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BROKEN CHIRAL SYMMETRIES AND LIGHT COMPOSITE FERMIONS *

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ABSTRACT

We elaborate on a mechanism for obtaining naturally light composite fermions in a strongly interacting gauge theory. A simple toy model is used to illustrate the basic ideas.

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1. Introduction

The weak interaction symmetry breaking scale (~ 250 GeV) can naturally arise as a consequence of a strongly interacting gauge theory at a scale $\sim$TeV [1]. There then exists the interesting possibility that in addition to the scalars, the quarks and leptons are also composite objects in the TeV range. A crucial problem in such a scenario is an understanding of how the composite fermions can be so light (few MeV--few GeV) in comparision to the binding scale. Recently, 't Hooft has described how global chiral symmetries can provide a natural explanation for the occurrence of massless composite fermions in a strongly interacting gauge theory [2]. Moreover he presented a set of consistency conditions which must be satisfied by these composite fermions. An unbroken global chiral symmetry (guardian symmetry) in essence protects the fermions from receiving a mass. Subsequently, using tumbling [3] and complimentarity [4], several solutions to the consistency conditions were found. The present paper extends the previous work on massless composite fermions and describes a general mechanism by which these fermions can obtain a small mass. We elaborate on some recent discussions on the weak generation of fermion masses by Peskin [5] and Dimopoulos and Susskind [6].

The essential feature of the mechanism involves two steps, of which the first is a partial spontaneous breakdown of the guardian symmetry. In general, a subset of the original fermions are left massless. They are still protected by the unbroken subgroup of the guardian symmetry. The others obtain mass either through instantons and the condensates which spontaneously break the symmetry or via the condensates alone.

The second step involves a weak explicit breaking of the guardian symmetry. The remaining massless fermions can then receive a mass of order α_W, the coupling constant of the feeble gauge interaction.

For the sake of simplicity we discuss a particular strong interaction model with a strong gauge group $SU(4)_s$. The model includes the following fermions: a $\overline{10}$ of $SU(4)_s$ ($\chi_{ij} = \chi_{ji}$; $i,j = 1, \ldots 4$) and 8 fields transforming as a 4 of SU(4) ($\psi^{i\alpha}$; $\alpha = 1, \ldots 8$). The fields χ_{ij} and ψ^i are left-handed two-component Weyl spinors. The full symmetry of the model is $SU(4)_s \times SU(8) \times U(1)_Q$ where the global SU(8) symmetry acts on the 8 ψ states and Q is given by $Q(\psi) = 6$ and $Q(\chi) = -8$. All or part of the global symmetry $SU(8) \times U(1)_Q$ may act as the guardian symmetry protecting some subset of fermions from obtaining mass.

In section 2 we present a discussion of the model in the so-called symmetric picture; i.e., along the original lines of 't Hooft [2]. We solve the anomaly consistency conditions for the two possibilities: (a) the full global symmetry remains unbroken, and (b) only a subgroup of G_{global} is unbroken. In both cases we discuss the subsequent massless composite fermion spectrum. In the case of the broken global symmetry we elaborate on several inequivalent breaking patterns and the corresponding multi-fermion condensates which cause this breaking. The discussion is designed to illustrate some representative cases of the many possibilities. In addition we describe explicitly the two mechanisms for mass generation for the unprotected fermions.

The symmetric picture appears very complicated as far as dynamical symmetry breaking is concerned. It does not seem possible to decide which of the many possibilities is dynamically favored. In order to obtain further insight we discuss the model in section 3 in the "broken picture" via tumbling [3]. We demonstrate the complimentarity [2,4] of the two pictures. We emphasize however that there is no broken scenario corresponding to the unbroken global symmetry case of section 2. Recall that the 't Hooft consistency conditions are essentially kinematical constraints which must be satisfied by the massless spectra as a result of an assumed unbroken global symmetry. If the constraints cannot be satisfied we conclude that our assumption was incorrect and the symmetry is in fact broken. However, if the constraints can be satisfied (as is the case for our example) we conclude that the full global symmetry may (or may not) remain unbroken. We then require additional input to determine which solution is dynamically favored. We note that the tumbling rules (<u>if correct</u>) provide some additional dynamical input which in this case is sufficient to rule out the unbroken case.

In sections 4 and 5 we gauge an $SU(4)_W$ subgroup of the $SU(8) \times U(1)_Q$ global symmetry. This new interaction is assumed to be weak on the scale Λ_S of the strong interaction $SU(4)_S$. We denote the weak gauge coupling by $\alpha_W = g_W^2/4\pi$. In some cases this weak interaction explicitly breaks the remaining guardian symmetry. The formerly massless fermions can and do then obtain small masses of order $\alpha_W \Lambda_S$ or $\alpha_W^2 \Lambda_S$. We discuss the symmetric picture in section 4 and the broken picture in section 5. The mass generation mechanism for the light fermions is shown to be more transparent in the symmetric picture. We discuss the complimentarity of the two pictures regarding the specific question of mass generation.

In section 6 we finally discuss the relevance of our analysis for model building.

2. Symmetric Picture

The considered model is based on an $SU(4)_S \times SU(8) \times U_Q(1)$ symmetry with the massless fermion content

$$\begin{aligned} \chi_{ij} = \chi_{ji} &= (\overline{10},\ 1,\ Q=-8) \\ \psi^{i\alpha} &= (4,\ 8,\ Q=6) \end{aligned} \tag{1}$$

All states are left-handed Weyl spinors, $i,j = 1, \ldots 4$ are $SU(4)_S$ indices and $\alpha = 1, \ldots 8$ is an SU(8) index. $SU(4)_S$ is an asymptotically free gauge symmetry which is considered to become strong at a scale Λ_S. Q is the charge that is conserved by the $SU(4)_S$ instanton process (fig. 1).

In the so-called symmetric picture one assumes that $SU(4)_S$ confines, leaving only $SU(4)_S$-singlet bound states in the physical spectrum. The first question we address is the possible existence of massless composite fermions. According to 't Hooft these massless composite fermions should necessarily give rise to the same anomalies in the chiral currents as the fundamental fermions from (1). In the model with the $SU(8) \times U(1)_Q$ chiral symmetry there exist three types of chiral anomalies from the triangle graphs shown in fig. 2. The fundamental particles from (1) exhibit the anomalies 4, 24 and 1792 from the processes in fig. 2a,b and c, respectively.

't Hooft's nontrivial consistency condition has group theoretical solutions in our model of which the simplest is the two index anti-symmetric representation of SU(8) with $Q = 4$: (1, 28, 4).

There is still the question of whether there exist fermionic $SU(4)_S$-singlet bound-states in our model which posess these transformation properties. These bound states consist of an odd number of fundamental fermions bound in an $SU(4)_S$ singlet state. There exists only one type of three particle bound state $\psi^{i\alpha}\chi_{ij}\psi^{j\beta}$. One recognizes immediately that the states antisymmetric in α and β

$$\xi^{\alpha\beta} = \psi^{i\alpha}\chi_{ij}\psi^{j\beta} - \psi^{i\beta}\chi_{ij}\psi^{j\alpha} \tag{2}$$

forms the 28-representation of SU(3) with charge $Q = 4$, and therefore solves 't Hooft's condition. We thus conclude that these 28 states are candidates for massless composite fermions, protected by the $SU(8) \times U(1)$ guardian symmetry. Other possible bound states which are not protected by the symmetry will acquire masses on a scale of order Λ_S.

We should, however, keep in mind that the anomaly condition is only a necessary kinematical constraint on the massless fermions in the case of an unbroken global symmetry. The symmetry could be broken dynamically and one would then in general expect less massless fermions. In the remainder of this section we will discuss several possibilities of a broken global symmetry. We still remain in the symmetric picture, where the strong $SU(4)_S$ remains unbroken.

The global symmetry could be broken by scalar $SU(4)_S$-singlet condensates that have nontrivial $SU(8) \times U(1)$ transformation properties. There do not exist any two-fermion condensates which fulfill these criteria. The following listed in order of increasing number of fermions are some of the simplest candidates:

$$\phi^{\alpha}_{\beta} = \left\langle \psi^{i\alpha}\chi_{ij}\psi^{*}_{k\beta}\chi^{*jk} \right\rangle \tag{3a}$$

$$\phi^{\alpha\beta\gamma\delta} = \left\langle \psi^{i\alpha}\psi^{j\beta}\psi^{k\gamma}\psi^{\ell\delta}\varepsilon_{ijk\ell} \right\rangle \tag{3b}$$

$$\tilde{\theta}^{\alpha\beta\gamma\delta} = \left\langle \psi^{i\alpha}\psi^{j\beta}\psi^{k\gamma}\psi^{\ell\delta}\chi_{ij}\chi_{k\ell} \right\rangle \tag{3c}$$

$$\theta^{\alpha\beta\gamma\delta} = \left\langle \psi^{i_1\alpha}\chi_{i_1j_1}\psi^{i_2\beta}\chi_{i_2j_2}\psi^{i_3\gamma}\chi_{i_3j_3}\psi^{i_4\delta}\chi_{i_4j_4}\varepsilon^{j_1j_2j_3j_4} \right\rangle \tag{3d}$$

The four-fermion condensates (a) and (b) can break the chiral symmetry. The formerly massless fermions, however, are still protected by this smaller guardian symmetry. In case (a) that is obvious since the condensate ϕ^{α}_{β} has charge $Q = 0$ and therefore leaves $U_Q(1)$ unbroken. Case (b) is more complicated since $Q(\phi^{\alpha\beta\gamma\delta}) = 24$, but in all the possible breaking patterns there remains enough symmetry to still protect the formerly massless particles.

