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# QUARK DYNAMICS IN QCD AT HIGH TEMPERATURE*

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ABSTRACT

Global symmetry realizations in QCD with two massless quark flavors are studied by semiclassical methods at high temperature. The response of QCD to external field theory probes gives an indication of symmetry realizations and their interdependence as the temperature is lowered. The semiclassical approximation of QCD is equivalent to the statistical mechanics problem of a quark and gluon plasma in a background field of correlated instanton fluctuations, and is shown to be described by an effective field theory. From the collective instanton effects with quarks some insight can be gained into how the dielectric properties of the medium affect chirality correlations responsible for the onset of the spontaneous chiral SU(2) symmetry breaking phase transition, and alternatively, how quarks affect the dielectric properties of the medium.

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## I. INTRODUCTION

An understanding of the hadronic physics of ordinary matter is expected to come from QCD with two massless quark flavors. In particular, the theory must explain the confinement of quarks into color singlet hadrons, and the symmetries of the strong interactions. The classical Lagrangian for this theory, besides being Lorentz, scale and C, P and T invariant has a local color SU(3) gauge symmetry and a global $U_L(2) \times U_R(2)$ symmetry of the massless left- and right-handed quark fields. There are many possible realizations of these symmetries. While the realization of some of the symmetries of the classical Lagrangian are understood in the quantum theory — for example, how the classical scale invariance is broken by the renormalization anomalies and how the chiral U(1) symmetry is broken by the axial anomaly and $\theta$-vacuum [1,2,3], other symmetry realizations are less well understood. For the other global symmetries, from an analysis of effective potentials for possible order parameters, built out of polynomials of color SU(3) and chiral SU(2) invariants of bilinear quark fields, it is possible that the chiral SU(2), isospin, and P and CP symmetries could all be spontaneously broken [4]. Also, the color gauge symmetry could in principle be realized as a spontaneously broken symmetry as opposed to either a confined or normal (Coulombic) symmetry [5].

We would like to better understand how QCD actually realizes its chiral SU(2) symmetry as spontaneously broken, its isospin, P and CP symmetries as manifest, and realizes its color SU(3) symmetry as a confined symmetry. We would also like to understand how these realizations are interdependent. An example of an interdependence of symmetry realization comes from the fact that any dimensional order parameter associated with spontaneous symmetry breaking can only exist because of the breaking of the classical scale invariance by quantum renormalization effects (spontaneous symmetry breaking cannot occur if all couplings are at their fixed points) [6]. Also, without the axial anomaly the realizations of chiral U(1) and chiral SU(2) symmetry are linked;[1] with the anomaly the realization of the chiral U(1) symmetry is linked to topological properties of color gauge field configurations [1,2,3], and consequently the realization of chiral SU(2) symmetry may also depend on topological properties of color gauge field configurations. Since these configurations may furthermore have something to do with the confinement realization [7,8], the color gauge symmetry realization and spontaneous chiral SU(2) symmetry breaking may also be linked.

In order to study symmetry realizations, and since QCD presumably has only a single phase, we consider the finite temperature theory which can have many phases. (While the Wilson lattice gauge theory is known to have only one phase for all values of its coupling, the finite temperature theory has a phase transition [9,10].) We can imagine heating the theory to a high temperature for which it is as symmetric as possible, and then lowering the temperature to see various phase transitions.

At moderately high temperature the theory can be analyzed semiclassically. Temperature serves as an infrared cutoff, and with it the contribution of quantum effects with large coupling strength can be controlled. It is reasonable to expect there to be an indication of the resulting phase structure of the low temperature theory even in the high temperature phase, since the kinds of correlations responsible for the phase transition begin to set in before the critical temperature. One indication of the resulting phase structure can be obtained from the high-temperature phase by coupling external fields to the order parameters, and exploring how the free energy changes. If the strength of an external field is increased, then the total free energy is lowered because this increase in free energy is more than compensated by the dynamics lowering its internal energy. The external field imposes order energetically preferred by the dynamics, but which is opposed at high temperature by the randomizing thermal fluctuations.

We will consider external fields that can exist in the vacuum because of spontaneous symmetry breaking of an external field theory. If the QCD response increases the magnitude of this vacuum field, then the internal energy of QCD is lowered by the imposed order parameter. If, on the other hand, the external vacuum field is decreased by the QCD response, then the QCD internal energy is raised by such an imposed order parameter, and so will not tend to spontaneously break the associated symmetry. It will also tend to restore the symmetry spontaneously broken in the external field theory.

Even for those external vacuum fields that are increased by the QCD response, though, there is no guarantee that the QCD dynamics will actually be able to create a phase transition as the temperature is lowered. For example, an external magnetic field will be increased by a paramagnetic material as well as by the high-temperature phase of a ferromagnetic material. However, in the fixed weak external magnitude field, as the temperature is lowered, there will be a rapid decrease in internal free energy of the potential ferromagnetic material compared to that of the paramagnetic material. We would therefore expect that those correlations responsible for the onset of the phase transition turn on very rapidly as the temperature is lowered.

Now studies of lattice QCD without quarks at zero temperature show that there is a very rapid transition from weak to strong coupling behavior [11,12]. The $\beta$ function that describes the change in effective coupling strength for different scales changes almost discontinuously from its weak coupling perturbative behavior to its strong coupling confining behavior at a certain small value of the coupling. This transition occurs over a range of couplings that are so small that 2-loop perturbative corrections are negligible. This indicates that there are very important nonperturbative weak coupling effects. Semiclassical tunneling fluctuations, instantons [13], are an example of such effects. They also have the property that at finite temperature their contribution turns on exponentially fast as the temperature is lowered [14], and so we expect they give a good description of the onset of the phase transition. We do not know if in fact the semiclassical approximation is valid at the critical temperature for the spontaneous chiral SU(2) symmetry breaking phase transition, but the methods we will describe can in principle be extended to answer this question.

This paper is organized as follows: in the next section, IIA, we review some of the symmetries of QCD with two massless flavor quarks. In IIB we show how some of the implications of spontaneous symmetry breaking follow from Goldstone-Ward identities. The consequences follow from nonzero-order parameters; whether or not the dynamics chooses to have nonzero order parameters is determined from the minima of the effective potential, which is discussed in Section IIC. In Section IID we show that QCD with massless quarks could potentially spontaneously break P and CP as a consequence of the unusual way it realizes its chiral U(1) symmetry. We also give a simple heuristic explanation of how the realization of this chiral U(1) symmetry follows from the θ-vacuum. In Section III we discuss 't Hooft's chirality selection rules [1] that govern the behavior of massless quarks in background color gauge fields with nontrivial topology. These selection rules are realized by way of zero-eigenvalues of the Dirac operator in such background gauge fields. From the dependence of the determinant of the Dirac operator in both background gauge and scalar fields we can simply understand the flavor structure of the 't Hooft interaction [1], as well as corrections in higher powers of the external fields. We can also simply understand the θ transformation property under chiral U(1) rotations of the quark fields [2]. In Section IVA we review the instanton contribution to the Euclidean functional integral [15], including higher order external scalar field dependence, and in IVB discuss some of the corrections to this contribution when finite temperature boundary conditions are included.

In Section V we consider QCD coupled to various scalar field theories, analogous to scalar sectors of weak interaction models. The classical potentials for these scalar models are chosen to realize global symmetries in various ways: (1) spontaneous chiral SU(2) and isospin breaking, and (2) spontaneous chiral SU(2) and CP breaking. We consider these models at finite temperature and heuristically show that the QCD corrections to the scalar effective potentials can induce various phase transitions. The QCD corrections enhance the tendency for spontaneous chiral SU(2) symmetry breaking in these models, and act to restore isospin and CP symmetry.

In Section VI we analyze chirality correlations in the semiclassical approximation to QCD at finite temperature. These effects are shown to be describable by an effective fermionic field theory, essentially the finite temperature version of the quantum field theory of 't Hooft interaction [2]. We physically motivate a transformation of this field theory to a form suggestive of a chiral SU(2) $\sigma$-model, the simplest approximation to which has a correspondence with previous analyses [7,16,17]. The QCD free energy in external scalar fields is briefly discussed in this approximation.

In Section VII we consider additional corrections to the semiclassical approximation. The collective effects of dipolar correlations of instantons and anti-instantons, represented as an effective field theory by Jevicki [18], is here generalized to include the effects of massless quarks. From an approximation to this field theory we show how to compute the quark corrections to the Callan-Dashen-Gross dielectric susceptibility [7,8]. Our final result is expressed as an effective Lagrangian added to the usual QCD Lagrangian. Perturbative evaluation of this field theory generates (besides the usual perturbative QCD graphs) an approximation to the effects of configurations with nontrivial topological field fluctuations — that is, a plasma of instantons and anti-instantons interacting through dipolar and chirality correlations, with quarks propagating in background instanton fields and interacting through gluons in background instanton fields. Finally, Section VIII is a summary.

