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SOME THOUGHTS ON THE MASS OF THE TOP QUARK*

Stuart Raby

Stanford Linear Accelerator Center

Stanford University, Stanford, CA 94305

ABSTRACT

We propose a new mechanism for up-down symmetry breaking within the context of a technicolor scenario. The experimentally determined ratio $M_W/M_Z\cos\theta_W \simeq 1$ is in addition preserved at the technicolor scale. If we assume that the mechanism works at the level of the heaviest generation we find $m_t = (f^{-1/2})38$ GeV. The parameter f depends on strong technicolor dynamics and can in principle be determined. Via a crude estimate we find f to be a number of order 1.

1. Introduction

We assume an Extended Technicolor (ETC) scenario [1,2] and three generations of ordinary quarks and leptons. We also assume that the generation hierarchy can be explained by a hierarchy of gauge interaction symmetry breaking scales (Tumbling) [3,4]. In this context it is natural to assume that the heaviest generation should be the simplest system to describe since it is closest in scale to the Technicolor (TC) [1,2] world (see fig. 1). In particular it should be possible to write down a closed,

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gauge invariant, system of interactions which includes Techni-fermions and the heaviest generation of ordinary fermions only. If we then confine ourselves to this system we must find a simple scenario to explain both top-bottom splitting and, of course, why $m_{\nu_\tau} \simeq 0$ on this scale. From experiments at PETRA [5] we also know that the ratio $m_t/m_b \geq 4$, i.e., it is not a small number. Finally, we recall that any ETC scenario which gives t-b splitting may be in danger of violating the experimentally determined ratio $M_W/M_Z \cos\theta_W \simeq 1$. A "natural" preservation of this ratio requires a "custodial" SU(2) symmetry at the TC scale [6].

Let us now briefly describe several mechanisms for obtaining t-b splitting. In the simplest scenarios both the t and b quarks obtain their mass through the first order feed down graph of fig. 2. Differences in the mass can arise if [7]:

(a) The right-handed t and b quarks are in inequivalent representations of ETC. This mechanism typically manifests itself by (i) different coupling strengths at the right-handed vertices, or (ii) a different number of TC condensates contributing in the top and bottom quark sectors. These two mechanisms occur for example in the SU(9) model of ref. [8]. The mass ratios obtained in this way are generally on the order of 2-3.

(b) Top-bottom symmetry is spontaneously broken at the TC level. This is possible as long as one preserves a "custodial" SU(2) which differs from the broken isospin symmetry acting on the Techni-fermions $\binom{T}{B}$ (see ref. 6). Unfortunately the order of magnitude of t-b splitting resulting from such a mechanism will be difficult to estimate.

(c) The same mechanism which removes right-handed neutrinos can in some cases renormalize the ETC coupling to right-handed top quarks [9] (see fig. 3).

(d) Finally, t and b quarks receive their mass in different orders of ETC. This last mechanism is the subject of this paper.

Let's elaborate. Suppose that in some TC model the top quark obtained mass from the usual feed down graph of fig. 2a. We thus obtain a mass

$$m_t = \frac{g^2}{2M^2} \langle \bar{T}T \rangle \tag{1.1}$$

where all strong, non-perturbative TC dynamics is implicit in the condensate

$$\langle \bar{T}T \rangle \equiv m^3 \tag{1.2}$$

m is a dynamical scale parameter set by the $W^{\pm}$ and Z^{o} boson masses to be of order 330 GeV [10]. Specifically this number is obtained by scaling up the ordinary QCD scale [11] $\langle \bar{q}q \rangle \sim (245\text{ MeV})^3$ with the dimensionless ratio F_T/f_π, where F_T is the Techni-pion decay constant and f_π is the ordinary pion decay constant. Finally, $g^2/2M^2$ is the effective 4-fermi coupling for broken ETC. Let's now suppose that within the same model the bottom quark cannot obtain mass via the same diagram, but can receive mass to lowest order in ETC by the graph of fig. 4. Then on simple dimensional grounds we expect

$$m_b = f\left(\frac{g^2}{2M^2}\right)^2 m^5 \tag{1.3}$$

where f is a constant which must be determined dynamically. We thus have

$$\begin{aligned} m_t &= xm \\ m_b &= fx^2 m \end{aligned} \tag{1.4}$$

with

$$x \equiv \frac{g^2 m^2}{2M^2} \quad .$$

Since m and m_b are known, then clearly m_t is determined once we evaluate f. We find

$$m_t = f^{-\frac{1}{2}} (38 \text{ GeV}) \quad . \tag{1.5}$$

In addition, if the approximations are to make any sense at all, we must satisfy the requirement that the ETC breaking scale M is much greater than the TC condensate scale m. We find

$$\frac{m}{M} = \left(f\alpha_{ETC}(M)\right)^{-\frac{1}{2}} \frac{1}{7.4} \tag{1.6}$$

where $\alpha_{ETC}(M) = g^2/4\pi$.

The paper is divided into two parts. In sect. 2 we introduce a one generation model. The model has t-b splitting as discussed in mechanism (d) with a "custodial" SU(2) which preserves the ratio $M_W/M_Z\cos\theta_W \simeq 1$. We find in addition $m_\tau = m_b$ and $m_{\nu_\tau} = 0$. In sect. 3 we try to estimate the constant f. We find f to be a number of order one. Unfortunately our estimate is too crude to be useful. We do, however, discuss how our estimate can in principle be improved.

