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HEAVY ONIUM TRANSITIONS AND OPERATOR PRODUCT EXPANSION*

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ABSTRACT

Attempts at multipole expansion for hadronic transitions in heavy onium states leads us to analyze the process using operator product expansion. We find two regimes where the process can be controlled using operator product expansion. For these processes, it is shown that a Callan Symanzik equation exists. Problems pertaining to consistent higher order α calculations are pointed out.

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1. Introduction

In atomic transitions, the first few terms of the multipole expansion are a good approximation to the decay rates, branching ratios and level widths. With the discovery of charmonium and upsilonium spectra and further expectations of heavier quarkonia, it has become of great interest if similar predictions can be made for these systems. Gottfried[1] first pointed out that in coulomb gauge such an expansion might be possible. In that paper, the various problems in making such an expansion and utilizing it for numerical predictions were pointed out. But all questions pertaining to renormalization were not dealt with. Later efforts were made to do the same with greater sophistication in a gauge invariant fashion.[2] The multipole expansion was related to the operator product expansion in a heuristic fashion and some estimates were made for various processes by Peskin and Bhanot[3] and Peskin.[4]

With the recent development of very powerful techniques of renormalization group, and Callan-Symanzik equations and their application to pion form factor,[5] large angle scattering and other exclusive and inclusive processes,[6,7] it has now become possible to make some precise statements and give a detailed prescription for a systematic calculation of various amplitudes. It has recently been applied to inclusive decay of quarkonia also.[6] In this paper we extend the analysis of Duncan and Mueller to discuss exclusive and semi-exclusive processes.

We intend to develop the formalism for the transition rates from an excited level of an onium state to a lower state. The formalism presented should provide a nice testing ground for QCD when toponium is discovered. The formalism is also applicable to all gauge groups and hence to hypercolor sector and its various extensions and its coupling to the low energy world. The calculation of the branching ratios in such cases would become independent of incalculable bound state dynamics and hence should be of interest.

We would like to show that the transition rates for a quarkonia state with heavy quark mass M can be factorized into two parts. The coefficient function in the operator product expansion in two process depends on the large four momentum exchanged in the bound state which is (Be, B_m) where the binding energy $Be \sim \alpha^2 M$ and $B_m \sim \alpha M$. The matrix element of the operators have a relevant scale p which is of the order of quark masses or rest masses of the pion.

We distinguish two cases where the amplitudes can be factorized.

(1) Transitions $\phi \to \phi' + x$ with $m_\phi^2 - m_{\phi'}^2 \approx Be(\phi')$. It is a transition from near the continuum to states near the ground state. This happens to be the case when the momentum K of the radiated particles is of the order of binding energy and can be treated as a hard part, and the soft scale p appears only in the form factors of the final particles.

(ii) Transitions $\phi \to \phi' + x$ with $m_\phi^2 - m_{\phi'}^2 \ll Be(\phi')$. It is a transition between two nearby states. Here the total four momentum radiated $K \ll Be$ and hence is the soft scale p.

We drop all terms suppressed by powers of p/Be, but demonstrate factorization to all orders in α and all logs. But, the scheme of calculation only indicates leading order in $\alpha \log Be$, that is leading logs.

In this paper we do not present any detailed calculations but content ourselves with just presenting the formalism. We begin with the inclusive decay of $\phi \to \phi' +$ anything where ϕ is our excited state which can decay to ϕ' a lower lying state with the emission of two gluons which finally hadronize. This section essentially sets the notation and clarifies the nature of calculations that have to be made in order to obtain numbers for various processes. Part of the calculation has already been done by Yan[2] and Peskin.[3] In the next section we deal with $\phi \to \phi' + 2\pi$'s where the energy carried by the π's $\equiv K$ is the order of the binding energy Be of the ϕ' system, i.e., $K \approx$ Be. This process has all the complications of factorization and the Callan-Symanzik equation. These tools show their utility here, clearly. To generalize it to semi-inclusive processes is straightforward. The calculation of the hard part in these processes is identical to the inclusive process. This region $K \approx$ Be is not the traditional multipole expansion region but presumably a generalization of it. Here it is simply the lowest twist expansion on the light cone.

Next we deal with the traditional multipole expansion regime, that is $k \ll$ Be and discuss the process $\phi \to \phi' + 2\pi$'s. Now the π's are essentially at rest and we obtain an expansion which is the local operator product expansion and operators with the lowest dimension appear. But, if a process cannot proceed by the lowest dimension operators as it might be prohibited by conservation laws then the expansion has to be made to higher dimensions; this can be done, but has not been done here. Some problems are pointed out.

