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THE DISTRIBUTION FUNCTIONS AND THE UNDERLYING STRING STRUCTURE OF HADRONIC REACTIONS*

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ABSTRACT

On the basis of the topological $1/N_c$ expansion, various types of the distribution function for the soft and the hard processes are obtained, which contain the freedoms pertaining to the underlying string structure of hadronic reactions. It is stressed that the distribution functions for the soft process refer to the energy fractions of jets and are distinct from those for the hard process in their physical meanings. In order to test the present model the average multiplicities are studied by using the distribution functions. For multi-string processes the freedom of partitioning the energy among jets causes the effective energy for each extended string to be largely reduced compared with the total energy. In deep inelastic lepton-hadron scattering, owing to the peculiar double-string structure of the singlet component the average multiplicity is predicted to increase with decreasing Q^2 at fixed W higher than ~20 GeV.

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I. Introduction

It has been proven that the quantum chromodynamics (QCD) is successful in understanding hadron structure at short distances.[1] The parton model has gained some theoretical underpinnings on the perturbative QCD and has achieved the phenomenological interpretation of hard reactions. However the difficulty occurs when the parton model is examined in the soft processes with the results obtained in the hard processes. The trouble, which encounters in explaining the particle distributions,[2] suggests the significant effect of the color confinement to the distribution functions and to the decay functions. Although there are several attempts in which these functions for the soft processes are supposed to be different from those in the hard processes,[2-6] we have not yet attained a convincing answer about how to take account of the non-perturbative effect to these functions. In this paper, with the emphasis on the underlying string structure pertaining to the color confinement mechanism we find various types of the distribution function. The calculations of the mean multiplicity which rely on these distribution functions exhibit that the freedoms of partitioning the energy among jets play important roles in understanding the characteristic features of hadronic reactions.

Hadrons are considered to be string-like objects in which the color flux is confined.[7] Therefore, in order to clarify the color confinement effect in the jet productions, it is of importance to notice the underlying string structure of reactions. On the basis of the topological $1/N_c$ expansion,[8] we assume that the jet production develops via two steps. The extended strings are produced in the first step and their independent breakups follow. By virtue of a quark counting rule we derive several types of the distribution function, which depend not only upon the number of strings but also upon whether the process is soft or hard. In the soft process accompanying multiple extended strings, the distribution functions represent the probability of sharing the energy among several jets. On the other hand, in the hard process the distribution functions mean the probability of finding a constituent with a certain energy fraction and are distinct from those for the soft process in their physical meanings. Especially, in deep inelastic lepton-hadron scattering the distribution functions do not refer to the energy fractions of jets. The independent breakups of the multiple strings result in the overlapping of hadron plateaus in the rapidity space. Therefore, the features of final states such as the mean multiplicity, the particle distribution and the correlation, are closely related to the underlying string structure of the process and also to the energy fractions carried by the jets. The distribution functions for jets imply that one of the jets is more likely to carry the large energy fraction, namely, the energy fractions of jets are predominantly far from the equal partition. Then, the freedom of sharing the energy among jets causes the effective energy of each string to be largely reduced compared with the total energy.

This article is organized as follows. In Section II we assume the two-step development of hadronic reactions. Several types of the string structure are illustrated. To compute various types of the distribution function, we make a family of the assumption about the scattering amplitude. For the amplitude of the first step, we introduce the quark counting rule which incorporates both the Reggeon effect in the soft region and the dimensional counting rule in the hard region. As for the breakups of strings, it is taken that the totally inclusive sum for each string becomes a constant probability irrespectively of the production mechanism. After studying the meson-meson collision with double strings, we obtain the meson distribution functions for the soft process in Section III. We proceed in Section IV to deal with the soft meson-baryon and baryon-antibaryon processes which possess double and triple strings, respectively. The baryon distribution functions are found for double strings and for triple ones. Considering the deep inelastic lepton-hadron scattering, we calculate the distribution functions for the hard process in Section V, which contain both the valence component and the sea one. The peculiar string structure for the sea component is emphasized. In the sea component there appears a freedom of distributing the energy among jets, which is independent of the energy fraction carried by the sea quark (antiquark). In Section VI, to test the present model we compute the mean multiplicity by using the distribution functions obtained here. We find that the large reduction of the effective energy for each string considerably affects the behaviors of the mean multiplicities. In deep inelastic lepton-hadron scattering, the average multiplicity is predicted to increase with decreasing Q^2 at higher $\sqrt{s}$ than the presently available region. It is also a consequence of the large reduction of the effective energy that the onset of the increase appears at such high $\sqrt{s}$. The comparison of the calculations with the data is made in Section VI. Section VII is devoted to summary and discussions.

II. The Two-Step Development of Reactions

Standing on the topological $1/N_c$ expansion inspired by QCD,[8] we take the viewpoint that jet productions take place through two steps. In the first step extended strings are produced along with the rearrangement of quarks, antiquarks and color fluxes. The hard subprocess of hard reactions is also involved in the step. The second step is the independent breakups of each extended string, which is linked to the suppression of the higher order terms in the topological $1/N_c$ expansion. The breakups mean the fragmentation into clusters via the creation of q-$\bar{q}$ pairs with the limited transverse momentum. Similar models have been proposed by several authors.[9-11]

Here we depict several types of the string structure. Figure 1 illustrates soft hadron-hadron collisions. In BB scattering no processes with a single string exist. Triple strings develop in $B\bar{B}$ annihilation. In the figure we consider only the case that each string has a linear structure without any loops. In deep inelastic lepton-hadron reactions there are two types as seen in Fig. 2. Thus it is cogent to classify the hadronic reactions according to their string structures. Due to the color confinement any extended strings should be eighter of q-$\bar{q}$, q-qq, $\bar{q}$-$\overline{qq}$ and qq-$\overline{qq}$ in the minimum structure.

In multi-string processes there appears the freedoms of distributing the energy among jets which control the behavior of hadron plateaus coming from extended strings. The freedoms are described in terms of the jet(j)-distribution function, which is distinct from the so-called distribution function hereafter referred to as the constituent(c)-distribution function. In the j-distribution function the sum of the energy fraction of the jet amounts to unity owing to the energy conservation. On the other hand, in the c-distribution function the sum of the energy fraction carried by the valence is less than unity granted that a hadron consists purely of the valence quarks (antiquarks), because some missing energy resides in the binding. Therefore, for the soft process we ought to make use of the j-distribution function but not of the c-distribution function. For example, one-particle distributions for the soft process are given by convoluting the j-distribution function with the fragmentation function. The c-distribution function is explored in the hard process in which a constituent is violently struck. Indeed, in deep inelastic lepton-hadron scattering the structure function exhibits the c-distribution function in itself. The sea component possesses the freedom of sharing the energy among jets but the valence component does not. The freedom in the sea component is distinguished from the c-distribution function. Here we present a family of the assumption about the amplitudes for jet productions. Then we will find the j- and c-distribution functions in the subsequent sections.

On the above viewpoint we assume <u>(A) the factorization of the scattering amplitude T into two pieces</u>

$$T = \mathscr{T} \cdot \tau \tag{2.1}$$

<u>for soft processes</u>, where $\mathscr{T}$ and τ are the amplitudes for the first step and for the second one, respectively. Spins of constituents are ignored here. For hard processes $\mathscr{T}$ is replaced by $\mathscr{T} \cdot \hat{T}$, where $\hat{T}$ for the hard subprocess is subject to the perturbative QCD. The separation of the perturbative part from the non-perturbative one is recognized in view of the fact that mass singularities can be factorized in the perturbative

calculation of QCD.[12] For the second step it is assumed that (B) the totally inclusive sum over breakups of each extended string gives rise to a constant probability irrespectively of the production mechanism.

Our starting concern is to find how $\mathcal{T}$ and τ depend on the underlying string structure of the process considered. Under the assumptions (A) and (B), we settle the amplitudes $\mathcal{T}$ and τ. Concretely, we apply the multiperipheral model[13] to MM scattering with a single extended string as shown in Fig. 3. The strong ordering approximation of a multiperipheral chain leads to the amplitude

$$T = g^{N+1} \cdot \prod_{i=1}^{N} \exp[\alpha_2 \cdot (y_i - y_{i+1})] \quad , \tag{2.2}$$

where g is an effective coupling and α_2 stands for the mesonic Reggeon. The rapidities of clusters with the momenta $k_i = (\omega_i, k_i)$ $(i = 1,2,\ldots N+1)$ are defined by $2y_i = \ln\{(\omega_i + k_{iz})/(\omega_i - k_{iz})\}$. Throughout this paper we take the c.m. system with the z-axis along the beam direction and take the ordering $y_1 \gtrsim y_2 \gtrsim \cdots \gtrsim y_{N+1}$. The total cross section is written as

$$\sigma = \frac{(2\pi)^4}{s} \sum_N \left\{ \prod_{i=1}^{N} \int \frac{d\vec{k}_i}{(2\pi)^3 2\omega_i} \right\} \cdot \delta^4 \left(p_a + p_b - \sum_i k_i \right) \cdot |T|^2 \tag{2.3}$$

with s being the c.m. energy squared and $s \propto \exp(y_a - y_b)$. Inserting Eq. (2.2) into Eq. (2.3) and integrating over the phase space with p_T-cutoff, we obtain

$$\sigma \propto \exp[\{2(\alpha_2 - 1) + \tilde{g}^2\} \cdot (y_a - y_b)] \tag{2.4}$$

for the single-string process with $\tilde{g}^2 = g^2\langle p_T^2\rangle/(16\pi^2)$. The topological unitarization[8,14] implies $\tilde{g}^2 = 1-\alpha_2$. We are now in a position to factorize the amplitude (2.2) as follows;

$$\mathscr{T} = \exp\left[\frac{1}{2}(\alpha_2-1)\cdot(y_1-y_{N+1})\right] \cong \exp\left[\frac{1}{2}(\alpha_2-1)\cdot(y_a-y_b)\right] \quad , \qquad (2.5)$$

$$\tau = g^{N+1}\cdot\prod_{i=1}^{N}\exp\left[\frac{1}{2}(\alpha_2+1)\cdot(y_i-y_{i+1})\right] \quad . \qquad (2.6)$$

Here τ satisfies the assumption (B). In fact, substituting $(\alpha_2+1)/2$ for α_2 in Eq. (2.4), we have an energy-independent cross section. From our viewpoint the multiperipheral model is irrelevant to other than single-string processes. Owing to the assumption (B), the production of multiple strings results in the overlapping of hadron plateaus. Whereas, when applied the multiperipheral model to double-string processes, no such overlapping occurs.

