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ORDER PARAMETERS IN A MODIFIED LATTICE GAUGE THEORY*

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ABSTRACT

We reexamine a modified lattice gauge theory, considered earlier by Mack and Petkova[1] and Yaffe[2], in which the 't Hooft order parameters behave in unexpected ways. We find that the situation becomes clear when one notes that the modified theory contains two pairs of dual order parameters. We discuss the order parameters, their commutation relations, their weak and strong coupling behavior, and the corresponding topological fluxes.

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I. INTRODUCTION

Recently, Mack and Petkova[1] and Yaffe[2] have considered a modified version of Wilson's lattice gauge theory.[3] Their work casts doubt on the usefulness of 't Hooft's loop operator[4] and topological fluxes[5] as order parameters.

Specifically,

(a) They show that the 't Hooft operator obeys an area law at weak coupling in the modified (Mack-Petkova, MP) theory, compared to the expected perimeter law in the Wilson theory, even though the MP theory should resemble the Wilson theory arbitrarily closely at small enough coupling.

(b) The Wilson loop should still satisfy an area law at weak coupling in the MP theory, thus violating the expected complementarity (one area law and one perimeter law) between the Wilson and 't Hooft operators.

(c) In Ref. 2 it is shown that the area law behavior of the 't Hooft loop in the MP theory need not imply the "heaviness" of 't Hooft's topological magnetic flux, violating the expected equivalence between these two order parameters.

This paper results from an attempt to understand these unusual features of the MP theory, and to clarify the nature of dual order parameters. We show that these features of the MP theory arise because it has two conserved magnetic fluxes, two kinds of 't Hooft operator, two conserved electric fluxes, and two kinds of Wilson operator, where the

Wilson theory has only one of each. At weak coupling, the 't Hooft operator of the Wilson theory should be identified with a different operator in the MP theory from that considered in Refs. 1 and 2. Further, taking into account all of the order parameters, we find complementarity between pairs of 't Hooft and Wilson operators, and equivalence between looplike order parameters and the corresponding topological fluxes.

In Sec. II we discuss the order parameters in the Wilson formulation of lattice gauge theory, and discuss the significance of the 't Hooft commutation relation. In Sec. III we describe the new order parameters for the MP theory and find their commutation relations. We then discuss the ways that different order parameters probe the physics at weak and strong coupling, and point out that the physics of the MP theory at strong coupling is surprisingly complicated. Finally, the corresponding topological fluxes are considered. In Sec. IV we discuss the implications for order parameters in other lattice and continuum gauge theories.

II. ORDER PARAMETERS IN THE WILSON THEORY

We consider a pure SU(n) lattice gauge theory defined on a d-dimensional cubic lattice. In this section we need not be concerned with boundary conditions, and may take the lattice to be infinite. We first review the terminology and formulation of lattice gauge theories. For more details, see Refs. 2, 3, or 6.

The lattice contains sites s, bonds b, plaquettes p, cubes c, etc., also designated as 0-cells, 1-cells, 2-cells, etc. Cells are defined with an orientation. There is a dual lattice, whose sites each correspond to a d-cell of the original lattice. Every set, S_r, of r-cells in the original lattice has a dual set, $*(S_r)$ of (d-r)-cells on the dual lattice, and vice versa. The boundary, ∂S_r, of the set S_r is a set of (r-1)-cells, defined in the natural way. The coboundary, ∇S_r, is a set of (r+1)-cells given by $*\partial*(S_r)$.

The dynamical variables are elements of SU(n), designated U[b], one for each bond. For any path C of bonds $b_1, b_2, \ldots, b_n$ define $U[C] = U[b_1]\ldots U[b_n]$. The action is the standard[3]

$$S(U) = \sum_p -\frac{1}{g^2} \text{Re tr } U[\partial p] \quad . \tag{1}$$

The Wilson lattice gauge theory defines

$$Z = \int \prod_b dU[b] \exp\left(-S(U)\right) \tag{2}$$

with dU the normalized Haar measure on SU(n).