2. The model

In addition to the standard strong and electroweak interactions $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ we assume there exists a new strong force $SU(5)_I \otimes SU(5)_{II}$. We also assume the following particle content under the combined group

$$SU(5)_I \otimes SU(5)_{II} \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \quad . \tag{2.2}$$

We have the states

$$\begin{array}{ll}
\begin{pmatrix} T_{ai} & \vert & t_{a5_2} \\ B_{ai} & \vert & b_{a5_2} \end{pmatrix} & (1,\ 5,\ 3,\ 2,\ \frac{1}{3}) \\
\begin{pmatrix} \bar{T}^{ai} & \vert & \bar{t}^{a5_2} \end{pmatrix} & (1,\ \bar{5},\ \bar{3},\ 1, -\frac{4}{3}) \\
\begin{pmatrix} \bar{B}^a_\alpha & \vert & \bar{b}^a_{5_1} \end{pmatrix} & (5,\ 1,\ \bar{3},\ 1,\ \frac{2}{3}) \\
\begin{pmatrix} V_\alpha & \vert & \nu_{5_1} \\ E_\alpha & \vert & \tau_{5_1} \end{pmatrix} & (5,\ 1,\ 1,\ 2,\ -1) \\
\begin{pmatrix} \bar{E}_i & \vert & \bar{\tau}_{5_2} \end{pmatrix} & (1,\ 5,\ 1,\ 1,\ 2)
\end{array} \tag{2.2}$$

where the indices

$$\left\{ (\alpha, \beta = 1,\ \ldots,\ 4),\ 5_1 \right\} \in SU(5)_I \quad ,$$

$$\left\{ (i, j = 1,\ \ldots,\ 4),\ 5_2 \right\} \in SU(5)_{II} \quad ,$$

and $(a = 1, \ldots, 3) \in SU(3)_C$. We emphasize the fifth index now in anticipation of the ETC breaking which we shall discuss shortly. Our one generation of quarks and leptons is identified as follows:

$$\begin{array}{ll}
\begin{pmatrix} t \\ b \end{pmatrix} \leftrightarrow \begin{pmatrix} t_{a5_2} \\ b_{a5_2} \end{pmatrix} & \begin{array}{l} \bar{t} \leftrightarrow \bar{t}^{a5_2} \\ \bar{b} \leftrightarrow \bar{b}^a{}_{5_1} \end{array} \\
\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \leftrightarrow \begin{pmatrix} \nu_{5_1} \\ \tau_{5_1} \end{pmatrix} & \bar{\tau} \leftrightarrow \bar{\tau}_{5_2} \quad .
\end{array} \tag{2.3}$$

Note that $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ is anomally free at this stage. However, in order to make $SU(5)_I \otimes SU(5)_{II}$ anomally free we also require the following additional states. These states are also necessary within the framework of tumbling ideas to explain the ETC breaking. We have

$$
\begin{array}{ll}
\left(Q^{\alpha}_{ia} \quad Q^{\alpha}_{5_2 a} \;\middle|\; Q^{5_1}_{ia} \quad Q^{5_1}_{5_2 a}\right) & (\bar{5},\ 5,\ 3,\ 1,\ -\tfrac{2}{3}) \\
\left(\bar{Q}^{ia} \quad \bar{Q}^{5_2 a}\right) & (1,\ \bar{5},\ \bar{3},\ 1,\ \tfrac{2}{3}) \\
\left(\bar{Q}^{ia}_{\alpha} \quad \bar{Q}^{ia}_{5_1} \;\middle|\; \bar{Q}^{5_2 a}_{\alpha} \quad \bar{Q}^{5_2 a}_{5_1}\right) & (5,\ \bar{5},\ \bar{3},\ 1,\ \tfrac{2}{3}) \\
\left(Q^{\alpha}_{a} \quad Q^{5_1}_{a}\right) & (\bar{5},\ 1,\ 3,\ 1,\ -\tfrac{2}{3}) \\
\left(L^{\alpha}_{i} \quad L^{\alpha}_{5_2} \;\middle|\; L^{5_1}_{i} \quad L^{5_1}_{5_2}\right) & (\bar{5},\ 5,\ 1,\ 1,\ 2) \\
\left(\bar{L}^{i} \quad \bar{L}^{5_2}\right) & (1,\ \bar{5},\ 1,\ 1,\ -2) \\
\left(\bar{L}^{i}_{\alpha} \quad \bar{L}^{i}_{5_1} \;\middle|\; \bar{L}^{5_2}_{\alpha} \quad \bar{L}^{5_2}_{5_1}\right) & (5,\ \bar{5},\ 1,\ 1,\ -2) \\
\left(L^{\alpha} \quad L^{5_1}\right) & (\bar{5},\ 1,\ 1,\ 1,\ 2) \\
\left(N^{\alpha\beta} \;\middle|\; N^{5_1 \alpha}\right) & (\overline{10},\ 1,\ 1,\ 1,\ 0) \quad .
\end{array}
\tag{2.4}
$$

Both $SU(5)_I \otimes SU(5)_{II}$ are asymptotically free and thus at some scale U_{ETC} they will become strong. Note that $\beta_{SU(5)_I} \neq \beta_{SU(5)_{II}}$ and thus they are not required to become strong at the same scale, although we assume for simplicity that this is the case. At U_{ETC} the following condensates form which break $SU(5)_I \otimes SU(5)_{II}$ down to $SU(4)_{T_1} \otimes SU(4)_{T_2}$. Technicolor in this scenario is a product group. We have

$$
\begin{aligned}
&\langle \bar{Q}^{ia} Q^{5_1}_{ia} + \bar{Q}^{5_2 a} Q^{5_1}_{5_2 a} \rangle \\
&= \langle \bar{Q}^{5_2 a}_{\alpha} Q^{\alpha}_{a} + \bar{Q}^{5_2 a}_{5_1} Q^{5_1}_{a} \rangle \\
&= \langle L^{5_1}_{i} \bar{L}^{i} + L^{5_1}_{5_2} \bar{L}^{5_2} \rangle \qquad (2.5) \\
&= \langle \bar{L}^{5_2}_{\alpha} L^{\alpha} + \bar{L}^{5_2}_{5_1} L^{5_1} \rangle \\
&= \langle N^{\alpha\beta} N^{\gamma\delta} \epsilon_{\alpha\beta\gamma\delta 5_1} \rangle \neq 0 \quad .
\end{aligned}
$$