For the amplitude $\mathscr{T}$ we next introduce a quark counting rule.[15] Namely, it is assumed that (C) $\mathscr{T}$ is the product of the factor attached to each quark (antiquark) which takes part in the production of the extended strings apart from the hard subprocess. In the process as shown in Fig. 3, for example, a spectator quark, a spectator antiquark and a pair-annihilated quark contribute to the production of the extended string. The s-dependence of $\mathscr{T}$ in Eq. (2.5) is brought about by the pair-annihilated quark, since only the quark undergoes high momentum change along the rearrangement. Then, in the soft region we have the counting rule such that the quark rearranging from the cluster i to the cluster j, gives rise to the factor $\exp[-\gamma_n(t)\cdot|y_i-y_j|]$. The factor diminishes in magnitude as the rapidity difference increases.

$\gamma_n(t)$, which represents the Reggeon effect to the quark, is assumed to be given by

$$\alpha_n(t) = 1 - n \cdot \gamma_n(t) \quad . \tag{2.7}$$

$\alpha_n(t)$ means the trajectory of n-body system of q's and/or $\bar{q}$'s. $\gamma_n(o)$ becomes independent of n if $1 - \alpha_n(o)$ obeys the additive rule with respect to the quark number, which is derived from the calculations based on the topological unitarization.[16] However, in this paper we assume the additive rule not for $1 - \alpha_n(o)$ but for the asymptotic value $1 - \alpha_n(-\infty)$. For the quark free from the Reggeon effect we take $\gamma_n = 1(\alpha_n = 1-n)$, which is supposed to be an asymptotic value of $\gamma_n(t)(\alpha_n(t))$ at large $|t|$. In the subsequent sections, this asymptotic behavior of $\gamma_n(t)$ will turn out to be consistent with the dimensional counting rule.[17] In connection with the unitarization for each extended string, the cross channel with respect to $\gamma_n(t)$ is given in the process where a single q-$\bar{q}$ pair is substituted for the breakup in each string.

Correspondingly to the expression of $\mathscr{T}$, τ is rewritten as

$$\tau \sim s \cdot \prod_{i,j} \exp[-\gamma_n(t) \cdot |y_i - y_j|] \quad , \tag{2.8}$$

where the product runs over all quarks in the breakups. In the case of multi-string processes, τ_α defined by

$$\tau_\alpha \sim s_\alpha \cdot \prod_{i,j}^{\{\alpha\}} \exp[-\gamma_n(t) \cdot |y_i - y_j|] \tag{2.9}$$

satisfies the assumption (B). Here s_α is the c.m. energy squared of the extended string labelled as α and the product goes over the quarks within the breakup of the string α. Then, we have the relation

$$\frac{\tau}{s} = \prod_{\alpha} \frac{\tau_\alpha}{s_\alpha} \quad . \tag{2.10}$$

The present model does not contain logarithmic corrections to Regge behavior and to scaling behavior.

III. Meson Distribution Functions in Soft Processes

Let us begin with the soft MM scattering with two extended strings, in which we have four spectators participating in the production of the strings. In this case there are freedoms of sharing the energy among jets, which are absent in the single-string process. Each string disintegrates into two jets moving to the opposite direction, which are labelled in order in Fig. 4. Due to the limited transverse momenta of final hadrons, four jets remain approximately in a collinear configuration. The amplitude is given by

$$\mathscr{T} = \prod_{i=1,3} \exp[-\gamma_2(t_i') \cdot (y_a - y_{i1})] \cdot \prod_{j=2,4} \exp[-\gamma_2(t_j') \cdot (y_{j1} - y_b)] \ , \tag{3.1}$$

$$\tau = s \cdot \frac{\tau_I}{s_I} \cdot \frac{\tau_{II}}{s_{II}} \quad . \tag{3.2}$$

Here we use the notations

$$\begin{aligned} t'_{1(3)} &= (p_a - p'_{3(1)})^2 \ , \qquad t'_{2(4)} = (p_b - p'_{4(2)})^2 \ , \\ s_I &= (p_1 + p_2)^2 \ , \qquad s_{II} = (p_3 + p_4)^2 \ , \end{aligned} \tag{3.3}$$

where p_i stands for the i-th jet momentum $\sum_j p_{ij}$ and p_i' is equal to p_i in $O(\sqrt{s})$ but $p_i'^2 = m^2 (= p_a^2 = p_b^2)$.

Introducing the scaling variables $x_i = 2E_i/\sqrt{s}$ $(i = 1,2,3,4)$, we have

$$\exp(|y_{a(b)} - y_{i1}|) \cong \frac{m_{CT}}{m r_i x_i} \tag{3.4}$$

with m_{CT} being the transverse mass of mesonic clusters. r_i means the ratio p_{i1z}/p_{iz}. The rapidity difference $|y_{a(b)} - y_{i1}|$ increases with decreasing x_i. We now make the approximation that all clusters produced from a string stand in a line with the averaged rapidity spacing η. Then the ratio r_i becomes $r_i \cong 1-e^{-\eta} \equiv r$. The arguments t_i' of γ_2 coming from the mesonic Reggeons are expressed as

$$t_1' \cong -m^2 \frac{(1-x_3)^2}{x_3} \quad , \quad t_3' \cong -m^2 \frac{(1-x_1)^2}{x_1} \tag{3.5}$$

and so forth. Therefore, $\mathscr{T}$ depends only upon the scaling variables x_i.

It is now convenient to transcribe Eq. (2.3) into the form proper to the four-jet production a + b → [1]+[2]+[3]+[4]. According to the scaling feature of $\mathscr{T}$ the integrals can be carried out over the breakups of two extended strings with the fixed jet momenta. Thus

$$\sigma_{MM}^{(2)} = \text{const} \cdot s \left\{ \prod_{i=1}^{4} \int \frac{d\vec{p}_i}{E_i} \right\} \cdot \delta^4\left(p_a + p_b - \sum_i p_i\right) |\mathscr{T}|^2 \quad . \tag{3.6}$$

In consideration with p_T-cutoff in the phase space, the integrals over jet momenta can be put into

$$s \left\{ \prod_{i=1}^{4} \int \frac{d\vec{p}_i}{E_i} \right\} \cdot \delta^4\left(p_a + p_b - \sum p_i\right) \to \text{const} \cdot \left\{ \prod_{i=1}^{4} \int_0^1 \frac{dx_i}{x_i} \right\} \cdot \tag{3.7}$$

$$\cdot\, \delta(x_1 + x_3 - 1)\, \delta(x_2 + x_4 - 1)$$

at large $\sqrt{s}$. The integrals in Eq. (3.7) accommodate the freedom of partitioning the energy $\sqrt{s}/2$ among two jets moving to the same direction. Defining the function

$$S_M^{(2)}(x_1) = \int_0^1 \frac{dx_3}{x_1 x_3} \delta(x_1 + x_3 - 1) \cdot \prod_{i=1,3} \exp[-2\gamma_2(t_i') \cdot (y_a - y_{i1})] \quad , (3.8)$$

we find

$$\frac{d^2\sigma_{MM}^{(2)}}{dx_1 dx_2} = \text{const} \cdot S_M^{(2)}(x_1)\, S_M^{(2)}(x_2) \quad . \tag{3.9}$$

The function $S_M^{(2)}(x)$ shows scaling behavior. This feature is related to the well-known result that double-string processes give rise to the Pomeron part of the total cross section. We are now able to derive the form

$$S_M^{(2)}(x_1) = \text{const} \cdot x_1^{-\alpha_2(t_1')} (1 - x_1)^{-\alpha_2(t_3')} \quad . \tag{3.10}$$

This behaves as

$$S_M^{(2)}(x_1) \propto \begin{cases} x_1^{-\alpha_2(0)} (1 - x_1) & \text{at } x_1 \sim 0 \quad , \\ \{x_1(1 - x_1)\}^{-\alpha_2(-m^2/2)} & \text{at } x_1 \sim \frac{1}{2} \quad , \\ x_1(1 - x_1)^{-\alpha_2(0)} & \text{at } x_1 \sim 1 \quad , \end{cases} \tag{3.11}$$

because $t_1'(t_3') \cong 0$ while $|t_3'|(|t_1'|)$ is large in the vicinity of $x_1 = 0$ ($x_1 = 1$). If we approximate Eq. (3.10) in the form of a single term, we get

$$S_M^{(2)}(x_1) \cong \text{const} \cdot x_1^{-\alpha_2(0)} (1-x_1)^{-\alpha_2(0)} \quad . \tag{3.12}$$

As seen from Eq. (3.8), $S_M^{(2)}(x)$ demonstrates the probability of distributing the energy among two jets with the fractions x and 1-x. Then, aside from its normalization $S_M^{(2)}(x)$ is identified with the j-distribution function for mesons in the soft process.