The power counting arguments that we have made use of are very similar to Gottfried's original arguments. Peskin[4] had tried to make an operator product expansion by showing a factorization to the lowest order in the hard and soft parts. But, in doing so he had made use of many heuristic arguments and assumptions. We have done so without the use of all such assumptions, by an analysis of Feynman diagrams. Also, we have analyzed the applicability of the Callan-Symanzik equation which was not investigated by Peskin. So, in this paper we have essentially put the idea of Gottfried on a firm basis, but differ with the previous work in details.

2. $\phi \rightarrow \phi'$ + Anything

In atomic systems, the excited states decay into a lower lying level by emission of a photon. For this process a multipole expansion is very good. If the decay is forbidden then it takes place by a higher multipole which can be thought of as a two photon decay and is the dominant decay. In QCD one gluon emission is forbidden as it leaves the system in an octet state. Hence the decay takes place by the emission of two gluons. These gluons later hadronize into soft hadrons. In the asymptotic regime decay by higher number of gluon intermediate state are suppressed as they have an extra power of running coupling constant. This is true as a factorization between the soft and hard parts occurs which is shown explicitly in the next section. The calculation of the level width then is just the calculation of $\phi \rightarrow \phi'$ + 2 gluons.

Therefore, for the process consider the amplitude shown in Fig. 1.

$$W(P,P',K,K',\ell,\ell') = \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 d^4x_5 d^4x_6 d^4x_7$$

$$\exp\Big[i(P'+K')x_1 + i(K-P)x_2 - i(K'-P')x_3 - i(P+\ell)x_4$$

$$+ i(P'+\ell')x_5 + i(\ell-P)x_6 - i(\ell'-P')x_7 \Big]$$

$$\times \left\langle 0 \right| \overline{T}\left\{ \psi(x_4)\psi(x_5)\overline{\psi}(x_6)\overline{\psi}(x_7) \right\} T \left\{ \psi(0)\psi(x_1)\overline{\psi}(x_2)\overline{\psi}(x_3) \right\} \left| 0 \right\rangle$$

$$\xrightarrow[\substack{4P^2 \to M_\phi^2 \\ 4P'^2 \to M_{\phi'}^2}]{} \frac{1}{|4P^2 - M_\phi^2 + i\varepsilon|^2} \times \frac{1}{|4P'^2 - M_{\phi'}^2 + i\varepsilon|^2}$$

$$\chi(P+K, K-P)\, \overline{\chi}(P+\ell, \ell-P)\, \widetilde{\chi}'(P'+K', K'-P')$$

$$\chi'(P'+\ell', \ell'-P') \sum_H (2\pi)^4 \delta^4(P_H - P + P') |\mathcal{M}_H|^2 \qquad (1)$$

The notation in the above closely follows that of Duncan and Mueller.[5] In Eq. (1) χ', χ, $\overline{\chi}'$ and $\overline{\chi}$ are the Bethe-Salpeter wave functions and $\mathcal{M}_H$ is the decay amplitude of the heavy quarkonia excited state ϕ to decay to a lower lying state ϕ' and real hadrons with momentum P_H. Figure 2 represents the part of the calculation which corresponds to the coefficient function in the OPE which would be dependent on the hard momentum flow in the system which is of the order of αM.

Hence

$$W = \int \frac{d^4q_1}{(2\pi)^4} G(P,P',K,K',q_1)\, \Delta(P-P'+q_1)\, \Delta(-P+P'+q_1) G^*(P,P',\ell,\ell',q_1) \qquad (2)$$

where $\Delta(p)$ is the cut propagator and

$$G(P,P',k,k',q_1) = \Delta(P'+k')\Delta(k'-P')\left\{\int \sum_{i,j} \left\langle P',k' \middle| O_{1j} \middle| \frac{P+P'-q_1}{2}, q_3 \right\rangle \right.$$

$$\times K\left(\frac{P+P'-q_1}{2}, q_2, q_3\right) \left\langle \frac{P+P'-q_1}{2}, q_2 \middle| O_{1i} \middle| P,k \right\rangle \frac{d^4q_2}{(2\pi)^4} \frac{d^4q_3}{(2\pi)^4} \tag{3}$$

$$\left. + \sum_i \left\langle P_1',k' \middle| O_{2i}(P,P',k,k',q_1) \middle| P,k \right\rangle \right\} \Delta(P+k)\Delta(k-P)$$

The operators O_{2i} and O_{1j} are defined in Figs. 3(a) and 3(b) respectively and are amputated. The wavy lines correspond to transverse as well as coulomb gluons, only coulomb gluons are represented by dashed lines. The kernel K is defined to be

$$K(P,k,k') = \left\langle 0 \middle| T \;\; \psi(P+k')\,\psi(P+k)\,\overline{\psi}(k-P)\,\overline{\psi}(k'-P) \middle| 0 \right\rangle \tag{4}$$

In the lowest order the kernel K goes over to $\hat{K}$ shown in Fig. 4. The kernel K is so defined that it is just the Green function for scattering of two massive fermions which are off-shell in an octet state. In the non-relativistic limit it would go over to the usual repulsive coulomb force. Besides the kernel $\hat{K}$ is essentially a non-singular kernel as it has no bound states. The best way to calculate it is as a scattering amplitude in a repulsive central coulomb field. The summation over infinite ladders is necessary as the perturbation series is in a parameter $\alpha M/B_m$ where B_m is the momentum flow and is of the order of αM.