## IIA. SYMMETRIES OF QCD

The Lagrangian for QCD with two massless flavor quarks is

$$\mathscr{L} = (\bar{u}\ \bar{d})\begin{pmatrix} i\not{D}(A) & 0 \\ 0 & i\not{D}(A) \end{pmatrix}\begin{pmatrix} u \\ d \end{pmatrix} - \frac{1}{2g^2}\,\mathrm{tr}\,F_{\mu\nu}(A)\,F^{\mu\nu}(A) \quad . \tag{2.1}$$

The color SU(3) gauge covariant derivatives, $D(A) = \partial_\mu + iA_\mu$, $A_\mu = A^a_\mu(\lambda^a/2)$, act on the three component color spinors u and d, each component of which is a four component Dirac spinor. The color curvature is

$$F_{\mu\nu}(A) = \partial_\mu A_\nu - \partial_\nu A_\mu - i\left[A_\mu, A_\nu\right] \quad . \tag{2.2}$$

This classical Lagrangian is invariant under local color gauge transformations of the quarks and gauge fields,

$$\begin{pmatrix} u(x) \\ d(x) \end{pmatrix} \rightarrow \begin{pmatrix} \Omega(x) & 0 \\ 0 & \Omega(x) \end{pmatrix}\begin{pmatrix} u(x) \\ d(x) \end{pmatrix} , \tag{2.3a}$$

and

$$A_\mu(x) \rightarrow \Omega(x)A_\mu(x)\Omega^\dagger(x) - i\Omega(x)\partial_\mu\Omega^\dagger(x) \quad , \tag{2.3b}$$

where $\Omega(x)$ is an element of color SU(3) associated with the space-time point x; associated with a path in space-time is a path on the SU(3) manifold. It is also invariant under P, C, T and global $U_L(2) \times U_R(2)$ transformations of the left- and right-handed quark flavor doublets: defining $\psi = \binom{u}{d}$, and the left- and right-handed projections

$\psi_{\binom{L}{R}} = [(1 \mp \gamma_5)/2]\psi$, then $\mathscr{L}$ is invariant under

$$\begin{pmatrix}\psi_L \\ \psi_R\end{pmatrix} \to \begin{pmatrix} e^{i\xi}\, e^{i\vec{\alpha}\cdot\vec{\tau}/2} & 0 \\ 0 & e^{i\eta}\, e^{i\vec{\beta}\cdot\vec{\tau}/2}\end{pmatrix}\begin{pmatrix}\psi_L \\ \psi_R\end{pmatrix} \quad , \qquad (2.4)$$

that is, under independent U(2) transformations of the left- and right-handed flavor spinors. The subgroup $\begin{pmatrix} e^{i\xi} & 0 \\ 0 & e^{i\xi}\end{pmatrix}$ is the $U_B(1)$ baryon number subgroup, implying the conservation of the baryon number current $\bar{\psi}\gamma_\mu\psi$. The subgroup $\begin{pmatrix} e^{i\vec{\alpha}\cdot\vec{\tau}/2} & 0 \\ 0 & e^{i\vec{\alpha}\cdot\vec{\tau}/2}\end{pmatrix}$ is the $SU_I(2)$ isospin subgroup, implying the conservation of the isospin currents, $J^a_\mu \equiv \psi\gamma_\mu(\tau^a/2)\psi$. Both of these symmetries are manifest symmetries of the strong interactions. That is, the strongly interacting particles fit into families associated with the group representations of these symmetries, and the interactions of these paraticles are governed by selection rules which follow from the local conservation of these currents. The transformations

$$\begin{pmatrix} e^{i\xi} & 0 \\ 0 & e^{-i\xi}\end{pmatrix}$$

are the chiral U(1) transformations; they rotate all left-handed fermion fields one way and all right-handed fermion fields in the opposite direction by the same amount. Associated with this symmetry is the classically conserved axial vector current $\bar{\psi}\gamma_\mu\gamma_5\psi$. This chiral U(1) symmetry is, however, neither a manifest symmetry of the strong interactions, nor a spontaneously broken symmetry in the usual way.

Finally, the transformations

$$\begin{pmatrix} e^{i\vec{\alpha}\cdot\vec{\tau}/2} & 0 \\ 0 & e^{-i\vec{\alpha}\cdot\vec{\tau}/2} \end{pmatrix}$$

are the chiral SU(2) transformations which rotate the left-handed $u_L$ and $d_L$ into a linear combination of one another, and the right-handed $u_R$ and $d_R$ into a linear combination of one another in the opposite direction, and associated with this symmetry are the axial vector isospin currents, $J^a_{5_\mu} \equiv \psi\gamma_\mu\gamma_5(\tau^a/2)\psi$. This chiral SU(2) symmetry seems to be a spontaneously broken symmetry. The understanding of these chiral symmetries must come from the quantum field theory.

## IIB. GOLDSTONE-WARD IDENTITIES

Chiral SU(2) spontaneous symmetry breaking is studied by means of the Ward identity

$$\int_x \partial^\mu \left\langle J^a_{5_\mu}(x)\, i\bar{\Psi}\gamma_5 \frac{\tau^b}{2}\Psi \right\rangle = \left\langle \left[Q^a_5, i\bar{\Psi}\gamma_5\frac{\tau^b}{2}\Psi\right]\right\rangle = -\delta^{ab}\langle\bar{\Psi}\Psi\rangle \quad . \qquad (2.5)$$

The last equality (which follows from the canonical anticommutation relations of the fermion field operators) implies that if $\bar{\Psi}\Psi$ has a nonvanishing vacuum expectation value, then $Q^a_5$ does not annihilate the vacuum state, but connects it to the same particle state as does the isovector pseudoscalar operator $i\bar{\Psi}\gamma_5(\tau^b/2)\Psi$. The theorem of Goldstone, Salam and Weinberg [19] applies here and implies that this contributing state is that of a massless particle. This is manifest as a massless pole in the term on the left of Eq. (2.5),

$$\int_x e^{iq\cdot x}\left\langle J^a_{5_\mu}(x)\, i\bar{\Psi}\gamma_5 \frac{\tau^b}{2}\Psi\right\rangle \underset{q\to 0}{\sim} \frac{q_\mu}{q^2}\langle\Psi\Psi\rangle\delta^{ab} \quad . \tag{2.6}$$

These three massless particles are identified with the pions [20]. Also, following from the Ward identity for the axial vector vertex function embedded in Eq. (2.5),

$$\begin{aligned}
&\int_x \partial^\mu \left\langle J^a_{5_\mu}(x)\bar{\Psi}(0)\gamma_5 \frac{\tau^b}{2}\Psi(0)\right\rangle \\
&= \int_x \int_{z,z'} \mathrm{tr}\ \gamma_5 \frac{\tau^b}{2} G(0,z)\left[\partial^\mu \Gamma^a_{5_\mu}(z,z';x)\right] G(z',0) \\
&= \int_x \int_{z,z'} \mathrm{tr}\ \gamma_5 \frac{\tau^b}{2} G(0,z)\left[G^{-1}(z,x)\gamma_5 \frac{\tau^a}{2}\delta^4(x-z') \right.\\
&\qquad \left. + \delta^4(z-x)\frac{\tau^a}{2}\gamma_5 G^{-1}(x,z')\right] G(z',0) \\
&= \delta^{ab}\int_p \mathrm{tr}\ G(p) \quad ,
\end{aligned} \tag{2.7}$$

and from the consequent behavior of the vertex function implied by Eq. (2.6),

$$\Gamma^a_{5_\mu}(p,-p';q) \underset{q\to 0}{\sim} \frac{q_\mu}{q^\nu}\left[G^{-1}(p)\gamma_5 \frac{\tau^a}{2} + \frac{\tau^a}{2}\gamma_5 G^{-1}(p')\right]\delta^4(p-p'+q) \ , \tag{2.8}$$

The u and d quarks acquire the same dynamical mass proportional to $\langle\Psi\bar{\Psi}\rangle$, since Eqs. (2.7) and (2.8) imply a nonvanishing anticommutator of $G^{-1}(p)$ and $\gamma_5$.

Spontaneous isospin breaking is governed by a Ward identity similar to Eq. (2.5) for spontaneous chiral SU(2) symmetry breaking,

$$\int_x \partial^\mu \left\langle J^a_{5_\mu}(x)\bar{\Psi}\tau^b\Psi \right\rangle = \left\langle \left[ Q^a, \bar{\Psi}\tau^b\Psi \right] \right\rangle = \varepsilon^{abc}\left\langle \bar{\Psi}\tau^c\Psi \right\rangle \quad . \tag{2.9}$$

If we pick the direction of spontaneous isospin breaking to be in the 3-direction, then

$$\left\langle \bar{\Psi}\tau^3\Psi \right\rangle = \langle \bar{u}u \rangle - \langle \bar{d}d \rangle \neq 0 \tag{2.10}$$

would lead to different dynamical masses for the u and d quarks. The generators $Q^1$ and $Q^2$ would be spontaneously broken and this would imply the existence of massless charged scalar particles, the neutral partner of which (created by $\bar{\Psi}\tau^3\Psi$) would not have to be massless. There seems to be no evidence in the real world, though, for this dramatic pattern of scalar particles [20].

## IIC. EFFECTIVE POTENTIALS

The question of whether or not $\langle\bar{\Psi}\Psi\rangle$, for example, is nonzero is a dynamical one. It can be answered by computing the effective potential $V(\langle\bar{\Psi}\Psi\rangle)$[6,21]. If V has a global minimum for $\langle\bar{\Psi}\Psi\rangle$ nonzero, then the ground state spontaneously breaks the chiral symmetry. In order to study the possible spontaneous symmetry breaking of other global symmetries we will consider the effective potential for the more general quark bilinear color singlet order parameter, $\left\langle \bar{\Psi}_i[(1+\gamma_5)/2]\Psi_j \right\rangle$ .

The computation of the effective potential proceeds as follows: First a source term

$$-\left( \bar{\psi}_L \Phi \psi_R + \bar{\psi}_R \Phi^\dagger \psi_L \right) \tag{2.11}$$

is added to the Lagrangian, Eq. (1.1). The $2\times 2$ flavor matrix scalar field $\Phi$ can be represented,

$$\Phi = \sigma + i\vec{\pi} \cdot \vec{\tau} + i\eta + \vec{\phi} \cdot \vec{\tau} \tag{2.12}$$

If $\Phi$ is considered a dynamical field (as we will do later), this Yukawa coupling is invariant under $U_L(2) \times U_R(2)$ transformation; under $SU_L(2) \times SU_R(2)$, $\Phi$ transform as the $(\frac{1}{2},\frac{1}{2})$ representation. This Yukawa coupling represents the interaction energy of the external source fields $\Phi_{ij}$, with the quark fields $\bar{\psi}_i[(1+\gamma_5)/2]\psi_j$.