It is possible to see flavor dependence of $S_M^{(2)}(x)$. The exponent $\alpha_2(0)$ of x in Eq. (3.12) arises from the spectator quark of the jet which carries the fraction x. Taking $\alpha_\rho(0) = 0.5$, $\alpha_{K*}(0) = 0.25$ and $\alpha_{D*}(0) = -1$, we have

$$S_{u/\pi^+}^{(2)}(x) \cong \text{const} \cdot x^{-0.5}(1-x)^{-0.5} \quad , \tag{3.13}$$

$$S_{s/K^-}^{(2)}(x) \cong \text{const} \cdot x^{-0.25}(1-x)^{-0.5} \quad , \tag{3.14}$$

$$S_{c/D}^{(2)}(x) \cong \text{const} \cdot x(1-x)^{-0.5} \tag{3.15}$$

for π, K and D, respectively. The average fractions are $\langle x \rangle_{u/\pi} = 0.50$, $\langle x \rangle_{s/K} = 0.60$ and $\langle x \rangle_{c/D} = 0.80$. However, one of the two jets is inclined to carry the most part of the fraction. In fact, the averaged value of min(x,1-x) is $\langle x_{min} \rangle_\pi = 0.18$. This means that the energy fractions of jets are predominantly far from the equal partition. Therefore, the effective energy of each string is considerably reduced compared with the total energy. The large reduction of the effective energies is of critical importance in understanding the behaviors of the mean multiplicities, as will be presented in Section VI. Furthermore, a heavy-quark jet is more likely to carry the large fraction. This feature qualitatively resembles the fragmentation function[18] and implies that the rapidity extent of the overlapping of two hadron plateaus varies depending on the flavor of the initial hadrons.

IV. Baryon Distribution Functions in Soft Processes

In the soft MB scattering with double strings, a qq-jet appears instead of the $\bar{q}$-jet [4] in Fig. 4. The amplitude $\mathscr{T}$ is given by the substitution $\gamma_2(t_4') \to 2\gamma_3(t_4')$ in Eq. (3.1). γ_3 is due to the baryonic Reggeon. The same manipulation demonstrated in the previous section leads to the j-distribution function

$$S_{q/B}^{(2)}(x_2) = \int_0^1 \frac{dx_4}{x_2 x_4}\,\delta(x_2 + x_4 - 1)\exp[-2\gamma_2(t_2')\cdot(y_{21} - y_b)]\cdot$$

$$\cdot\exp[-4\gamma_3(t_4')\cdot(y_{41} - y_b)] \quad ,$$

$$= \text{const}\cdot x_2^{-\alpha_2(t_2')}(1 - x_2)^{(1-4\alpha_3(t_4'))/3} \tag{4.1}$$

for baryons in double-string processes, where

$$t_2' = -M^2\frac{x_2^2}{(1 - x_2)} \quad , \quad t_4' = -(1 - x_2)\left(\frac{m^2}{x_2} - M^2\right) \tag{4.2}$$

with M being the baryon mass. $S_{q/B}^{(2)}(x)$ also exhibits scaling behavior. Considering the properties at the kinematical limits, we obtain

$$S_{q/B}^{(2)}(x) \propto \begin{cases} x^{-\alpha_2(0)}(1 - x)^3 & \text{at } x \sim 0 \quad , \\ x(1 - x)^{(1-4\alpha_3(0))/3} & \text{at } x \sim 1 \quad , \end{cases} \tag{4.3}$$

which is of the approximate form

$$S^{(2)}_{q/B}(x) \cong \text{const} \cdot x^{-\alpha_2(0)} (1-x)^{(1-4\alpha_3(0))/3} \quad . \tag{4.4}$$

Since $\alpha_3(0) \le 1/4$, $S^{(2)}_{q/B}(x)$ is singular at $x = 0$ but not at $x = 1$. By using $\alpha_3(0) = 0.2 \sim -0.2$, the average fraction of u-quark jets from the proton becomes

$$\langle x \rangle_{u/p} = 0.24 \sim 0.31 \quad . \tag{4.5}$$

It should be noted that the average fraction of the q-jet is smaller than 1/3. Similarly one can find

$$\langle x \rangle_{s/\Lambda,\Sigma} = 0.32 \sim 0.41 \quad , \tag{4.6}$$

$$\langle x \rangle_{c/\Lambda_c,\Sigma_c} = 0.56 \sim 0.64 \quad , \tag{4.7}$$

taking $\alpha_{K*}(0) = 1/4$ and $\alpha_{D*}(0) = -1$.

Next let us take up the triple-string process, which appears in $B\bar{B}$ annihilation. The process results in the production of six jets $a+b \to [1]+[2]+\ldots+[6]$ as illustrated in Fig. 5. The amplitude is written as

$$\mathscr{T} = \prod_i^{\text{odd}} \exp[-\gamma_3(t_i') \cdot (y_a - y_{i1})] \cdot \prod_j^{\text{even}} \exp[-\gamma_3(t_j') \cdot (y_{j1} - y_b)] \quad , \tag{4.8}$$

which comes from six spectators. Equation (3.4) remains valid also in this case. The arguments t_i' are

$$\begin{aligned} t_1' &\equiv (p_a - p_3' - p_5')^2 \cong -(1 - x_3 - x_5)\left\{m^2\left(\frac{1}{x_3} + \frac{1}{x_5}\right) - M^2\right\} \quad , \\ t_3' &\equiv (p_a - p_5' - p_1')^2 \cong -(1 - x_5 - x_1)\left\{m^2\left(\frac{1}{x_5} + \frac{1}{x_1}\right) - M^2\right\} \end{aligned} \tag{4.9}$$

and so on. Thus $\mathscr{T}$ is a scaling function again.

One can obtain the expression of $\sigma^{(3)}_{B\bar{B}}$ similarly to Eq. (3.6). Employing the transformation

$$s\left\{\prod_{i=1}^{6}\int\frac{d\vec{p}_i}{E_i}\right\}\cdot\delta^4\left(p_a+p_b-\sum p_i\right)\rightarrow \text{const}\cdot\left\{\prod_{i=1}^{6}\int_0^1\frac{dx_i}{x_i}\right\}\cdot$$

$$\cdot\,\delta(x_1+x_3+x_5-1)\,\delta(x_2+x_4+x_6-1)\quad, \tag{4.10}$$

one finds

$$\frac{d^4\sigma^{(3)}_{B\bar{B}}}{dx_1dx_2dx_3dx_4}=\text{const}\cdot S_B^{(3)}(x_1,x_3)\;S_B^{(3)}(x_2,x_4) \tag{4.11}$$

with the function

$$S_B^{(3)}(x_1,x_3) = \int_0^1\frac{dx_5}{x_1x_3x_5}\;\delta(x_1+x_3+x_5-1)\cdot\prod_{i}^{\text{odd}}\;\exp[-2\gamma_3(t_i')\cdot(y_a-y_{i1})]\;,$$

$$=\text{const}\cdot x_1^{-(1+2\alpha_3(t_1'))/3}\;x_3^{-(1+2\alpha_3(t_3'))/3}\cdot$$

$$\cdot\,(1-x_1-x_3)^{-(1+2\alpha_3(t_5'))/3}\quad. \tag{4.12}$$

$S_B^{(3)}(x_1,x_3)$ also shows scaling behavior and $\sigma^{(3)}_{B\bar{B}}$ is a constant. Two variables x_1 and x_3 represent the partition of the energy among three $\bar{q}$-jets. Therefore, $S_B^{(3)}(x_1,x_3)$ is identified with the j-distribution function for baryons in triple-string processes and is approximated as

$$S_B^{(3)}(x_1,x_3)\cong\text{const}\cdot\{x_1x_3(1-x_1-x_3)\}^{-(1+2\alpha_3(0))/3}\quad. \tag{4.13}$$

By taking $\alpha_3(0) = 0.2 \sim -0.2$, the exponents become $-(1+2\alpha_3(0))/3 = -0.2 \simeq -0.5$ for the proton. The integral of $S_B^{(3)}(x_1,x_3)$ with respect to x_3 leads to

$$S_B^{(3)}(x_1) \equiv \int_0^{1-x_1} S_B^{(3)}(x_1,x_3)dx_3 \quad ,$$

$$\cong \text{const} \cdot x_1^{-(1+2\alpha_3(0))/3} (1-x_1)^{(1-4\alpha_3(0))/3} \quad . \tag{4.14}$$

Although we have $\langle x \rangle_{u/p} = 1/3$ owing to the symmetric treatment of three strings, the energy fractions of three jets are predominantly far from the equal partition.

V. Distribution Functions in Hard Processes

As an example of hard processes, we refer to deeply inelastic e-p scattering. There are two types of string structure (the single-string process and the double-string one).[10] Summing over final states of each string and using the notations $M\nu = pq$, $Q^2 = -q^2$ and $s = (p+q)^2$, one finds the structure function[19]

$$\nu W_2 = \text{const} \cdot \frac{s}{(1-x)} \left\{ \prod_{i=1}^{n} \int \frac{d\vec{p}_i}{E_i} \right\} \cdot \delta^4\left(p+q-\sum_i p_i\right) |\mathcal{T}|^2 \tag{5.1}$$

for the jet production "γ" + p $\rightarrow$ [1]+[2]+...+[n] at fixed $x = Q^2/(2M\nu)$.

