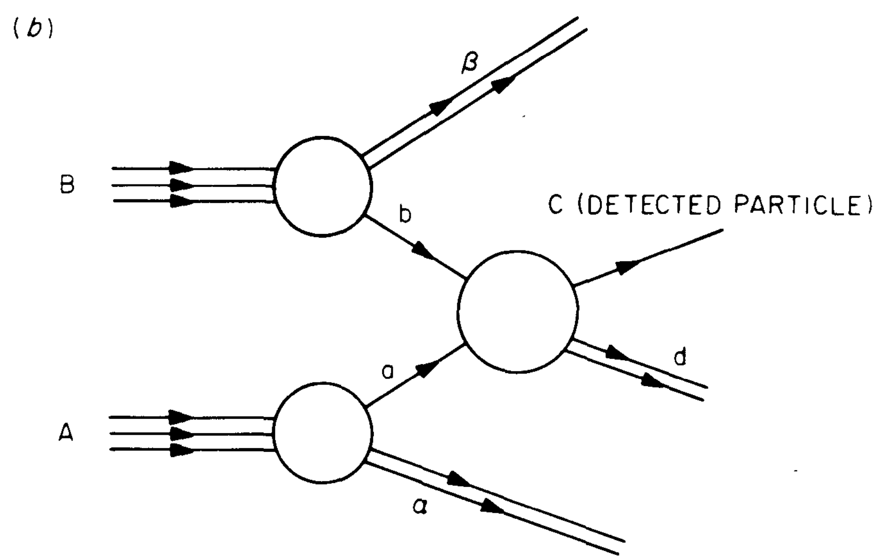

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Fig. 1

distribution (see Fig. 1) will be proportional to $G_{C/B}(x,C_T)$ in B-fragmentation region. Clearly x is predicted to ge a scaling variable[5,6] where

$$x = \frac{C_o + C_z}{B_o + B_z} = x_L\, x_{max}(W) \quad , \tag{4}$$

where

$$\begin{aligned} x_L &= \frac{C_o + C_z}{\left(C_o + C_z\right)_{max}} \quad , \\ x_{max} &= \frac{\left(C_o + C_z\right)_{max}}{B_o + B_z} \quad . \end{aligned} \tag{5}$$

Now x_{max} depends only on the center-of-mass energy W and the minimum "missing mass", M, of the reaction $A + B \to C + X$, and x_L clearly must be between 0 and 1. It is easy to see that as $W \to \infty$, $x_{max} \to 1$. For finite energies (W includes the rest masses) one finds the approximate results for the forward and backward directions (the exact expressions are not very transparent):

$$\begin{aligned} x_{max}(\theta \sim 0^o) &\simeq \left(W^2 - M^2\right)\Big/\left(W^2 - A^2\right) \quad , \\ x_{max}(\theta \sim 180^o) &\simeq \left(W^2 - M^2\right)\Big/\left(W^2 - B^2\right) \quad . \end{aligned} \tag{6}$$

Hence at moderate energies, for a light beam particle B incident on a heavy target a, one finds

$$x_{max}(0^o) = \frac{W^2 - B^2}{W^2 - A^2}\; x_{max}(180^o) \;\gg\; x_{max}(180^o) \quad . \tag{7}$$

Thus kinematics tells us that x_L scaling may look very different in the forward and backward direction. Note that x_L is not the Feynman scaling variable $x_F = |C_z| \,/\, |C_z|_{max}$, but approaches it for large C_z (>> C).

Let us now briefly look at an example of a "counting rule". The object here is to relate the behavior of $G(x,C_T)$ for $x \to 1$ or for $C_T \to \infty$ to some simple property of the nucleon-nucleon force at short distances.[5,7] Note that for $x \to 1$, all the other particles in the bound state must be stopped (the sum of all the x's must be 1). It is intuitively clear that the "softer" the N-N force, the faster G must vanish in these limits. For the probability of pulling a nucleon or a bound state C out of the state B, one finds

$$\begin{aligned} G_{C/B} &\sim \left(1 - x\right)^g \\ &\sim \left(C_T^2\right)^{-g+1} \end{aligned} \tag{8}$$

where $g = 2T(B - C) - 1$, and T depends on the nucleon-nucleon force. For example, if nucleons interacted point-like with the exchange of vector gluons, then $T = 1$. If the N-N force were due to exchange of rho's and omega's with monopole form factors, then $T = 3$. Likewise, $T = 3$ if the quark degrees of freedom are fully excited. In general, however, T must be considered to be a parameter that effectively describes the N-N force in a certain regime. Rough fits to the data yield $T \sim 3$-4.