The condensates (c) and (d) in general break the symmetry. As a result some of the formerly massless fermions are no longer protected and can receive a mass. We will discuss these cases in more detail. The condensate $\tilde{\theta}^{\alpha\beta\gamma\delta}$ is symmetric under the interchange $(\alpha\beta)\leftrightarrow(\gamma\delta)$. It can provide several inequivalent possibilities for a breakdown of the global symmetry out of which we will discuss two. The first is

$$\tilde{\theta}^{\alpha\beta\gamma\delta} = \tilde{v}\eta^{\alpha\beta}\eta^{\gamma\delta} \tag{4}$$

where

$$\tilde{v} \sim O\left(\Lambda_S^9\right)$$

and

$$\eta = \left(\begin{array}{cc|cc|cc|cc} 0 & 1 & & & & & & \\ -1 & 0 & & & & & & \\ \hline & & 0 & 1 & & & & \\ & & -1 & 0 & & & & \\ \hline & & & & 0 & 1 & & \\ & & & & -1 & 0 & & \\ \hline & & & & & & 0 & 1 \\ & & & & & & -1 & 0 \end{array}\right)$$

is the symplectic metric. The vacuum expectation values (v.e.v.) in (4) break SU(8) to SP(8). $U_Q(1)$ is also broken and there is no other remaining U(1)-symmetry. The fermions $\xi^{\alpha\beta}$ decompose into a 1 + 27 representation of SP(8). None of the 28 states is protected by this symmetry and all of them receive a mass through the $\tilde{\theta}$ condensate as displayed in fig. 3.

A second possibility for the v.e.v. is

$$\tilde{\theta}^{abcd} = \tilde{v}\eta^{ab}\eta^{cd} \tag{5}$$

where a,b,c,d = 1, ... 6. Here SU(8) breaks down to SU(2) × SP(6) where the SU(2) acts on the components $\alpha = 7$ and 8 and

$$\eta = \left(\begin{array}{cc|cc|cc} 0 & 1 & & & & \\ -1 & 0 & & & & \\ \hline & & 0 & 1 & & \\ & & -1 & 0 & & \\ \hline & & & & 0 & 1 \\ & & & & -1 & 0 \end{array}\right)$$

$U_Q(1)$ is broken but the charge

$$Z = Q - 2\begin{pmatrix} 1 & & & & & & & 0 \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & 1 & & & & \\ & & & & 1 & & & \\ & & & & & 1 & & \\ & & & & & & -3 & \\ 0 & & & & & & & -3 \end{pmatrix} \tag{6}$$

remains unbroken. We are therefore left with the unbroken symmetry SU(2) × SP(6) × $U_Z(1)$ under which the states $\xi^{\alpha\beta}$ decompose in the following way

$$\begin{aligned} \xi^{ab} &= (1,\ 1+14,\ Z=0) \\ \begin{pmatrix} \xi^{a7} \\ \xi^{a8} \end{pmatrix} &= (2,\ 6,\ Z=8) \\ \xi^{78} &= (1,\ 1,\ Z=16) \end{aligned} \tag{7}$$

Masses for ξ^{a7}, ξ^{a8} and ξ^{78} are forbidden by the symmetry and only the 15 states ξ^{ab} are unprotected. They indeed receive a mass through the process in fig. 3.

Other breakdown patterns with the $\tilde{\theta}$ condensate include SU(4) × SP(4) and SU(6) × SU(2) and can be discussed along the same lines.

We finally want to discuss the completely antisymmetric $\theta^{\alpha\beta\gamma\delta}$ condensate. There are again several possible breakdown patterns depending on the vacuum expectation values. We first consider

$$\theta^{1234} \neq 0 \tag{8}$$

SU(8) breaks to SU(4) × SU(4). $U_Q(1)$ is broken since $Q(\theta^{1234}) = -8$ but

$$Z = Q + 2 \begin{pmatrix} 1 & & & & & & & 0 \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & 1 & & & & \\ & & & & -1 & & & \\ & & & & & -1 & & \\ & & & & & & -1 & \\ 0 & & & & & & & -1 \end{pmatrix} \tag{9}$$

is conserved. The decomposition of $\xi^{\alpha\beta}$ with respect to SU(4) × SU(4) × $U_Z(1)$ reads (r,s = 1,2,3,4; x,y = 5,6,7,8)

$$\begin{aligned} \xi^{rs} &= (6,\ 1,\ 8) \\ \xi^{xr} &= (4,\ 4,\ 4) \\ \xi^{xy} &= (1,\ 6,\ 0) \end{aligned} \tag{10}$$

ξ^{rs} and ξ^{xr} are still protected by the SU(4) × SU(4) × $U(1)_Z$ symmetry, whereas a Majorana mass term is allowed for ξ^{xy}. The eight particle condensate can however not directly give a mass to ξ^{xy} as was the case in the $\tilde{\theta}$ example. We need in addition the instanton process (fig. 1).

Note that in this case if we ignored the instanton there would be an additional conserved quantum number that would forbid a mass for ξ^{xy}. In the previous $\tilde{\theta}$ case there would also have been an additional conserved quantum number, but its value for ξ^{ab} was zero. The process that gives mass to ξ^{xy} is shown in fig. 4. Observe that due to the SU(8) antisymmetry of the instanton process this graph does not give mass to ξ^{rs} and ξ^{xr}.

Another possibility is

$$\theta^{78ab} \propto \eta^{ab} \tag{11}$$

where a,b = 1, ... 6. The symmetry is broken to SU(2) × SP(6) × $U_Z(1)$ where

$$Z = Q - 2 \begin{pmatrix} 1 & & & & & & & 0 \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & 1 & & & & \\ & & & & 1 & & & \\ & & & & & 1 & & \\ & & & & & & -3 & \\ 0 & & & & & & & -3 \end{pmatrix} , \tag{12}$$

and the decomposition of $\xi^{\alpha\beta}$ is

$$\begin{aligned} \xi^{ab} &= (1,\ 1+14,\ 0) \\ \begin{pmatrix} \xi^{a7} \\ \xi^{a8} \end{pmatrix} &= (2,\ 6,\ 8) \\ \xi^{78} &= (1,\ 1,\ 16) \end{aligned} \tag{13}$$

Again the instanton process is necessary for the mass generation. ξ^{ab} gets mass through the process in fig. 4, whereas ξ^{a7}, ξ^{a8} and ξ^{78} remain massless.

We would like to note here that in all of these cases of a broken chiral symmetry, 't Hooft's consistency conditions are fulfilled for the unbroken subgroup. To give an example, we consider the anomaly of three $U_Z(1)$ currents in the last case. $A\ (\xi^{a7} + \xi^{a8} + \xi^{78}) = 16^3 + (12 \times 8^3) = 10240$. The fundamental particles decompose into

$$\begin{aligned} \chi_{ij} &= (1,\ 1,\ -8) \times 10 \\ \begin{pmatrix} \psi^{i7} \\ \psi^{i8} \end{pmatrix} &= (2,\ 1,\ 12) \times 4 \\ \psi^{ia} &= (1,\ 6,\ 4) \times 4 \end{aligned} \tag{14}$$

$A = (-8)^3 \cdot 10 + 8 \cdot (12)^3 + 24 \cdot (4)^3 = 10240$ is the anomaly of the fundamental particles.

We hope that we have convinced the reader that a discussion of the possibilities of a breakdown of the chiral symmetries opens a Pandoras box. The possibilities include:

(a) an unbroken symmetry with 28 massless states (no condensate),

(b) a broken symmetry with 28 massless states (4 particle condensates),

(c) a broken symmetry with no massless states (e.g., SP(8) case), and

(d) broken symmetry where some of the states receive a mass and others remain massless.

The mass generation is possible sometimes directly through the condensate (fig. 3) or indirectly through the instanton process (fig. 4).

With our poor knowledge of confining strong interactions it is impossible to decide at this stage which particular breakdown is most likely to be chosen by the dynamics of the system. In the next section we make a modest attempt to narrow the possibilities.

3. The Broken Picture

This picture shows the same model from a different point of view. It is here no longer assumed that the strong interaction $SU(4)_S$ remains unbroken. At first sight it seems that the two pictures have nothing in common; however, it has been shown that the spectrum of light fermions can exhibit surprising similarities. This is a consequence of the phenomenon called complimentarity [2,4]. In this section we will discuss the model along the lines given in ref. 3 and investigate how far the phenomenon of complimentarity can be extended. Complimentarity could then be used to obtain some constraints on the possibilities in the symmetric picture. The basic assumption in this picture is the formation of condensates in the most attractive channel (MAC), which in our model would be

$$\phi_j^\alpha = \psi^{i\alpha}\chi_{ij} \quad . \tag{15}$$

This condensate has nontrivial transformation properties with respect to SU(8) as well as $SU(4)_S$, and therefore in general will break both symmetries. We consider two inequivalent patterns of condensation. The first one is

$$\left\langle \phi_j^\alpha \right\rangle_0 \propto \delta_j^\alpha \qquad j = 1,2,3,4 \tag{16}$$

and the second one is the "tumbling" solution, where in the first step only one component develops a vacuum expectation value, e.g.,

$$\left\langle \phi_4^8 \right\rangle_0 \neq 0 \quad . \tag{17}$$

We will in the remainder of this section discuss these possibilities in detail. But before we do that let us point out that with the assumption of condensates in the MAC the case of an unbroken SU(8) is no longer possible.