Below the scale U_{ETC} we are left with the uncondensed fermions. Those carrying Technicolor include

$$
\begin{array}{cc}
\begin{pmatrix} T_{ai} \\ B_{ai} \end{pmatrix} & \begin{array}{c} \bar{T}^{ai} \\ \bar{Q}^{ai}_{5_1} \end{array} \\
\bar{E}_{i} & \bar{L}^{i}_{5_1}
\end{array}
\qquad (2.6a)
$$

$$
\begin{array}{cc}
\begin{pmatrix} V_{\alpha} \\ E_{\alpha} \end{pmatrix} & \begin{array}{c} N^{5_1 \alpha} \\ L^{\alpha}_{5_2} \end{array} \\
\bar{B}^{a}_{\alpha} & Q^{\alpha}_{5_2 a}
\end{array}
\qquad (2.6b)
$$

$$
\begin{array}{cc}
\bar{Q}^{ia}_{\alpha} & Q^{\alpha}_{ia} \\
\bar{L}^{i}_{\alpha} & L^{\alpha}_{i}
\end{array}
\qquad (2.6c)
$$

They have been placed in three separate groups to make explicit the global symmetry [12]

$$SU(7)_L \otimes SU(7)_{\bar{L}} \otimes SU(5)_L \otimes SU(5)_{\bar{L}} \otimes SU(4)_L \otimes SU(4)_{\bar{L}} \tag{2.7}$$

which exists at the TC scale when we ignore strong and electroweak interactions. This global symmetry includes as a subgroup an $SU(2)_L \otimes SU(2)_{\bar{L}}$ symmetry which will be instrumental in enforcing the result $M_W/M_Z\cos\theta_W = 1$. We also have the remaining TC singlet states

$$\begin{array}{ll} \begin{pmatrix} t_{a5_2} \\ b_{a5_2} \end{pmatrix} & \begin{array}{l} \bar{t}^{a5_2} \\ \bar{b}^{a}_{5_1} \end{array} \\ \begin{pmatrix} \nu_{5_1} \\ \tau_{5_1} \end{pmatrix} & \bar{\tau}_{5_2} \end{array} \quad . \tag{2.8}$$

Although $SU(4)_{T_1} \otimes SU(4)_{T_2}$ is explicitly non-asymptotically free with the fermions of eq. (2.6) we shall assume that a second condensate forms at the scale U_{TC}. It includes all the remaining TC non-singlet fermions in horizontal pairs in eq. (2.6). These condensates preserve $SU(4)_{T_1} \otimes SU(4)_{T_2}$, but break $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ down to $SU(3)_C \otimes U(1)_{EM}$ giving mass to the $W^{\pm}$ and Z^o. They also break the global symmetry of eq. (2.7) down to the diagonal subgroup

$$SU(7) \otimes SU(5) \otimes SU(4) \quad . \tag{2.9}$$

This group contains the necessary "custodial" SU(2) [6] and we thus have $M_W/M_Z\cos\theta_W = 1$. The scales U_{TC} and U_{ETC} must be very close together for this scenario to make any sense. We discuss the scales further at the end of this section.

For the moment let us continue with our scenario. The remaining uncondensed TC singlet fermions of eq. (2.8) now receive mass from the TC condensates [eq. (2.6)] via ETC gauge bosons. The top quark obtains its mass from the first order graph of fig. 5. The bottom quark and tau on the other hand obtain their mass to lowest order in ETC exchange through the second order graph of fig. 6. Finally the tau neutrino remains massless. To summarize we have

$$\begin{aligned} m_t &= xm \\ m_b = m_\tau &= fx^2 m \\ m_{\nu_\tau} &= 0 \end{aligned} \tag{2.10}$$

with

$$x = \frac{g^2 m^2}{2M^2} \quad .$$

There are two problems with our scenario. The model contains several light and massless axions. Perhaps this problem can be avoided when the model is extended to include three generations. Then there is the question of the two distinct scales U_{TC} and U_{ETC}. We shall now show that they are really quite close together.

We associate the scale U_{TC} with the Techni-pion decay constant

$$F_T = \frac{1}{\sqrt{N_{TD}}} \text{ (250 GeV)} \tag{2.11}$$

F_T is determined by the mass of the $W^\pm$ and Z^o bosons. N_{TD} is the number of Techni-doublets of $SU(2)_L$ and $N_{TD} = 4$ in our case. U_{ETC} is associated with

$$F_{ETC} = \frac{2M}{g}\frac{1}{\sqrt{N_E}} \tag{2.12}$$

where N_E is the number of ETC condensates which give mass M to the ETC gauge boson. $N_E = 5$ for $SU(5)_I$ and $N_E = 4$ for $SU(5)_{II}$ as can be seen in eq. (2.5). We thus have

$$m \simeq \frac{5}{2} F_{TC}$$

$$m \equiv \frac{M}{g}\sqrt{2x} = \sqrt{2x}\, F_{ETC} \tag{2.13}$$

with $N_E = 4$, or

$$F_{ETC} \simeq \frac{5}{2}\frac{1}{\sqrt{2x}} F_{TC} \simeq 5F_{TC}$$

with $x \sim \frac{1}{8.6}$. Since the scales are so close it might be reasonable to assume that below U_{ETC}, strong coupling dynamics is driving the beta function for the TC theory. Thus we would not rely on the perturbative β function for $SU(4)_{T_1} \otimes SU(4)_{T_2}$.