For the single-string process as shown in Fig. 6, two jets [1] and [2] are produced collinearly along the z-axis. Since the energy fractions of the jets are settled as $x_i = 2E_i/\sqrt{s} = 1$ $(i = 1,2)$, Eq. (5.1) is reduced to

$$\nu W_2 = \text{const} \cdot \frac{1}{(1-x)} |\mathcal{T}|^2 \quad . \tag{5.2}$$

The amplitude $\mathscr{T}$ comprises three factors, i.e., $\exp[-(y_{q2} - y_{11})] \cong m_q r/m_{CT}$, $\exp[-2(y_{21} - y_p)] \cong \{Mr(1-x)/m_{CT}\}^2$ and $\exp[-\gamma(t)(y_{q1} - y_p)] \cong (Mx/m_q)^{\gamma(t)}$. The rapidities y_{q1}, y_{q2} and y_p belong to the momenta $q_1(=xp)$, $q_2(=q_1+q)$ and p, respectively. The difference $y_{21} - y_p$ becomes large with increasing x. The Reggeon has an influence only on the quark rearranged in the crossed channels. The argument t is given by $t = (p - p_2')^2 \cong -M^2x^2/(1-x)$, where p_2' is equal to p_2 in $O(\sqrt{s})$ but $p_2'^2 = M^2$. The structure function νW_2 is expressed in terms of the c-distribution function

$$F_B^{(v)}(x) \cong \text{const} \cdot x^{-\alpha_2(t)}(1-x)^3 \tag{5.3}$$

for the valence part of baryons. The exponent of $(1-x)$ amounts to $2n_s - 1$ with n_s being the number of the spectator quarks. Equation (5.3) is attained by replacing the exponent α_3 of $(1-x)$ in Eq. (4.1) by its asymptotic value. It should be noticed that the c-distribution function does not correspond to the freedom of sharing the energy among jets. In fact, in the valence component there are no such freedoms. But the freedom emerges in the sea component.

We now proceed to consider the double-string processes relevant to the sea component. There are two diagrams according to whether the virtual photon scatters with a quark or with an antiquark, as illustrated in Fig. 7. In the processes we have not only the freedom mentioned above but also peculiar string structures. In Fig. 7a, for instance, the string I ranges from $y_1 \cong \ell n(\sqrt{s}/m_{CT})$ to $y_2 \cong -\ell n(x_2\sqrt{s}/M_{CT})$ with the length

$$y_1 - y_2 \cong \ell n\left(\frac{x_2 s}{m_{CT} M_{CT}}\right) \tag{5.4}$$

and disintegrates into two jets [1] and [2]. On the other hand, the string II shrinks into the rapidity interval between $y_{3\ell}$ and $y_3 \cong -\ell n(x_3\sqrt{s}/m_{CT})$, where the cluster (3ℓ) has the largest rapidity on the string II and the averaged value of $w \equiv m_{CT}\exp(-y_{3\ell})/\sqrt{s}$ turns out to be $\sim 4m_{CT}x/(3m_q(1-x))$ $(\cong 4m_{CT}Q^2/(3m_q s))$. Then, the string II becomes a single jet [3] on the negative rapidity side except for the case $w < m_{CT}/\sqrt{s}$ $(y_{3\ell} > 0)$. Since $y_{3\ell}$ possibly varies under the kinematical constraint $-y_{3\ell} \leq -y_3 \cong \ell n(x_3\sqrt{s}/m_{CT})$, the integration of $y_{3\ell}$ is left throughout taking inclusive sum. We have to insert the integral $\int dy_3 = \int dw/w$ in the r.h.s. of Eq. (5.1) and the variable w is limited to $0 < w \leq x_3$. The case of $w = x_3$ means the threshold of the jet [3]. Consequently, the string II remains $\sim\ell n(x_3/w)$ in rapidity length.

In the event of $w < m_{CT}/\sqrt{s}$, the string II expands also into the positive rapidity region. Therefore, the energy conservation should be taken into account on the positive rapidity side and for the string I $y_1 \cong \ell n(\sqrt{s}/m_{CT})$ is replaced by $y_1 \cong \ell n(u\sqrt{s}/m_{CT})$ with $u \cong 1 - m_{CT}^2/(ws)$. However, we have $u \cong 1 - 3m_{CT}m_q/(4Q^2)$ in the dominant contribution because $\langle w\rangle \cong 4m_{CT}x/(3m_q)$. For this reason, on the positive rapidity side the effect of the energy conservation to the string I is not only restricted within $Q^2 \lesssim 3m_q\sqrt{s}/4$ but also small in the region $Q^2 \gtrsim 8m_{CT}m_q$ in which we have $u \gtrsim 0.9$. To illustrate the situation mentioned above, in Fig. 8 we show the behaviors of hadron plateaus from the strings I and II.

The structure function is put into

$$\nu W_2 = \text{const}\cdot\frac{1}{(1-x)}\int_0^1 dx_3 \int_0^{x_3} dw\, f_a(x,x_3,w) \quad , \tag{5.5}$$

where

$$f_a(x,x_3,w) = \frac{|\mathscr{T}|^2}{x_3(1-x_3)w} \tag{5.6}$$

for the case of Fig. 7a. $\mathscr{T}$ is of the form

$$\mathscr{T} \cong \exp[-|y_{11}-y_{q2}| - |y_{q1}-y_{3\ell}| - 2\gamma_3(t_2')(y_p-y_{21}) - \gamma_2(t_3')(y_p-y_{31})] \quad , \tag{5.7}$$

where

$$\exp(-|y_{11} - y_{q2}|) \cong r \quad , \quad \exp(-|y_{q1} - y_{3\ell}|) \cong \min\left(\frac{w}{\Delta}, \frac{\Delta}{w}\right) \quad ,$$
$$\exp(y_{21}-y_p) \cong (1-x)(1-x_3)rM_{CT}/M \quad , \quad \exp(y_{31}-y_p) \cong (1-x)x_3rM/m_{CT} \tag{5.8}$$

with $\Delta = xm_{CT}/((1-x)m_q)$. The arguments of $\gamma_{2,3}$ are given by

$$t_2' = -\left\{1 - (1-x)x_3\right\} \cdot \left\{\frac{m^2}{(1-x)x_3} - M^2\right\} \quad ,$$
$$t_3' = -M^2 \frac{\left\{1 - (1-x)(1-x_3)\right\}^2}{(1-x)(1-x_3)} \quad . \tag{5.9}$$

The variable x_3 represents the freedom of sharing the energy among jets. Then, the j-distribution function is involved in the expression of f_a. As the Reggeon does not come into effect at large x, we have

$$f_a(x,x_3,w) \cong \text{const} \cdot x^{-4}(1-x)^8 x_3(1-x_3)^3 w \tag{5.10}$$

at large x. Then the distribution function is proportional to $(1-x)^7$ at large x and the exponent of $(1-x)$ amounts to $2n_s - 1$ with $n_s = 4$. This result is in accord with the spectator counting rule.[20] On the other hand, at small x ($x \lesssim 0.2$) f_a is approximated as

$$f_a = \text{const} \cdot \left\{(1-x)x_3\right\}^{-\alpha_2(0)} \cdot \left\{(1-x)(1-x_3)\right\}^{(1-4\alpha_3(0))/3} \cdot$$
$$\cdot \min\left\{\left(\frac{w}{\Delta}\right)^2, \left(\frac{\Delta}{w}\right)^2\right\} . \tag{5.11}$$

For the process in Fig. 7b the calculation is parallel. The difference is attributed to the rapidity location of the qq-jet. f_b is obtained by the substitution $x_3 \leftrightarrow (1-x_3)$ in Eqs. (5.10) and (5.11) and in Eq. (5.5) the upper limit x_3 with respect to w is replaced by $m_{CT}x_3/M_{CT}$. The distribution functions for Fig. 7a and for Fig. 7b are almost equal to each other. However, it should be noted that the favored energy fractions carried by the qq-jet are different among two cases. If we consider only the effect of the valence quark, the probability of finding the qq-jet with the energy fraction v is given by

$$\sim \text{const} \cdot v^{(1-4\alpha_3(t_q))/3} (1-v)^{-\alpha_2(t_{qq})} , \tag{5.12}$$

where

$$t_q = -\left\{1-(1-x)(1-v)\right\}\left\{\frac{m^2}{(1-x)(1-v)} - M^2\right\} ,$$
$$t_{qq} = -M^2 \frac{\left\{1-(1-x)v\right\}^2}{(1-x)v} . \tag{5.13}$$

As a result of (5.12), it becomes that the larger v is more likely with decreasing x. The function (5.12) is hardly influenced by the sea-quark at small x but fairly affected at large x where the string II is short in rapidity length. The configuration with the large v is suppressed in Fig. 7a, whereas enhanced in Fig. 7b. This situation is also shown in Fig. 8.

VI. Mean Multiplicities

In order to test the present model, in this section we calculate the mean multiplicities $\langle n\rangle$ of final hadrons and compare with the data. Our calculations rely on the distribution functions derived in the preceding sections. It will become evident that the underlying string structure has a significant influence on the mean multiplicities and that not only the number of strings but also the large reduction of their effective energies are of critical importance in understanding the behaviors of $\langle n\rangle$.