Let us now discuss the condensate (16). SU(8) breaks to $SU(4)_D \times SU(4)$, where $SU(4)_D$ is the diagonal subgroup of $SU(4)_S$ and the SU(4) subgroup of SU(8) that acts on the first four components. There is no remaining strong group. $U_Q(1)$ is broken but

$$Z = Q + 2\begin{pmatrix} 1 & & & & & & & 0 \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & 1 & & & & \\ & & & & -1 & & & \\ & & & & & -1 & & \\ & & & & & & -1 & \\ 0 & & & & & & & -1 \end{pmatrix} \tag{18}$$

is conserved. The decomposition of the fundamental particles with respect to $SU(4)_D \times SU(4) \times U_Z(1)$ is as follows: (r,s = 1,2,3,4 are $SU(4)_D$ indices and x,y = 5,6,7,8 are SU(4) indices).

$$\begin{aligned} \chi_{rs} &= (\overline{10},\ 1,\ -8) \\ \psi^{rs} &= (10,\ 1,\ 8) + (6,\ 1,\ 8) \\ \psi^{rx} &= (4,\ 4,\ 4) \end{aligned} \tag{19}$$

$(\overline{10}, 1, -8)$ and $(10, 1, 8)$ are massive through the condensation process and $\psi^{[rs]}, \psi^{rx}$ remain massless. There is a one-to-one correspondence between these massless states and the massless bound states in (10). This correspondence is guaranteed by complimentarity, which in this case is realized at the level of a broken global symmetry. Note that the massive composite fermions ξ^{xy} in (10) do not exist in the broken picture (19). This just points out the fact that complimentarity does not necessarily require one-to-one correspondence between massive states.

The second possibility of condensation is the tumbling pattern. The v.e.v. $\langle\phi_4^8\rangle_0$ breaks $SU(4)_S \times SU(8) \times U_Q(1)$ to $SU(3)_S \times SU(7) \times U_X(1) \times U_Y(1)$ where

$$\begin{aligned} X &= 7E_4 + 3E_8 \\ Y &= 3Q + 2E_4 \end{aligned} \tag{20}$$

$$E_4 \text{ is } \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -3 \end{pmatrix} \text{ of } SU(4)_S$$

and

$$E_8 = \begin{pmatrix} 1 & & & & & & & \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & 1 & & & & \\ & & & & 1 & & & \\ & & & & & 1 & & \\ & & & & & & 1 & \\ & & & & & & & -7 \end{pmatrix} \text{ of } SU(8)\ .$$

Eight states become massive through the condensate and the remaining massless particles in $SU(3) \times SU(7) \times U_X(1) \times U_Y(1)$ notation are ($i = 1,2,3,$; $\alpha = 1,2,3,4,5,6,7$)

$$\begin{aligned} \psi^{i\alpha} &= (3,\ 7,\ 10,\ 20) \\ \psi^{4\alpha} &= (1,\ 7,\ -18,\ 12) \\ \chi_{ij} &= (\bar{6},\ 1,\ -14,\ -28) \end{aligned} \tag{21}$$

We are however still left with a strong $SU(3)_S$ interaction. The MAC in SU(3) is $\phi_i^\alpha = \chi_{ij}\psi^{j\alpha}$ ($i = 1,2,3$; $\alpha = 1,2,3,4,5,6,7$).

We then assume

$$\left\langle \phi_3^7 \right\rangle_0 \neq 0$$

which causes the breakdown to $SU(2)_S \times SU(6) \times U_A(1) \times U_B(1) \times U_C(1)$

$$\begin{aligned} A &= 3E_3 + E_7 \\ B &= 3X - 2E_7; \quad E_3 = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -2 & \\ & & & 0 \end{pmatrix}_S ; \quad E_7 = \begin{pmatrix} 1 & & & & & & & \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & 1 & & & & \\ & & & & 1 & & & \\ & & & & & 1 & & \\ & & & & & & -6 & \\ & & & & & & & 0 \end{pmatrix}_8 \\ C &= Y - 2X \end{aligned} \tag{22}$$

and the remaining uncondensed fermions are

$$\begin{aligned} \psi^{ia} &= (1,\ 6,\ 4,\ 28,\ 0) \\ \psi^{3a} &= (1,\ 6,\ -5,\ 28,\ 0) \\ \psi^{4a} &= (1,\ 6,\ 1,\ -56,\ 48) \\ \psi^{47} &= (1,\ 1,\ -6,\ -42,\ 48) \quad i,\ j = 1,2 \\ \chi_{ij} &= (3,\ 1,\ -6,\ -42,\ 0) \quad a,b, = 1,2,3,4,5,6 \end{aligned} \tag{23}$$

$SU(2)_S$ condensation finally occurs and leads to the condensates

$$\begin{aligned} \phi^{ab} &= \left\langle \psi^{ia} \psi^{jb} \varepsilon_{ij} \right\rangle_0 \propto \eta^{ab} \\ \omega &= \left\langle \chi_{ij} \chi_{k\ell} \varepsilon^{ik} \varepsilon^{j\ell} \right\rangle \end{aligned} \tag{24}$$

which do not break $SU(2)_S$ but break the chiral symmetry. We are left with the symmetry $SU(2)_S \times SP(6) \times U_D(1) \times U_Z(1)$ where all the uncondensed massless fermions are $SU(2)_S$ singlets

$$\begin{aligned}\psi^{3a} &= (1,\ 6,\ 1,\ 8)\\ \psi^{4a} &= (1,\ 6,\ -1,\ 8)\\ \psi^{47} &= (1,\ 1,\ 0,\ 16)\end{aligned} \tag{25}$$

and

$$D = \frac{1}{9}\left(\frac{B}{7} - A\right) = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 1 & \\ & & & -1 \end{pmatrix}_{SU(4)_S} + \begin{pmatrix} 0 & & & & & & & \\ & 0 & & & & & & \\ & & 0 & & & & & \\ & & & 0 & & & & \\ & & & & 0 & & & \\ & & & & & 0 & & \\ & & & & & & 1 & \\ & & & & & & & -1 \end{pmatrix}_{SU(8)}$$

$$Z = \frac{C}{3} + 8D = Q - 4\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}_{SU(4)_S} - 2\begin{pmatrix} 1 & & & & & & & \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & 1 & & & & \\ & & & & 1 & & & \\ & & & & & 1 & & \\ & & & & & & -3 & \\ & & & & & & & -3 \end{pmatrix}_{SU(8)}$$

We thus remain with 2 massless SP(6) sextets and one massless singlet similar to the massless particles in (7) or (13). There, however, we had an additional global SU(2) symmetry. Note that if $\langle\phi_4^8\rangle_0 = \langle\phi_3^7\rangle_0$, there would then be a one-to-one correspondence between (7) and (25). D is then the third generator of the SU(2) symmetry and the states ψ^{3a} and ψ^{4a} form an SU(2) doublet. If $\langle\phi_4^8\rangle_0 \neq \langle\phi_3^7\rangle_0$, as is the case discussed here, then the corresponding symmetric picture must include the condensate (3a)

$$\phi^\alpha_\beta = \begin{pmatrix} 1 & & & & & & & \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & 1 & & & & \\ & & & & 1 & & & \\ & & & & & 1 & & \\ & & & & & & 1 & \\ & & & & & & & -7 \end{pmatrix}$$

in addition to $\tilde{\theta}^{\alpha\beta\gamma\delta}$ (3c) or $\theta^{\alpha\beta\gamma\delta}$ (3d).

Finally we can in this case identify the massive composites ξ^{ab} in (7) or (13) with the $SU(2)_S$ singlet states

$$\xi^{ab} = \psi^{ia}\chi_{ij}\psi^{jb} - \psi^{ib}\chi_{ij}\psi^{ja} \; ; \quad i,j = 1,2 \tag{26}$$

We remark that there are at least two scales involved in this breaking pattern. They correspond to

$$\begin{array}{c} SU(8) \\ \Big\downarrow \; \Lambda_S \\ SU(6) \\ \Big\downarrow \; \Lambda'_S \\ SP(6) \end{array} \tag{27}$$

where the first step occurs at the $SU(4)_S$ scale and the second occurs at a scale Λ'_S where the group $SU(2)_S$ becomes strong. The states ξ^{ab} have mass of order Λ'_S. Note that complimentarity doesn't require this ratio of scales to persist in the symmetric picture. It is nevertheless suggestive. It corresponds in the symmetric picture to having one scale as the binding scale of the massless or light composites and a second lower scale associated with the multi-fermion condensate.

4. Light Fermion Masses (Symmetric Picture)

So far we have kept $SU(8) \times U(1)$ strictly as a global symmetry. The only gauge symmetry $SU(4)_S$ in the model became strong at the scale Λ_S. In the following we will regard part of the global symmetry as a weak gauge symmetry (weak in the sense that $\alpha_W = g_W^2/4\pi$ is small at the scale Λ_S). This is motivated by the fact that in the real world there exist gauge symmetries with these properties. Specifically we will gauge an $SU(4)_W$ subgroup of SU(8) in the way that the 8-dimensional representation of SU(8) decomposes into a quartet (4) and antiquartet ($\bar{4}$) with respect to $SU(4)_W$. With this decomposition we are left with the anomaly free fermion spectrum $[SU(4)_S \times SU(4)_W]$

$$\chi_{ij} = (\overline{10}, 1)$$
$$\psi^{i\alpha} = \begin{cases} (4, 4) \\ (4, \bar{4}) \end{cases} \tag{28}$$

Gauging this $SU(4)_W$ subgroup breaks the SU(8) chiral symmetry explicitly, and could lead to a generation of light fermion masses, as we will show in the remainder of this section.

We will first investigate this model in the symmetric picture where this mechanism is most transparent. According to the discussion in the preceeding two sections, we will confine ourselves to the cases where the global symmetries are spontaneously broken either to SU(4) × SU(4) or SU(2) × SP(6), corresponding to the $\theta^{\alpha\beta\gamma\delta}$ and $\tilde{\theta}^{\alpha\beta\gamma\delta}$ condensates occuring in the symmetric picture.