The ground state energy in the presence of the external source fields $\Phi_{ij}$ is proportional to $W(\Phi) = -i \ \ell n \ Z(\Phi)$, where $Z(\Phi)$ is the vacuum-to-vacuum amplitude in the presence of $\Phi$. The computation of $W(\Phi)$ can be formulated in terms of the determinant of the u and d quark inverse propagators in the background $\Phi$ field,

$$\det\left[\begin{pmatrix} i\not{D}(A) & 0 \\ 0 & i\not{D}(A) \end{pmatrix} - \Phi\,\frac{1+\gamma_5}{2} - \Phi^{\dagger}\,\frac{1-\gamma_5}{2}\right] \tag{2.13}$$

The $2 \times 2$ matrix of color covariant derivatives is diagonal in the u and d flavor space, while the matrices $\Phi$ and $\Phi^{\dagger}$ are not, in general. Expressing the determinant as the exponential of the trace of the logarithm, this logarithm can be expanded in powers of $\Phi$ and $\Phi^{\dagger}$, and corresponds to the sum of all graphs, with arbitrary numbers of external $\Phi$ and $\Phi^{\dagger}$ fields, of a single quark loop, with the quark propagating in a background color gauge field (see Fig. 1). Thus the $\Phi$ dependent part of the determinant can then be expressed in the form

$$\exp\left\{i\int\left[Z\ \mathrm{tr}\left|\partial_\mu\Phi\right|^2 - V_A(\Phi)\right]\right\} \tag{2.14}$$

where the kinetic energy term comes from the local contribution of the vacuum polarization graph, and $V_A(\Phi)$ is a nonlocal polynomial in all powers of $\Phi$ and $\Phi^\dagger$ and their derivatives, the coefficients of which depend on A. The determinant is weighted by the amplitude for each color gauge field configuration, $\exp\{iS(A)\}$, where

$$S(A) = \frac{1}{2g^2} \int \mathrm{tr}\, F_{\mu\nu}(A)\, F^{\mu\nu}(A) \tag{2.15}$$

and is summed over all possible configurations, giving for $W(\Phi)$,

$$W(\Phi) = -i\,\ell n \left\{ \int \mathscr{D}A\; e^{iS(A)} \det\left[\begin{pmatrix} i\not{D}(A) & 0 \\ 0 & i\not{D}(A) \end{pmatrix} - \Phi\,\frac{1+\gamma_5}{2} - \Phi^\dagger\,\frac{1-\gamma_5}{2}\right]\right\} \tag{2.16}$$

$$= -i\,\ell n \left( \int \mathscr{D}A\; \exp\left\{i\left(S(A) - i\,\mathrm{tr}\,\ell n\, \not{D}(A)\right)\right\} \exp\left\{i\int\left[Z\,\mathrm{tr}\left|\partial_\mu\Phi\right|^2 - V_A(\Phi)\right]\right\}\right)$$

The effect of coupling the scalar fields $\Phi_{ij}$ to the quarks is almost like coupling to QCD the scalar sector of a weak interaction model, since QCD induces the dynamics for such a model, apart from infinite renormalization. All that is needed is to make $\Phi$ a dynamical degree of freedom, partly just to carry out the renormalization. In order to get an indication of the symmetry realizations in QCD, we could ask what effects QCD has on the symmetry realizations of various weak interaction models. $W(\Phi)$ would contribute to the effective quantum action of the weak interaction sector, and thus to its effective potential. Models of this kind will be pursued in Section IV.

From $W(\Phi)$ a Legendre transformation is performed to obtain

$$\Gamma\left(\left\langle \bar\Psi_i\,\frac{1+\gamma_5}{2}\,\Psi_j \right\rangle\right) \equiv W(\Phi) - \int \left\langle \Psi\left(\Phi\,\frac{1+\gamma_5}{2} + \Phi^\dagger\,\frac{1-\gamma_5}{2}\right)\Psi \right\rangle , \tag{2.17}$$

where

$$\left\langle \bar{\Psi}_i \frac{1+\gamma_5}{2} \Psi_j \right\rangle = i \frac{\delta W(\Phi)}{\delta \Phi_{ij}} \quad . \tag{2.18}$$

In performing this transformation the source dependence of $\langle \bar{\Psi}_i [(1+\gamma_5)/2] \Psi_j \rangle$ must be inverted; all the $\Phi_{ij}$ dependence in $W(\Phi)$ must be transformed to $\langle \bar{\Psi}_i [(1+\gamma_5)/2] \Psi_j \rangle$ dependence. $\Gamma$ is the generating functional for all one-particle irreducible vertex functions with $\langle \bar{\Psi}_i [(1+\gamma_5)/2] \Psi_j \rangle$ vertices. It has the structure

$$\Gamma\left(\left\langle \bar{\Psi}_i \frac{1+\gamma_5}{2} \Psi_j \right\rangle\right) = \int \left[ Z \ \mathrm{tr} \left| \partial_\mu \left\langle \Psi_i \frac{1+\gamma_5}{2} \Psi_j \right\rangle \right|^2 - V\left(\left\langle \bar{\Psi}_i \frac{1+\gamma_5}{2} \Psi_j \right\rangle\right) \right] \tag{2.19}$$

The kinetic energy comes from the local structure of all $\langle \bar{\Psi}_i [(1+\gamma_5)/2] \Psi_j \rangle$ vacuum polarization graphs; and V is, in general, a nonlocal polynomial in all powers of $\langle \bar{\Psi}_i [(1+\gamma_5)/2] \Psi_j \rangle$ and their derivatives. $\Gamma$ is thus the full quantum action for the $\langle \bar{\Psi}_i [(1+\gamma_5)/2] \Psi_j \rangle$ fields; that is, it contains the full QCD dynamics of these composite fields. In the long-wavelength limit, it presumably reduces to the nonlinear $\sigma$-model which contains the content of current algebra chiral dynamics [23].

The ground state is characterized by constant $\langle \bar{\Psi}_i [(1+\gamma_5)/2] \Psi_j \rangle$ in which case all derivatives of $\langle \bar{\Psi}_i [(1+\gamma_5)/2] \Psi_j \rangle$ vanish. V is then a polynomial in all powers of the constant $\langle \bar{\Psi}_i [(1+\gamma_5)/2] \Psi_j \rangle$ . It is the quantum potential which has a minimum for the background field $\langle \bar{\Psi}_i [(1+\gamma_5)/2] \Psi_j \rangle$ of the vacuum. The ground state energy could be lowered by such a background field due to the consequent spontaneous symmetry breaking dynamical correlations.

IID. VACUUM COSETS, θ-VACUUA, CHIRAL U(1) SYMMETRY AND CP VIOLATION

In our discussion of the Goldstone-Ward identities for spontaneous chiral SU(2) symmetry breaking we had considered a particular frame. In the massless theory there are an infinite number of possible degenerate vacuua, and $\langle\bar{\Psi}\Psi\rangle$ is a choice of one of these directions (analogous to choosing the direction of magnetization of a ferromagnet in, say, the Z direction). The different possible directions are characterized by the elements of the coset space[2] $U_L(2) \times U_R(2)/SU_I(2) \times U_B(1) \times Z_2\big(\text{chiral SU(2)}\big) \times Z_2\big(\text{chiral U(1)}\big)$, where $Z_2\big(\text{chiral U(1)}\big)$ is the discrete subgroup of chiral U(1) rotations $\left\{\begin{pmatrix} e^{i\pi} & 0 \\ 0 & e^{-i\pi}\end{pmatrix}, 1\right\}$ and the $Z_2\big(\text{chiral SU(2)}\big)$ is $\left\{\begin{pmatrix} e^{i\pi} & 0 \\ 0 & e^{-i\pi}\end{pmatrix}, 1\right\}$. $SU_I(2) \times U_B(1) \times Z_2 \times Z_2$ is the invariance group of the vacuum apart from the usual space-time and gauge symmetries (analogous to rotations about the axis of magnetization of a ferromagnteic); that is, $\langle\bar{\Psi}\Psi\rangle$ is invariant under $SU_I(2) \times U_B(1) \times Z_2 \times Z_2$.[3] Different elements of the coset space are obtained from $\langle\bar{\Psi}\Psi\rangle$ by chiral U(2) transformations,

$$\begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \to \begin{pmatrix} e^{i\theta/4} e^{i\vec{\alpha}\cdot\vec{\tau}/2} & 0 \\ 0 & e^{-i\theta/4} e^{-i\vec{\alpha}\cdot\vec{\tau}/2} \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \tag{2.20}$$

$$\langle\bar{\Psi}\Psi\rangle \to e^{i\theta/2}\left\langle \bar{\Psi}_R e^{i\vec{\alpha}\cdot\vec{\tau}} \Psi_L \right\rangle + e^{-i\theta/2}\left\langle \bar{\Psi}_L e^{-i\vec{\alpha}\cdot\vec{\tau}} \Psi_R \right\rangle . \tag{2.21}$$

Under this transformation it appears that isospin symmetry is also spontaneously broken; however, the isospin transformation must be conjugated

$$\begin{pmatrix} e^{i\vec\beta\cdot\vec\tau/2} & 0 \\ 0 & e^{i\vec\beta\cdot\vec\tau/2} \end{pmatrix} \rightarrow \begin{pmatrix} e^{i\vec\alpha\cdot\vec\tau/2} & 0 \\ 0 & e^{-i\vec\alpha\cdot\vec\tau/2} \end{pmatrix} \begin{pmatrix} e^{i\vec\beta\cdot\vec\tau/2} & 0 \\ 0 & e^{i\vec\beta\cdot\vec\tau/2} \end{pmatrix} \begin{pmatrix} e^{-i\vec\alpha\cdot\vec\tau/2} & 0 \\ 0 & e^{i\vec\alpha\cdot\vec\tau/2} \end{pmatrix}, \tag{2.22}$$

and this conjugated isospin is an invariance of the vacuum.

An important point about this chiral U(2) transformation is that the functional integral not only transforms in the covariant way just described, but has another change as well. Fujikawa [24] has shown that the fermionic integrations measure is not invariant under chiral U(1) rotations but transforms under exp $\{i(\theta/4)\gamma_5\}$

$$\mathcal{D}\psi\,\mathcal{D}\bar\psi \rightarrow \mathcal{D}\psi\,\mathcal{D}\bar\psi\; e^{i\theta\nu(A)} \quad , \tag{2.23}$$

where

$$\nu(A) = \frac{1}{16\pi^2}\int \mathrm{tr}\; F_{\mu\nu}(A)\;\tilde F^{\mu\nu}(A) \tag{2.24}$$

and is nonzero for gauge field configurations with nontrivial topology. This derivation effectively assumes there is no chiral U(1) massless particle. There is thus an extra term induced in the color gauge field action. This term is odd under P and T, and so it would appear that CP is violated, but just as in the case of isospin there is a conjugated CP operation under which the theory is invariant. Thus physical quantities cannot depend on $\theta$.