This theory has a pair of dual order parameters. The first is the Wilson loop, associated with a closed path C

$$A^r[C] = \chi^r(U[C]) \quad , \tag{3}$$

where r designates a representation of SU(n), and χ^r is the character in that representation. The second is the 't Hooft operator[4], $B^\tau[Q,S]$, associated with an element τ of Z(n), a co-closed set of cubes, Q, and a set of plaquettes, S, such that $\nabla S = Q$. The replacement

$$\text{tr } U[\partial p] \to \text{tr}(\tau U[\partial p]) \text{ for } p \in S \tag{4}$$

in (1) is equivalent to inserting $B^{\tau}[Q,S]$ into the functional integral.[6] The operators $B^{\tau}[Q,S]$ and $B^{\tau}[Q,S']$ differ only by the change of variables

$$U[b] \rightarrow \tau U[b] \text{ for } b \in T \qquad (5)$$

where T is any set of bonds such that $S'-S = \nabla T$. S is therefore a Dirac surface: $B^{\tau}[Q,S]$ is a local operator associated with the surface Q.[7]

The operator product $B^{\tau}[Q,S]A^{r}[C]$ is not invariant under (5), but change by a factor $\omega(r,\tau)$ (the value of τ in representation r) for every link in $T \cap C$. The number of such links is equal to $W[C;S'-S]$, the number of times the curve C links the closed surface S'-S. We have[6]

$$B^{\tau}[Q,S']A^{r}[C] = B^{\tau}[Q,S]A^{r}[C]\omega(r,\tau)^{W[C;S'-S]} \quad . \qquad (6)$$

One may give (6) a Schrödinger interpretation when Q, S, and C lie perpendicular to a chosen time axis.[1,6] When Q and C change time orderings (and therefore operator orderings), S crosses C, leading to a phase factor. The result is the 't Hooft commutation relation[4]

$$B^{\tau}[Q,S]A^{r}[C] = A^{r}[C]B^{\tau}[Q,S]\omega(r,\tau)^{\bar{W}[C;Q]} \quad . \qquad (7)$$

In (7), A and B are to be considered as operators in the Hilbert space sense; therefore the order matters. The bar on W indicates that C and Q are to be interpreted as lying in the d-1 space dimensions only.

The existence of local operators B, on Q, and A, on C, satisfying the nonlocal commutator (6) or (7), is a nontrivial feature of gauge theories. Equations (6) or (7) may be interpreted as a conservation law. They state that one operator introduces a disturbance into the system which may be detected by a motion of the dual operator.[8] The magnitude of this disturbance, essentially τ for B^{τ} or the n-ality of r for A^{r} [thus taking values in Z(n)] is independent of whether Q and C pass very close together or remain separated by a distance large compared to the characteristic scales of the theory. The roles of source and detector are quite interchangeable; there is really only one "flux", which appears as electric or magnetic depending on which operator is regarded as the source.

The vacuum expectation values of $A^{r}[C]$ and $B^{\tau}[Q,S]$ for large curves C and surfaces Q are determined by the behavior of these fluxes. These expectation values are expected to be associated with the possible phases of the system as follows[4]:

In an ordered (Higgs) phase,

$$\begin{aligned} \ln\langle A^{r}[C]\rangle &\propto -\mathrm{Per}[C] \\ \ln\langle B^{\tau}[Q,S]\rangle &\propto -\mathrm{Area}[{}^{*}Q] \quad . \end{aligned} \tag{8a}$$

In a massless (perturbative) phase,

$$\begin{aligned} \ln\langle A^{r}[C]\rangle &\propto -\mathrm{Per}[C] \\ \ln\langle B^{\tau}[Q,S]\rangle &\propto -\mathrm{Per}[{}^{*}Q] \quad . \end{aligned} \tag{8b}$$

In a disordered (confining) phase,

$$\begin{aligned} \ln\langle A^{r}[C]\rangle &\propto -\mathrm{Area}[C] \\ \ln\langle B^{\tau}[Q,S]\rangle &\propto -\mathrm{Per}[{}^{*}Q] \quad . \end{aligned} \tag{8c}$$

For a closed surface S_r, Per[S_r] is the number of r-cells in S_r, and Area[S_r] is the number of (r+1)-cells in the smallest surface S_{r+1} such that $\nabla S_{r+1} = S_r$. We are assuming that $\tau \neq 1$ and that r is a representation of non-zero "n-ality".