3. Estimate of f

The constant f includes all the strong TC corrections to the naive graph of fig. 6, just as $\langle\bar{T}T\rangle$ includes all the strong interaction corrections to fig. 5. In order to evaluate f we must first isolate the strong interaction contributions. Figure 7 is topologically equivalent to fig. 6a. It is easy to verify that in the limit M >> m the bottom quark mass is given by the expression

$$m_b = \left(\frac{g^2}{2M^2}\right)^2 \frac{1}{2}\,\mathrm{Tr}\left[-iS_{ab}(q=0)\right] \tag{3.1}$$

where the amplitude $S_{ab}(q)$ is the Fourier transform of

$$S_{ab}(x) = \left\langle T\left\{\left(B_i^{*T}\bar{\sigma}^{\mu}\right)_a\left(Q_{5_2}^{*\alpha^T}\bar{\sigma}_{\mu}Q_i^{\alpha}\right)(x)\right.\right.$$

$$\left.\left.\left(\bar{B}_{\alpha}^{*T}\bar{\sigma}^{\nu}\sigma^2\right)_b\left(\bar{Q}_{5_1}^{i*T}\bar{\sigma}_{\nu}\bar{Q}_{\alpha}^{i}\right)(0)\right\}\right\rangle \quad . \tag{3.2}$$

We have suppressed the color indices and a,b are two component spin indices. The two component matrices $\bar{\sigma}^{\mu}$ are defined in the Appendix. Using the definition of the four component Dirac fields ψ^1, ψ^2, ψ^3, as defined in eq. (A.10), our notation can be greatly simplified. Equation (3.2) is then equivalent to

$$S_{ab}(x) = -\left\langle T\left\{\left(\psi_L^{1\dagger}\bar{\sigma}\mu\right)_a\left(\psi_L^{3\dagger}\bar{\sigma}_{\mu}\psi_L^2\right)(x)\right.\right.$$

$$\left.\left._b\left(\sigma^{\nu}\psi_R^3\right)\left(\psi_R^{2\dagger}\sigma_{\nu}\psi_R^1\right)(0)\right\}\right\rangle \tag{3.3}$$

or finally

$$S_{ab}(x) = -4\left\langle T\left\{_a\left(\sigma^2\psi_L^2\right)\left(\psi_L^{1\dagger}\sigma^2\psi_L^{3^*}\right)(x)\right.\right.$$

$$\left.\left.\left(\psi_R^{3^T}\sigma^2\psi_R^1\right)\left(\psi_R^{2\dagger}\sigma^2\right)_b(0)\right\}\right\rangle \quad . \tag{3.4}$$

We can now derive a spectral decomposition of $S_{ab}(x)$ where we insert a complete set of Techni-baryonic states between the two TC singlet 3 quark operators as illustrated in fig. 8. We then obtain

$$S_{ab}(x) = -4\delta_{ab}\int_{Th}^{\infty} dm^2 \rho(m^2)\left(i\Delta_F(x,m)\right) \tag{3.5}$$
$$+ \text{ other traceless terms}$$

where

$$\rho(q^2) = \frac{1}{2} \operatorname{Tr} \rho_{ab}(q)$$

$$= \frac{(2\pi)^3}{2} \sum_n \delta^4(p_n - q)\left\langle\left(\psi_L^2\right)_a\left(\psi_L^{1\dagger}\sigma^2\psi_L^{3*}\right)(0)\,|n\right\rangle \tag{3.6}$$
$$\times \left\langle n\left|\left(\psi_R^3\sigma^2\psi_R^1\right)\left(\psi_R^{2*}\right)_a(0)\right.\right\rangle$$

and

$$\Delta_F(x,m) \equiv \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip\cdot x}}{p^2 - m^2 + i\varepsilon} \quad . \tag{3.7}$$

We finally have

$$m_b = \left(\frac{g^2}{2M^2}\right)^2 \left[4\int_{Th}^{\infty} \frac{dm^2 \rho(m^2)}{m^2}\right] \quad . \tag{3.8}$$

We now assume that the sum over intermediate states is dominated by the lowest lying T baryon state $|B\rangle$ with mass m_B, i.e.,

$$\sum_n \to \int \frac{d^4p_n}{(2\pi)^3}\, \delta\left(p_n^2 - m_B^2\right) \tag{3.9}$$

and we have

$$m_b = \left(\frac{g^2}{2M^2}\right)^2 \left[\frac{2C_B^2}{m_B^2}\right] \tag{3.10}$$

where

$$C_B\delta_{a1} \equiv \left\langle \left(\psi_L^2\right)_a \left(\psi_L^{1\dagger}\sigma^2\psi_L^{3*}\right)(0)\,|B\right\rangle \tag{3.11}$$

is the T Baryon wavefunction evaluated at the origin. Recall from eq. (1.3) that the quantity f is defined by

$$fm^5 \equiv \frac{2C_B^2}{m_B^2} \tag{3.12}$$

where m is defined by the condensate scale eq. (1.2). The only approximation that we have made up till this point has been to neglect the higher radial excitations of the T Baryon (B). This, however, should be a fairly good approximation since the wavefunction at the origin should necessarily decrease for the more extended states.

In order to obtain f we must now evaluate both m_B and C_B. In principle these quantities can be obtained by scaling up similar quantities from QCD. For example, B is a scaled up version of the proton. Thus C_B is a scaled up version of the amplitude for this local twist 4 three-quark operator to create a proton [13,14]. Such an operator contributes in principle to the protons electromagnetic form factor at large Q^2, but is unfortunately not the leading contribution. The leading contribution is given by the amplitude of the twist 3 operator $u_L u_L d_R$ [13,14].

