SLAC-PUB-2654
November 1980
(T/E)

PHASE TRANSITION IN YANG-MILLS THEORY FOR θ NEAR π*

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ABSTRACT

We infer the existence of a phase transition in pure non-Abelian gauge theories at zero temperature for θ near π. Our analysis assumes that instantons are responsible for the onset of the deviation from perturbative behavior in the theory. We show that this leads to a weak coupling zero of the β-function for θ near π, implying the loss of confinement.

Submitted to Physical Review D
(Comments and Addenda)

*Work supported by the Department of Energy, contract DE-AC03-76SF00515.

From topological arguments 't Hooft[1] and Mandelstam[2] have enumerated the possible phases of the $SU(2)/Z_2$ Yang-Mills gauge theory: (1) a confining phase; (2) a superconducting phase (dual to the confining phase); (3) a compact QED phase, and (4) a Coulomb phase. The lattice version of this theory strongly indicates that the theory confines and has no phase transition for any value of its gauge coupling.[3] The lattice gauge theory, though, corresponds to the $\theta = 0$ version of the continuum theory. The Yang-Mills theory is actually a class of theories characterized by a gauge angle θ,[4,5] with period 2π, which acts like an additional coupling in the Lagrangian,

$$\frac{\theta}{16\pi^2} \mathrm{tr}\, F_{\mu\nu}\tilde{F}^{\mu\nu} = \frac{\theta}{8\pi^2} \mathrm{tr}\, \vec{E} \cdot \vec{B} \tag{1}$$

As a function of θ, the theory can exhibit different kinds of behavior; for example, for $\theta \neq 0$ or π, there is CP violation. We will give evidence to indicate that there is phase structure in the Yang-Mills theory for θ greater than a critical value. Our analysis indicates that for θ near π the asymptotically free phase has a small infrared fixed point. It is possible that this fixed point separates this phase, which is a weak coupling Coulombic phase, from a superconducting phase.

Our analysis is based on the assumption that instanton effects[6,7] are responsible for the onset of the deviation from perturbative behavior at weak coupling.[8] We will compute the effects of instantons on coupling renormalization for arbitrary θ. Because of infrared problems associated with instanton calculations we consider the theory at high temperature. We will use temperature as a scale with respect to which the effective coupling is determined. From the change of this effective coupling with scale, the β-function as a function of the effective coupling will be determined.

The behavior of the $\theta = 0$ finite temperature theory is known from the lattice theory.[9,10] At high temperature the theory is in a (Coulomb) plasma phase, and as the temperature is reduced reaches a phase boundary at a point that depends on g. On the other side of this boundary the theory is in the confining phase.

In the continuum theory at finite temperature the momentum-dependent effective coupling is also temperature dependent, $\bar{g}^2(Q^2/\Lambda^2, T^2/\Lambda^2)$. As $Q^2 \to 0$, for $T^2 >> \Lambda^2$, this coupling does not continue to increase, but becomes independent of Q^2,

$$\bar{g}^2\left(\frac{Q^2}{\Lambda^2}, \frac{T^2}{\Lambda^2}\right) \to \bar{g}^2\left(\frac{Q^2 \sim T^2}{\Lambda^2}, \frac{T^2}{\Lambda^2}\right) \equiv \bar{g}^2\left(\frac{T^2}{\Lambda'^2}\right). \quad (2)$$

Physically, this is because quantum fluctuations with energy that is small compared to the temperature are washed out by the thermal fluctuations, even if the low energy fluctuations have large coupling. This implies that in the sum over all energies in Feynman graphs, the low energy contributions are dominated by thermal fluctuations rather than quantum fluctuations with large coupling. We therefore assume that at high temperature the semiclassical, $\hbar$, approximation, is valid. The temperature-dependent effective coupling can then be computed from perturbative and instanton fluctuations.