Assuming a mass gap, in the above phases one operator satisfies a perimeter law and one an area law; this is "complementarity". 't Hooft has argued that (with the mass gap) simultaneous perimeter law behavior is impossible. We shall see this in Sec. IV. Simultaneous area laws were also unexpected. We shall see below why they occur in the Mack-Petkova model.

III. ORDER PARAMETERS IN THE MACK-PETKOVA THEORY

In order to define the MP theory we need a function $\eta(U)$ from SU(n) into its center Z(n). It is defined by

$$-\frac{\pi}{n} < \arg\, \mathrm{tr}\left(U\eta^{-1}(U)\right) \leq \frac{\pi}{n} \quad . \tag{9}$$

This fixes $\eta(U)$ as the element of Z(n) which is "nearest" to U. The MP theory is obtained from the Wilson theory by adding a constraint on the U-integrations,

$$\prod_{p\in\partial c} \eta(U[\partial p]) = 1 \tag{10}$$

for all cubes c. This preserves gauge invariance and, in the weak coupling limit, any configuration which would be excluded by (10) is already heavily penalized in the action, so the theories are expected to be equivalent in this limit.[1]

In the MP theory we still have $A^r[C]$ defined by (3) and $B^\tau[Q,S]$ defined by (4), and Eqs. (6) and (7) still hold. There are also new order parameters. Equation (10) may be interpreted as conservation of a flux. The flux, $\eta(U[\partial p])$, is defined on plaquettes and takes values in Z(n). The constraint (10) states that the total flux leaving any cube, and therefore any closed 3-surface, is zero. A new Wilson operator may be defined in terms of the flux through a surface,

$$A_Z^\rho[C,D] = \omega\Big(\rho, \prod_{p\in D} \eta(U[\partial p])\Big) \quad . \tag{11}$$

ρ designates a representation of Z(n), C is a closed curve, and D is any 2-surface such that $\partial D = C$. Owing to (10), $A_Z^\rho[C,D]$ depends only on C : D is a Dirac surface. When the MP theory is written as a Z(n) theory with fluctuating couplings constants, $A_Z^\rho[C,D]$ is the Wilson loop for that theory; hence the subscript Z. The other Wilson loop (3), associated with SU(n) gauge invariance, will henceforth be designated with the subscript S.

We may also form a source for the new flux by replacing the constraint (10) with the constraint

$$\prod_{p\in\partial c} \eta(U[\partial p]) = \sigma \ , \quad c \in Q \tag{12}$$

where σ is an element of Z(n) and Q is a set of cubes. Q must be co-closed or else the constraints cannot all be satisfied at once. We designate this source $\tilde{B}^\sigma[Q]$. Like $B^\tau[Q, S]$, it is a source of magnetic flux, associated with an element of Z(n) and a co-closed set of cubes. $\tilde{B}^\sigma[Q]$ was not considered in the earlier work on this theory; it is needed to complete the set of order parameters.

$A_S^\rho[C]$ commutes with $\tilde{B}^\sigma[Q]$ (for C and Q disjoint) because both are local in terms of the U's. $A_Z^\rho[C,D]$ does not:

$$\tilde{B}^\sigma[Q]A_Z^\rho[C,D'] = \tilde{B}^\sigma[Q]A_Z^\rho[C,D]\,\omega(\rho,\sigma)^{W[Q;D'-D]} \quad . \tag{13}$$