Let us first consider the SU(4) × SU(4) case. SU(8) is spontaneously broken through the θ-condensate. The question of whether or not this breakdown affects the gauge symmetry as well has been termed the subgroup alignment problem [7]. The favored alignment corresponds to the case of a maximally unbroken gauge symmetry [7], which in our case would mean a conserved $SU(4)_W$ acting on the 4 and $\bar{4}$ of (28). The 22 massless fermions ξ^{rs} and ξ^{xr} given in (10) will still remain massless.

The remainder of this section will be devoted to the $SU(2) \times SP(6)$ case. This breakdown can occur with the θ as well as the $\tilde{\theta}$ condensate [compare (5) and (11)]. The fermions ξ^{ab} in (7) and (13) receive masses from the graphs of figs. 3 and 4, respectively. We now switch on the $SU(4)_W$ interaction. It is obvious that the spontaneous breakdown of SU(8) to $SU(2) \times SP(6)$ will induce a breakdown of the $SU(4)_W$ gauge symmetry. Following ref. 7 we remain with the symmetry

$$SU(3)_W \times U_E(1) \times U_I(1)$$

where $SU(3)_W \times U_E(1)$ is a gauge symmetry and $U_I(1)$ is an additional global symmetry: the only remaining exact global symmetry. Explicitly, E is the $SU(4)_W$ generator

$$E = \begin{pmatrix} 1 &&&&&&& \\ & -1 &&&&&& \\ && 1 &&&&& \\ &&& -1 &&&& \\ &&&& 1 &&& \\ &&&&& -1 && \\ &&&&&& -3 & \\ &&&&&&& 3 \end{pmatrix}$$

and I is (29)

$$I = \begin{pmatrix} 1 &&&&&&& \\ & -1 &&&&&& \\ && 1 &&&&& \\ &&& -1 &&&& \\ &&&& 1 &&& \\ &&&&& -1 && \\ &&&&&& 1 & \\ &&&&&&& -1 \end{pmatrix}$$

where we have used SU(8) notation. The generator Z of (25) is explicitly broken by gauging $SU(4)_W$. The quartet of $SU(4)_W$ is given by $\psi^{i\alpha}$ $(\alpha = 1,3,5,7)$ and the antiquartet by $\psi^{i\alpha}$ $(\alpha = 2,4,6,8)$. Under the symmetry $SU(3)_W \times U_E(1) \times U_I(1)$ the fundamental states of (1) transform as follows

$$\begin{pmatrix} \psi^{i1} \\ \psi^{i3} \\ \psi^{i5} \end{pmatrix} = (3,\ 1,\ 1)\ ; \qquad \begin{pmatrix} \psi^{i2} \\ \psi^{i4} \\ \psi^{i6} \end{pmatrix} = (\bar{3},\ -1,\ -1)$$

$$\psi^{i7} = (1,\ -3,\ 1)\ ; \qquad \psi^{i8} = (1,\ 3,\ -1) \tag{30}$$

where the direction of SU(3) in SU(8) has been determined by the condensate of (5). Note that the SU(2) × SP(6) × U(1) guardian symmetry is explicitly broken by the $SU(4)_W$ gauge interaction.

Let us now write down explicitly the fermion spectrum (7) in an $SU(3)_W \times U_E(1) \times U_I(1)$ notation.

$$\begin{aligned} \xi^{ab} &= (1,\ 0,\ 0) + (8,\ 0,\ 0) + (3,\ 2,\ 2) + (\bar{3},\ -2,\ -2) \\ \xi^{a7} &= (3,\ -2,\ 2) + (\bar{3},\ -4,\ 0) \\ \xi^{a8} &= (3,\ 4,\ 0) + (\bar{3},\ 2,\ -2) \\ \xi^{78} &= (1,\ 0,\ 0) \quad . \end{aligned} \tag{31}$$

The states ξ^{ab} were already massive through the processes in fig. 3 and 4. ξ^{ab} thus consists of a massive Dirac triplet, and massive Majorana singlet and octet. The inspection of the remaining thirteen states shows that they are no longer protected by the remaining SU(3) × U(1) × U(1) symmetry. They could in principle form two massive Dirac triplets and a Majorana singlet. Indeed they do receive mass due to a feed-down of the ξ^{ab} mass via the massive $SU(4)_W/SU(3)_W$ gauge bosons as displayed in figs. 5 and 6. Observe that the triplets obtain a mass of order α_W whereas the singlet receives a mass of order α_W^2. This is transparent from (31) since ξ^{78} requires two gauge bosons to change its indices from 78 to ab.

Do we expect these light triplets to be degenerate? It is clear from figs. 5 and 6 that the triplet with $E=4$, $I=0$ as well as the singlet receive their mass through the singlet and octet of ξ^{ab}, whereas the massive triplet of ξ^{ab} feeds its mass down to the triplet with charge $E = -2$ and $I = 2$.

We note that in the SP(6) symmetric limit the massive fermion bound states ξ^{ab} have mass $m_1 = 3m_{14}$ for the process in fig. 3 and $m_1 = 2m_{14}$ for fig. 4. We expect that the weak gauge interaction will not significantly change this result. Thus $m_8 \simeq m_3$ and $m_1 \simeq 3m_8$ for fig. 3 and $m_1 \simeq 2m_8$ for fig. 4. Hence we do not expect these light triplets to be degenerate.

Finally there are additional mass generation processes for the light fermions. These are given in fig. 7. They have the following simple imterpretation. Light pseudo-Goldstone bosons are produced as a result of the spontaneous breaking of the global symmetry SU(8) × U(1). These bosons appear in the channel $\theta^{\alpha\beta\gamma\delta}$ or $\tilde{\theta}^{\alpha\beta\gamma\delta}$. They have off-diagonal matrix elements between the massless and massive composite fermions. A weak exchange can then take the pseudo into the condensate. Such new graphs will always appear in any effective low energy Lagrangian which includes the composite fermions and the pseudo-Goldstone bosons.

5. Light Fermions (Broken Picture)

The case of the global SU(8) breaking down to SU(4) × SU(4) is straightforward in the broken picture. However, in complete analogy with the previous discussion in the symmetric picture (section 4) none of the massless fermions obtain a mass by gauging $SU(4)_W$. It is thus an uninteresting case. We therefore shall only present a discussion of the broken picture for the SP(6) case of section 4. In section 3 we discussed the broken picture before gauging $SU(4)_W$. The massless fermion states were given in (25) in terms of their global $SP(6) \times U(1)_D \times U(1)_Z$ quantum numbers. After gauging $SU(4)_W$ we saw in section 4 in the symmetric picture that the remaining conserved symmetry is

$$SU(3)_W \times U(1)_E \times U(1)_I \quad .$$

The same is essentially true here. The remaining conserved symmetry in the broken picture is

$$SU(2)_S \times \left[SU(3)_W \times U(1)_{\tilde{E}} \times U(1)_{\tilde{I}}\right]$$

where

$$\tilde{E} = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & -3 & \\ & & & 3 \end{pmatrix}_{SU(4)_S} + \begin{pmatrix} 1 & & & & & & & \\ & -1 & & & & & & \\ & & 1 & & & & & \\ & & & -1 & & & & \\ & & & & 1 & & & \\ & & & & & -1 & & \\ & & & & & & -3 & \\ & & & & & & & 3 \end{pmatrix}_{SU(8)}$$

$$\tilde{I} = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 1 & \\ & & & -1 \end{pmatrix}_{SU(4)_S} + \begin{pmatrix} 1 & & & & & & & \\ & -1 & & & & & & \\ & & 1 & & & & & \\ & & & -1 & & & & \\ & & & & 1 & & & \\ & & & & & -1 & & \\ & & & & & & 1 & \\ & & & & & & & -1 \end{pmatrix}_{SU(8)}$$

Note that if we ignore the $SU(4)_S$ contribution to $\tilde{E}$ and $\tilde{I}$ they are exactly equivalent to E and I defined in (29). The remaining massless fermion states (25) transform as singlets under $SU(2)_S$ and under $SU(3)_W \times U(1)_{\tilde{E}} \times U(1)_{\tilde{I}}$ as follows

$$
\left.\begin{aligned}
\psi^{3\alpha} &= (3,\ -2,\ 2)\\
\psi^{4\alpha} &= (3,\ 4,\ 0)
\end{aligned}\right\}\qquad \alpha = 1,3,5
$$

$$
\left.\begin{aligned}
\psi^{3\alpha} &= (\bar{3},\ -4,\ 0)\\
\psi^{4\alpha} &= (\bar{3},\ 2,\ -2)
\end{aligned}\right\}\qquad \alpha = 2,4,6 \tag{33}
$$

$$
\psi^{47} = (1,\ 0,\ 0) \quad .
$$

Note the one-to-one correspondence in the spectrum of massless states of (33) and (31) in the two complimentary pictures. Clearly the states $(3,-2,2)$ and $(\bar{3},2,-2)$ as well as $(3,4,0)$ and $(\bar{3},-4,0)$ can obtain Dirac masses without breaking the unbroken symmetries. The state $(1,0,0)$ can obtain a Majorana mass. We expect from our knowledge of the symmetric picture that the Dirac masses will be of order α_W and the Majorana mass of order α_W^2. This was easily seen in the symmetric picture by index counting [see the discussion following (31)]. There was, however, another way of making this observation which is equally true in both pictures. Recall from (25) that the charge Z was the guardian symmetry. This symmetry is explicitly broken by $SU(4)_W$. In fact the broken gauge generators in $SU(4)_W/SU(3)_W$ can change the value of Z by ± 16. This could easily be seen by considering for example the $W_1^7 W_2^8$ gauge mass term. We finally note that, in order to give the triplets mass, Z must change by ± 16 [see (25)] and is thus consistent with one W exchange. However the singlet state has $Z = 16$ and can only get mass to order α_W^2.