If there is very strong momentum clustering in the nucleus,[8] then one will find that $(B - C)$ is replaced by the $(\bar{B} - C)$, where $\bar{B}$ is the number of nucleons in the average cluster and the G function vanish at $x = \bar{B}/B$ rather than at $x = 1$ (if one gives the clusters some fermi momentum then this point is averaged over).

When one extracts the parameter g from data by fitting the inclusive momentum distribution it is very important to use the correct variable x rather than x_L. The factor of $x_{max}(W)$ can have a large effect on the value of g, especially when comparing the beam and target fragmentation region.

Note that we are not claiming that x is the "best" scaling variable. Indeed, it is not, since clearly there will exist arbitrarily chosen scaling functions that fit the data better than any arising from a given theory (which necessarily will yield correction and extra nonscaling terms), even the correct theory!

One's first reaction to a formula such as Eq. (7) is that it probably is nonsense for nuclei, especially for large atomic number. However, this is not necessarily the case. Consider the variable x in the limit $B \to \infty$, then $(C_+ \equiv C_o + C_z)$

$$x = C_+\Big/\left(B_o + B_z\right) \sim C_+\Big/BM \quad , \tag{9}$$

where M is the nucleon mass, and

$$\begin{aligned} G &= \left(1 - x\right)^{2TB\cdots} \sim \left(1 - C_+\big/BM\right)^{2TB} f\left(C_T\right) \\ G &\sim f\left(C_T\right) \ \exp\left(-\frac{2T}{M}\,C_+\right) \quad . \end{aligned} \tag{10}$$

This takes the familiar form of a thermodynamic spectrum but with the variable C_+ rather than C_o. The dependence on C_+ [and the factor $f(C_T)$] produce an angular variation which is quite similar to that seen in the data. Furthermore, the dependence of C_+ on the mass produces[6] a difference between the effective temperature for pions (60 MeV) and nucleons (40 MeV) in the same kinetic energy range (0.3-1 GeV) which is again not unlike the data for $T \sim 3.5$.

Let us now turn to QCD, its associated model for hadrons and some possible ramifications for nuclear physics. It is very easy to get a physical understanding of the effects of QCD and confinement. Perhaps the easiest way is to imagine that QCD is an ordinary field theory that was designed by a government committee. Everything works as expected but in reverse.

As an example, one has a picture that the nucleon-nucleon form is due to meson (pion, rho, omega, two-pion, etc) exchange. Since these contributions fall off exponentially at large distances, the longest range part of the force is due to single pion exchange which is easily evaluated. At shorter distances, these more massive exchanges become more and more complicated and an accurate computation is more and more difficult.

In asymptotically free theories such as QCD, things work the same after a sign change. At short distances (higher momentum transfers) the coupling gets weaker and weaker (as $1/\ln Q^2/\Lambda^2$) and hence perturbation theory is valid. One can expand in the number of gluons involved and even sum the leading terms in this series. At large distances, the coupling constants increase; they increase so fast that the force actually starts to increase as a power of the distance. This is the "confining" potential between colored objects that is expected to grow $\approx$ linearly with the separation. The detailed behavior of the theory in this strong coupling regime and its transition to the perturbative regime is under intense study. It goes without saying that the behavior of the hadronic bound states at large distances is controlled by the strong coupling behavior of the theory.

The potential between a quark-antiquark pair, each of which is a color triplet, has a simple behavior at large and small distances in a color singlet state:

$$\begin{aligned} V(r) &\sim r \qquad (r \text{ large}) \quad , \\ &\sim -\frac{1}{r \ln r} \quad (r \text{ small}) \quad . \end{aligned} \tag{11}$$

This potential is relevant for mesons and for heavy quark bound states such as the psi , psi-prime, epsilon, and hopefully more. In the nucleon, a bound state of three quarks, two of the quarks form a $\bar{3}$ state ($3 \times 3 = \bar{3} + 6$) which then combines with the third quark to form an overall singlet.