The commutator of $A_Z^\rho[C, D]$ with $B^\tau[Q,S]$ is tricky because both operators have Dirac surfaces. We should consider only products such that $D \cap S = \phi$, else the phase is somewhat arbitrary due to ordering ambiguity. This implies that we should consider only deformations of D and S such that

$$W[C ; S'-S] = W[Q;D'-D] \quad . \tag{14a}$$

Another way to see (14a) is to consider all the surfaces to be perpendicular to the time axis; under a reversal of time-ordering

$$W[C ; S'-S] = W[Q;D'-D] = \bar{W}[C ; Q] \quad . \tag{14b}$$

Under such deformations there is no change of phase when D crosses Q, as (10) still holds everywhere. When S crosses C , however, the operator (11) is not invariant under the change of variables (5). It follows that

$$B^\tau[Q,S']A_Z^\rho[C , D'] = B^\tau[Q,S]A_Z^\rho[C , D]\omega(\rho,\tau)^{W[C ; S'-S]} \quad . \tag{15}$$

The operator $B^\tau[Q,S]$ thus creates <u>both</u> "SU(n)" and "Z(n)" magnetic flux. We can define an operator $B_S^\tau[Q,S]$ which creates only SU(n) flux, if we twist the action by τ [Eq. (4)] and the constraint by τ^{-1} [Eq. (12)

with $\sigma = \tau^{-1}$]. $B_S^\tau[Q,S]$ can be considered as the product $B^\tau[Q,S]\tilde{B}^{\tau^{-1}}[Q]$. Designating $\tilde{B}^\tau[Q]$ as $B_Z^\tau[Q]$, we have[9]

$$B_a^\tau[Q]A_b^r[C] = A_b^r[C]B_a^\tau[Q]\,\omega(r,\tau)^{\delta_{ab}\bar{W}[C;Q]} \qquad (16)$$

where a and b take values "S" or "Z", r refers to either a representation of SU(n) or Z(n), depending on b, and we have dropped explicit reference to the Dirac surfaces. We see that the MP model contains two pairs of dual order parameters. Note that both the "SU(n)" and "Z(n)" fluxes take their values in Z(n).

The constraint (10) forces the action in the presence of $B^\tau[Q,S]$, Eq. (4), to be large on a set of size at least Area[*Q]. At sufficiently weak coupling one can then prove an area law for this operator.[1,2] The same proof applies to $B_Z^\tau[Q]$. The expectation value of the operator $B_S^\tau[Q,S]$, with both action and constraint twisted, evades this. In fact, any configuration which gives a significant contribution to the 't Hooft operator in the weak-coupling Wilson theory, also contributes to $B_S^\tau[Q,S]$ in the MP theory. We conclude that B_S^τ, not the operator B^τ, is the operator in the MP model which corresponds to the 't Hooft operator in the Wilson theory.

In all, we expect the following at weak coupling:

$$\ln\langle A_S^r[C]\rangle \propto -\mathrm{Area}[C] \qquad (17a)$$

$$\ln\langle B_S^\tau[Q,S]\rangle \propto -\mathrm{Per}[{}^*Q] \qquad (17b)$$

$$\ln\langle A_Z^\rho[C',D]\rangle \propto -\mathrm{Per}[C'] \qquad (17c)$$

$$\ln\langle B_Z^\tau[Q']\rangle \propto -\mathrm{Area}[{}^*Q'] \;. \qquad (17d)$$

The expectation value of a product of different operators from (17a-d) will satisfy an area law if any factor does. Equations (17a,b) follow from the expected behavior of the corresponding operators in the Wilson theory. Equation (17d) is proven as in Refs. 1 and 2, and (17c) follows from the same expansion[1,2] used for (17d), plus a Peierls argument.[10] Equations (17) reflect the fact that at weak coupling, Z(n) electric flux is shielded and Z(n) magnetic flux is confined, and vice versa for the SU(n) fluxes. We conclude that there is complementarity separately between the "SU(n)" order parameters, and the "Z(n)" order parameters, and that there is an operator, B_S^τ, which probes the same physics as the 't Hooft operator in the Wilson theory.

It is interesting to note that if we take a model which interpolates between the Wilson and MP theories, wherein (10) is replaced in the functional integral by a function of $\prod_{p \in \partial c} \eta(U[\partial p])$ which is highly peaked at the value 1, there is only one pair of order parameters. As soon as (10) is less than exact, $A_Z^\rho[C,D]$ is no longer independent of D; D is then a real, not a Dirac, surface, and the operator satisfies an area law due to purely local effects. Further, it now becomes possible to shield $B_Z^\tau[Q]$; it is easy to prove that $B_Z^\tau[Q]$ will now always satisfy a perimeter law, and the proof is purely local, involving only configurations within one or two lattice spacings of Q. Neither operator, then, probes the vacuum structure. This is an illustration of the point that these Z(n)-valued order parameters defined on surfaces always come (and go) in pairs.