The simplest graphs are given in fig. 8. They are very complicated. For the triplets they involve five condensates, two massive strong gauge boson exchanges and a weak gauge boson exchange. In addition, since $SU(2)_S$ is unbroken, these graphs must be dressed by strong $SU(2)_S$ exchanges. Although the situation appears quite complicated, we note that there exists a simple one-to-one correspondence between the graphs in the broken and symmetric pictures. In fig. 9 we have redrawn the graphs of fig. 8. They clearly correspond to those of fig. 5. In fact given any graph in the symmetric picture, one can find the corresponding graph in the broken picture simply by extracting from the multi-fermion condensates and massless composites the two particle condensates. For example, in fig. 10 we give the corresponding graph to fig. 6a.

6. Reconstruction

The preceding analysis can be summarized most succinctly by the schematic diagram of fig. 11. We have a full global symmetry group G and we weakly gauge a subgroup $H \subset G$. G, in general, contains a maximal subgroup G_t for which there exists a solution to 't Hooft's consistency conditions and a set of massless fermions. In fact in our case, $G_t = G$. The dynamics may not favor an unbroken G_t. In that case G_t further breaks to G_D, the final guardian symmetry. The composite massless fermions of G_t consequently decompose into representations of G_D, some of which obtain mass; the others remaining massless. The gauge symmetry will in the process break down to the final unbroken subgroup H' lying in the intersection of G_D and H. This breaking can in principle involve several scales. H' will include all the known conserved gauge symmetries, i,e., Q_{EM}, $SU(3)_{COLOR}$, etc.

For example we have discussed the case where $G = G_t = SU(8) \otimes U(1)_Q$ and $H = SU(4)$. For the breaking pattern

$$G' \to G_D \equiv SP(6) \otimes SU(2) \otimes U(1)_Z$$

we found

$$H' = SU(3) \otimes U(1)_E$$

which we may try to interpret as SU(3) color and U(1) electromagnetism, where $Q_{EM} = E/6$ [see (29) and (31)]. The fermions of (31) form four irreducible representations under the extended color interaction SU(4). They are as follows: (see fig. 12)

$$\begin{array}{lll}
6: & (\xi^{13}, \xi^{15}, \xi^{35}) & \bar{D} \\
 & (\xi^{17}, \xi^{37}, \xi^{57}) & d \\
6: & (\xi^{24}, \xi^{26}, \xi^{46}) & D \\
 & (\xi^{28}, \xi^{48}, \xi^{68}) & \bar{d} \\
15 \oplus 1: & \frac{1}{\sqrt{3}}(\xi^{12} + \xi^{34} + \xi^{56}) & N \\
 & (\xi^{12}, \xi^{14}, \xi^{16}, \ldots) & Q \\
 & (\xi^{18}, \xi^{38}, \xi^{58}) & u \\
 & (\xi^{27}, \xi^{47}, \xi^{67}) & \bar{u} \\
 & \xi^{78} & \nu
\end{array} \tag{34}$$

where $\bar{D}$, D, Q, N are massive composite quarks, quaits and leptons and $\bar{d}$, d, $\bar{u}$, u, ν are massless until SU(4) extended color is turned on. Then $\bar{u}$, u, $\bar{d}$, d obtain mass of order $\alpha_c(\Lambda_S)\Lambda_S$ and ν obtains mass of order $\alpha_c^2(\Lambda_S)\Lambda_S$. Clearly this is not a realistic model. We have no electrons and moreover we cannot imbed $SU(2)_{W-S}$ into G in such a way as to have only left-handed interactions. In a realistic model we must be able to take all the low-energy fermions and think of them as massless composites which get mass via extended weak or color forces through massive composites. The major problem of reconstruction is to satisfy the constraint that all these composite states can be a realization of 't Hooft's consistency conditions. This naturally guarantees that the binding scale Λ_S can in principle be arbitrarily greater than the quark and lepton mass scales. Of course for extended weak or color forces the mass ratios are fixed in practice by the value of α_2 or α_c at Λ_S. This is in contrast to scenarios based on extended Technicolor and tumbling [8] where the ratios of scales cannot in principle be made arbitrarily large.

We have not discussed the many pseudo-Nambu-Goldstone bosons which abound in such a scheme. They will in general contribute to the low-energy phenomenology. We feel however that such a discussion will best wait until we have a more realistic model.

Finally, how heavy do we expect the massive composites D, $\bar{D}$, Q and N to be? Their mass will typically break the electroweak group $SU(2)_L \otimes U(1)_Y$. We thus expect that the constituent fermion mass is of order ~ 300 GeV corresponding to a mass of order ~ 1 TeV for these composite states.

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Figure Captions

Fig. 1. The $SU(4)_S$ - instanton process that breaks global U(8) but conserves SU(8). It annihilates eight ψ's and six χ's.

Fig. 2. The triangle graphs that give rise to the chiral anomalies involving a) three SU(8) currents; b) two SU(8) and one $U_Q(1)$ current; and c) three $U_Q(1)$ currents.

Fig. 3. The $\tilde{\theta}$ - condensate gives mass to some of the composite fermions directly.

Fig. 4. The θ - condensate combines with the instanton process to provide a mass for the composite fermions.

Fig. 5. Masses for the light composite fermions. The notation is that of equation (31). The W's are massive weak gauge bosons.

Fig. 6. Masses for the light fermions through the θ - condensate and the instanton process. The notation is that of (31) and fig. 5.

Fig. 7. Additional graphs for the masses of the light fermions. The graphs on the right-hand side show the corresponding graphs in an effective low-energy theory of fermions and pseudo-Goldstone bosons.

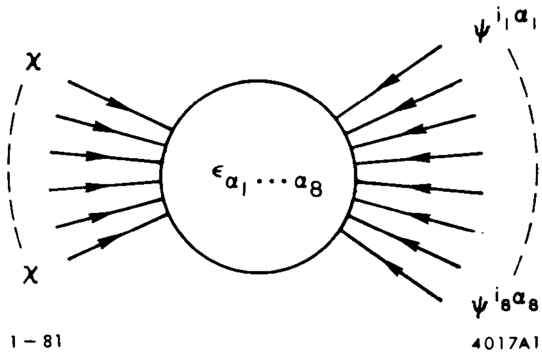
Fig. 8. The masses of the light triplets in the broken picture. W(S) denote weak(strong) massive gauge bosons. The condensates are defined in (17), (22) and (24). The notation for the light fermions refers to (33).

Fig. 9. The distortion of the graphs of fig. 8 that shows the correspondence to the symmetric picture (compare fig. 5). The three condensates $\phi^{ab}\ \omega\ \phi^{ab}$ correspond to the $\tilde{\theta}$ - condensate, and $\psi^{3\alpha}\ \phi_3^7\ (\psi^{4\alpha}\ \phi_4^8)$ correspond to the composite fermions $\xi^{7\alpha}\ (\xi^{8\alpha})$ [compare (33) and (31)].

Fig. 10. The graph in the broken picture that corresponds to fig. 6a. $\phi_4^8\ \phi_3^7\ \omega\ \phi^{cd}$ replaces θ^{87cd}.

Fig. 11. A schematic drawing of the general patterns of symmetry breaking. $G \supseteq G_t \supseteq G_D$ where G is the full global symmetry, G_t is the maximal subgroup for which 't Hooft's consistency conditions are satisfied, and G_D is the final guardian symmetry. $G \supseteq H \supseteq H'$ where H is the weak gauge group and H' is the final unbroken weak gauge symmetry. Clearly $G_D \cap H \supseteq H'$.

Fig. 12. Decomposition of $\xi^{\alpha\beta}$ with respect to the unbroken gauge group $H' = SU(3)_c \times U(1)_{E.M.}$ in the notation of equation (34).



χ
χ
$\epsilon_{\alpha_1 \cdots \alpha_8}$
$\psi^{i_1 \alpha_1}$
$\psi^{i_8 \alpha_8}$
1 − 81
4017A1