Let us first consider the Euclidean theory and then include the modifications due to the periodic boundary conditions. The instanton is a nontrivial minimum of the classical action with $\nu = 1$, which is approximately the field of a 4-D magnetic dipole,[7]

$$A_\mu(x) = 2\rho^2 \bar{\eta}^a_{\mu\nu} \Omega \frac{\lambda^a}{2} \Omega^\dagger \frac{(x-X)_\nu}{(x-X)^2} \frac{1}{(x-X)^2 + \rho^2} \simeq M_{\mu\nu} \frac{(x-X)_\nu}{(x-X)^4} \quad . \tag{3}$$

Although $M_{\mu\nu}$ is antiself-dual, $F_{\mu\nu}$ is self-dual. For an anti-instanton with $\nu = -1$, the duality properties are reversed. The instanton interaction energy with a field is proportional to $M_{\mu\nu}F_{\mu\nu}$, so instantons interact with anti-instantons through a dipole-dipole interaction, but do not interact with other instantons to the same order. The grand partition function for this dipole plasma can be expressed as a field theory with Lagrangian,[11]

$$\begin{aligned} \mathscr{L}_{\text{eff}} &= \frac{1}{2g^2} \text{tr } F^2_{\mu\nu} - \int \frac{d\rho}{\rho^5} \lambda(\rho) \\ &\times \int d\Omega \left[e^{i\theta} \exp\left\{ \left(\frac{2\pi}{g}\right)^2 \text{tr } M_{\mu\nu}F_{\mu\nu} \right\} + e^{-i\theta} \exp\left\{ \left(\frac{2\pi}{g}\right)^2 \text{tr } \bar{M}_{\mu\nu}F_{\mu\nu} \right\} \right] , \end{aligned} \tag{4}$$

where $\lambda(\rho)$ is the instanton density. This quantum field theory is to be integrated over all gauge fields other than instantons.

For finite temperature $\mathscr{L}_{\text{eff}} \to \mathscr{H}_{\text{eff}}$. This effective field theory is modified by periodic boundary conditions, and by an explicit temperature dependence in λ. The dipolar interaction between widely separated instantons and anti-instantons is preserved at finite temperature.[12] At high temperature,[12]

$$\lambda(\rho,T) \simeq \lambda(\rho) \exp\left\{ -\frac{4}{3}(\pi\rho T)^2 \right\} \quad . \tag{5}$$

The free energy of the gauge theory at high temperature is that of a gluon plasma in the background field of correlated topological fluctuations. The net effect of this background field is to modify the dielectric properties of the gluon plasma. The leading contribution comes from expanding the exponentials in $\mathscr{L}_{\rm eff}$ in Eq. (4) in powers of $F_{\mu\nu}$. The $\mathcal{O}(\hbar)$ high temperature effective Hamiltonian density appropriate for weak slowly varying fields is,

$$\mathscr{H}_{\rm eff} \simeq \frac{\varepsilon(T)}{2\bar{g}^{2}(T)} \, {\rm tr}\, \vec{E}^{2} + \frac{{\rm tr}\, \vec{B}^{2}}{2\bar{g}^{2}(T)\mu(T)} + \frac{T^{2}}{3} \, {\rm tr}\, A_0^{2} - \frac{\pi^{2}}{15} T^{4} - \cos\theta \int_0^{\infty} \frac{d\rho}{\rho^{5}} \, \lambda(\rho,T) \quad , \tag{6}$$

where

$$\varepsilon = \frac{1}{\mu} \simeq 1 - \cos\theta \; \eta \quad , \tag{7}$$

and where

$$\eta(T) = \frac{4\pi^{2}}{3} \int \frac{d\rho}{\rho} \, \frac{8\pi^{2}}{g^{2}} \, \lambda(\rho,T) \quad . \tag{8}$$

(We have neglected a $\sin\theta \; {\rm tr}\, \vec{E}\cdot\vec{B}$ term which is irrelevant for the perturbative analysis to be considered.) The instanton effects have renormalized the gauge field coupling strength; the temperature-dependent effective coupling becomes

$$g^{2}_{\rm eff}(T) \equiv \bar{g}^{2}(T)\mu(T) = \frac{1}{\frac{11}{12\pi^{2}} \ln \frac{T}{\Lambda}} \; \frac{1}{1-\cos\theta \; \eta(T)} \simeq \frac{1+\cos\theta \; \eta(T)}{\frac{11}{12\pi^{2}} \ln \frac{T}{\Lambda}} \quad . \tag{9}$$