The total decay amplitude is easily seen to obey the usual renormalization group equation

$$\left[\mu^2 \frac{\partial}{\partial \mu^2} + \beta(g) \frac{\partial}{\partial g}\right] W = 0 \tag{5}$$

In this equation β depends on the parameters m/μ, μ/M and also in principle on Be/μ. Here m is the mass of the light quarks.

The Be dependence is completely related to M and α and Be is of order $O(\alpha^2 M)$. The dependence on m/μ is smooth in the limit $m \to 0$ and as μ is going to be chosen of the size $\alpha^2 M$, M/μ is very small and hence the limit can be taken. The renormalization for the heavy quark propagator is done on the mass shell (Appendix B).

Once the limit $m/\mu \to 0$ is taken, W is just a function of μ/M and $\alpha(M)$. So, in Eq. (5) the derivative $\mu^2 \frac{\partial}{\partial \mu^2} = -M^2 \frac{\partial}{\partial M^2}$. This equation then allows us to relate the decay rates of two different mass onia between two levels of the same quantum numbers.

The transitions that cannot proceed by two gluon intermediate states can proceed by 3 gluon intermediate states. This can happen due to conservation laws. Such processes can then take place by 3 gluon intermediate states. All the considerations stated above would still be valid except that O_{1i}'s will now be interserted thrice, or O_{2i} and O_{1j} once. But this complicates the intermediate kernel K as it is now not purely an octet repulsive kernel and takes on other representations.

Now we would like to make a short remark on the calculation of higher order corrections to processes. The higher order corrections come firstly from more gluon intermediate states which can be calculated as above. But, besides to this order there are also corrections to wavefunctions and to the kernel K. For wave function these can be calculated as done by Duncan[8] by inserting certain kernels L sandwiched between undressed ladders. Kernel L are either 3 particle irreducible kernels or are kernels that have transverse gluons but are two particle irreducible. The kernel K has also to be calculated to higher order and it can again be done as insertions of kernels L in kernel $\hat{K}$.

But all this makes the calculation extremely hard and presumably not very useful.

Higher order corrections to the ladder may be very important to phenomenology of lighter bound states like the J/ψ and its spectra. It may even be very important for the bottom spectrum. The higher order corrections are down only by powers of α. In this regime α is of the order of 0.1 to 0.2. Also, the number of diagrams that contribute are large and this may make the $O(\alpha)$ correction comparable to the leading contribution.

3. $\phi \to \phi' + 2$ for $k \approx Be$

Consider the amplitude $W(P_1,K_1,P_2,K_2,p_1,k_1,p_2,k_2)$ shown in Fig. 5 with all external fermion legs amputated. The amplitude obeys the usual renormalization group equation similar to Eq. (5) where the differential operator contains $-(4\gamma_\psi + 4\gamma_\psi)$ also. On going to the bound state poles for ϕ' and ϕ the amplitude takes the form shown in Fig. 6.

$$W(P_1,P_2,p_1,k_1,p_2,k_2) = \frac{1}{16}(\gamma_5\gamma_+)_{\beta_1\alpha_1}(\gamma_5\gamma_-)_{\beta_2\alpha_2} \int d^4x_1\, d^4x_2\, d^4x_3\, d^4x_4\, e^{i(p_1+k_1)x_1}\, e^{i(p_2+k_2)x_2} \cdot e^{-i(k_1-p_1)y_1}\, e^{-i(k_2-p_2)y_2} \left\langle P_2 \right| T\,\psi_{\alpha_1}(x_1)\,\psi_{\alpha_2}(x_2)\,\psi_{\beta_1}(y_1)\,\bar\psi_{\beta_2}(y_2) \left| P_1 \right\rangle_{tr} \tag{6}$$

and now obeys the equation

$$\left(\mu^2 \frac{\partial}{\partial\mu^2} + \beta\frac{\partial}{\partial g} - 4\gamma_\psi\right) W = 0 \tag{7}$$