For the production of a single q-$\bar{q}$ string with the c.m. energy $\sqrt{s}$ we take the parametrization

$$\langle n\rangle_1(\sqrt{s}) = a(\ell n\sqrt{s})^2 + b\,\ell n\sqrt{s} + 2c_q \quad , \tag{6.1}$$

where a, b and c_q are numerical constants. This form gives a good fit to the pp data.[21] The process $e^+e^- \to$ hadrons is one of the typical processes with a single string. Then, we have

$$\langle n\rangle_{e^+e^-}(\sqrt{s}) = \langle n\rangle_1(\sqrt{s}) \quad , \tag{6.2}$$

supposing the universal property of extended strings both for the soft reactions and for the hard ones. In pp scattering accompanying double strings, $\langle n\rangle$ is given by

$$\langle n\rangle_{pp}(\sqrt{s}) = \frac{\int_0^1 dx \int_0^1 dx'\, S_p^{(2)}(x)\; S_p^{(2)}(x')(\langle n_I\rangle + \langle n_{II}\rangle)}{\int_0^1 dx \int_0^1 dx'\; S_p^{(2)}(x)\; S_p^{(2)}(x')} \quad , \tag{6.3}$$

where $\langle n_I \rangle$ and $\langle n_{II} \rangle$ stand for the average multiplicities of hadrons from the strings I and II, respectively. They are of the forms

$$\begin{aligned} \langle n_I \rangle &= \langle n \rangle_1 \left(\sqrt{sx(1-x')} \right) + c_{qq} - c_q \quad , \\ \langle n_{II} \rangle &= \langle n_I \rangle (x \leftrightarrow x') \quad . \end{aligned} \tag{6.4}$$

Since each string of this process consists of a q-jet and a qq-jet, the constant terms of $\langle n_I \rangle$ and $\langle n_{II} \rangle$ are put into $c_q + c_{qq}$. Equation (6.3) is transcribed as

$$\langle n \rangle_{pp}(\sqrt{s}) = 2 \left\{ \langle n \rangle_1 \left(\frac{\sqrt{s}}{\lambda_2} \right) + c_{qq} - c_q \right\} + \frac{1}{2}\, a d_2 \quad , \tag{6.5}$$

where

$$\lambda_2 = \exp\left[-\frac{1}{2} \langle \ln x(1-x) \rangle_2 \right] \quad , \tag{6.6}$$

$$d_2 = \langle (\ln x)^2 \rangle_2 + \langle (\ln(1-x))^2 \rangle_2 - (\langle \ln x \rangle_2)^2 - (\langle \ln(1-x) \rangle_2)^2 \quad . \tag{6.7}$$

The symbol $\langle \ldots \rangle_2$ means the averaged value with the distribution function $S_p^{(2)}(x)$. It should be noticed that the effective energy in the r.h.s. of Eq. (6.5) is reduced by the factor λ_2 compared with Eq. (6.1). This is essentially a reflection of the multi-string structure. The value of λ_2 becomes fairly larger than the minimum value $\lambda_2 = 2$ because $\langle x_{min} \rangle$ is small.

Similarly, for $\bar{p}p$ annihilation the average multiplicity is expressed as

$$\langle n \rangle_{\bar{p}p\,annih}(\sqrt{s}) = 3\langle n \rangle_1 \left(\frac{\sqrt{s}}{\lambda_3} \right) + \frac{1}{2}\, a d_3 \quad , \tag{6.8}$$

where

$$\lambda_3 = \exp[-\langle \ell n\, x \rangle_3] \quad , \tag{6.9}$$

$$d_3 = 3\left\{\langle (\ell n\, x)^2 \rangle_3 - (\langle \ell n\, x \rangle_3)^2\right\} \quad . \tag{6.10}$$

In this case the symbol $\langle \ldots \rangle_3$ stands for the average value with $S_p^{(3)}(x,x')$. The minimum value of λ_3 is 3. From Eqs. (6.2), (6.5) and (6.8) we obtain the relations

$$\langle n \rangle_{\bar{p}p\,annih}(\sqrt{s}) = \frac{3}{2}\cdot \langle n \rangle_{pp}\left(\frac{\lambda_2}{\lambda_3}\sqrt{s}\right) + 3\left(c_q - c_{qq} - \frac{1}{4}\, ad_2\right) + \frac{1}{2}\, ad_3 \quad , \tag{6.11}$$

$$\langle n \rangle_{e^+e^-}(\sqrt{s}) = \frac{1}{2}\cdot \langle n \rangle_{pp}(\lambda_2\sqrt{s}) + \left(c_q - c_{qq} - \frac{1}{4}\, ad_2\right) \quad . \tag{6.12}$$

Four constants λ_2, λ_3, d_2 and d_3 vary depending on the Regge intercepts $\alpha_2(0)$ and $\alpha_3(0)$. As the leading trajectories, we take the values

$$\alpha_2(0) = 0.55 \quad , \quad \alpha_3(0) = 0.15 \quad , \tag{6.13}$$

which lead to

$$\lambda_2 = 4.1 \quad , \quad \lambda_3 = 6.5 \quad , \quad d_2 = 5.6 \quad , \quad d_3 = 9.6 \quad . \tag{6.14}$$

It is worth noting that λ_2 and λ_3 are about two times larger than their minimum values which were taken in Ref. 10. λ_2 and λ_3 are slightly sensitive to $\alpha_2(0)$ and $\alpha_3(0)$, respectively. For example, at $\alpha_3(0) = 0.20$ we have $\lambda_3 = 6.9$. But the calculations are insensitive to our approximations of the j-distribution functions which are made in Eqs. (4.4) and (4.13).

Here we take the pp data as an input. The data on the mean charged multiplicity can be fitted in terms of the function[21]

$$\langle n_{ch}\rangle_{pp}(\sqrt{s}) = 0.472 \cdot (\ln\sqrt{s})^2 + 0.88 \cdot \ln\sqrt{s} + 0.88 \qquad (6.15)$$

with GeV unit in the region $3 \leq \sqrt{s} \leq 152$ GeV. In this case we get

$$a = 0.236 \quad , \quad b = 1.108 \quad , \quad c_q + c_{qq} = 1.21 \quad . \qquad (6.16)$$

Only one adjustable parameter $c_q - c_{qq}$ is now left. In Fig. 9 we show the calculations with $c_q - c_{qq} = 0.55$ for $\bar{p}p$ annihilation and for $e^+e^- \to$ hadrons together with the data. As seen in Fig. 9, the calculation is close to the asymptotic value[24] $\langle n\rangle_{\bar{p}p\,annih}/\langle n\rangle_{pp} = 1.5$ as low as $\sqrt{s} \sim 10$ GeV, because the ratio λ_3/λ_2 is rather close to unity. And our calculations are brought into line with the data. While another asymptotic relation[24] $\langle n\rangle_{e^+e^-} = 1/2\langle n\rangle_{pp}$ is kept away in the calculations even at $\sqrt{s} = 100$ GeV. This is interpreted as due to the large value of λ_2. Furthermore, the fit for e^+e^- is in good agreement with the data at $\sqrt{s} \lesssim 9$ GeV but above 9 GeV the calculation is small compared to the data. This discrepancy may be attributable to the bottom production and to the hard-gluon jets which are suppressed in hadron-hadron collisions. It is necessary to extract the data on two-jet events without heavy quarks. We cannot expect the universal property of strings including the hard processes in the naive form. And also the presently available energy is not high enough to test the present model.

Next we compute the mean multiplicity in deep inelastic lepton-hadron scattering, which has been discussed by several authors.[25,10] Since the valence component, for example, in e-p scattering implies a single string which comprises a hard q-jet and a soft qq-jet, we simply have

$$\langle n\rangle^{(v)}_{ep}(\sqrt{s}) = \frac{1}{2}\left\{\langle n\rangle_1(\sqrt{s}) + \langle n\rangle_{e^+e^-}(\sqrt{s})\right\} + c_{qq} - c_q \quad , \qquad (6.17)$$

which is independent of Q^2 at fixed s. On the other hand, for the sea component the final hadrons come from two strings. In order to cancel the contributions from the hard q-jet, it is adequate to consider the difference of the mean multiplicities among the valence component and the sea component

$$\langle \Delta n \rangle_{ep} = \langle n \rangle_{ep}^{(s)} - \langle n \rangle_{ep}^{(v)} \quad . \tag{6.18}$$

$\langle \Delta n \rangle_{ep}$ is described as

$$\langle \Delta n \rangle_{ep} = \frac{\int_0^1 dx_3 \int_0^{x_3} dw\, f(x,x_3,w)\, (\langle n_I \rangle + \langle n_{II} \rangle)}{\int_0^1 dx_3 \int_0^{x_3} dw\, f(x,x_3,w)} - \langle n \rangle_1 (\sqrt{s}) \quad , \tag{6.19}$$

where

$$\langle n_I \rangle + \langle n_{II} \rangle = \langle n \rangle_1 \left(\sqrt{s(1-x_3)} \right) + \langle n \rangle_1 \left(m_{CT} \sqrt{x_3/w} \right) + c_{qq} - c_q \tag{6.20}$$

and $f = f_a + f_b$. As the sea component dominates over the valence one only at small x, it is sufficient to refer to the region $x \lesssim 0.2$. The approximate expression becomes

$$\langle \Delta n \rangle_{ep} \cong -\frac{1}{2} (a \ln s + b) \cdot \ln \lambda(x) + \langle n \rangle_1 \left(\frac{m_{CT}}{\sqrt{\xi(x) \cdot \Delta}} \right) - 0.4 \quad . \tag{6.21}$$

The comparison of Eq. (6.21) with Eqs. (6.19) and (6.20) shows that the first term of Eq. (6.21) is attributable to the reduction of the string I and that the second term is the contribution of the string II. $\lambda(x)$ and $\xi(x)$ represent the reduction factors of the strings I and II, i.e., $\exp(-\langle \ln(1-x_3) \rangle)$ and $\exp(-\langle \ln(x_3 \Delta/w) \rangle)$, respectively. In the manipulation of taking the average w is converted into $\sim\Delta = m_{CT} x/(m_q(1-x))$.