We shall henceforth define

$$C_p \delta_{b2} \equiv \left\langle 0 \middle| u_L^a u_L^a d_R^b \middle| \text{proton} \right\rangle \quad . \tag{3.13}$$

This is <u>not</u> the quantity we want, but we shall use it anyway in the hope that it can nevertheless give us an order of magnitude estimate for f. Using the results of Brodsky et al. [14] we find

$$C_p = \sqrt{6}\,\sqrt{\frac{2}{3}} \int [dx] \phi_s(x_i, Q) (p^+)^{3/2} \tag{3.14}$$

where

$$\phi_s(x_i,Q) \equiv \bar{C}x_1x_2x_3\left(\ln \frac{Q^2}{\Lambda^2}\right)^{-\gamma_m} \tag{3.15}$$

is the proton's wave function integrated over the transverse momenta of the 3 quark constituent up to momentum Q; x_i are the fraction of longitudinal momentum carried by quark i; and $p^+ = p^o + p^3$ is the proton's longitudinal momentum in light cone coordinates. The factors $\sqrt{6}$ and $\sqrt{2/3}$ arise from the color and flavor normalizations of the proton, respectively. Finally, the integration measure is

$$[dx] \equiv \prod_{i=1}^{3} dx_i \delta\left(1- \sum_{i=1}^{3} x_i\right) \quad . \tag{3.16}$$

Upon integration we find

$$C_p = \sqrt{6}\sqrt{\frac{2}{3}}\frac{\bar{C}}{120}\left(\ln \frac{Q^2}{\Lambda^2}\right)^{-\gamma_m}(p^+)^{3/2} \quad . \tag{3.17}$$

The constant $\bar{C}$ has been determined from the asymptotic behavior of the proton's electromagnetic form factor in ref. [14]. They find using

$$G_M(Q^2) \rightarrow \frac{32\pi^2}{9}\bar{C}^2\frac{\alpha_s^2(Q^2)}{Q^4}\left(\ln \frac{Q^2}{\Lambda^2}\right)^{-2\gamma_m}(e_{\parallel} - e_{-\parallel}) \tag{3.18}$$

$$\times\left[1 + O\left(\alpha_s(Q^2)\right)\right]$$

$$\bar{C} = (.26 - 1.4)\ \mathrm{GeV}^2 \tag{3.19}$$

where the first value is obtained neglecting the $O(\alpha_s(Q^2))$ corrections and the second value is obtained with these corrections accounted for. Clearly the constant $\bar{C}$ is only known to within an order of magnitude. Finally let's discuss the factor $\left(\ln \frac{Q^2}{\Lambda^2}\right)^{-\gamma_m}$. This factor results from the anomalous dimensions of the 3 quark operator in eq. (3.13). We must in principle evaluate it at Q=M, the point at which the logarithmic divergences of the 3 quark operator are cut off by the ETC bosons. This factor is then of order one and shall henceforth be neglected. We finally obtain

$$f = \frac{2C_p^2}{m_p^2 m^5} = \frac{2\left[\sqrt{6}\,\sqrt{\frac{2}{3}}\,\frac{\bar{C}}{120}\,(m_p)^{3/2}\right]^2}{m_p^2 m^5} \tag{3.20}$$

where m is the QCD condensate scale, i.e., m ~ 245 MeV. We find

$$f = .04 - 1.2 \quad . \tag{3.21}$$

Conclusions

We have discussed a new mechanism for up-down symmetry breaking within the context of a TC scenario. The mechanism can in principle give large ratios (i.e., $m_t/m_b \sim 10$) which may in fact be necessary (and which are already required to describe m_c/m_s). At the same time the experimentally observed ratio $M_W/M_Z\cos\theta_W \simeq 1$ can be preserved. An evaluation of the exact splitting is unfortunately greatly dependent on our knowledge of strong interaction processes. We have made a crude estimate of these effects. This estimate can in principle be improved by measuring non-leading contributions to the proton's electromagnetic form factor at large Q^2, thus obtaining the matrix element of the relevant twist four operator to the proton state.

Finally the model of sect. 2 can be trivially generalized to a three generation system by extending the ETC group to $SU(7)_I \otimes SU(7)_{II}$. It is clear that the model contains a non-trivial spectrum of fermions. Unfortunately it does not seem possible within this simple framework to avoid problems with $\Delta G = 2$ processes (where G is the generation number) mediated by the ETC bosons [15]. A solution to this problem is necessary before further progress can be made in constructing realistic models with Extended Technicolor.

Appendix

A two component massless left-handed field χ satisfied the equation

$$-\vec{\sigma} \cdot \vec{p}\chi = E\chi \tag{A.1}$$

where $\vec{\sigma}$ are Pauli spinors in the representation

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} .$$

Given two massless left-handed fields χ_1, χ_2, we can construct the four component Dirac field

$$\psi = \begin{pmatrix} \chi_1 \\ \sigma^2 \chi_2^* \end{pmatrix} \equiv \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \tag{A.2}$$

where ψ_R satisfies the equation

$$\vec{\sigma} \cdot \vec{p}\psi_R = E\psi_R \quad . \tag{A.3}$$

The Dirac spinor ψ satisfies the equation

$$\vec{\alpha} \cdot \vec{p}\psi = E\psi \tag{A.4}$$

where we work in the chiral basis

$$\begin{gathered} \vec{\alpha} = \gamma^o \vec{\gamma} \qquad \beta = \gamma^o \\ \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \qquad \gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\ \sigma^\mu \equiv (1, \vec{\sigma}) \qquad \bar{\sigma}^\mu \equiv (1, -\vec{\sigma}) \quad . \end{gathered} \tag{A.5}$$

In this basis we define the parity operator $\mathscr{P}$

$$\mathscr{P}\psi_L(x^o,\vec{x})\mathscr{P}^{-1} = \psi_R(x^o,-\vec{x}) \tag{A.6}$$

and charge conjugation C

$$C\psi_L C^{-1} = \sigma^2\psi_R^* \tag{A.7}$$

or

$$C\chi_1 C^{-1} = -\chi_2 \quad .$$

A massive Dirac field satisfies the equation

$$(\vec{\alpha}\cdot\vec{p} + \beta m)\psi = E\psi \tag{A.8}$$

with the mass operator given by

$$\begin{aligned} \bar{\psi}\psi &\equiv \psi_L^\dagger\psi_R + \psi_R^\dagger\psi_L \\ &\equiv \chi_1^{*T}\sigma^2\chi_2^* + \chi_2^T\sigma^2\chi_1 \quad . \end{aligned} \tag{A.9}$$