Fig. 1

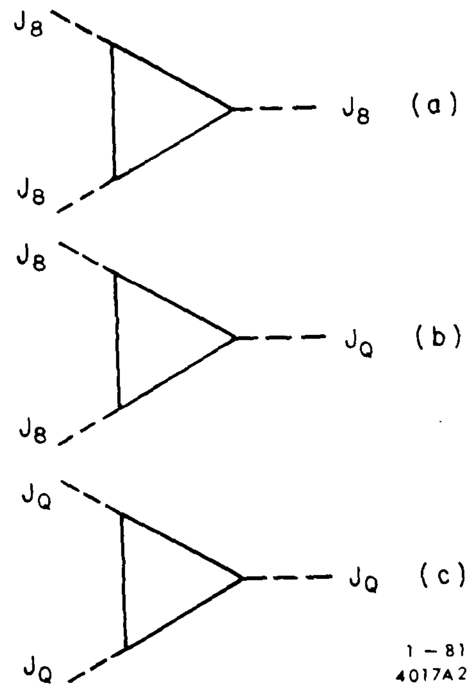


Fig. 2

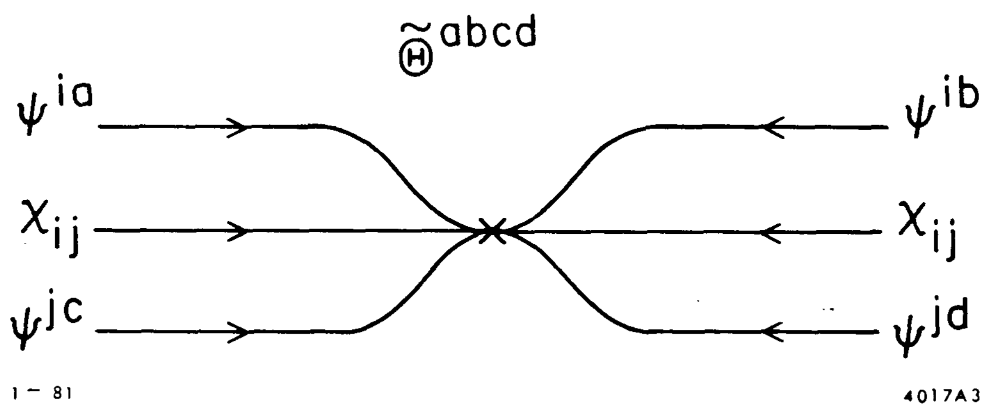
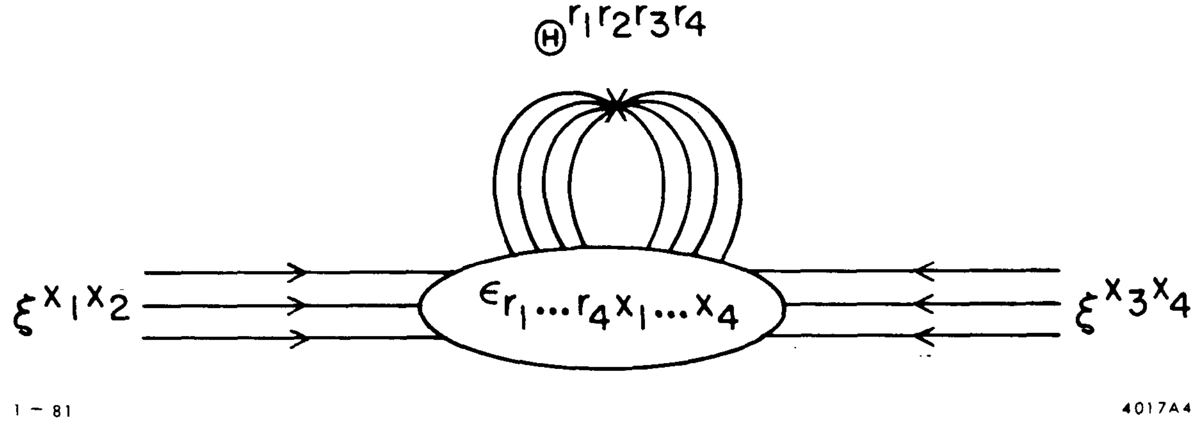
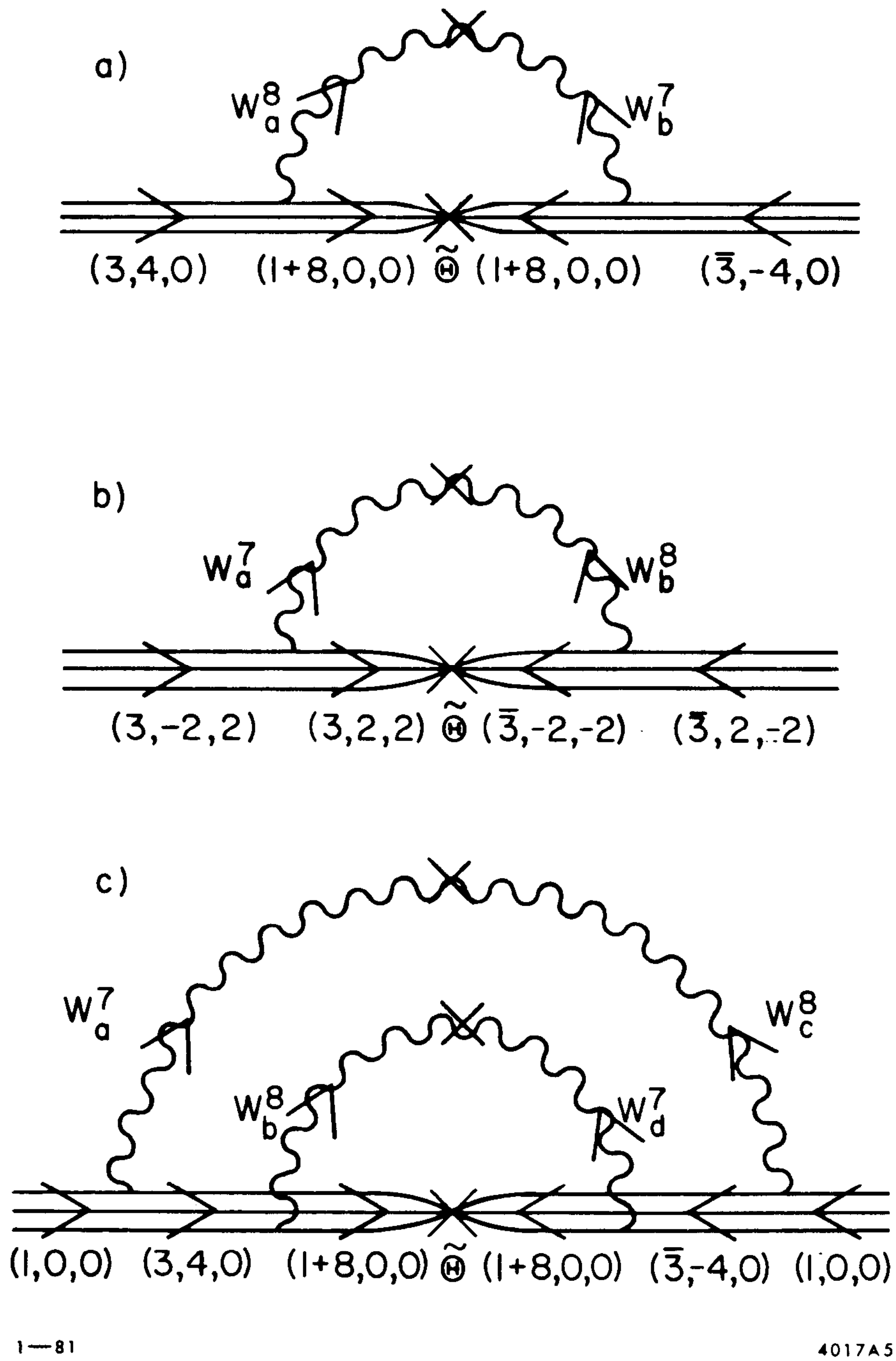

$\tilde{\Theta}^{abcd}$
ψ^{ia}
ψ^{ib}
χ_{ij}
χ_{ij}
ψ^{jc}
ψ^{jd}
1 – 81
4017A3

Fig. 3



$\Theta^{r_1 r_2 r_3 r_4}$
$\xi^{x_1 x_2}$
$\epsilon_{r_1 \dots r_4 x_1 \dots x_4}$
$\xi^{x_3 x_4}$
1 − 81
4017A4

Fig. 4



a)
W^8_a
W^7_b
(3,4,0) (1+8,0,0) Θ̃ (1+8,0,0) (3̄,-4,0)
b)
W^7_a
W^8_b
(3,-2,2) (3,2,2) Θ̃ (3̄,-2,-2) (3̄,2,-2)
c)
W^7_a
W^8_c
W^8_b
W^7_d
(1,0,0) (3,4,0) (1+8,0,0) Θ̃ (1+8,0,0) (3̄,-4,0) (1,0,0)
1—81
4017A5