λ and ξ approach $\lambda_2 = 4.1$ at sufficiently large s with fixed Q^2, but $\lambda > \lambda_2 > \xi$. Especially, the reduction of the string I is considerably large. In fact, we have $\lambda \sim 15$ while $\xi \sim 1$ at $x \sim 0.2$ and $\lambda \sim 4.8$ and $\xi \sim 2.4$ at $x \sim 0.01$, when we set $m_{CT} = 2M_{CT}/3 = 2m_q$ and $m_{CT}^2 = 0.6\ (\mathrm{GeV})^2$.

In Fig. 10 our calculations are exhibited. Although $\langle \Delta n \rangle_{ep}$ in Fig. 10 (the dotted curves) is roughly a linear function of $\ell n Q^2$, it is only in considerably small x region that $\langle \Delta n \rangle_{ep}$ keeps a positive value. For example, the regions in which $\langle \Delta n \rangle_{ep}$ remains positive are $Q^2 \lesssim 6\ (\mathrm{GeV})^2$ ($x \lesssim 0.06$) at $\sqrt{s} = 10$ GeV and $Q^2 \lesssim 200\ (\mathrm{GeV})^2$ ($x \lesssim 0.02$) at $\sqrt{s} = 100$ GeV. This results from the large reduction of the effective energies for the strings together with the shrinkage of the string II. In order to compare the calculations with the data, we define

$$\langle\langle \Delta n \rangle\rangle_{ep} \equiv \langle \Delta n \rangle_{ep} \cdot \frac{F_2^{(s)}}{F_2^{(v)} + F_2^{(s)}} \quad , \tag{6.22}$$

where $F_2^{(v),(s)}$ stand for the structure functions of the valence and the sea, respectively. The quantity is related as

$$\langle\langle \Delta n \rangle\rangle_{ep} = \langle n \rangle_{ep} - \langle n \rangle_{ep}^{(v)} \quad , \tag{6.23}$$

where $\langle n \rangle_{ep}$ is the observed mean multiplicity and is given by

$$\langle n \rangle_{ep} = \frac{\langle n \rangle_{ep}^{(v)} \cdot F_2^{(v)} + \langle n \rangle_{ep}^{(s)} \cdot F_2^{(s)}}{F_2^{(v)} + F_2^{(s)}} \quad . \tag{6.24}$$

Since the sea contribution is negligibly small at large x, say $x \gtrsim 0.3$, $\langle n \rangle_{ep}^{(v)}$ in Eq. (6.23) can be replaced by the averaged value of $\langle n \rangle_{ep}$ at $x \gtrsim 0.3$. Inserting the structure functions given in Ref. 26 into

Eq. (6.22), we obtain $\langle\langle \Delta n \rangle\rangle_{ep}$, which is shown by the solid curves in Fig. 10 together with the νp data at $\sqrt{s}$ = 8-10 GeV.[27] However, the present data[27,28] are not enough to test our calculations and more data at higher $\sqrt{s}$ are desirable.

VII. Summary and Discussions

In this paper we have derived the j- and the c-distribution functions corresponding to the respective string structure. The j-distribution functions, which represent the freedom of sharing the energy among jets, imply that the energy fractions of jets are predominantly far from the equal partition. Therefore, the freedom causes the large reduction of the effective energy of each string. It is the j-distribution function to which we ought to refer in the calculations for the soft processes. In order to compare the present model with experiments, we have calculated the mean multiplicities for several processes and found that the multiplicities are significantly affected by the number of the strings and also by the reduction of their effective energies. The reduction factors are ~4 and ~6.5 in the soft processes with double and triple strings, respectively. In the deep inelastic lepton-hadron scattering the c-distribution function does not correspond to the freedom of distributing the energy among jets, which is described by another variable (x_3 in Section V). Indeed, the valence component belongs to the single-string production, in which the freedom is absent. In the sea component we have a peculiar double-string structure and one of the strings shrinks compared with another one. The difference of the mean multiplicity $\langle n \rangle - \langle n \rangle^{(v)}$, which is free from the effect of the hard jet, has been predicted to increase with decreasing Q^2 at fixed $\sqrt{s}$ higher than ~20 GeV. It is also a consequence of the large reduction of effective energies that the increase of $\langle n \rangle - \langle n \rangle^{(v)}$ emerges at long last in such high energy region. Then, the more data at higher energies than the presently available region are needed to test the present model.

To reveal the underlying string structure, it is also useful to study the one-particle distributions. In multi-string processes the hadron plateaus coming from each string overlap in the rapidity space. The detailed studies of the overlapping for BB and for $\overline{B}B$ have been already made by Capella et al.[9] and by Sukhatme,[29] respectively. However, they used δ-functions for the distribution functions. The present model implies that the hadron plateaus overlap almost only in the small x_F region because of the rather small value of $\langle x_{min} \rangle$. The expected one-particle distributions are similar in shape to those obtained in Refs. 9 and 29 in the central region. At large $|x_F|$ there appears only a slight difference between the one-particle distributions for multi-string processes and those for single-string processes. This feature in the fragmentation region is consistent with the experiments.[2]

As for two-particle distributions in the multi-string processes, it is predicted that a positive correlation for the exotic combination associated with the initial hadron emerges in a specific region. On a single string we have a negative short-range correlation for $\pi^+ - \pi^+$ but a positive one for $\pi^+ - \pi^-$. In the multi-string processes, owing to the independent breakups of each string there are no correlations between hadrons decaying from the different strings except for the leading particles, which get a positive correlation. For instance, in the double-string process of π^+-beam we have a u-jet and a $\bar{d}$-jet in the beam fragmentation region. In both the jets positively charged hadrons appear predominantly as the fastest particles compared with negative ones. Since $\langle x_{min} \rangle = 0.18$, the rapidity difference of two leading particles becomes $\langle |\Delta y| \rangle = \ln (0.82/0.18) \cong 1.5$ in average. Therefore, in this case it is expected that when the fastest π^+ is triggered, the $\pi^+ - \pi^+$ correlation on the beam side changes its sign from negative to positive around $|\Delta y| \cong 1.5$ and then disappears as $|\Delta y|$ increases.

Furthermore, it is important to study the overlapping of hadron plateaus in the sea component in deep inelastic lepton-hadron scattering. In charm productions of e-p collisions D-$\bar{D}$ correlations will also inform us about the string structure. When D ($\bar{D}$) is produced as a leading particle in the current fragmentation region, the rapidity location of $\bar{D}$ (D) varies depending on Q^2 at large fixed s unlike D ($\bar{D}$).

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FIGURE CAPTIONS

1. Examples of the soft hadron-hadron collisions; (a) BB, (b) $\bar{B}B$ with double strings and (c) $\bar{B}B$ annihilation.
2. The string structure of deep inelastic lepton-hadron scattering; (a) the valence component and (b) the sea component.
3. The soft MM scattering with a single string.
4. The soft MM scattering with double strings, which results in the four-jet production a+b → [1]+[2]+[3]+[4]. The jets [1] and [3] ([2] and [4]) go along the direction of the initial hadron a (b).
5. The soft $\bar{B}B$ scattering with triple strings, which implies the six-jet production a+b → [1]+...+[6].
6. The string structure of the valence component for deep inelastic e-p scattering, which shows the two-jet production "γ"+p → [1]+[2].
7. Two types of the string structure for the sea component, where mostly three jets are produced. The jet [3] (the string II) comprises a q-jet in (a) and a qq-jet in (b).
8. The averaged behaviors of hadron plateaus from the strings I and II. The cases (a) and (b) correspond to Fig. 7a while (c) and (d) to Fig. 7b. The locations and the overlappings of the hadron plateaus from the strings strongly depend upon x.
9. The comparison of the calculations of $\langle n_{ch} \rangle$ with the experimental data. The fit of $\langle n_{ch} \rangle$ for pp in Ref. 21 are used as an input. The data for $\bar{p}p$ annihilation and for e^+e^- → hadrons are taken from Refs. 22 and 23, respectively.

10. The difference of the charged mean multiplicity $\langle\langle \Delta n_{ch} \rangle\rangle = \langle n_{ch} \rangle - \langle n_{ch} \rangle^{(v)}$. The solid curves and the dotted curves show the calculations of $\langle\langle \Delta n_{ch} \rangle\rangle$ and $\langle \Delta n_{ch} \rangle$, respectively. The data are from Ref. 27 and $\langle n_{ch} \rangle^{(v)}$ is taken as the averaged value of $\langle n_{ch} \rangle$ at $x \gtrsim 0.3$.