The commutation relation (16) does not uniquely identify B_Z with A_Z and B_S with A_S. Equation (16) is unchanged, for example, if we replace B_S^τ with $B_S^\tau B_Z^\tau$ and A_Z^ρ with $A_Z^\rho A_S^r$, with n-ality(r) = - n-ality(ρ).

A more general statement of the expected complementarity is the following, which assumes a mass gap: A magnetic (electric) operator will satisfy an area law if and only if it has a nontrivial commutator with some electric (magnetic) operator that satisfies a perimeter law. To apply the "only if" half of this we need to assume that we have identified all of the conserved fluxes; to apply the "if" half we do not. More will be said about this in Sec. IV.

At weak coupling, B_S happens to probe the same physics as A_S, and B_Z as A_Z, but at strong coupling there is a different alignment. The infinite coupling Wilson theory is trivial: it factorizes into separate U-integrations. In the MP theory, these integrations are still constrained and coupled by (10). Thus, for example, the Osterwalder-Seiler cluster expansion[11] does not follow, since integrals over disjoint sets of bonds do not factor. As a result, there is no immediate proof of analyticity at large g^2, or even of a mass gap. The constraint (10) has one simple property: if it is satisfied by a configuration U[b] it is satisfied by $\tau[b]U[b]$ for an arbitrary Z(n) valued function $\tau[b]$. Equation (10) is thus a constraint only on the cosets

$$\hat{U}[b] = U[b]Z(n) \quad . \tag{18}$$

Using this property one may prove a perimeter law for $B_S^\tau B_Z^\tau = B^\tau$, and an area law for any electric operator with which it does not commute (one separates the $\tau[b]$ integration and uses Griffiths inequalities to relate this to a Z(n) gauge theory[1]).

The operators $B_Z^\tau[Q]$ and $A_S^r[C]A_Z^\rho[C,D]$ with n-ality(r) = - n-ality(ρ) probe different physics. B_Z^τ, given by (12), inserts a twist into the <u>coset</u> variables $\hat{U}[b]$. To the extent that the constraint (10) permits this twist to spread out, $\langle B_Z^\tau \rangle$ will be large and $\langle A_S^r A_Z^\rho \rangle$ will be small. It appears that any of the combinations (8a-c) might arise. Note that any of these combinations, taken with the expectation values found for the operators that probe the Z(n) physics, satisfies the generalized complementarity. We will not pursue this further, as the MP model at strong coupling may be as difficult as the weak coupling gauge theory in which our interest really lies.

The topological fluxes[5] which correspond to the various order parameters may be developed in a similar way; we shall summarize the result. In an MP theory in periodic spacetime, we may form a sourceless SU(n) magnetic flux σ in direction (jk) by taking $B_S^\sigma[Q,S]$ with S orthogonal to the j and k directions, and expanding S until the opposite edges meet across spacetime. We then have $Q = \nabla S = 0$. We may project out a sourceless SU(n) electric flux ρ_i by inserting the projection operator

$$P(\vec{\rho}) = \prod_i \left(\frac{1}{n} \sum_{\sigma_i} \omega(\rho_i, \sigma_i) G_i(\sigma_i) \right) \tag{19}$$

into the functional integral, where i runs over spatial directions, ρ_i is a representation of Z(n), the sum runs over Z(n), and $G_i(\sigma_i)$ is a gauge transformation aperiodic by σ_i in direction i. $P(\vec{\rho})$ commutes with all Z(n) Wilson loops, as the latter are products of local gauge invariant quantities; it therefore projects only according to SU(n) flux.