We have gone to the frame where ϕ' and ϕ are essentially at rest and $P_{1-} \sim k_{1-} \sim P_{2+} \sim k_{2+} \sim$ Be and the other components are small. Again, it is easy to see that the arguments presented in Duncan and Mueller for factorization of π wave functions can be applied here. The soft gluons near mass shell, collinear to the fast pions will give zero contribution when inserted in all possible ways into the $\phi \to \phi'$ block by Ward identities. Here all the large momentum are of the order of Be and the softer region is essentially characterized by the π mass. μ is chosen such that $M_\pi < \mu <$ Be renormalization at μ^2 and following the arguments of Duncan and Mueller we get

$$W(P_1,P_2,p_1,k_1,p_2,k_2) \underset{Be \gg \mu, m}{\sim} \sum_{n_1,n_2} \frac{1}{4p_{1-}^{\,n_1}} \, tr\,(\gamma_5\gamma_+ V^{n_1}(p_1,k_1))$$

$$\cdot\, W_{n_1n_2}(Be) \frac{1}{4p_{2+}^{\,n_2}} \, tr\,(\gamma_5\gamma_- V^{n_2}(p_2,k_2)) \tag{8}$$

Where the $V^n(p,k)$ satisfy the same equation as in Ref. 5. Following the same steps and taking the limit $m/\mu \to 0$ we finally obtain

$$\left(\mu^2 \frac{\partial}{\partial\mu^2} + \frac{\beta\partial}{\partial_y} - 4\gamma_\psi\right) W_{n_1n_2} = \sum_{n_1'} \gamma_{n_1n_1'} \; W_{n_1'n_2'} \; (Be)$$

$$+ \sum_{n_2'} \gamma_{n_2n_2'} \; W_{n_1n_2'} \; (Be) \tag{9}$$

In the above we have shown the dependence to be on Be rather than M as Be is the relevant scale for the momentum flow, though it is, of course, determined by M and $\alpha(M)$.

In order to go any further we need to calculate the hard part shown in Fig. 7. This is in fact the same calculation that one needs to do for the inclusive decay. So, the branching ratio would be a number which would not involve the details of this part and is given by the same formula as in Ref. 5 with t now being $t = \ell n\left(\frac{B_e^2}{\mu^2}\right)$ which is $t \approx \ell n \left(\frac{\alpha^4(M)M^2}{\mu^2}\right)$.

It is also easy to see that the factorization would take place for all such processes. For instance it would go for a $\phi \to \phi' + \pi$ + anything with the obvious cut vertices appearing as the soft part.

The cross section for the decay $\phi \to \phi' + \pi$ + anything is as shown in Fig. 8. The cross section factors into a time-like cut-vertex for a gluon to π times a hard part. Therefore

$$W = \int \frac{d^4 q_1}{(2\pi)^4} G(P,P',k,k',q_1)_\mu \, A\!\!\!/ \, (P - P' + q_1) \, G^* (P,P',\ell,\ell',q_1)_\nu \, \Gamma_{\mu\nu}(q_1) \quad (10)$$

The integral over q can be easily made into convolution using factorization and the evolution carried out using the usual arguments about Callen-Symanzik equation.

4. <u>$\phi \to \phi' + 2\pi$ for $k/Be \ll 1$</u>

Again consider the amplitude shown in Fig. 7, but now we restrict ourselves to soft pions that will be observed when a transition takes place between two nearby levels well below the threshold for the continuum.

The discussion of factorization given below is in a theory with scalar gluons. In the vector gluon theory, the collinear gluons have to be added in as done usually. And, they would make the operators gauge invariant as usual. So, consider the decomposition as in Fig. 9 of the amplitude W as in Eq. (6). In Fig. 9, the decomposition is such that large momentum flow is in the graph τ and λ is the soft momentum part where finally it hadronizes to 2 pions. In this case, such a separation is possible as seen through the following argument, and all contributions that have more than two particle bridges between τ and λ are down by powers of Be as the phase space integration gained by adding an extra gluon cannot sufficiently compensate for the large denominator in the τ. This follows due to the usual power counting arguments and hence the bridges are two particle fermion or gluon bridges. Also, all the propagators in τ even the one near the gluon emissions range all the way up to Be the only scale the τ part of the graph knows about, which is set by the "off shellness" of the heavy quarks and gluons in the ϕ' and ϕ bound states, which is the same for the kernel K.

The topology is what allows us a factorization of the soft and hard parts. This factorization is really the operator product expansion where the Be sets the large scale and the momentum of the pions the soft momentum scale. In this approximation the assumption is that the binding energy and the momentum flow in the bound state are much larger than four momentum of the emitted system. This regime is the conventional OPE regime. The conventional OPE requires that in the limit all components of q_u must approach infinity together, that is as the same power. Here the Be $\sim \alpha^2 M$ and the binding momentum as αM but in the limit that both go to infinity the difference is only a power of α and not M which varies only logarithmically. So, the operators that occur are the usual lowest dimension operators that are gauge invariant. The lowest dimension operators are $N_4 F_{\mu\nu} F_{\mu\nu}$, $N_4 \bar{\psi} \not{D} \psi$ and $N_3 \bar{\psi} \psi$.