Because no physical quantities depend on chiral U(1) rotations of the quark fields, there is a chiral U(1) symmetry of the quantum theory. Nevertheless, this chiral U(1) symmetry is not realized explicitly; different $\theta$ values correspond to different possible superselection sectors such that no QCD[4] perturbations can change $\theta$ [2,3]. Thus there are an infinite number of possible degenerate vacuua, related by chiral U(1) rotations. The chiral U(1) symmetry is therefore spontaneously broken. However, this situation is different from usual spontaneous symmetry breaking in two respects. First, this spontaneous chiral U(1) symmetry breaking is independent of the obvious order parameter — it occurs whether or not chiral SU(2) symmetry is spontaneously broken by $\langle\bar{\Psi}\Psi\rangle \neq 0$. Second, there is no associated massless particle. This spontaneous chiral U(1) symmetry breaking is somewhat analogous to spontaneous breaking of a gauge symmetry. There the gauge fields define a frame at each space-time point which can be chosen so that relative to this frame the phase of the order parameter does not oscillate — the Nambu-Goldstone mode can be gauged away. Here $\exp\{i(\theta/16\pi^2)\int \mathrm{tr}\, F_{\mu\nu}(A)\, \tilde{F}^{\mu\nu}(A)\}$ is a topological phase shift of the amplitude for a given gauge field configuration $\exp\{iS(A)\}$ — it is a phase shift proportional to the topological charge of the configuration A. This phase defines a frame. Under a chiral U(1) rotation of the quark fields (or change of phase of the order parameter) $\theta$ is transformed; but for $\theta$ constant throughout space-time the phase of the order parameter cannot oscillate — it is locked to the constant direction given by the topological phase, so the Nambu-Goldstone boson cannot get excited.

As we have said, physical quantities cannot depend on which element of the coset space is chosen to describe spontaneous chiral SU(2) symmetry breaking; associated with any chiral U(2) rotation of $\langle\bar{\Psi}\Psi\rangle$ there is a conjugated isospin and CP invariance of the vacuum.[5]

Nevertheless, these isospin and CP symmetries could in principle be spontaneously broken. Spontaneous isospin breaking would be described by (the conjugated version of) the order parameter $\langle\bar{\Psi}\tau^3\Psi\rangle$. As we will now discuss, spontaneous CP violation is only in principle possible because of the particular way the chiral U(1) symmetry is spontaneously broken.

Spontaneous CP violation would arise if both $\langle\bar{\Psi}\Psi\rangle$ and $\langle i\bar{\Psi}\gamma_5\Psi\rangle$ were nonzero in a $\theta = 0$ vacuum (or any chiral U(2) rotation of this situation).[6] If the theory had a chiral U(1) symmetry either explicitly realized or spontaneously broken in the usual way, one could choose a frame by making a chiral U(1) rotation so that $\langle i\bar{\Psi}\gamma_5\Psi\rangle$ would be zero. Now, though, under a chiral U(1) rotation $\langle i\bar{\Psi}\gamma_5\Psi\rangle$ could still be rotated away, but in that frame we would have spontaneous chiral SU(2) symmetry breaking driven by $\langle\bar{\Psi}\Psi\rangle_\theta \neq 0$, where this expectation value is defined by[7]

$$\langle\bar{\Psi}\Psi\rangle_\theta = \lim_{\Phi\to 0} \int \mathcal{D}A\, e^{iS(A)}\, e^{i\theta\nu(A)} \det\left[\begin{pmatrix} i\not{D}(A) & 0 \\ 0 & i\not{D}(A)\end{pmatrix} - \Phi\,\frac{1+\gamma_5}{2} - \Phi^\dagger\,\frac{1-\gamma_5}{2}\right] \mathrm{tr}\left\langle x\left|\left[\begin{pmatrix} i\not{D}(A) & 0 \\ 0 & i\not{D}(A)\end{pmatrix} - \Phi\,\frac{1+\gamma_5}{2} - \Phi^\dagger\,\frac{1-\gamma_5}{2}\right]^{-1}\right|x\right\rangle , \tag{2.25}$$

with the explicit additional phase factor $\exp\{i\theta\nu(A)\}$ accompanying the functional integral over all color fields. This phase could now lead to CP violation; it can only be rotated away by making $\langle i\bar{\Psi}\gamma_5\Psi\rangle$ nonzero. Thus from symmetry considerations, and from the link between chiral U(1) global transformations of quark fields and the color gauge field topological phase, spontaneous CP violation as well as spontaneous isospin and chiral SU(2) symmetry breaking are possible. It is a question of dynamics how these symmetries are in fact realized.

We comment that the dynamics that governs spontaneous global symmetry breaking can be translated to a problem in gauge field correlations. Both the determinant and tr $G^A$ in Eq. (2.25) can be expressed in terms of gauge field invariants, as exemplified by Schwinger's [27] formulas for expressions of this kind (in QED, and for $A_\mu$ that gives a constant $F_{\mu\nu}$). For example, for $\Phi = M\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$, the leading term in the weak field expansion of $G^A(M)$ is proportional to (1/M) tr $F^2_{\mu\nu}(A)$. Since this relation is an approximation to the trace anomaly [28], implying a further relation between the breaking of scale symmetry and spontaneous symmetry breaking, we expect an exact formula is possible.) Alternatively, $\langle\bar{\Psi}\Psi\rangle$ can be expressed in terms of topological fluctuations through Crewther's formula [29],

$$\langle\bar{\Psi}\Psi\rangle_\theta = \lim_{\substack{q\to 0\\ M\to 0}} \frac{1}{M} \int_x e^{iq\cdot x} \left\langle \frac{1}{4\pi^2}\,\mathrm{tr}\,F_{\mu\nu}(A)\,\tilde{F}^{\mu\nu}(A)\;\frac{1}{4\pi^2}\,\mathrm{tr}\,F_{\alpha\beta}(A)\,\tilde{F}^{\alpha\beta}(A)\right\rangle_\theta \tag{2.26}$$

When expressed in terms of a functional integral, this formula and Eq. (2.25) imply an association

$$\mathrm{tr}\; G^A(M) \;:\; \frac{1}{M}\,\mathrm{tr}\; F_{\mu\nu}(A)\;\tilde{F}^{\mu\nu}(A)\;\; \nu(A) \quad ; \tag{2.27}$$

The averages (over all A weighted by $e^{iS(A)}e^{i\theta\nu(A)}$ det$[i\not{D}(A)-M]$) of both sides of this relation are equal to $M \to 0$. For the right-hand side, a mass dependence proportional to M must be induced from the determinant by the topological factors. Crewther's formula implies that tr $F\tilde{F}$ correlations, which are dominated by glueballs and chiral

U(1) mesons [30], are important for QCD to spontaneously break its chiral SU(2) symmetry.

## III. CHIRALITY SELECTION RULES

In order to explicitly demonstrate the kind of effects that follow from color gauge field configurations with nontrivial topology, we review 't Hooft's chirality selection rules [1] which govern configurations with $\nu(A) \neq 0$. These selection rules follow from the relation between the anomaly in the U(1) axial vector current and $\nu(A)$. In a background field of any color gauge field configuration A, there must be a chirality change for each massless quark flavor of

$$\Delta\left\langle Q_5\right\rangle_A = -2\nu(A) \tag{3.1}$$

For two flavors the total chirality charge is $-4\nu(A)$. This chirality change can be expressed in terms of changes in the numbers of quarks and antiquarks of a particular chirality,

$$\Delta Q_5 = \Delta\left(N_R + \bar{N}_L - N_L - \bar{N}_R\right) \quad . \tag{3.2}$$

Thus the necessary chirality change in a background field with $\nu(A) \neq 0$ can be achieved by the creation or annihilation of massless quarks. In the presence of a color gauge field configuration (in Euclidean space-time) with $\nu(A) = 1$, there must be a chirality change of -2 for each flavor. This can be achieved, for example, by the creation from the vacuum (with zero-chirality) of quark pairs, $u_L + \bar{u}_R + d_R + \bar{d}_L$. If there are sources present to absorb these quarks (and the chirality changes), this Euclidean space-time event can be represented as in Fig. 2. In this figure the sources that absorb the quarks can have color as well as flavor. Any field configuration with $\nu(A) = -1$ could be a source of quarks of opposite chirality, or a sink for the quarks created by a configuration with $\nu(A) = +1$, as represented in Fig. 3.

For a configuration with many fluctuations, with relatively localized fields with $\nu(A) = \pm 1$, the chirality selection rules can be satisfied by exchanging massless quarks between them in many possible ways. When there are not enough regions with $\nu(A) = -1$ to absorb the quarks created from regions with $\nu(A) = +1$, they must be absorbed by sources.

The way these selection rules are explicitly asserted is that the functional integral contains the factor det $\not{D}(A)$; this determinant is the product of eigenvalues obtained from

$$\not{D}(A)\ \psi_n = \varepsilon_n\ \psi_n \tag{3.3}$$

for each flavor. For any background field configuration with $|\nu(A)| = N$, there are N zero eigenmodes for each flavor. Such a field configuration can only contribute to Green's functions with chirality 2N times the number of massless quarks [1,2,3].

Now let us consider the Euclidean vacuum-to-vacuum amplitude in the presence of the color singlet source fields $\Phi$ of Eq. (2.12), and the analogous color octet fields $\Theta$, where $\Theta = \Theta^a\lambda^a$ can be expressed in terms of Hermitian fields analogous to Eq. (2.12) for $\Phi$,

$$\Theta^a = \xi^a + i\vec{\beta}^a \cdot \vec{\tau} + i\zeta^a + \vec{\rho}^a \cdot \vec{\tau} \tag{3.4}$$

We consider these fields, which couple to quarks like

$$\bar{\psi}_i\lambda^a \frac{1+\gamma_5}{2}\psi_j\Theta^a_{ij} + \bar{\psi}_i\lambda^a \frac{1-\gamma_5}{2}\psi_j\Theta^{\dagger a}_{ij} \quad , \tag{3.5}$$

since chirality changes can be absorbed by scalars coupled to quarks and antiquarks in either the color singlet or color octet channels (since $3\times\bar{3} = 1+8$). The vacuum amplitude can be expressed in terms of a functional integral,

$$Z(\Phi,\Theta) = \int \mathscr{D}A e^{-S(A)} e^{i\theta\nu(A)} \det\left[\begin{pmatrix} \not{D}(A) & 0 \\ 0 & \not{D}(A) \end{pmatrix} + (\Phi+\Theta)\frac{1+\gamma_5}{2} + \left(\Phi^\dagger+\Theta^\dagger\right)\frac{1-\gamma_5}{2}\right] . \tag{3.6}$$

Consider the contribution of a color gauge field configuration with $\nu(A) = -1$. From the chirality selection rules the first nonvanishing term in the functional expansion of the determinant in powers of the external source fields is the second-order term,

$$\begin{aligned}\det\left[\not{D}(A) + (\Phi+\Theta)\frac{1+\gamma_5}{2} + \left(\Phi^\dagger+\Theta^\dagger\right)\frac{1-\gamma_5}{2}\right] &= \det(\not{D}(A)+\varepsilon)\,\frac{1}{2}\Bigg\{ \mathrm{tr}\left[(\Phi+\Theta)\right.\\ &\times \left.\frac{1+\gamma_5}{2}\,\frac{1}{\not{D}(A)+\varepsilon}\right]\mathrm{tr}\left[\frac{1}{\not{D}(A)+\varepsilon}\,\frac{1+\gamma_5}{2}\,(\Phi+\Theta)\right] \\ &- \mathrm{tr}\left[(\Phi+\Theta)\frac{1+\gamma_5}{2}\,\frac{1}{\not{D}(A)+\varepsilon}\,(\Phi+\Theta)\frac{1+\gamma_5}{2}\,\frac{1}{\not{D}(A)+\varepsilon}\right]\Bigg\} + \ldots\end{aligned} \tag{3.7}$$

where $\varepsilon$ is infinitesimal, and the trace is a functional space-time trace as well as one over color, flavor and Dirac indices. This term is represented graphically in Fig. 4.