Fig. 5

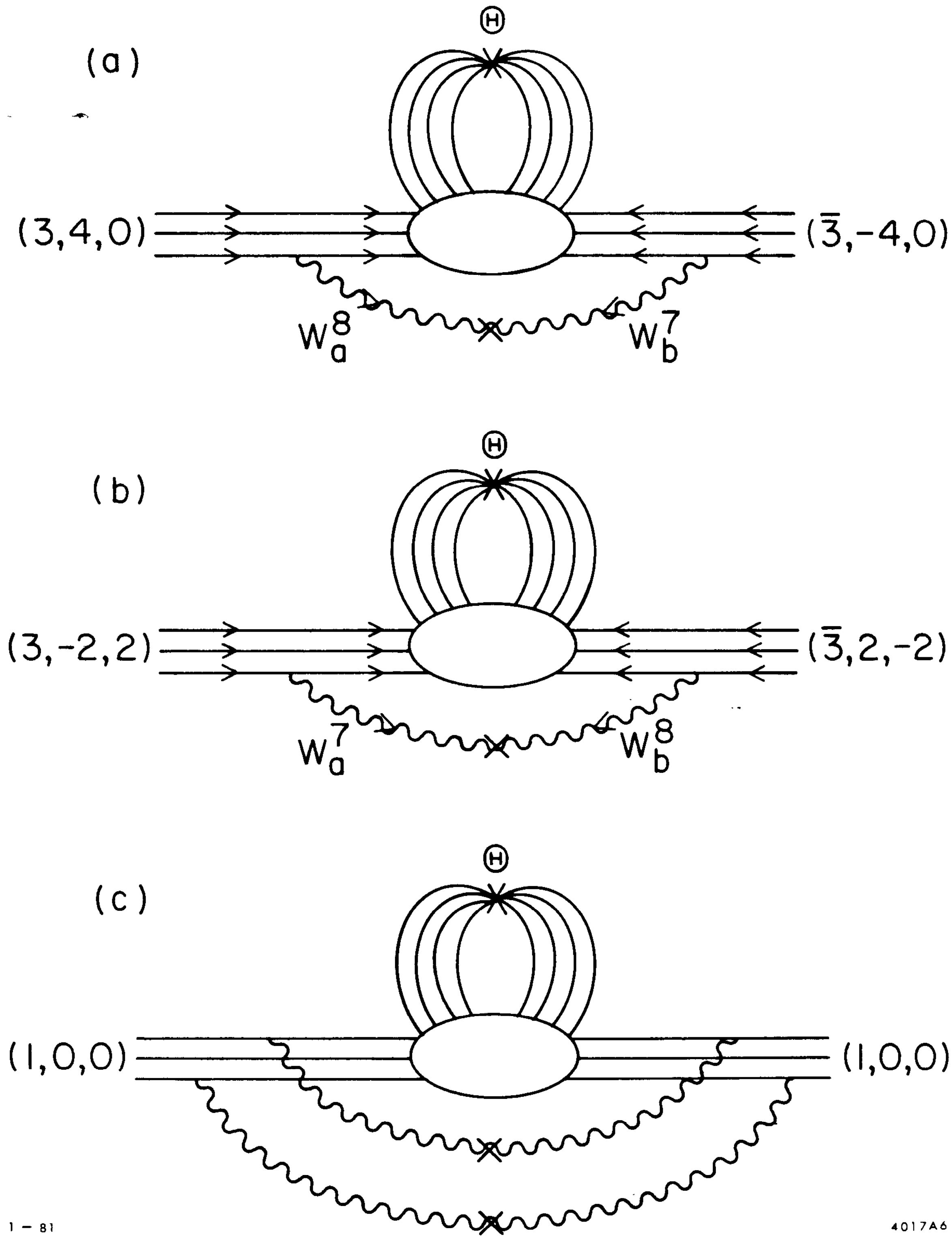


Fig. 6

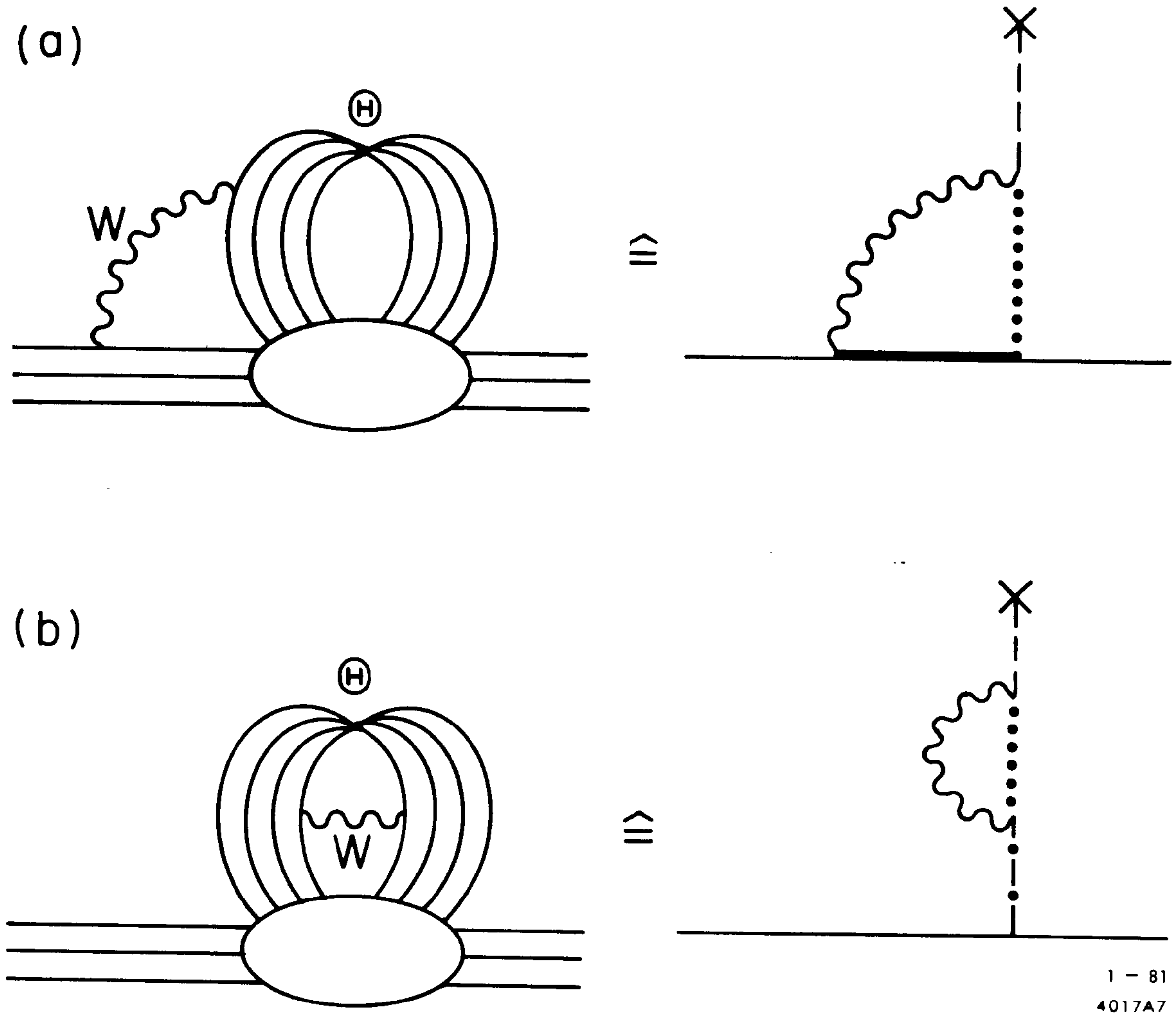


Fig. 7

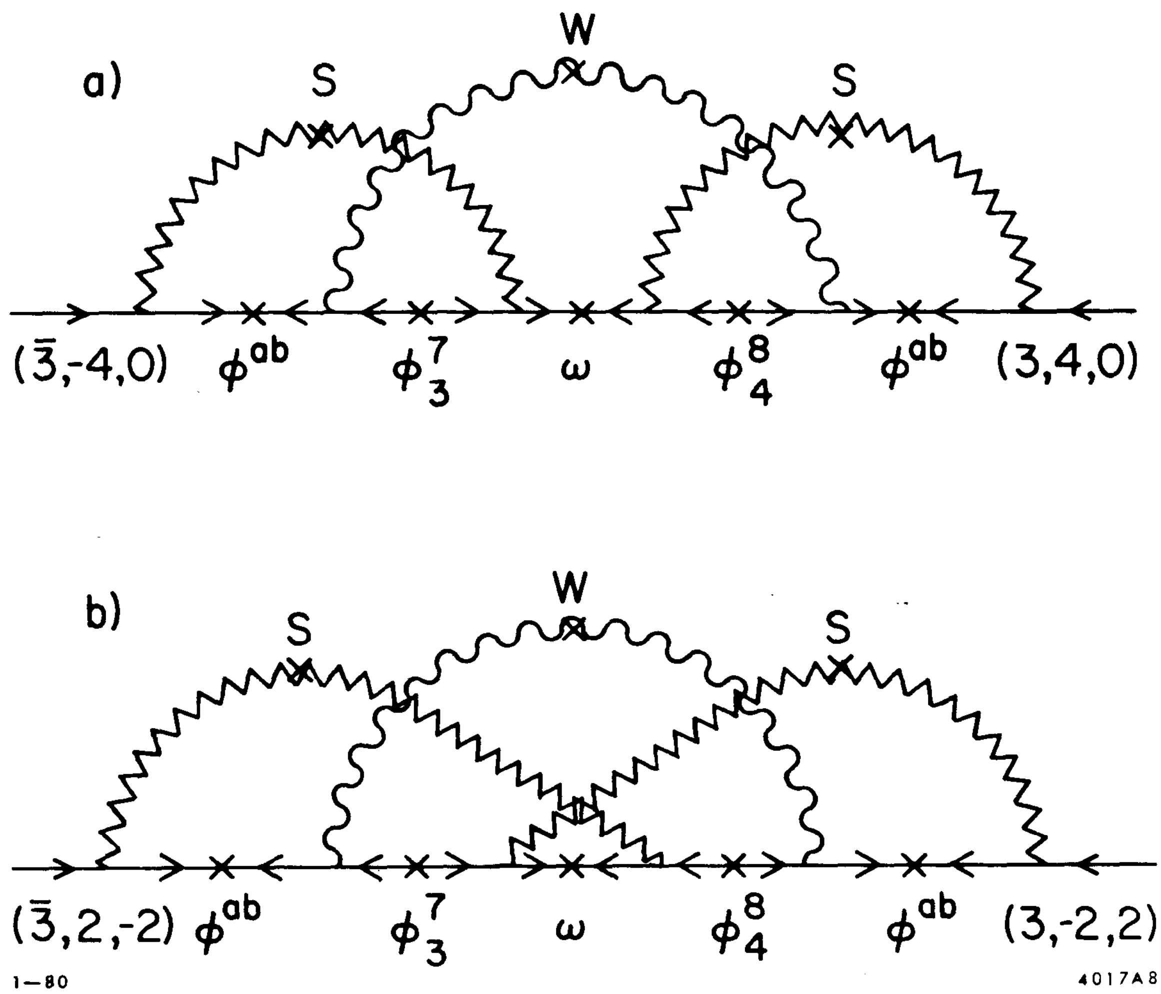
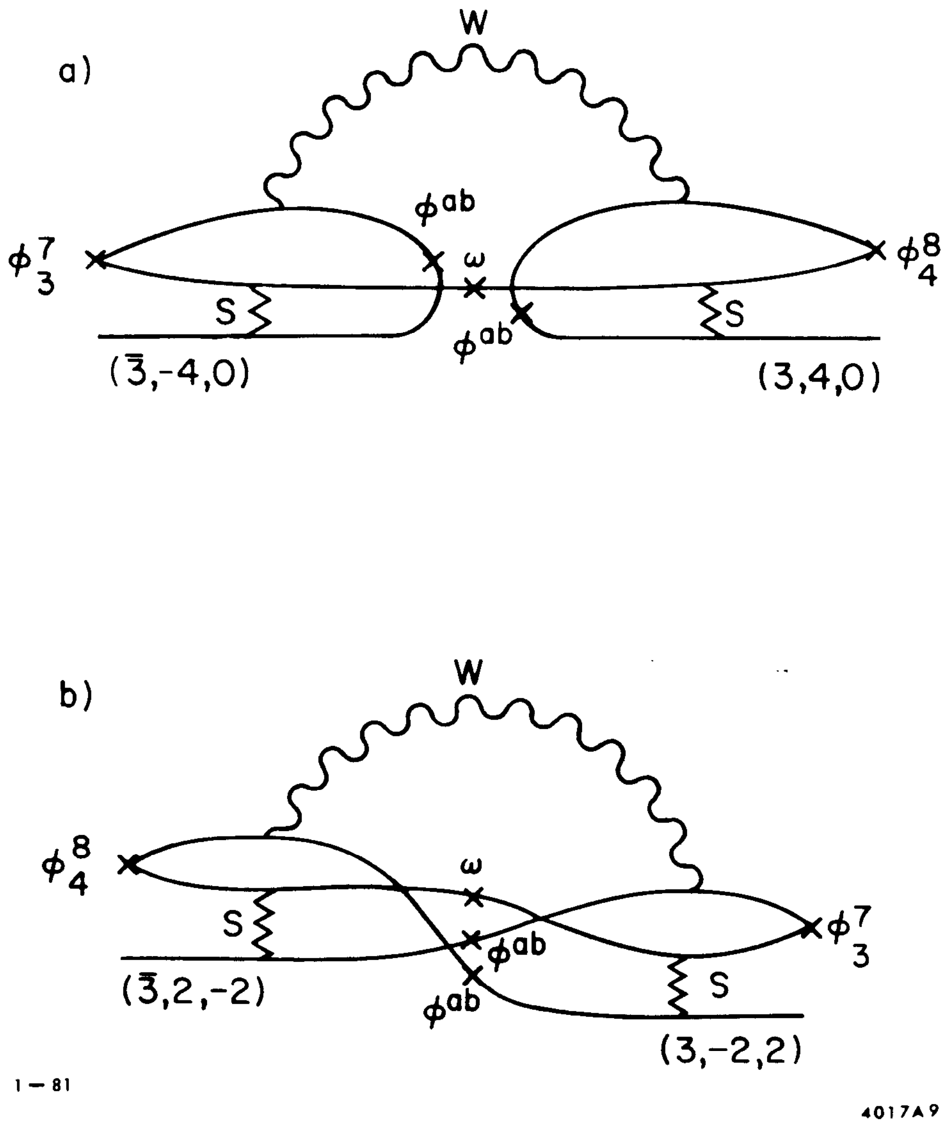
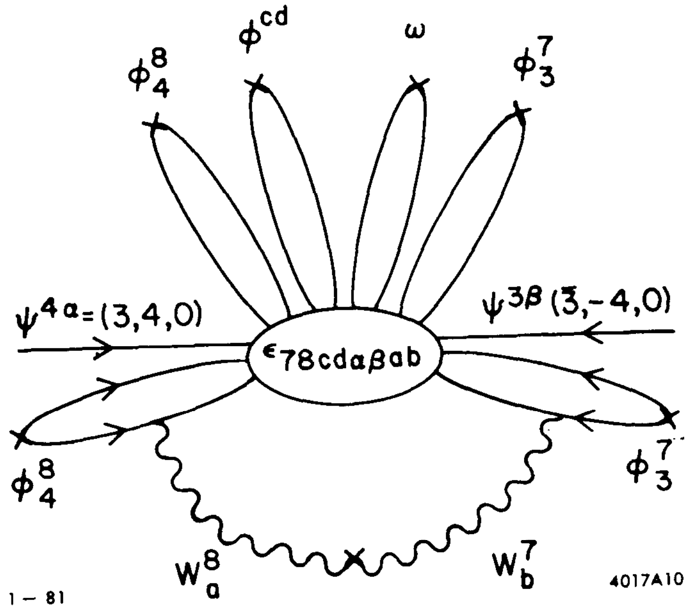