Therefore

$$W(P_1,P_2, p_1,k_1,p_2,k_2) = \sum_i \langle \pi(p_1,k_1)\, \pi(p_2,k_2) | N\, O_i | 0 \rangle \langle \phi'(P'), i_{p'-p} | \phi(P) \rangle \qquad (11)$$

$$= \sum_i \langle \text{final hadrons} \mid N\, O_i \mid 0 \rangle \; W_i(P', P) \qquad (12)$$

In vector gluon theories $i_{p'-p}$ is to be understood as the various possible decompositions that is quarks or gluons evaluated at the momentum $(P'-P,0)$, $W_i(P',P)$ is the singular function.

Now applying Callen-Symanzik equations we get

$$D\, W_i = \gamma_{ji}\, W_j \qquad (13)$$

where γ_{ij} are the various anomalous dimension matrix.

So when one calculates the W_i to the lowest order the only term that is non-zero comes from the i corresponding to the gluon operator. The fermion operators only occur through mixing. So, that the mixing of $N_3 \bar{\psi}\psi$ to $N_4 F_{\mu\nu} F_{\mu\nu}$ only comes as $m\, N_3 \bar{\psi}\psi$ by simple power counting and observing that the m comes from the light quark mass in the numerator, so when the limit $m/\mu \to 0$ is taken, this operator would not mix at all with the rest of the operators. Peskin has in fact calculated the coefficient function[4] where he has made a certain approximation for the kernel K. W_i, the two gluon coefficient function, is just the $G(P,P',k,k',0)$ as defined in Eq. (3) sandwiched between the wavefunctions of ϕ and ϕ'.

DISCUSSION

In this paper we have shown that the hadronic transitions of heavy onium states are in fact calculable using the ideas of the renormalization group. The results are analogous to the multipole expansion in the regime $k/Be \ll 1$, but it is not the multipole expansion, as it makes the expansion for the gluons to be emitted independently whereas in OPE the two gluons are emitted from a small space time region. In the other regime $k/Be \approx 1$, we are able to separate the hard and soft regions. The expansions are different in the two regimes.

We have refrained from doing explicit calculations as it is not clear if the leading order calculation is going to be phenomenologically relevant. For the J/ψ and upsilonium the value of α is more like 0.1-0.2. And the coefficient of the higher order terms here is expected to be large as the number of diagrams is large. In all potential models the spectrum of J/ψ extends to the linear part of the potential. Clearly, our formalism demands the spectrum to be inside the coulomb part and further requires the difference in the binding energy to be sufficiently large as compared to the rest mass of light baryons. This condition is certainly not satisfied by charmonium spectrum. But we have clearly shown that for heavy enough onia the level widths can in fact be predicted and in some sense brought the program started by Gottfried to a completion.

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APPENDIX A

Consider the amplitude $\mathcal{T}$ shown in Fig. 10. It is renormalized in the BPHZ scheme by the following forest formula

$$\mathcal{T}_R = \sum_{\mu\varepsilon\mathcal{M}_\mu} (-t^\gamma)\, \mathcal{T}_u \tag{14}$$

In Eq. (14) the $\mathcal{T}_R$ is the integrand of the Feynman diagram and $\mathcal{M}_\mu$ is the set of all forests μ. We are following Zimmermann's[8] notation. Again, as in Zimmermann we want to find the asymptotic behavior of the $\mathcal{T}_R$ as the mass of the heavy quark $M \to \infty$ keeping other variables fixed. The Be $\sim \alpha^2 M$ and hence Be is the large variable in which we are interested, k is the total momentum of the outgoing particles π's and is equal to P-P', and here is assumed to be small so as to satisfy $k/\mathrm{Be} \ll 1$. Now, in this regime we can make the usual over subtractions to find the dominant behavior in powers of k/Be. We are interested in finding the leading behavior so we oversubtract once. For this we need the definition of Δ-forests of Zimmermann. These are defined as (i) τ is the set of all graphs that connect the remainder of the graph λ by two gluon lines as in Fig. 10(a); (ii) τ is the set of graphs that connect the remains of the graph by two fermions (of course, they will only be the light fermions in the theory of mass $m \ll M$) as in Fig. 10(b).

The oversubtraction operator when acting on these graphs τ evaluates them at the point $k^2 = \mu^2 \ll M^2, \mathrm{Be}^2$. Therefore

$$\mathcal{T}_{reg} = \mathcal{T}_R - \sum_\tau \sum_{U_1\varepsilon\mathcal{M}_\tau} \sum_{U_2\varepsilon\mathcal{U}(\tau)} \prod_{\gamma_1\varepsilon U_1} (-t_1^{\gamma_1})(-t_1^\tau) \prod_{\gamma_2\varepsilon U_2} (-t^{\gamma_2})\, \mathcal{T}_R \tag{15}$$

In the above we have followed Zimmermann closely and our notation is the same as his.