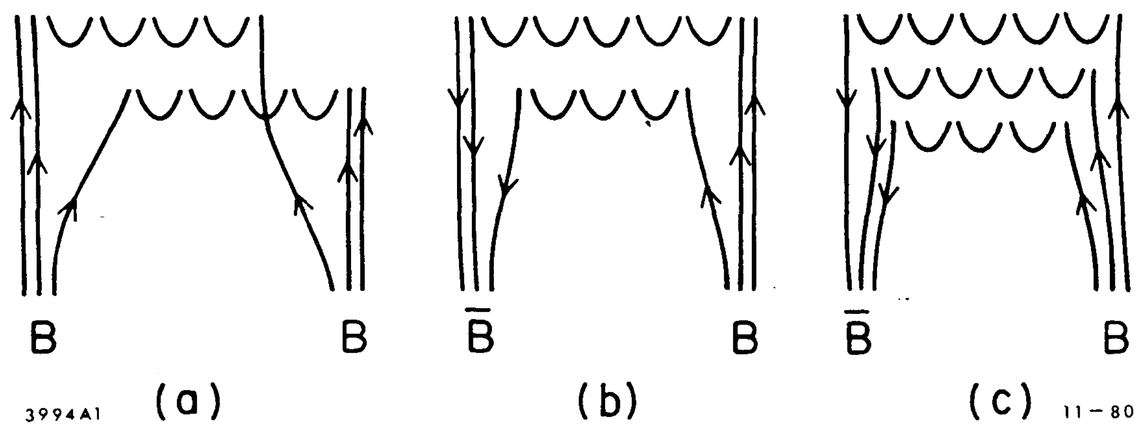


Fig. 1

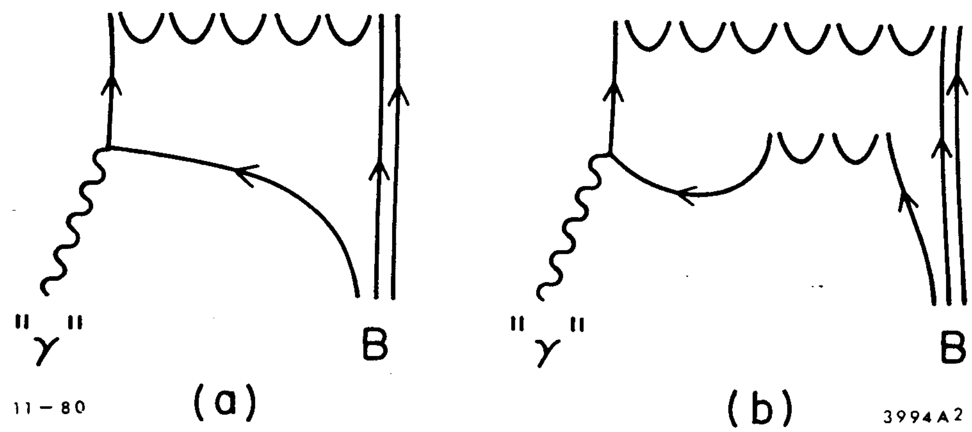


Fig. 2

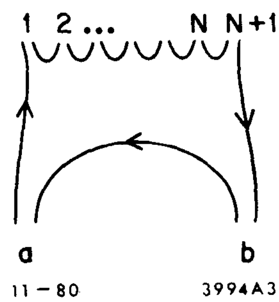
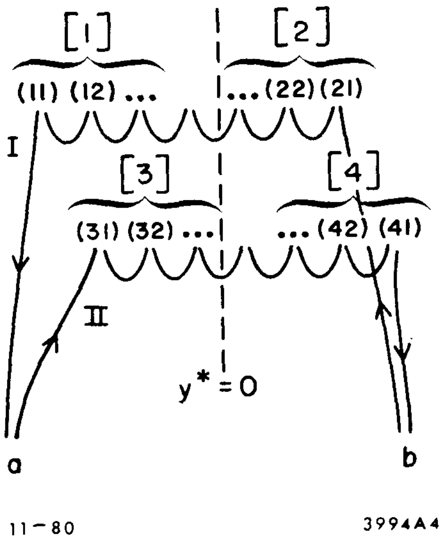


Fig. 3



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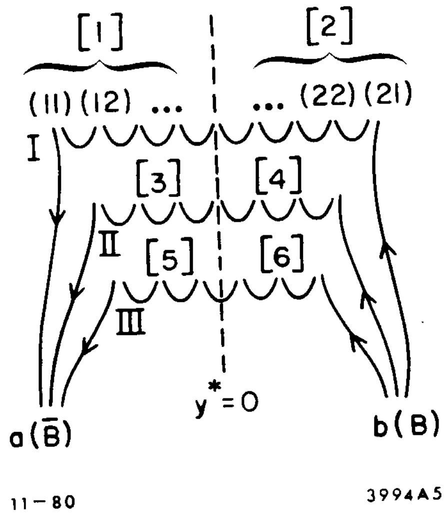
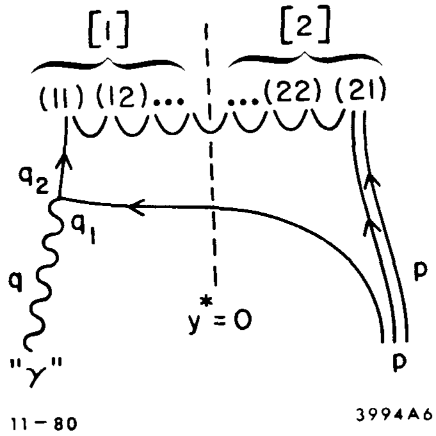


Fig. 5



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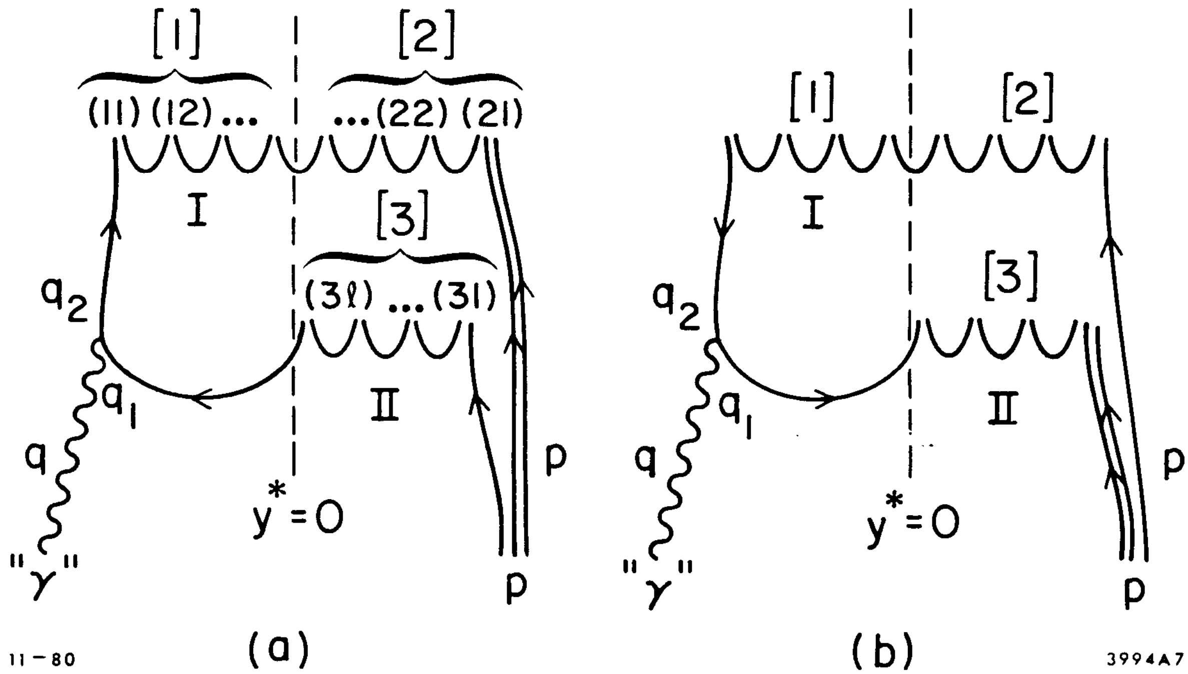
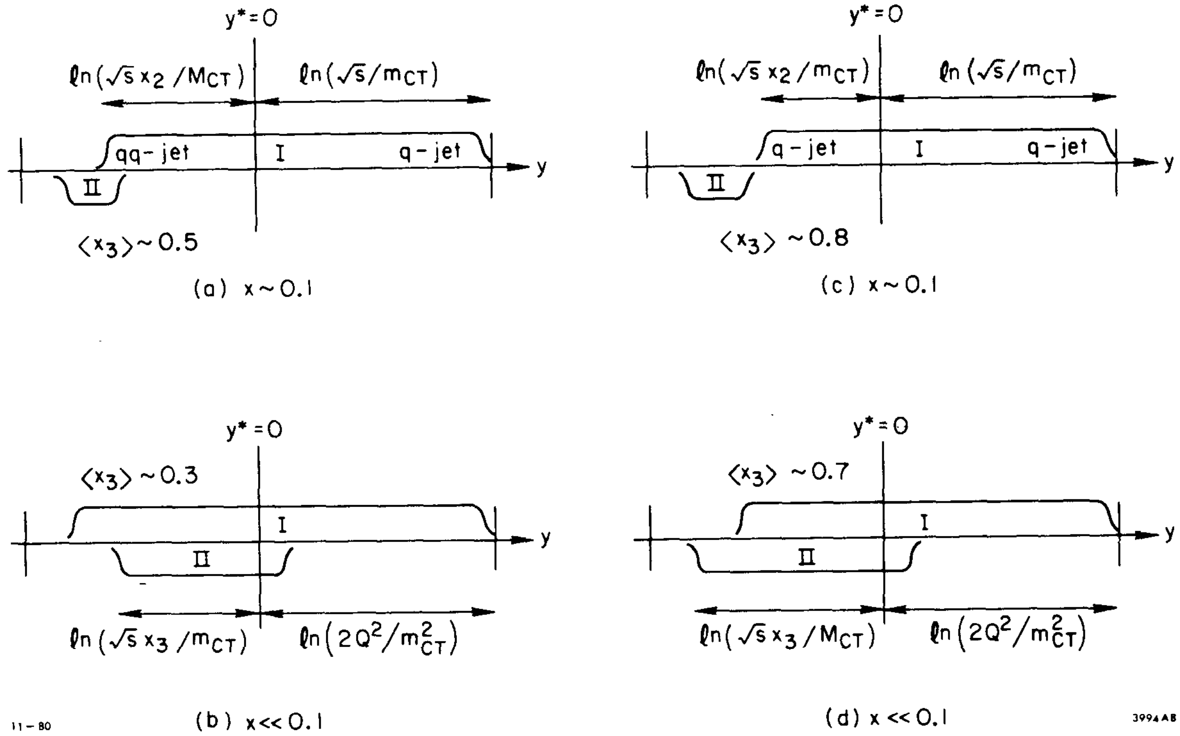