Expressing the space-time matrix element of the operator $(\not{D}(A)+\varepsilon)^{-1}$ in terms of eigenfunctions of $\not{D}(A)$,

$$\left\langle x' \left| \frac{1}{\not{D}(A)+\varepsilon} \right| x \right\rangle = \frac{\psi_0(x')\psi_0^\dagger(x)}{\varepsilon} + \sum_{n\neq 0} \frac{\psi_n(x')\psi_n^\dagger(x)}{\varepsilon_n} , \tag{3.8}$$

with the zero-mode eigenfunctions being a consequence of $|\nu(A)| = 1$, the determinant becomes

$$\det{}'\not{D}(A) \Bigg\{ \int_x \mathrm{tr}\left[\Big(\Phi(x)+\Theta(x)\Big)\frac{1+\gamma_5}{2}\begin{pmatrix} u_0(x)u_o^\dagger(x) & 0 \\ 0 & d_0(x)d_0^\dagger(x)\end{pmatrix}\right]$$

$$\times \int_x \mathrm{tr}\left[\begin{pmatrix} u_0(x')u_0^\dagger(x') & 0 \\ 0 & d_0(x')d_0^\dagger(x')\end{pmatrix}\frac{1+\gamma_5}{2}\Big(\Phi(x')+\Theta(x')\Big)\right]$$

$$- \int_{x,x'} \mathrm{tr}\Bigg[\Big(\Phi(x)+\Theta(x)\Big)\frac{1+\gamma_5}{2}\begin{pmatrix} u_0(x)u_0^\dagger(x') & 0 \\ 0 & d_0(x)d_0^\dagger(x')\end{pmatrix}$$

$$\times \Big(\Phi(x')+\Theta(x')\Big)\frac{1+\gamma_5}{2}\begin{pmatrix} u_0(x')u_0^\dagger(x) & 0 \\ 0 & d_0(x')d_0^\dagger(x')\end{pmatrix}\Bigg]\Bigg\}, \qquad (3.9)$$

The $1/\varepsilon^2$ from the zero-mode piece of the propagators canceling the $\varepsilon^2$ from $\det\,(\not{D}(A)+\varepsilon)$ due to the zero-modes. The prime on the determinant refers to the product of nonzero eigenvalues, and $u_o$ and $d_0$ are the zero-mode eigenfunctions of the Dirac operator in a background color gauge field with $\nu(A) = -1$ and have positive chirality. Since the zero-mode propagator is diagonal in flavor (but not in color), the flavor traces for the external fields $\Phi$ can be immediately done; they have the flavor structure,

$$\frac{1}{2}\left[(\mathrm{tr}\ \Phi)^2 - \mathrm{tr}\ \Phi^2\right] = \det\ \Phi \quad . \qquad (3.10)$$

(For $N_f$ flavors, this structure easily generalizes. For example, for three flavors $\Phi$ is a $3\times 3$ matrix; the leading term from a $\nu(A) = -1$ configuration will involve 3 scalar fields. Functionally expanding

Eq. (3.7) to third order gives a flavor structure,

$$\frac{1}{3!}\left[(\mathrm{tr}\ \Phi)^3 - 3\,\mathrm{tr}\ \Phi^2\ \mathrm{tr}\ \Phi + 2\,\mathrm{tr}\ \Phi^3\right] = \det \Phi\ .) \qquad (3.11)$$

For a gauge field configuration with $\nu(A) = 1$ the expression has the replacements: $(1+\gamma_5)/2 \to (1-\gamma_5)/2$, $\Phi \to \Phi^\dagger$, $\Theta \to \Theta^\dagger$. The zero-mode eigenfunctions of the Dirac operator now have negative chirality. Again the flavor trace can be trivially done, giving for the $\Phi^\dagger$ fields

$$\frac{1}{2}\left[\left(\mathrm{tr}\ \Phi^\dagger\right)^2 - \mathrm{tr}\left(\Phi^\dagger\right)^2\right] = \det \Phi^\dagger \qquad (3.12)$$

For $\theta \neq 0$, each of these determinants has a phase factor $e^{\pm i\theta}$; the sum of these terms has a flavor structure proportional to

$$e^{-i\theta}\det \Phi + e^{i\theta}\det \Phi^\dagger = 2\cos\theta\left(\pi^2 - \phi^2\right) - 4\sin\theta\ \pi\cdot\phi\ \cdot\cdot\ , \qquad (3.13)$$

where $\pi_\mu = (\sigma,\vec{\pi})$ and $\phi_\mu = (\eta,-\vec{\phi})$. This general structure follows simply from the chirality selection rules and $SU_L(2) \times SU_R(2) \times U_B(1)$ symmetry (as will be shown more explicitly later); therefore, the $\Theta$ terms must also have the same flavor structure. On the other hand, the coefficients of the $\Phi$ and $\Theta$ terms will not be equal, in general.

Of course this is only the lowest approximation to the external scalar field dependence of the determinants. In general there are terms of all higher powers in the scalar fields. To see this, we consider another way of deriving the $\Phi$ dependence of the determinants which follows simply from linear algebra.[8] For all configurations with $\nu(A) = -N$, for each flavor there are N zero-mode eigenfunctions of $\not{D}(A)$

with positive chirality, and for $\nu(A) = +N$ there are N zero-mode eigenfunctions of negative chirality

$$\left[\not{D} + \left(\Phi \frac{1+\gamma_5}{2} + \Phi^\dagger \frac{1-\gamma_5}{2}\right)\right]\psi_{oi} = \begin{cases} \Phi\ \psi_{0i}\ , & \nu = -N \\ \Phi^\dagger\ \psi_{0i}\ , & \nu = +N \end{cases} \tag{3.14}$$

where i = 1, ..., N. Also, since for all nonzero-eigenmodes,

$$\not{D}\left(\gamma_5\psi_n\right) = -\varepsilon_n\left(\gamma_5\psi_n\right) \quad , \tag{3.15}$$

it follows that

$$\left[\not{D} + \left(\Phi \frac{1+\gamma_5}{2} + \Phi^\dagger \frac{1-\gamma_5}{2}\right)\right] \frac{1+\gamma_5}{2}\psi_n = \varepsilon_n \frac{1-\gamma_5}{2}\psi_n + \Phi \frac{1+\gamma_5}{2}\psi_n \tag{3.16a}$$

and

$$\left[\not{D} + \left(\Phi \frac{1+\gamma_5}{2} + \Phi^\dagger \frac{1-\gamma_5}{2}\right)\right] \frac{1-\gamma_5}{2}\psi_n = \varepsilon_n \frac{1+\gamma_5}{2}\psi_n + \Phi^\dagger \frac{1-\gamma_5}{2}\psi_n \quad . \tag{3.16b}$$

For constant $\Phi$, the determinant then becomes

$$\det\left[\begin{pmatrix}\not{D} & 0\\ 0 & \not{D}\end{pmatrix} + \Phi\frac{1+\gamma_5}{2} + \Phi^\dagger\frac{1-\gamma_5}{2}\right] = \begin{Bmatrix}(\det \Phi)^N \\ (\det \Phi^\dagger)^N\end{Bmatrix}$$

$$\times \prod_n \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \otimes \begin{bmatrix} & \Phi & \begin{pmatrix}\varepsilon_n & 0\\ 0 & \varepsilon_n\end{pmatrix} \\ \begin{pmatrix}\varepsilon_n & 0\\ 0 & \varepsilon_n\end{pmatrix} & \Phi^\dagger & \end{bmatrix} \quad , \tag{3.17}$$

where the first terms in the curly brackets are determinants of the $2\times 2$ flavor matrices, the top term is for $\nu(A) = -N$, and the bottom term for $\nu(A) = +N$; and where the product over n is a product of determinants in the Dirac, flavor and color spaces — the direct product with the unit matrix is itself a direct product; the unit $2\times 2$ matrix counts the particle and antiparticle modes, and $1$ is a $3\times 3$ color unit matrix. For each n,

$$\det \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{bmatrix} \Phi & \begin{pmatrix} \varepsilon_n & 0 \\ 0 & \varepsilon_n \end{pmatrix} \\ \begin{pmatrix} \varepsilon_n & 0 \\ 0 & \varepsilon_n \end{pmatrix} & \Phi^\dagger \end{bmatrix} = \left(\varepsilon_n^4 - \varepsilon_n^2 \operatorname{tr} \Phi^\dagger\Phi + \det \Phi^\dagger\Phi\right)^6 . \tag{3.18}$$

Let us consider the first few terms in the expansion of the determinant for weak $\Phi$. We write

$$\det{}'\left[\not{D} + \Phi\,\frac{1+\gamma_5}{2} + \Phi^\dagger\,\frac{1-\gamma_5}{2}\right] = \exp\left\{6\sum_n{}' \ln\left(\varepsilon_n^4 - \varepsilon_n^2 \operatorname{tr}\Phi^\dagger\Phi + \det \Phi^\dagger\Phi\right)\right\}$$

$$= \exp\left\{6\left(\sum_n{}' \ln\,\varepsilon_n^4\right)\right\} \exp\left\{6\sum_n{}' \ln\left(1 - \frac{1}{\varepsilon_n^2}\operatorname{tr}\Phi^\dagger\Phi + \frac{1}{\varepsilon_n^4}\det\,\Phi^\dagger\Phi\right)\right\} \tag{3.19}$$