The precise meaning of oversubtractions closely follows the usual oversubtractions in gauge theories. This involves first making a tensor analysis and then retaining gauge invariant tensor and evaluating the coefficients of the structure at the point μ^2. In the case of two gluon, the dominant structure is just $F_{\mu\nu}F_{\mu\nu}$ which gives the operator $N_4 F_{\mu\nu}F_{\mu\nu}$ and for the fermion this gives the operator $N_4 \bar{\psi} \not{D} \psi$ and $N_3 \bar{\psi}\psi$. The last operator does not finally enter as the coefficient function always has a mass m_q term by power counting and hence vanishes as $m_q \to 0$. This leads to the Eq. (11).

APPENDIX B

In the text we have been using the words mass-shell and off-mass shell ness of heavy quarks. In some sense this is a crucial definition as all scales are set by heavy quark mass M. But, this quantity is not well defined as the quarks do not exist as asymptotic states. The definition of quark mass we need is such that it does not give large logarithms by log (M/μ) when the heavy quark propagator and insertions of gluon vertex on it are considered. The definition best suited to our purposes is therefore the Georgi and Politzer[9] definition of quark mass. Here, they define mass M of the quark to be the value M

$$S^{-1}(\not{p})_{p^2=-\mu_o^2} = \not{p} - M \qquad (16)$$

We choose a value of μ_o^2 of the order of M^2 itself. And this is what we choose to call the mass of the heavy quark. Also notice the equal definition and choice is not very important as long as the

$\log(M/\mu)$ terms are avoided which this definition does. Also we do not need to define the light quark mass with any precision as we take $m_q \to 0$.

REFERENCES

1. K. Gottfried, Phys. Rev. Lett. 40, 598 (1978).
2. T. M. Yan, Phys. Rev. D22, 1652 (1980).
3. M. Peskin and G. Bhanot, Nucl. Phys. B156, 391 (1979).
4. M. Peskin, Nucl. Phys. B156, 365 (1979).
5. A. Duncan and A. H. Mueller, Phys. Rev. D21, 1636 (1980).
6. A. Duncan and A. H. Mueller, Phys. Lett. 93B, 118 (1980).
   A. H. Mueller, Phys. Lett. 90B, 159 (1980).
7. A. Duncan, Phys. Rev. D13, 2866 (1976).
8. W. Zimmermann, in Lectures on Elementary Particles and Quantum Field Theory, edited by S. Deser, M. Grisaru, and H. Pendelton (MIT Press, Cambridge, Mass., 1971).
9. H. Georgi and H. Politzer, Phys. Rev. D14, 1828 (1976).