[1]
[2]
(11) (12)...
...(22) (21)
I
[3]
(3ℓ) ... (31)
q_2
q_1
II
q
p
$y^*=0$
"γ"
p
(a)
11-80
[1]
[2]
I
[3]
q_2
q_1
II
q
p
$y^*=0$
"γ"
p
(b)
3994A7

Fig. 7



y* = 0
ℓn(√s x2 /MCT)
ℓn(√s/mCT)
qq - jet
I
q - jet
y
II
⟨x3⟩ ~ 0.5
(a) x ~ 0.1
y* = 0
ℓn(√s x2 /mCT)
ℓn(√s/mCT)
q - jet
I
q - jet
y
II
⟨x3⟩ ~ 0.8
(c) x ~ 0.1
y* = 0
⟨x3⟩ ~ 0.3
I
y
II
ℓn(√s x3 /mCT)
ℓn(2Q²/m²CT)
(b) x << 0.1
y* = 0
⟨x3⟩ ~ 0.7
I
y
II
ℓn(√s x3 /MCT)
ℓn(2Q²/m²CT)
(d) x << 0.1
11 - 80
3994A8

Fig. 8

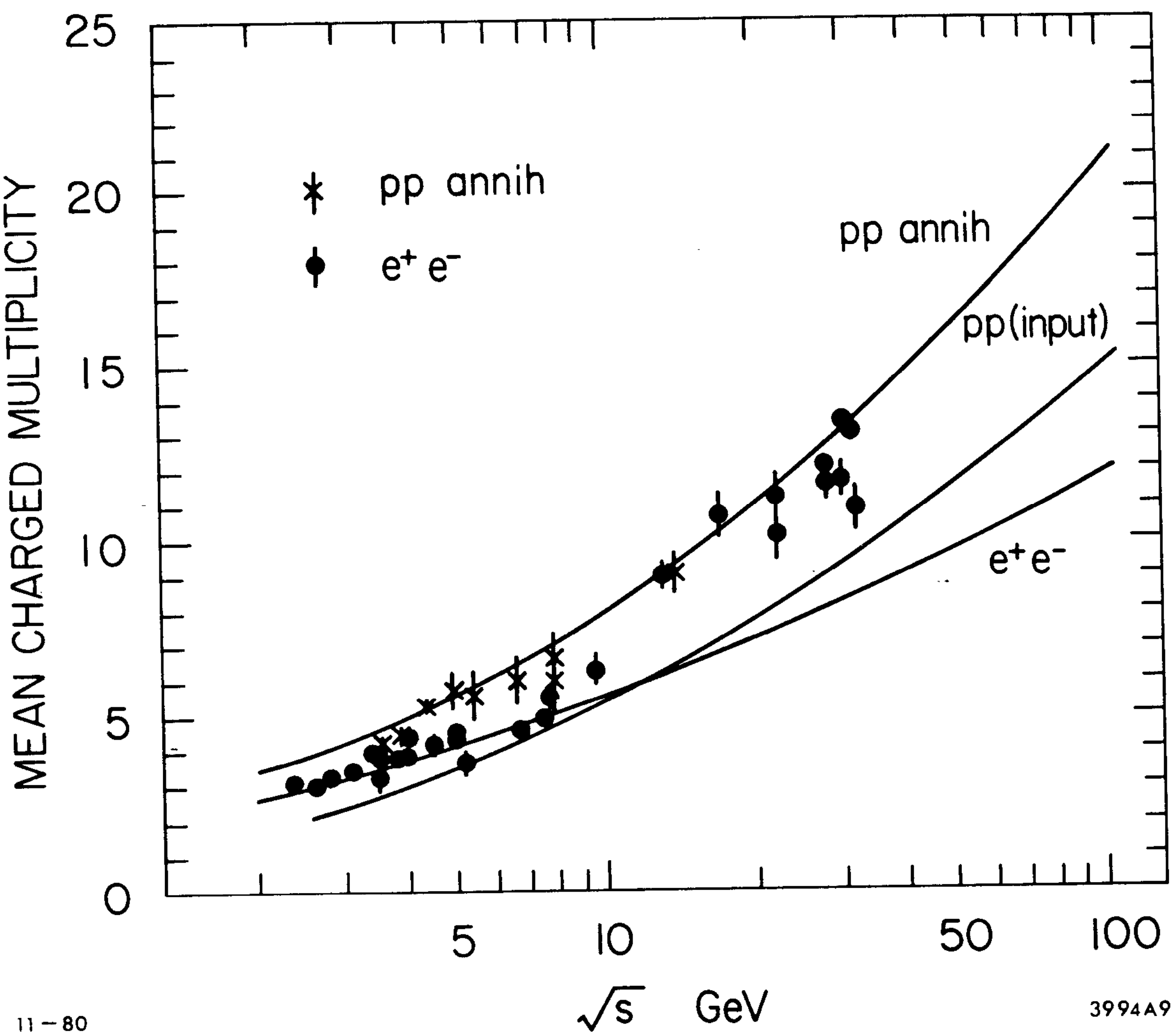

25
20
15
10
5
0
MEAN CHARGED MULTIPLICITY
pp annih
e+ e-
pp annih
pp(input)
e+e-
5
10
50
100
√s GeV
11-80
3994A9

Fig. 9

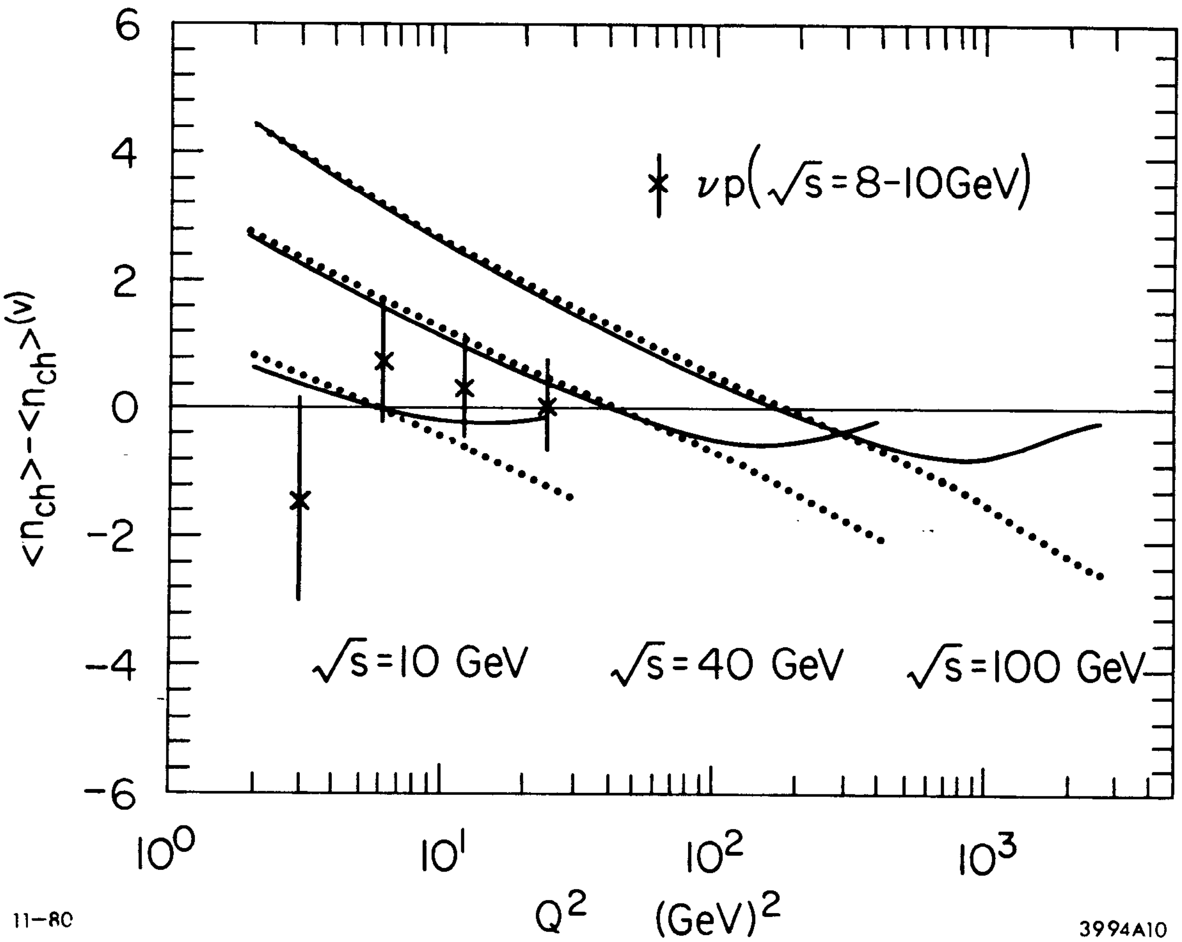

6
4
2
0
-2
-4
-6
$\nu p(\sqrt{s}=8\text{-}10\text{GeV})$
$\langle n_{ch}\rangle - \langle n_{ch}\rangle^{(\nu)}$
$\sqrt{s}=10$ GeV
$\sqrt{s}=40$ GeV
$\sqrt{s}=100$ GeV
10^0
10^1
10^2
10^3
Q^2 $(\text{GeV})^2$
11-80
3994A10

Fig. 10