Expanding the second log, the first few terms give,

$$\det{}'\not{D}\,\exp\left\{-\,6\sum_n{}'\left[\frac{1}{\varepsilon_n^2}\operatorname{tr}\Phi^\dagger\Phi - \frac{1}{\varepsilon_n^4}\det\,\Phi^\dagger\Phi + \frac{1}{2\varepsilon_n^4}\left(\operatorname{tr}\,\Phi^\dagger\Phi\right)^2 + \ldots\right]\right\}$$

$$= \det{}'\not{D}\left[1 - 6\left(\sum_n{}' \frac{1}{\varepsilon_n^2}\right)\operatorname{tr}\,\Phi^\dagger\Phi - 3\left(\sum_n{}' \frac{1}{\varepsilon_n^4}\right)\right. \tag{3.20}$$

$$\left.\times\ \left[\left(\operatorname{tr}\,\Phi^\dagger\Phi\right)^2 - 2\det\,\Phi^\dagger\Phi\right] + 18\left(\sum_n{}' \frac{1}{\varepsilon_n^2}\right)^2\left(\operatorname{tr}\,\Phi^\dagger\Phi\right)^2 + \ldots\right] .$$

Since

$$\operatorname{tr}\,\Phi^\dagger\Phi\Phi^\dagger\Phi = \left(\operatorname{tr}\,\Phi^\dagger\Phi\right)^2 - 2\det\,\Phi^\dagger\Phi \quad , \tag{3.21}$$

and since the nonzero-mode propagator is

$$G(A) = \sum_n{}' \frac{\psi_n \psi_n^\dagger}{\varepsilon_n} , \tag{3.22}$$

the determinant, for A such that $\nu(A) = \pm N$, and for constant $\Phi$, can be expressed

$$\det\left[\begin{pmatrix} \not{D} & 0 \\ 0 & \not{D} \end{pmatrix} + \Phi \frac{1+\gamma_5}{2} + \Phi^\dagger \frac{1-\gamma_5}{2}\right]$$

$$= \left\{ \begin{matrix} (\det \Phi)^N \\ (\det \Phi^\dagger)^N \end{matrix} \right\} \det{}'\not{D} \left[ 1 + \mathrm{tr}\left( G(A) \frac{1+\gamma_5}{2} G(A) \frac{1-\gamma_5}{2} \right) \mathrm{tr}\, \Phi^\dagger \Phi \right. \tag{3.23}$$

$$- \frac{1}{2} \mathrm{tr}\left( G(A) \frac{1+\gamma_5}{2} G(A) \frac{1-\gamma_5}{2} G(A) \frac{1+\gamma_5}{2} G(A) \frac{1-\gamma_5}{2} \right) \mathrm{tr}\, \Phi^\dagger \Phi \Phi^\dagger \Phi$$

$$\left. + \frac{1}{2} \mathrm{tr}\left( G(A) \frac{1+\gamma_5}{2} G(A) \frac{1-\gamma_5}{2} \right) \mathrm{tr}\left( G(A) \frac{1+\gamma_5}{2} G(A) \frac{1-\gamma_5}{2} \right) \left(\mathrm{tr}\, \Phi^\dagger \Phi\right)^2 + \ldots \right] .$$

The terms in square brackets can be obtained from an expansion of

$$\exp\left\{ \mathrm{tr}\, \ln \left[ 1 + G(A) \left( \Phi \frac{1+\gamma_5}{2} + \Phi^\dagger \frac{1-\gamma_5}{2} \right) \right] \right\} \tag{3.24}$$

If $\Phi$ is considered a constant mass matrix that does not transform under chiral U(1) rotations, then when $\psi \to \exp\{i\alpha\gamma_5\}\psi$, the mass terms in the original Lagrangian is transformed to

$$\bar{\psi}\Phi\, e^{2i\alpha} \frac{1+\gamma_5}{2} \psi + \bar{\psi}\Phi^\dagger e^{-2i\alpha} \frac{1-\gamma_5}{2} \psi . \tag{3.25}$$

In the determinants there will then be extra phases,

$$\det\left(e^{2i\alpha} \Phi\right) = e^{4i\alpha} \det \Phi \tag{3.26a}$$

$$\det\left(e^{-2i\alpha} \Phi^\dagger\right) = e^{-4i\alpha} \det \Phi^\dagger . \tag{3.26b}$$

Since a configuration with $\nu(A)$ is weighted by exp $\{i\theta\nu(A)\}$, the effect of this chiral U(1) rotation of the quark fields is $\theta \to \theta - 4\alpha$. This derivation of the $\theta$ transformation property shows it remains valid for arbitrary mass quarks [31]. (The Fujikawa [24] derivation of the transformation of the fermionic measure of the functional integral is only valid for massless quarks.) Of course the existence of the zero-modes of the Dirac operator $\not{D}(A)$, which imply the $\left\{(\det\Phi)^{|\nu(A)|}, (\det\Phi^\dagger)^{|\nu(A)|}\right\}$ terms, requires the assumption of no chiral U(1) massless pole.

For nonconstant $\Phi$, the product of eigenvalues in the function space does not diagonalize; there are now nonzero matrix elements of the form $\psi_0^\dagger\Phi\psi_n$ and $\phi_{n'}^\dagger\Phi[(1+\gamma_5)/2]\psi_n$. For a background field with $\nu(A) = -N$ so that the zero-modes are labeled by $i = 1, \ldots, N$, the determinant in function, Dirac and flavor space becomes

$$\det\begin{bmatrix} \psi_{0i}^\dagger\ \Phi\ \psi_{0j} & \psi_{0i}^\dagger\ \Phi\ \psi_n \\ \psi_{n'}^\dagger\ \Phi\ \psi_{0j} & \psi_{n'}^\dagger\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\otimes\begin{bmatrix}\Phi\,\frac{1+\gamma_5}{2} & \begin{pmatrix}\varepsilon_n & 0\\ 0 & \varepsilon_n\end{pmatrix}\frac{1+\gamma_5}{2} \\ \begin{pmatrix}\varepsilon_n & 0\\ 0 & \varepsilon_n\end{pmatrix}\frac{1-\gamma_5}{2} & \Phi^\dagger\,\frac{1-\gamma_5}{2}\end{bmatrix}\psi_n \end{bmatrix} \tag{3.27}$$

The eigenfunctions are flavor doublets of four component Dirac spinors, with matrix elements in flavor space implied; also implied are the diagonal color contractions.

The new matrix elements of the form $\psi_{n'}^\dagger\Phi\psi_n$ do not produce new structure, they only imply the $\Phi$'s cannot be pulled out of the integrals in Eq. (3.23) and so the traces over the G(A)'s do not simply contract to sums of products of inverse eigenvalues. Further discussion of the structure of these determinants, including the mixing of zero and nonzero models, will be discussed for a particular background gauge field in the next section.

## IVA. INSTANTON CONTRIBUTION

Let us review the contribution to the Euclidean functional integral of the instanton and anti-instanton configurations [15], first without the periodic boundary conditions; later the effect of these boundary conditions will be included. The instanton and anti-instanton configurations are the first of a class of nontrivial minima of the Euclidean action with all integer values of $\nu(A)$ and $S(A) = [(8\pi^2)/g^2]|\nu(A)|$.

The action is expanded in fluctuations about these configurations,

$$S(A + \delta A) \simeq S(A) + \frac{1}{2}\,\mathrm{tr}\;\delta A_\mu\;\mathscr{D}^{-1}_{\mu\nu}\;(A)\;\delta A_\nu \tag{4.1}$$

where $\mathscr{D}^{-1}_{\mu\nu}(A)$ is the inverse gluon propagator in the background field A. The fluctuation fields are expanded in eigenfunctions of the fluctuation operator,

$$\delta A_\mu(x) = \sum_n a_n\, A_{n\mu}(x) \quad , \tag{4.2}$$

where

$$\mathscr{D}^{-1}_{\mu\nu}(A)\; A_{n\nu}(x) = \lambda_n\, A_{n\mu}(x) \quad . \tag{4.3}$$

The contribution to the functional integral $Z(\Phi,\Theta)$ is given by a sum of the weights of these eigenstates. However, the operator $\mathscr{D}^{-1}_{\mu\nu}(A)$ has zero eigenvalues due to fluctuations associated with transformations of A that leave the action invariant. For the instanton background field,[9]

$$A_\mu(x) = -\bar{\eta}^a_{\mu\nu}\,\Omega\,\frac{\lambda^a}{2}\,\Omega^\dagger\partial_\nu\,\ell n\left(1 + \frac{\rho^2}{(x-X)^2}\right) \quad , \tag{4.4}$$

the action does not depend on $\rho$, $X_\mu$ or $\Omega$. The position and scale parameters are due to translation and scale invariance of the classical action, while the global gauge rotation is due to a mixing of color and Lorentz properties. In Euclidean space-time the Lorentz group is $O(4) \simeq SU(2) \times SU(2)$. Acting on Dirac spinors, the Lorentz generators are $\sigma_{\mu\nu}[(1\pm\gamma_5)/2] = \mp\frac{1}{2}\,\varepsilon_{\mu\nu\alpha\beta}\sigma_{\alpha\beta}[(1\pm\gamma_5)/2]$; that is, the left- and right-handed spinors are Lorentz rotated by self-dual and antiself-dual SU(2) subgroups of the spacial O(4). On the other hand, $\lambda^a$ associates with pairs of Dirac spinors in the fundamental representation of color SU(3), a color SU(2) vector. This color SU(2) index and the spacial SU(2) vector index are to be contracted, but the color and spacial SU(2) vectors can have an arbitrary relative orientation. The integration over $\Omega$ averages over all these relative orientations, as well as over all embeddings of this color SU(2) subgroup within color SU(3). The sums over the weights of these modes are converted to collective coordinate integrations over Euclidean space-time positions, scale sizes and global gauge orientations.[10] The result is

$$Z_{\substack{1\text{ instanton }+\\ 1\text{ anti-instanton}}} = \int d^4X \frac{d\rho}{\rho^5} d\Omega\, 2\left(\frac{2\pi}{g^2}\right)^6 e^{-8\pi^2/g^2} \frac{\det - D_\mu^2(A)}{\sqrt{\det' \mathscr{D}_{\mu\nu}^{-1}(A)}}$$

$$\times \left\{ e^{-i\theta} \det\left[\not{D}(A) + (\Phi+\Theta)\frac{1+\gamma_5}{2} + \left(\Phi^\dagger+\Theta^\dagger\right)\frac{1-\gamma_5}{2}\right]\right.$$

$$\left. + \; e^{i\theta} \det\left[\not{D}(\bar{A}) + (\Phi+\Theta)\frac{1+\gamma_5}{2} + \left(\Phi^\dagger+\Theta^\dagger\right)\frac{1-\gamma_5}{2}\right]\right\} \quad (4.5)$$

The prime on det' refers to the product of nonzero eigenvalues, and the term det-$D_\mu^2(A)$ is the Faddeev-Popov gauge fixing determinant.