FIGURE CAPTIONS

1. $\phi \to \phi$ + anything

2. Diagrams contributing to W

3. Operators O_1, O_2

4. Kernel K

5. $W(P_1,K_1,P_2,K_2,p_1,k_1,p_2,k_2)$

6. $W(P_1,P_2,p_1,k_2,p_2,k_2)$ for $k \ll Be$

7. $W_{n_1 n_2}$

8. $\phi \to \phi' + \pi$ + anything

9. W of Figure 6 in the regime $k \ll Be$

10. Various possibilities showing gluon and fermion bridges between λ and τ.

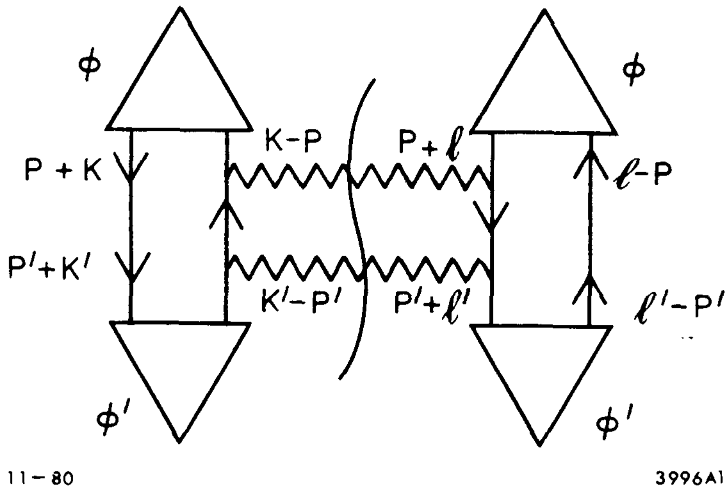

Fig. 1

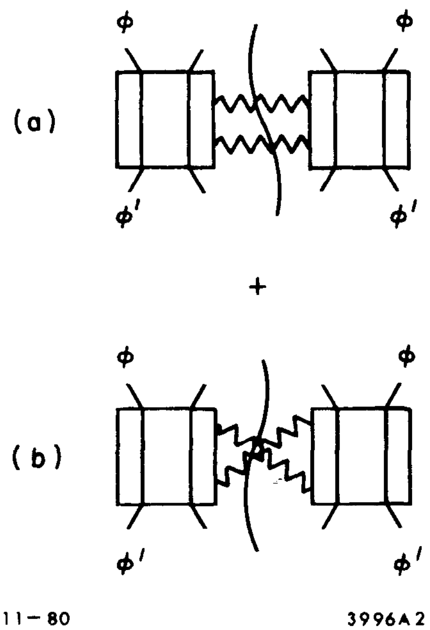

Fig. 2

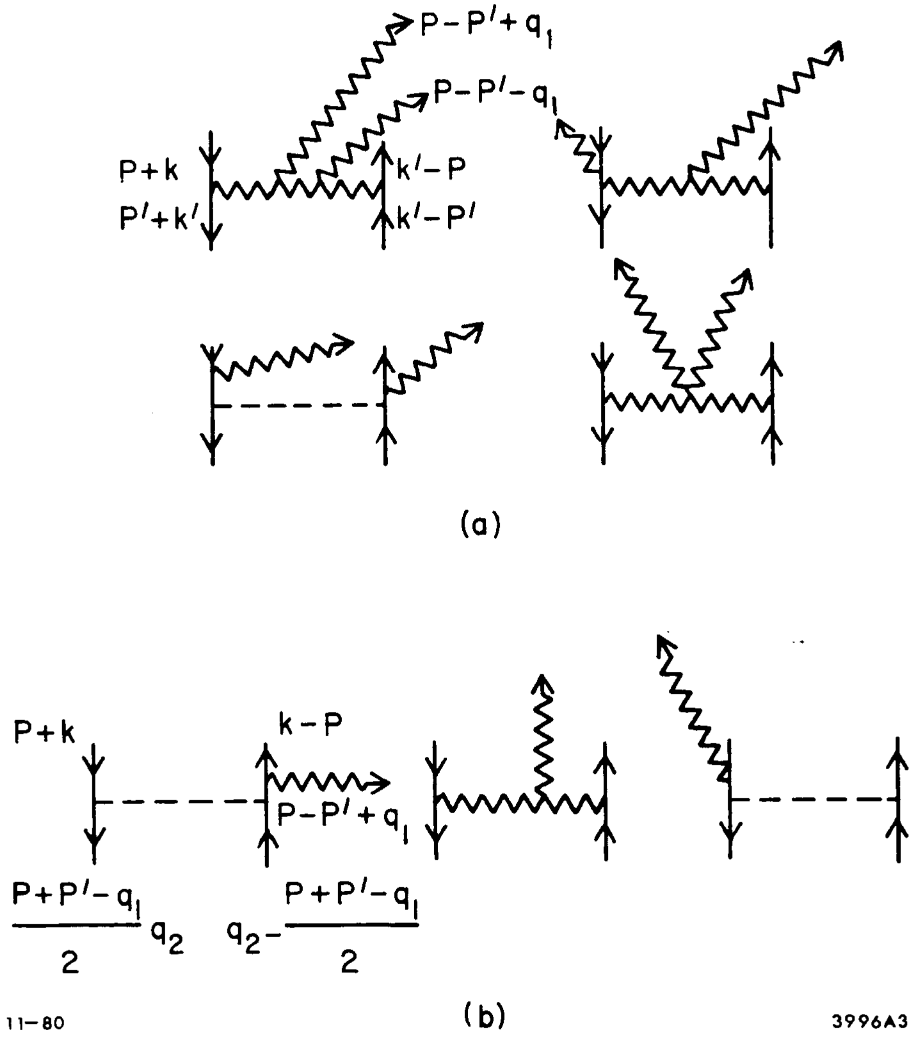
P−P′+q₁
P−P′−q₁
P+k
P′+k′
k′−P
k′−P′
(a)
P+k
k−P
P−P′+q₁
P+P′−q₁
2
q₂
q₂−
P+P′−q₁
2
(b)
11−80
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Fig. 3