Considering the condensates $\left\langle B_i\bar{Q}^i_{5_1}\right\rangle = \left\langle \bar{B}_\alpha Q^\alpha_{5_2}\right\rangle = \left\langle \bar{Q}^i_\alpha Q^\alpha_i\right\rangle \neq 0$ of eqs. (2.6) we can now define the four component Dirac fields

$$\psi_1 = \begin{pmatrix} B_i \\ \sigma^2\bar{Q}^{i*}_{5_1} \end{pmatrix} \qquad \psi_b = \begin{pmatrix} b_{5_2} \\ \sigma^2\bar{b}^*_{5_1} \end{pmatrix}$$

$$\psi_2 = \begin{pmatrix} Q^\alpha_i \\ \sigma^2\bar{Q}^{i*}_\alpha \end{pmatrix} \tag{A.10}$$

$$\psi_3 = \begin{pmatrix} Q^\alpha_{5_2} \\ \sigma^2\bar{B}^*_\alpha \end{pmatrix} \quad .$$

Finally we shall use the following identities

$$\sigma^2 \bar{\sigma}^\nu \sigma^2 = \sigma^{\nu T}$$

$$(\bar{\sigma}^\mu)_{ab} (\bar{\sigma}_\mu)_{cd} \equiv 2\varepsilon_{ac}\varepsilon_{bd} = -2\sigma^2_{ac}\sigma^2_{bd} \tag{A.11}$$

$$\bar{\sigma}^\mu \equiv \sigma_\mu \quad .$$

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I would like to thank G. Bodwin, S. Brodsky, S. Dimopoulos, E. Farhi, S. Gupta, K. Lane, H. P. Nilles, and L. Susskind for many fruitful discussions during the course of this work. This work was supported by the Department of Energy, contract DE-AC03-76SF00515.

References

[1] L. Susskind, Phys. Rev. D20 (1979) 2619;
S. Weinberg, Phys. Rev. D19 (1978) 1277.

[2] S. Dimopoulos and L. Susskind, Nucl. Phys. B155 (1979) 237;
E. Eichten and K. Lane, Harvard preprint HUTP 79/A002 (1979).

[3] S. Raby, S. Dimopoulos, and L. Susskind, Tumbling gauge theories, Stanford preprint ITP-653 (1979).

[4] S. Dimopoulos, S. Raby, and P. Sikivie, Problems and virtues of scalarless theories of electroweak interactions, Stanford preprint ITP-664 (1980).

[5] For a review, see contribution of B. Wiik in Proc. of the international Conf. on high energy physics, Madison, Wisconsin (1980).

[6] P. Sikivie, L. Susskind, M. Voloshin, and V. Zakharov, Isospin breaking in technicolor models, Stanford preprint ITP-661 (1980).

[7] We assume in the following discussion that the ETC group commutes with the electroweak group $SU(2)_L$.

[8] E. Farhi and L. Susskind, A technicolored G.U.T., SLAC-PUB-2361 (1979);
See in addition S. Dimopoulos and L. Susskind, Ref. [2], and also Ref. [6].

[9] An example of this phenomenon is discussed in B. Holdom, A realistic model with dynamically broken symmetries, Harvard preprint HUTP-80/A010 (1980).

[10] See for example, S. Dimopoulos, S. Raby, and G.L. Kane, Experimental

predictions from technicolor theories, Univ. of Michigan preprint UM-HE 80-22 (1980).

[11] See for example, M.A. Shifman, A.I. Vainshtein, and V.I. Zakharov, Nucl. Phys. B147 (1979) 385.

[12] There are of course additional U(1) symmetries which are extremely important when discussing axions.

[13] M. Peskin, Phys. Lett. 88B (1979) 128.

[14] G.P. Lepage and S.J. Brodsky, SLAC-PUB-2348 (1979); also SLAC-PUB-2478 (1980).

[15] S. Dimopoulos and J. Ellis, Challenges for extended technicolor theories, CERN preprint TH-2949-CERN (1980).

Figure captions

Fig. 1. An illustration of the different scales involved in a tumbling scenario of the generation hierarchy.

Fig. 2. The standard mass feed down graph for ETC.

Fig. 3. An ETC vertex renormalization due to the exchange of a lepto-quark boson of an assumed Pati-Salam symmetry. Such a graph may contribute to up-down splitting.

Fig. 4. A second order feed down graph.

Fig. 5. The first order feed down graph for the top quark.

Fig. 6. The second order feed down graphs for the bottom quark and tau.

Fig. 7. Figure 6a redrawn. It illustrates the operator product expansion analysis which we have implicitly assumed.

Fig. 8. An illustration of the spectral decomposition.