We may also form a sourceless Z(n) electric flux, by inserting $A_Z^\rho[C,D]$ into the functional integral, with D spanning spacetime in the

(0i) direction and $C = \partial D = 0$. Finally, we may project for sourceless Z(n) magnetic flux σ in direction (jk) by inserting the constraint

$$\delta\left(\prod_{p\in D_{jk}} \eta(U[\partial p])\ ,\ \sigma\right) \tag{20}$$

into the functional integral, where D_{jk} is a closed set of plaquettes spanning spacetime in the (jk) direction. Recalling (11), we see that Z(n) electric and magnetic flux, like the SU(n) fluxes, differ only by a 90° Euclidean rotation plus a Fourier transform with respect to Z(n).

The full expression for the flux free energy is

$$\exp - F(\rho_{Si},\sigma_{Sjk},\rho_{Zi},\sigma_{Zjk})$$

$$= \prod_m \left(\frac{1}{n^2} \sum_{\sigma_{S0m}} \sum_{\sigma_{Z0m}} \omega(\rho_{Sm},\sigma_{S0m})\omega(\rho_{Zm},\sigma_{Z0m})\right) Z(\sigma_{S\mu\nu},\sigma_{Z\mu\nu}) \tag{21}$$

where

$$Z(\sigma_{S\mu\nu},\sigma_{Z\mu\nu}) = \int \prod_b dU[b] \prod_c \delta\left(\prod_{p\in\partial c} \eta(U[\partial p]), 1\right)$$

$$\prod_{\mu>\nu} \delta\left(\prod_{p\in D_{\mu\nu}} \eta(U[\partial p]), \sigma_{Z\mu\nu}(\sigma_{S\mu\nu})^{-1}\right)$$

$$\exp \frac{1}{g^2} \sum_p \mathrm{Re\ tr}\left(U[\partial p] \prod_{\mu'>\nu'} (\sigma_{S\mu'\nu'})^{S_{\mu'\nu'}[p]}\right) . \tag{22}$$

Here the $D_{\mu\nu}$ are arbitrary closed sets of plaquettes spanning spacetime in the $(\mu\nu)$ direction and the $S_{\mu'\nu'}$ are arbitrary co-closed sets of plaquettes spanning spacetime orthogonal to the $(\mu'\nu')$ direction.

$S_{\mu'\nu'}[p]$ is the characteristic function

$$S_{\mu'\nu'}[p] = \begin{cases} 1 \text{ if } p \in S_{\mu'\nu'} \\ -1 \text{ if } -p \in S_{\mu'\nu'} \\ 0 \text{ otherwise.} \end{cases} \tag{23}$$

To illustrate the definitions, note that

$$\sum_{p \in D_{\mu\nu}} S_{\mu'\nu'}[p] = \delta_{\mu\mu'}\delta_{\nu\nu'} - \delta_{\mu\nu'}\delta_{\nu\mu'} \quad . \tag{24}$$

The appearance of $(\sigma_{S\mu\nu})^{-1}$ in the second delta function of (22) requires some explanation. In the space-time delta functions, it appears because the change of variables which carries the twist from G_i [in (19)] into the action also changes $A_Z^{\rho_i}[C,D_{0i}]$ by a phase. In the space-space delta functions it appears because when we expand $B_S^{\sigma}[Q,S]$ to form the sourceless flux in the (jk) direction, Q passes through D_{jk} once and the constraint (20) then flips because the constraint (12) is twisted on Q.

The covariance of the twisted functional integral (22) under 90° Euclidean rotations implies, as when there is only one pair of fluxes,[5] a duality equation for the flux free energy (21).

When any of the σ_{Zjk} is not the identity, the constraints force the action to be large on at least a co-closed set of plaquettes spanning space orthogonal to the (jk) direction. At sufficiently weak coupling, one can then prove by using the expansion of Ref. 2 that

$$F(\rho_{Si},\sigma_{Sjk},\rho_{Zi},\sigma_{Zjk}) - F(1,1,1,1) \to \infty \quad \text{for} \quad \sigma_{Zjk} \neq 1 \tag{25}$$