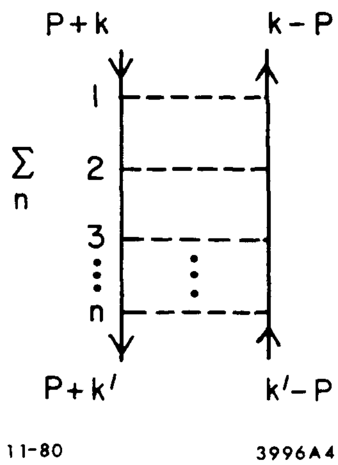

Fig. 4

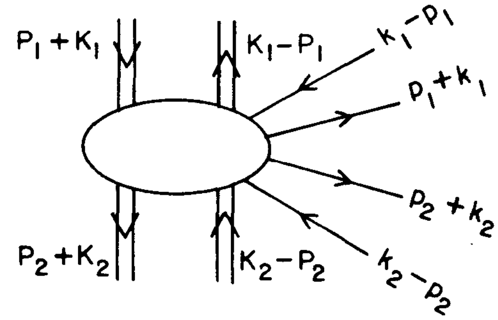

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Fig. 5

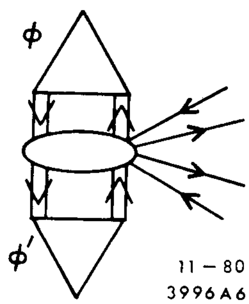


Fig. 6

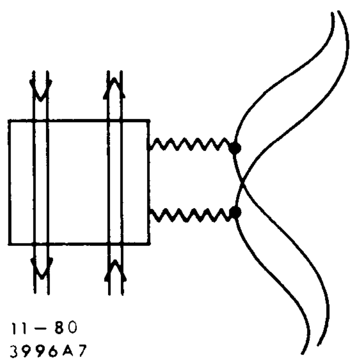


Fig. 7

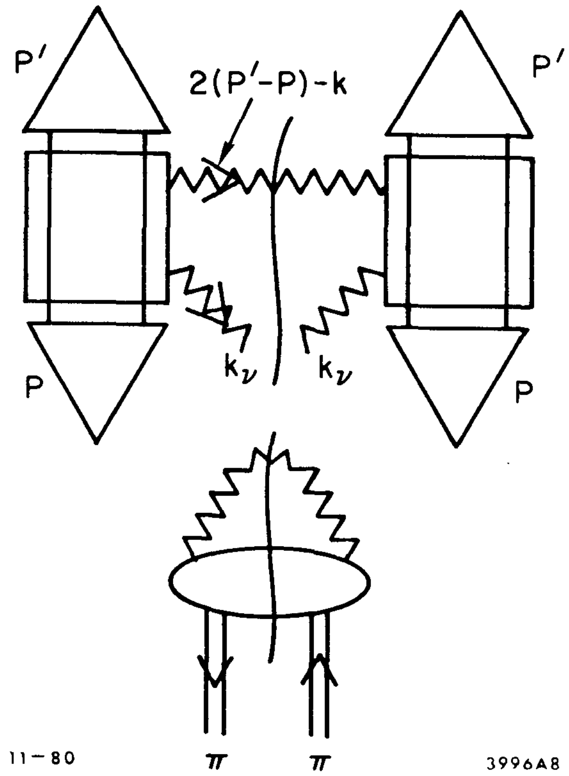


Fig. 8

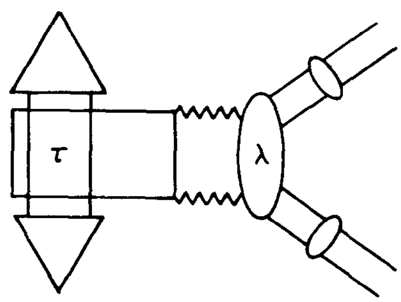
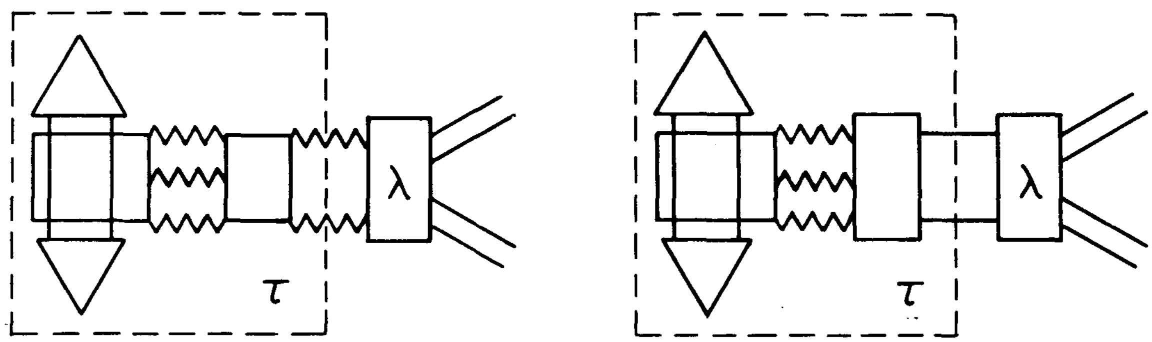


Fig. 9



11-80 (a) (b) 3996A10

Fig. 10