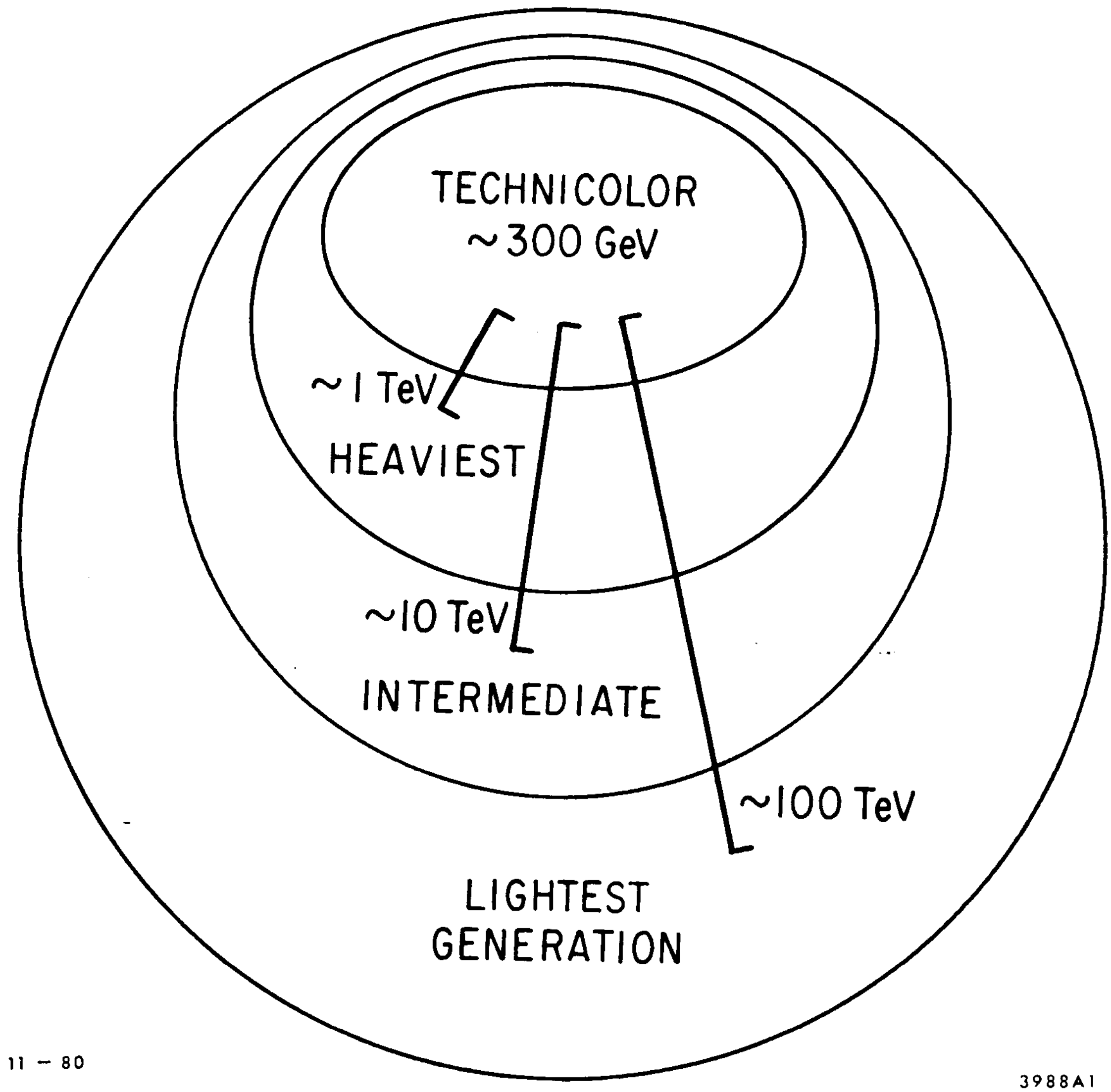

TECHNICOLOR
~300 GeV
~1 TeV
HEAVIEST
~10 TeV
INTERMEDIATE
~100 TeV
LIGHTEST
GENERATION
11 – 80
3988A1

Fig. 1

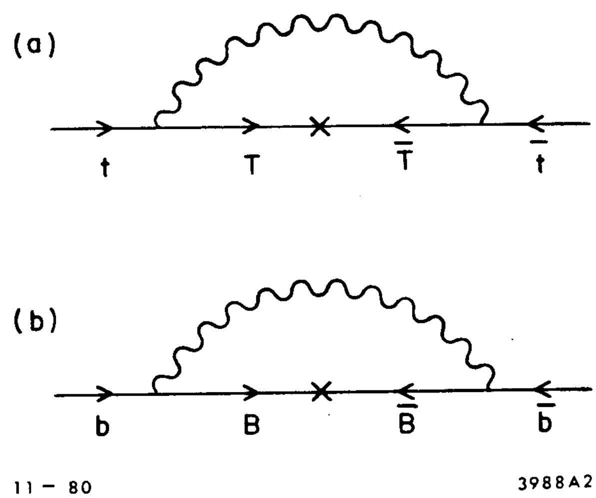


Fig. 2

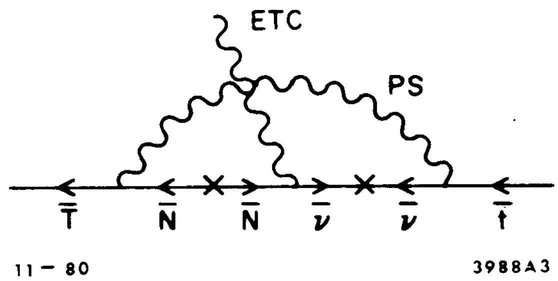
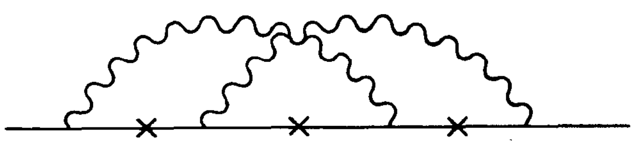