Let us examine some familiar hadronic bound states in the QCD picture[9] (pion, proton, neutron, deuteron, triton) and their basic contents:

$$|\pi\rangle = (q\bar{q})_1 + (q\bar{q} + \text{gluon})_1 + \ldots$$

$$|p\rangle = (uud)_1 + (uud + \text{gluon})_1 + \ldots$$

$$|n\rangle = (udd)_1 + (udd + \text{gluon})_1 + \ldots$$

$$|d\rangle = (uud)_1(udd)_1 + (uud + \text{gluon})_1(udd)_1 + \ldots$$

$$|t\rangle = (uud)_1(udd)_1(udd)_1 + \ldots$$

where the subscript 1 indicates a color singlet state.

The behavior of the structure function for these particles follows from our previous discussion with T = 1 except for an additional spin effect:

$$G_{q/\pi} \sim (1-x)^{1+1}$$

$$G_{q/p} \sim (1-x)^{3}$$

$$G_{q/d} \sim (1-x)^{9+1}$$

$$G_{q/t} \sim (1-x)^{15} \qquad ,$$

where the extra power of $(1 - x)$ arises in those cases in which the initial bound state is bosonic (has an even number of quarks in its basic wave function).

Now as Q^2(or Q_T^2) increases, where Q is the momentum transfer to the struck quark, the increase in final state phase space allows more and more gluons to be emitted while at the same time the gluon-quark coupling constant is decreasingly logarithmically with Q^2. The momentum taken up by the emitted gluons means that less is available to the quarks so that as Q^2 increases the quark distribution function increases at low x and decreases at high x. The radiative effects of the gluons introduce $\ln Q^2$ and $\ln \ln Q^2$ nonscaling effects in the distribution function also.

In addition to these log Q^2 effects, there are also a myriad of "higher twist" correction terms which behave as $1/Q^2$ and $1/Q^4$, etc., in addition to the $\ln Q^2$ terns, These arise from mass corrections, M^2/Q^2, finite size corrections $1/R^2Q^2 = \langle k_T^2\rangle/Q^2$, and coherence effects in the initial and final states. These higher twist terms are not due to some negligible, unphysical, esoteric effects. I remind you that all exclusive scattering and all elastic scattering-scattering processes are pure higher twist.

I would like to finish with mention of a few topics that might prove to be of some interest:

(1) There exists evidence[10] that there is a nonnegligible charm component in the nucleon carrying a reasonable amount of the momentum fraction x. It might be expected that the power law fermi motion in light nuclei, if they were used in a beam, could be a rather copious source of fast forward charm particles.

(2) There has been a recent letter[11] pointing out that photodisintegration of the deuteron in the "classical" energy range below 100 MeV is still not well understood, either experimentally or theoretically. In this note, an ad hoc modification of the deuteron wave function for $r < 1.5$ fermi is used to get agreement. I have not examined this problem in great detail but it is clear that a consistent and proper relativistic treatment has not given (one that explains also electron elastic and inelastic scattering from the deuteron at large Q^2) and the data does not seem all that great either!

(3) The relativistic formulation of the bound state problem allows a proper and invariant treatment of kinematic effects without losing the physical input from the nonrelativistic limit. These kinematic and threshold effects have been well discussed[12] and I shall omit any further consideration here.

(4) Shadowing and rescattering are a subject that still require considerable study in the relativistic case. As far as I know, a general, useful, and convenient formalism to discuss these effects has not yet been given. A relativistic version of the distorted wave born approximation (DWBA) should be very useful. It should take on a quite simple form if one uses a mixed representation for the wave function, i.e., use $\psi^{\pm}(b_T,x)$ the two-dimensional transform of $\psi^{\pm}(k_T,x)$.

(5) The A-dependence and particle production are subjects that have received considerable attention[13] but they are too complicated to adequately review here. I would just remark that the QCD-inspired quark model with color separation and confinement play an important qualitative role in these approaches.

(6) If we want to study the nuclear wave function at extremely small distances, this can be done by studying the effects of the weak interactions and in particular parity violation. If rho and omega exchange play an important role in the nucleon-nucleon force, then since the $W^{\pm}$ and Z^{0} mix with these ordinary vector mesons, there should be a small admixture of opposite parity states in the nucleus. The Compton wave length of the W - Z is $\sim 3 \times 10^{-3}$ fermi so that if this can be studied in detail (such as in polarization experiments with photons) one is examining rather short distances indeed!