The fermionic determinants have been discussed formally in the last section. The zero-mode matrix elements can now be evaluated with the explicit eigenfunctions [17],

$$\psi_0(x-X) = -\left(8\pi^2\right)^{3/2} \rho^{3/2}\, \mathscr{G}(x-X)\, \psi\;, \quad (4.6)$$

where

$$\mathscr{G}(x) \equiv \frac{G(x)}{\left(1+\frac{\rho^2}{x^2}\right)^{3/2}}\;, \quad (4.7)$$

and where

$$G(x) \equiv -\frac{1}{2\pi^2}\frac{\gamma_\mu x_\mu}{\left(x^2\right)^2} \quad (4.8)$$

is the Euclidean free massless fermion propagator, and $\chi$ is a constant color and Dirac spinor (for each flavor) of definite chirality; $\gamma_5\chi = \pm\chi$ for an instanton (anti-instanton), with the property

$$\mathrm{tr}_{color}\chi\chi^\dagger = \frac{1\pm\gamma_5}{2}\;. \quad (4.9)$$

This singular-gauge zero-mode wave function is obtained from 't Hooft's regular gauge result [15] by a color gauge transformation with $\Omega(x) = (\lambda_\mu x_\mu)/\sqrt{x^2}$, where $\lambda_\mu \equiv (1, i\lambda^a)$, $a = 1,2,3$, and by using the fact that the zero-mode wave function has definite chirality and zero angular momentum,

$$J^a \psi_0 = \left(S^a + \frac{\lambda^a}{2}\right)\psi_0 = 0 \quad , \tag{4.10}$$

where the spin is $S^a \equiv \frac{1}{4}\eta^a_{\mu\nu}\sigma_{\mu\nu}$ for instantons or $\frac{1}{4}\bar{\eta}^a_{\mu\nu}\sigma_{\mu\nu}$ for anti-instantons. The zero angular momentum condition implies a color rotation has the same effect on right- (left-) handed spinors as Lorentz rotations; this implies $\lambda_\mu x_\mu$ and $\gamma_\mu x_\mu$ have the same action on right- (left-) handed spinors.

The evaluation of the nonzero-mode determinants has been performed by 't Hooft [15], and has the essential effect of renormalizing the action. This is heuristically seen by expressing the nonzero-mode determinants in Eq. (4.5)

$$\exp\left\{-\frac{1}{2}\,\mathrm{tr}'\,\ln\,\mathscr{D}^{-1}_{\mu\nu}(A) + \mathrm{tr}\,\ln - D^2_\mu(A) + \mathrm{tr}'\,\ln\,\not{D}(A)\right\} \tag{4.11}$$

and functionally expanding the terms in the exponent in powers of A. This gives all one-loop graphs with external A fields; there is a local contribution with the structure $\int \mathrm{tr}\, F^2_{\mu\nu}(A)$ to renormalize the classical action. Defining the coupling $\lambda(\rho)$ (essentially the partition function for a single instanton),

$$\lambda(\rho) \equiv \frac{2}{(4\pi)^6}\left(\sqrt{S(A)}\right)^{12} \frac{e^{-S(A)}\det - D^2_\mu(A)}{\sqrt{\det'\ \mathscr{D}^{-1}_{\mu\nu}(A)}} \tag{4.12}$$

it therefore has the form

$$\lambda(\rho) = \text{const.} \left(\frac{8\pi^2}{\bar{g}^2(\rho\mu)}\right)^6 e^{-8\pi^2/\bar{g}^2(\rho\mu)} \quad . \tag{4.13}$$

where $\bar{g}^2(\rho\mu)$ is the usual one-loop effective gauge coupling, for scale size $\rho$, and subtracted at the momentum scale $\mu$.

The instanton and anti-instanton contribution to $Z(\Phi,\Theta)$ becomes

$$\begin{aligned} Z_{\substack{1\text{ instanton }+\\ 1\text{ anti-instanton}}} &= \int d^4X \frac{d\rho}{\rho^5} d\Omega\, \lambda(\rho)\Bigg(\Bigg[e^{-i\theta} \det \int_x \psi^\dagger_{f0}(X-x) \\ &\times \Omega^\dagger\big(\Phi(x) + \Theta(x)\big) \frac{1+\gamma_5}{2} \Omega\, \psi_{f'0}(x-X) + e^{i\theta} \det \int_x \psi^\dagger_{fo}(X-x) \\ &\times \Omega^\dagger\big(\Phi^\dagger(x) + \Theta^\dagger(x)\big) \frac{1-\gamma_5}{2} \Omega\, \psi_{f'0}(x-X)\Bigg] \exp \operatorname{tr} \ln\Bigg\{1 + G(A)\Bigg[\big(\Phi+\Theta\big) \\ &\times \frac{1+\gamma_5}{2} + \big(\Phi^\dagger + \Theta^\dagger\big)\frac{1-\gamma_5}{2}\Bigg]\Bigg\} + \text{mixing of zero- and nonzero-modes}\Bigg) \quad . \end{aligned} \tag{4.14}$$

The zero-mode determinant is in the $2\times 2$ flavor space. The integration over all global color gauge transformation $\Omega$ will not affect the $\Phi$ terms but will give a different coefficient to the $\Theta$ zero-mode terms. The result is

$$\begin{aligned} Z_{\substack{1\text{ instanton }+\\ 1\text{ anti-instanton}}} &= \left(8\pi^2\right)^2 \int d^4X\rho d\rho\, \lambda(\rho) \int_{x,x'} \operatorname{tr}\, \mathcal{G}(X-x)\, \mathcal{G}(x-X) \\ &\times \operatorname{tr}\, \mathcal{G}(X-x')\, \mathcal{G}(x'-X)\; 2\,\mathrm{Re}\, e^{-i\theta}\Big\{\Big[\big(\sigma(x) + i\eta(x)\big)\big(\sigma(x') + i\eta(x')\big) \\ &- \big(\vec{\phi}(x) + i\vec{\pi}(x)\big)\cdot\big(\vec{\phi}(x') + i\vec{\pi}(x')\big)\Big] - \frac{3}{16}\Big[\big(\xi^a(x) + i\zeta^a(x)\big) \\ &\times \big(\xi^a(x') + i\zeta^a(x')\big) - \big(\vec{\rho}^a(x) + i\vec{\beta}^a(x)\big)\cdot\big(\vec{\rho}^a(x') + i\vec{\beta}^a(x')\big)\Big]\Big\} \end{aligned}$$

$$\times \left( 1 + \int \mathrm{tr}\; G^A(x-X,x'-X) \frac{1+\gamma_5}{2} G^A(x'-X,x-X) \frac{1-\gamma_5}{2} \right.$$

$$\times \left\{ \left[ \sigma(x)\sigma(x') + \vec{\pi}(x)\cdot\vec{\pi}(x') + \eta(x)\eta(x') + \vec{\phi}(x)\cdot\vec{\phi}(x') \right] \right.$$

$$\left. \left. + \left[ \xi^a(x)\xi^a(x') + \vec{\beta}^a(x)\cdot\vec{\beta}^a(x') + \zeta^a(x)\zeta^a(x') + \vec{\rho}^a(x)\cdot\vec{\rho}^a(x') \right] \right\} + \ldots \right) \tag{4.15}$$

The nonzero-model propagators $G^A$ are those of Brown, Carlitz, Creamer and Lee [32]. These terms are shown graphically in Fig. 5. The first term is the 't Hooft term[11] [1,15], and the second term has nonzero-mode propagators in the same background instanton or anti-instanton of the first term.

The structure of the remaining terms not shown in Eq. (4.14) can be simply expressed graphically (Fig. 6). In the first term of Fig. 6, the instanton serves as a quark source; these quarks interact with the external scalar fields, and between scatterings propagate in the background field of the instanton. The second term represents a quark loop (in general there are many such loops), interacting with external scalar fields and propagating in the same background instanton field of the first term.

In the constant external scalar field limit, the terms involving the mixing of zero- and nonzero-modes vanish by orthogonality. The flavor and color contractions of the zero-mode terms can also be expressed in the form

$$\left(e^{-i\theta}\det\Phi + e^{+i\theta}\det\Phi^\dagger\right) - \frac{3}{16}\left(e^{-i\theta}\det\Theta^a + e^{+i\theta}\det\Theta^{\dagger a}\right)$$

$$= \left\{ 2\cos\theta\left[\left(\sigma^2+\vec{\pi}^2\right) - \left(\eta^2+\vec{\phi}^2\right)\right] + 4\sin\theta\left(\sigma\eta - \vec{\pi}\cdot\vec{\phi}\right)\right\} . \qquad (4.16)$$

$$- \frac{3}{16}\left\{2\cos\theta\left[\left(\xi^{a2}+\vec{\beta}^{a2}\right) - \left(\zeta^{a2}+\vec{\rho}^{a2}\right)\right] + 4\sin\theta\left(\xi^a\zeta^a - \vec{\beta}^a\cdot\vec{\rho}^a\right)\right\} .$$

The color and flavor contractions of the nonleading terms multiply the first term by a polynomial in all powers of invariants of the form

$$\mathrm{tr}\,\Phi^\dagger\Phi, \qquad \mathrm{tr}\,\Phi^\dagger\Phi\Phi^\dagger\Phi, \qquad \mathrm{tr}\,\Theta^\dagger\Theta,$$

$$\mathrm{tr}\,\Theta^\dagger\Theta\Theta^\dagger\Theta, \qquad \mathrm{tr}\,\Phi^\dagger\Phi\Theta^\dagger\Theta, \qquad \mathrm{tr}\,\Phi^\dagger\Theta\Phi^\dagger\Theta, \qquad \mathrm{tr}\,\Theta^\dagger\Phi\Theta^\dagger\Phi,$$

and for color SU(3),

$$\mathrm{tr}\,\Phi^\dagger\Theta\Theta^\dagger\Theta \quad , \qquad \text{and} \qquad \mathrm{tr}\,\Theta^\dagger\Theta\Theta^\dagger\Phi \ .$$

Consider the leading term in the external fields. For fixed $\rho$, for that part of the position integrations for which $(x-X)^2 \gg \rho^2$ and $(x'-X)^2 \gg \rho^2$, the effective propagators $\mathcal{G}$ are essentially free fermion propagators, and our amplitude is essentially that of lowest order perturbation theory from a four-fermi interaction. For short distances, however, the extra factors of $[1+\rho^2/(x-X)^2]^{3/2}$ in $\mathcal{G}$ act to dramatically soften the short distance behavior of the effective four-fermi vertex.