Fig. 3



11 − 80 3988A4

Fig. 4

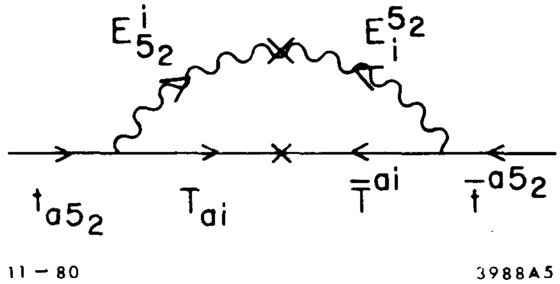
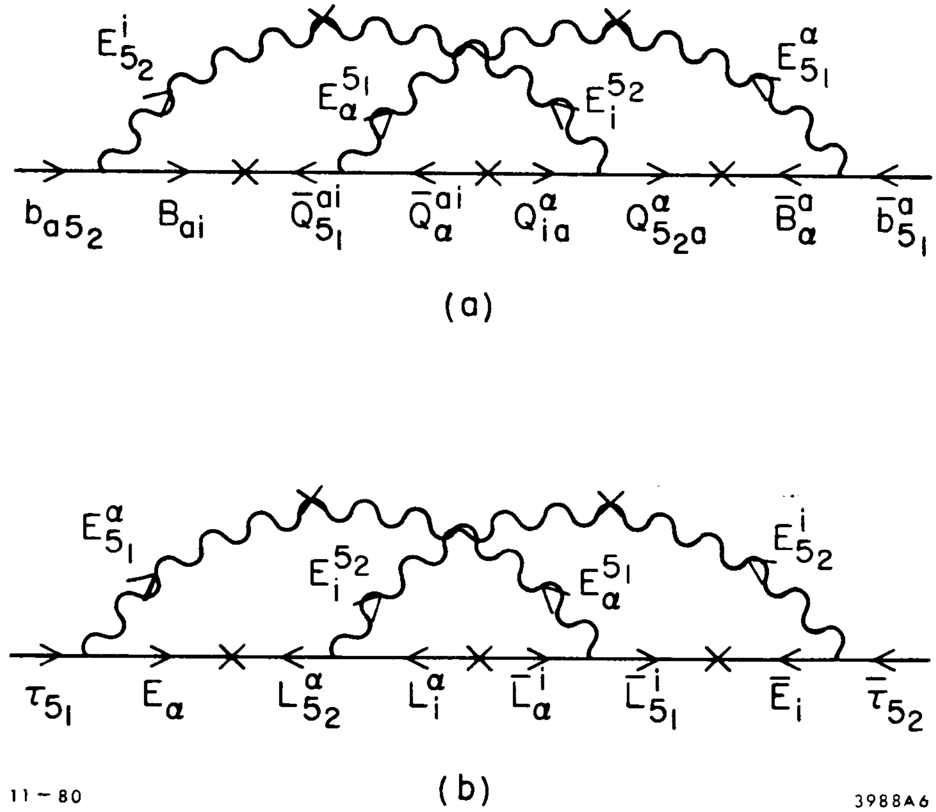


Fig. 5



$E^{i}_{5_2}$
$E^{5_1}_{\alpha}$
$E^{5_2}_{i}$
$E^{\alpha}_{5_1}$
b_{a5_2}
B_{ai}
$\bar{Q}^{ai}_{5_1}$
$\bar{Q}^{ai}_{\alpha}$
Q^{α}_{ia}
$Q^{\alpha}_{5_2 a}$
$\bar{B}^{a}_{\alpha}$
$\bar{b}^{a}_{5_1}$
(a)
$E^{\alpha}_{5_1}$
$E^{5_2}_{i}$
$E^{5_1}_{\alpha}$
$E^{i}_{5_2}$
τ_{5_1}
E_{α}
$L^{\alpha}_{5_2}$
L^{α}_{i}
$\bar{L}^{i}_{\alpha}$
$\bar{L}^{i}_{5_1}$
$\bar{E}_{i}$
$\bar{\tau}_{5_2}$
11-80
(b)
3988A6

Fig. 6

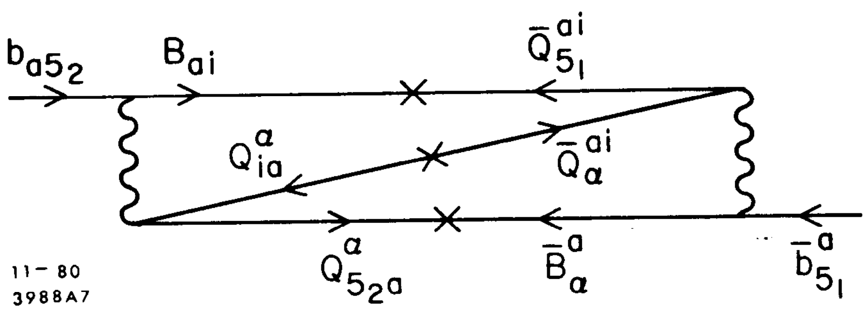

b_{a5_2}
B_{ai}
$\bar{Q}^{ai}_{5_1}$
Q^{α}_{ia}
$\bar{Q}^{ai}_{\alpha}$
$Q^{\alpha}_{5_2a}$
$\bar{B}^{a}_{\alpha}$
$\bar{b}^{a}_{5_1}$
11-80
3988A7

Fig. 7

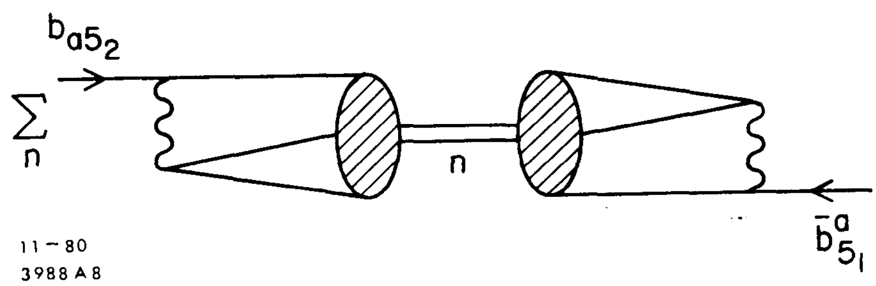


Fig. 8