(7) The proper treatment of the deuteron will require a treatment of the 6-quark problem. A start has been made in a discussion of this problem in the bag model.[14] The relevant diagrams for the 3-quark (nucleon) and 6-quark states are illustrated in Fig. 2. However, note that the 6-quark state is unstable against the decay into two separated 3-quark bound systems as illustrated in Fig. 3. This separated configuration will dominate the behavior of the wave function at large distances and hence will control the large r properties of the deuteron. We know that in the deuteron the nucleons are outside the range of the force for most of the time. At smaller distances they can interact by ordinary meson exchange, which in this model is the interchange of two quarks as shown in Fig. 4. At much smaller distances, the two bound states "fuse" into the 6-quark configuration and can no longer be cleanly separated into two objects called "nucleons". Thus we see that a full discussion of the deuteron will require (at the very least) a relativistic treatment of configuration mixing.

(8) Penultimately, let me point out an interesting possibility of new types of excited states for the deuteron (or any other nuclei). If the two nucleon configurations (they are color singlets) interact by the exchange of a gluon (which form a color octet) then one gets a new configuration in the deuteron[9] which is composed of two colored octet "nucleons":

$$|d\rangle = a_1 (uud)_1 (udd)_1 + a_8 (uud)_8 (udd)_8 + \ldots \quad .$$

Now these two colored objects will be confined; they will interact via a linearly rising potential (it probably should rise about twice as fast as the $q\bar{q}$ potential). Therefore they should exist only at intermediate distances, within the confining potential. Excited states of the "deuteron" can be formed by these colored baryon objects rattling around in the potential. The excitations should have a larger energy separation than those that are typical of a mesonic $q\bar{q}$ system. It would be very exciting if these new types of excited states that arise from the hidden color degrees of freedom are actually confirmed experimentally.

Finally, I would like to close by quoting Cato's advise to all reviewers, "I think the first wisdom is to hold the tongue".

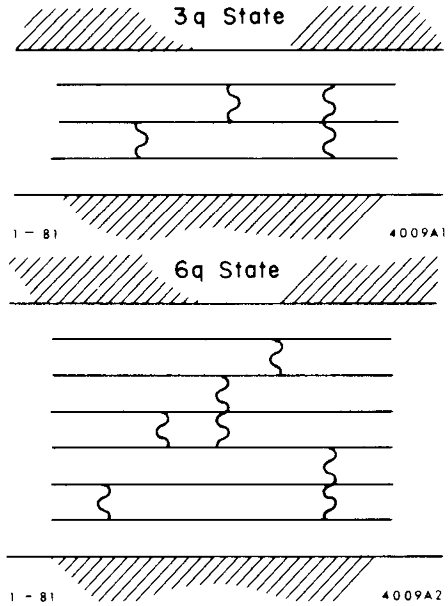

Fig. 2

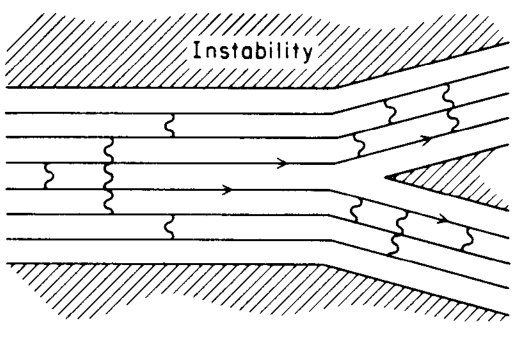

Fig. 3

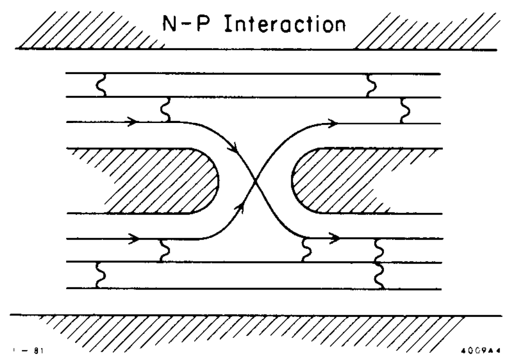

Fig. 4

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