a)
W
S
S
$(\bar{3},-4,0)$
ϕ^{ab}
ϕ_3^7
ω
ϕ_4^8
ϕ^{ab}
$(3,4,0)$
b)
W
S
S
$(\bar{3},2,-2)$
ϕ^{ab}
ϕ_3^7
ω
ϕ_4^8
ϕ^{ab}
$(3,-2,2)$
1—80
4017A8

Fig. 8



a)
W
ϕ^{ab}
ϕ_3^7
ω
ϕ_4^8
S
S
ϕ^{ab}
$(\bar{3},-4,0)$
$(3,4,0)$
b)
W
ϕ_4^8
ω
S
ϕ^{ab}
ϕ_3^7
$(\bar{3},2,-2)$
S
ϕ^{ab}
$(3,-2,2)$
1 — 81
4017A9

Fig. 9



ϕ^{cd}
ω
ϕ_4^8
ϕ_3^7
$\psi^{4\alpha}=(3,4,0)$
$\psi^{3\beta}(\bar{3},-4,0)$
$\epsilon_{78cd\alpha\beta ab}$
ϕ_4^8
ϕ_3^7
W_a^8
W_b^7
1 − 81
4017A10

Fig. 10

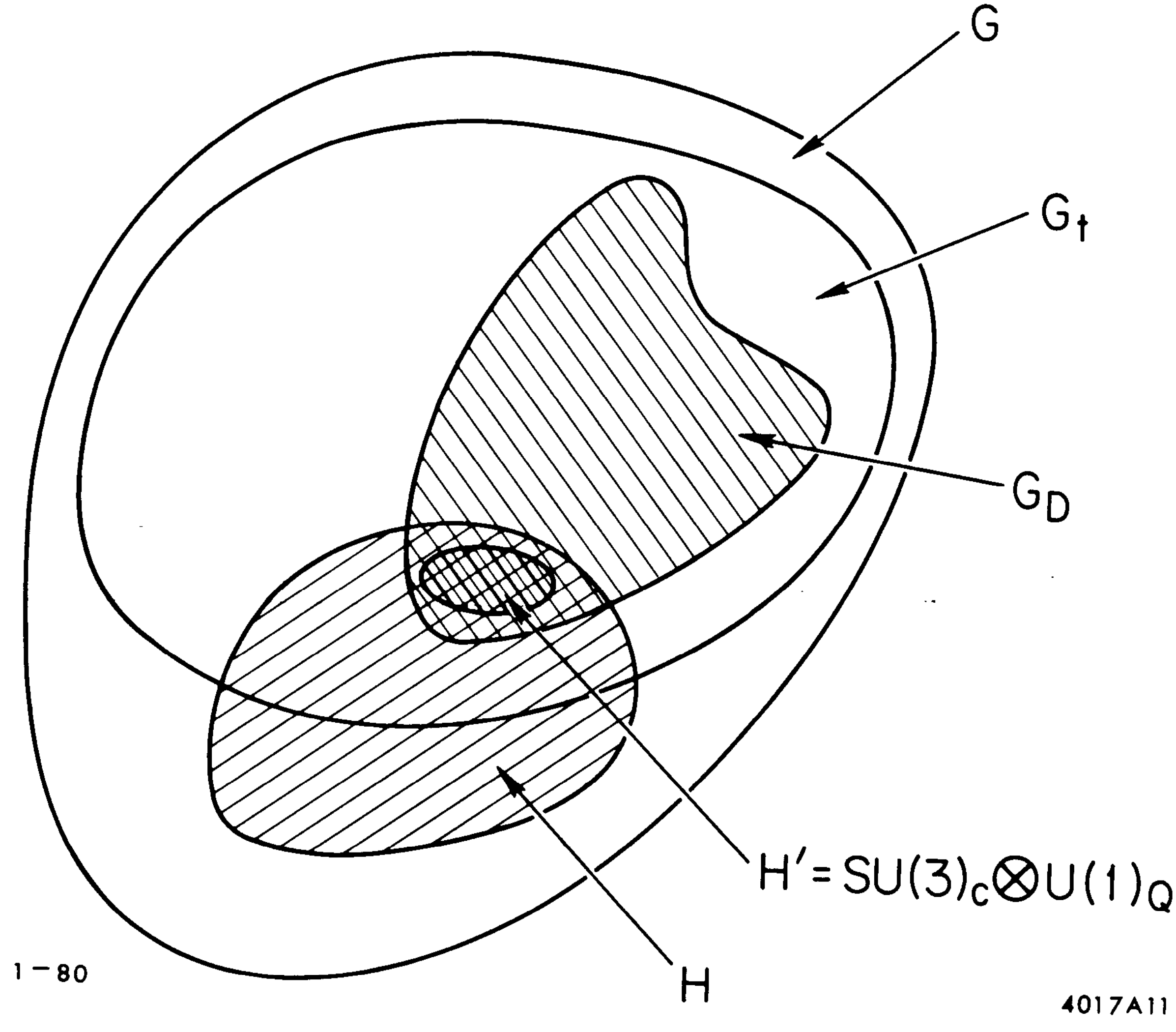


Fig. 11

$$
\begin{array}{c}
 \\ 1 \\ 3 \\ 5 \\ 7 \\ 2 \\ 4 \\ 6 \\ 8
\end{array}
\begin{array}{c}
\begin{array}{cccccccc}
1 & 3 & 5 & 7 & 2 & 4 & 6 & 8
\end{array} \\
\left(\begin{array}{cccc|cccc}
0 & \bar{D} & \bar{D} & d & & & & u \\
 & 0 & \bar{D} & d & \multicolumn{3}{c}{Q+N} & u \\
 & & 0 & d & & & & u \\
 & & & 0 & \bar{u} & \bar{u} & \bar{u} & \nu \\
\hline
 & & & & 0 & D & D & \bar{d} \\
 & & & & & 0 & D & \bar{d} \\
 & & & & & & 0 & \bar{d} \\
 & & & & & & & 0
\end{array}\right)
\end{array}
$$

1-81 4017A12

Fig. 12