This second form is of course valid for $\eta \ll 1$, although a more sophisticated analysis[7] shows it is in fact a better approximation for larger η (but still $\lesssim 1$). This formula implies that dimensionless temperature-dependent functions have the temperature dependence,

$$f\left(\bar{g}^2\left(\frac{T}{\Lambda}\right), \frac{\Lambda}{T}\right) \simeq f\left(g^2_{eff}\left(\frac{T}{\Lambda}\right), 0\right) \quad , \tag{10}$$

valid for $\mathcal{O}(\hbar)$. $\eta(T)$ has the approximate temperature dependence,

$$\left(\frac{\Lambda}{T}\right)^{11} \quad \left(\ell n \frac{T}{\Lambda}\right)^5 \quad , \tag{11}$$

and this kind of temperature dependence can only be generated in perturbation theory from terms of higher order in $\hbar$.

We now consider the behavior of the effective coupling, Eq. (9), for $\theta = 0$. For $T >> \Lambda$, η is negligible, and as T is decreased the effective coupling increases logarithmically; when the instanton effects rapidly turn on as T is further decreased, they further increase the coupling.

In contrast to this behavior, consider $\theta = \pi$. Now when the instanton contribution turns on it slows the rate of increase of the effective coupling;[16] as the temperature is lowered slightly further the rate of increase is slowed further until the coupling no longer increases. This occurs when

$$T \frac{\partial}{\partial T} g_{eff} \equiv \beta(g_{eff}) = 0 \quad . \tag{12}$$

This fixed point occurs for $\eta \sim \mathcal{O}(1/10)$, small enough so that the dilute gas formula is valid. This is because the perturbative contribution to the effective coupling changes very slowly with temperature, while the instanton contribution changes very rapidly; the instanton contribution therefore requires a much smaller magnitude in order to compensate the perturbative contribution in the rate of change. Also the coupling g at the fixed point is $\mathcal{O}(1)$, so $g^2/8\pi^2$ is small.

This analysis is also valid for a wedge around $\theta = \pi$, but as the magnitude of $\cos\theta$ gets smaller, the dilute gas gives less of a contribution. Higher order instanton correlations give corrections to all powers in $\cos\theta$, and our simple analysis breaks down.

While we have determined $\beta(g_{eff})$ by varying the temperature, this same β-function should govern the change in effective coupling with respect to all other single external scales, such as field strength or Q^2. Since (at least in perturbation theory) the zeros of the β-function are known to be independent of the prescription used to determine the effective coupling, we conclude that in the $T = 0$ theory the effective coupling as a function of Q^2 has an infrared fixed point for θ near π. (See Figs. 1 and 2.) If instantons are responsible for the onset of non-perturbative behavior in the $\theta = 0$ theory, then for θ near π in the asymptotically free phase there is no confinement. (In the high temperature theory the zero of the β-function was reached while still in the plasma phase.)

Our simple analysis cannot be extended past the zero of the β-function, but it is reasonable to suppose there is no singularity that prevents extrapolation past this zero. If there is another phase to the right of this fixed point, we offer two heuristic arguments why this could be the superconducting phase.[13] First, the long distance behavior of that phase is governed by a small I.R. fixed point, while confinement is associated with I.R. slavery. Secondly, at $T = 0$, the lowest order θ dependent term in the effective Lagrangian is proportional, at $\theta = 0$, to tr $(\vec{E}^2 - \vec{B}^2)$ and at $\theta = \pi$, to tr $(\vec{B}^2 - \vec{E}^2)$. Electric and magnetic fields have been interchanged. On one hand, the weak coupling phase is governed by both perturbative and instanton effects, the perturbative effects being independent of θ. On the other side of the fixed point, though, these perturbative effects are not relevant. We therefore expect this phase to be one with electric and magnetic fields interchanged. Since the possibilities, implied by 't Hooft's and Mandelstam's analysis,[14,15] for this phase are either the confined or spontaneous gauge symmetry breaking phase, it is most likely that it is the superconducting phase that confines magnetic charge, rather than the one that confines electric charge.

Acknowledgments

We wish to thank Joe Polchinski, Helen Quinn and Lenny Susskind for their helpful comments. This work was supported by the Department of Energy, contract DE-AC03-76SF00515.