when the dimensions of the lattice are all taken to infinity together (the particular case treated in Ref. 2 is seen to be $\sigma_{Sjk} = \sigma_{Zjk} \neq 1$). Z(n) magnetic flux is therefore "heavy". It follows by Fourier transformation that the flux free energy is independent of ρ_{Zi} in the infinite volume limit: Z(n) electric flux is "light". Finally, $F(\rho_i, \sigma_{jk}, 1, 1)$ is seen to be equivalent, at weak coupling, to $F(\rho_i, \sigma_{jk})$ in the Wilson theory, in the sense that any configuration contributing to the latter which is not heavily penalized in the action is permitted by the constraints to contribute to the former as well. These conclusions are entirely parallel to those for the order parameters, Eq. (17). A similar comparison can be made at strong coupling.

IV. CONCLUSIONS

We should note that we agree entirely with the physical conclusions of Refs. 1 and 2, that the important physics lies in the smooth spreading of SU(n) magnetic flux, not in the more often and more easily studied Z(n) variables. In fact, one could replace (10) with the stronger constraint

$$\eta(U[\partial p]) = 1, \quad \text{all } p \tag{26}$$

thus eliminating not just Z(n) monopoles but Z(n) flux as well, and expect the same weak coupling limit. (One can check that the theory defined by (26) has only one nontrivial 't Hooft operator, with both action and constraint twisted.)

Our goal here has been a deeper understanding of dual order parameters in gauge theories. The earlier work concluded that different order parameters in the MP model probe magnetic flux spreading in different ways. We see that this is possible because there are two magnetic fluxes, and different order parameters probe different combinations of the two. In particular, there is still a 't Hooft operator, $B_S^T[Q,S]$, which probes the physics of interest.

The lesson of the MP model is that the analysis of the order parameters becomes more complicated when there are new conserved quantities. This is just a special case of the general statement that, in any system, if one neglects any conserved quantity, one will underestimate the complexity of the phase structure. Fortunately, several statements may be made about general gauge theories without assuming that there are no unknown conserved quantities.

First, the topological fluxes defined by 't Hooft are exactly conserved. Therefore, any operator which creates a "heavy" 't Hooft flux will satisfy an area law, regardless of what other fluxes it creates. The converse need not be true: an operator may create only light 't Hooft fluxes, but a heavy unknown flux. Thus, to prove confinement it would be sufficient, but not necessary, to prove heaviness of 't Hooft's electric flux.

Second, the duality equation of 't Hooft is exact. It follows from the first point that if the 't Hooft operator satisfies a perimeter law, 't Hooft's magnetic flux is light (assuming, of course, a mass gap, so that fluxes must be light or heavy). From the duality equation it follows that electric flux is heavy, and that the Wilson loop satisfies an area law.

Thus, simultaneous perimeter law is ruled out, whereas to rule out simultaneous area law, we would first have to assume the absence of any unknown conserved flux.

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References and Footnotes

1. G. Mack and V. B. Petkova, Ann. Phys. (N.Y.) 123, 442 (1979).
2. L. Yaffe, Phys. Rev. D21, 1574 (1980).
3. K. Wilson, Phys. Rev. D10, 2445 (1975).
4. G. 't Hooft, Nucl. Phys. B138, 1 (1978).
5. G. 't Hooft, Nucl. Phys. B153, 141 (1979).
6. A. Ukawa, P. Windey, and A. Guth, Phys. Rev. D21, 1013 (1980).
7. Q is of dimension d-3: a point in three dimensions and a loop in four.
8. This motion is described in Ref. 4.
9. $B_Z^\tau[Q]$ does not have a simple Schrödinger interpretation, as it is not defined at a single time but is mixed up with the transfer matrix. Equation (16) should be interpreted not in the Schrödinger sense, but in the sense of the net phase picked up in the 't Hooft motion.[8] This is not disturbing: we may smear out any order parameter over a small distance in spacetime, destroying the Schrödinger interpretation, and it will still probe the vacuum in the same way.
10. R. Griffiths, in Phase Transitions and Critical Phenomena, ed. by C. Domb and M. Green (Academic, New York, 1972), Vol. I.
11. K. Osterwalder and E. Seiler, Ann. Phys. (N.Y.) 110, 440 (1978).