Note added:

After completing this paper we learned of the work of 't Hooft reported in the 21st Scottish Universities Summer School in Physics, August 1980, where he argues on topological grounds that confinement may occur for only a finite interval of θ excluding π. 't Hooft has also communicated to us that his previous analysis of the possible phases of Yang-Mills theory was valid only for $\theta = 0$. For $\theta \neq 0$, a confined phase would have a combination of electric and magnetic charge confined and an orthogonal combination screened.

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13. Roger Dashen had remarked to one of us (N.S.) some time ago that he had reason to believe the Yang-Mills theory at $\theta = \pi$ could be spontaneously broken.

14. The 't Hooft and Mandelstam analysis of possible phases assumes that the only independent order parameters are the Wilson loop and its dual. If other independent order parameters exist, then a more complicated phase structure can exist. The phase to the right of the fixed point could then be different from either the confining or spontaneous gauge symmetry breaking phase.

15. See, however, note added.

16. The observation that for $\theta > \pi/2$ instanton effects cause the coupling to decrease for increasing distance scale has been made by Callan, Dashen, and Gross in Ref. 8.

Figure Captions

Fig. 1. g^2 versus Q^2 at $T = 0$ for $\theta = 0$ and π.

Fig. 2. $\beta(g)$ for $\theta = 0$ and π, and possible phases.

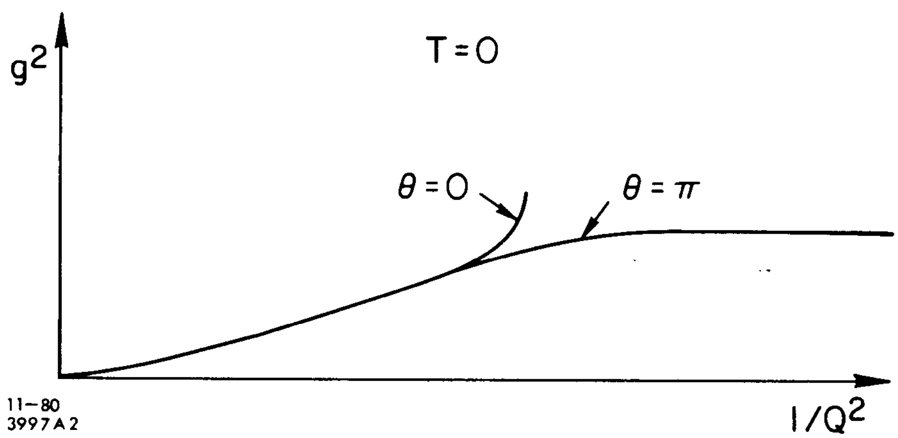

Fig. 1

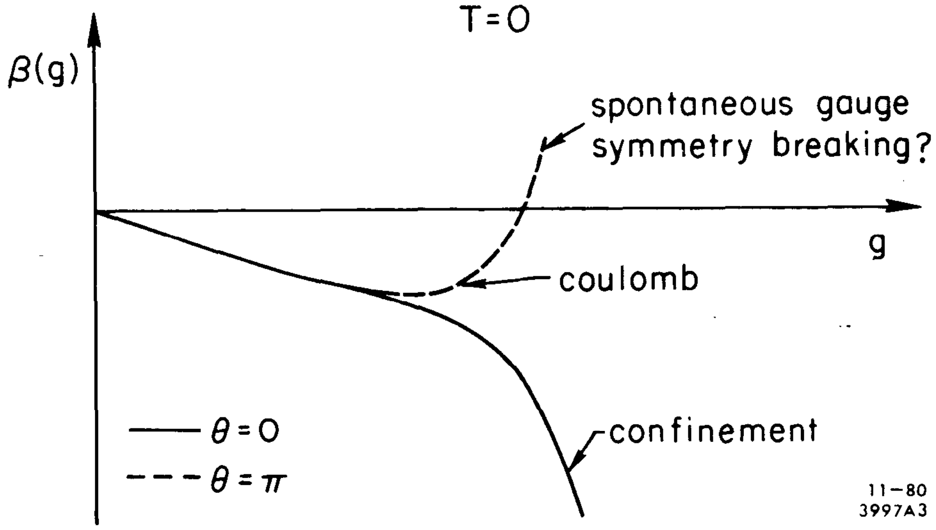
T=0
β(g)
spontaneous gauge
symmetry breaking?
g
coulomb
confinement
θ=0
θ=π
11-80
3997A3

Fig. 2