In our expression for Z, we must also integrate over all scale sizes $\rho$. For large $\rho$ the gauge coupling $\bar{g}^{2}(\rho\mu)/8\pi^2$ in the effective coupling $\lambda(\rho)$ gets large, invalidating the semiclassical approximation. On the other hand, if we consider the theory at finite temperature, for high enough temperature the thermal fluctuations will surpress the contribution of the large-scale strong quantum fluctuations. This is because any quantum fluctuation with energy that is small compared to the temperature is washed out by thermal fluctuations, independent of whether the low-energy quantum fluctuations have large coupling. Correspondingly, any quantum fluctuation with large energy relative to the temperature is unaffected by thermal fluctuations, so temperature serves as essentially just an infrared cutoff.

## IVB. EFFECT OF FINITE TEMPERATURE

We will now consider the theory at finite temperature and show how the leading high-temperature effect of thermal fluctuations is to exponentially cut off the contributions of large instantons with strong coupling.[12] The result valid for all temperatures has been given by Gross, Pisarski and Yaffe [14].

Finite temperature gauge fields are obtained from the Euclidean fields by requiring them to be periodic in their time variables with period $\beta$ (inverse temperature). The Harrington-Shepard [35] finite temperature instanton corresponds to a multi-instanton configuration with the infinite set of instantons spaced in time with the interval $\beta$, but at the same position in space and with the same scale size. For the 't Hooft form of the multi-instanton configuration it is

$$A^a(t,\vec{x}) = -\bar{\eta}^a_{\mu\nu}\partial_\nu \ln\left[1+\sum_{n=-\infty}^{\infty}\frac{\rho^2}{(t-T-n\beta)^2+(\vec{x}-\vec{X})^2}\right]$$

$$= -\bar{\eta}^a_{\mu\nu}\partial_\nu \ln\left[1+\frac{\pi^2\rho^2}{\beta|\vec{x}-\vec{X}|^2}\;\frac{\sinh\frac{2\pi}{\beta}|\vec{x}-\vec{X}|}{\cosh\frac{2\pi}{\beta}|\vec{x}-\vec{X}|+\cos\frac{2\pi}{\beta}(t-T)}\right] \tag{4.17}$$

Although this configuration is made up on an infinite number of instantons, it still has $\nu(A) = 1$ because the time integration goes only from 0 to $\beta$. As the temperature goes to zero, this reduces to a single instanton in singular gauge.

The thermal fluctuations about the finite temperature instanton are obtained along with the quantum fluctuations by imposing periodic boundary conditions in the Euclidean fields and propagators in the calculation of Feynman graphs for the quantum fluctuations. On the other hand, the thermal and quantum fluctuation effects can be separated in these graphs by transforming back to a Minkowski space-time description where the free propagators can be split into temperature independent and dependent terms, as for example [36], this massive scalar propagator,

$$\frac{1}{k^2 - m^2} - \frac{2\pi i \ \delta\left(k^2 - m^2\right)}{e^{\beta E} - 1} \tag{4.18}$$

where $E = \sqrt{\vec{k}^2 + m^2}$. The effects of these fluctuations can be described by an effective action. The leading high-temperature contributions to this effective action are determined by the graphs, most divergent by power counting [37], that arise in the functional background field expansion of terms like $\frac{1}{2}\,\mathrm{tr}'\,\ln\,\mathscr{D}^{-1}_{\mu\nu}(A)$. This is because, in the temperature dependent terms, the high energy contributions to the graphs are cutoff by Boltzmann factors, $\exp\{-\beta E\}$, essentially replacing the ultraviolet cutoff in the temperature independent terms by $1/\beta$. The seagull graph is quadratically divergent; it contributes to the effective action a term proportional to

$$\int_0^\beta dt \int d^3x \ \frac{1}{\beta^2} \ \mathrm{tr}\ A_\mu^2(x) \tag{4.19}$$

While in the temperature independent piece, this quadratic divergence in the seagull term is cancelled by a Schwinger term; this temperature dependent mass term is not cancelled[13] because of the lack of manifest Lorentz invariance of the temperature dependent terms. This term is similar to the photon mass term in a plasma. The integration over the finite temperature instanton is proportional to $\rho^2$, and so this correction to the classical action of $8\pi^2/g^2$ is proportional to $\rho^2/\beta^2$. The thermal fluctuations therefore cutoff the effects of large scale sizes like exp $\{-\text{const.}\ \rho^2/\beta^2\}$. The constant, evaluated by Gross, Pisarski and Yaffe [14], is $2\pi^2/3$ times the number of colors.

Finite temperature quark fields are obtained from the Euclidean fields by making them antiperiodic in their time variables,

$$\psi(t+n,\vec{x}) = (-1)^n \psi(t,\vec{x}) \quad . \tag{4.20}$$

These boundary conditions must be imposed on the eigenfunctions of the Dirac operator in Eq. (3.3). Similar to the gluon fluctuations just considered, there is a thermal correction to the instanton density coming from the quark contribution to the action, $-\text{tr}'\ \ell n\ \not{D}(A)$. The leading term at high-temperature comes from the most divergent graph, the vacuum polarization graph with external thermal instanton fields. Again, in the temperature dependent terms the seagull and Schwinger terms do not cancel, the quadratic divergence (photon mass) being replaced by $1/\beta^2$. Integrating over the instanton fields, the quarks thus contribute to the constant in the exponential suppression of large scale sizes in the instanton density; the constant is $\pi^2/3$ times the number of flavors [14].

The zero-mode eigenfunction of the Dirac operator in a background instanton field must also be corrected to include the antiperiodic boundary conditions. $\psi_0$ is now an antiperiodic solution of

$$\left(\not\partial + \not A(x)\right) \psi_0(x) = 0 \quad , \tag{4.21}$$

where $A_\mu$ is a finite temperature instanton. It can be obtained from Grossman's [38] analysis of the zero-mode eigenfunctions in a background multi-instanton configuration, and has the form

$$\psi_0(x) = -\frac{1}{\sqrt{2\pi^2\rho}} \Pi^{\frac{1}{2}}(x) \not\partial \left(\frac{\phi(x)}{\Pi(x)}\right) \chi \tag{4.22}$$

where

$$\Pi(x) \equiv 1 + \frac{\rho^2}{(x-X)^2} \quad , \tag{4.23a}$$

and where

$$\phi(x) \equiv \sum_{n=-\infty}^{\infty} (-1)^n \frac{\rho^2}{(t-T+n\beta)^2 + (\vec{x}-\vec{X})^2} \quad . \tag{4.23b}$$

As $1/\beta \to 0$, this reduces for the singular gauge zero-mode wavefunction, Eq. (4.6).

## V. MODELS OF PHASES

In order to get an indication of the symmetry realizations in QCD we will couple to QCD various scalar field theories, analogous to scalar sectors of weak interaction models. We choose the parameters of the potentials for these scalar models so that different combinations of symmetries are spontaneously broken, and then study the effect of QCD corrections on their effective potentials, and thus on their symmetry realizations. We consider these theories at high temperature, and lower temperature to explore phases. Two scalar models will be discussed with the following spontaneously broken symmetries: (A) both spontaneous chiral SU(2) symmetry breaking and spontaneous isospin breaking; (B) both spontaneous chiral SU(2) symmetry breaking and spontaneous P and CP violation. These models of flavor symmetry breaking are described by the Lagrangian

$$\mathscr{L} = -\frac{1}{2g^2}\,\mathrm{tr}\,F^2_{\mu\nu}(A) + (\bar{u}\;\bar{d})\begin{pmatrix} i\not{D}(A) & 0 \\ 0 & i\not{D}(A)\end{pmatrix}\begin{pmatrix} u \\ d\end{pmatrix}$$

$$- f(\bar{u}\;\bar{d})\left[\Phi\,\frac{1+\gamma_5}{2} + \Phi^\dagger\,\frac{1-\gamma_5}{2}\right]\begin{pmatrix} u \\ d\end{pmatrix} - \frac{1}{4}\,\mathrm{tr}\left|\partial_\mu\Phi\right|^2 - V(\Phi) \tag{5.1}$$

A.

In the first model we consider $V(\Phi)$ to be invariant under $U_L(2) \times U_R(2)$. The potential is chosen to be a quartic polynomial of the two possible $U_L(2) \times U_R(2)$ invariants,

$$\tfrac{1}{2}\,\mathrm{tr}\,\Phi^\dagger\Phi = \left(\sigma^2 + \vec{\pi}^2\right) + \left(\eta^2 + \vec{\phi}^2\right) = \pi^2 + \phi^2\,, \tag{5.2}$$

and

$$\mathrm{tr}\,\Phi^\dagger\Phi\Phi^\dagger\Phi = \left(\mathrm{tr}\,\Phi^\dagger\Phi\right)^2 - 2\left[\left(\pi^2 - \phi^2\right)^2 + 4\left(\pi\cdot\phi\right)^2\right]\,, \tag{5.3}$$

where again $\pi_\mu = (\sigma,\vec{\pi})$ and $\phi_\mu = (\eta,-\vec{\phi})$. Under chiral U(1) transformations these four-vectors transform into linear combinations of one another, and under chiral SU(2) transformations the components of $(\sigma,\vec{\pi})$ are transformed into one another so that $\sigma^2+\vec{\pi}^2$ is invariant, and similarly for $(\eta,-\phi)$.

We choose V to be[14]

$$V(\Phi) = -\frac{\mu^2}{4}\,\mathrm{tr}\,\Phi^\dagger\Phi + \frac{\lambda}{16}\left(\mathrm{tr}\,\Phi^\dagger\Phi\right)^2 - \frac{h}{16}\,\mathrm{tr}\,\Phi^\dagger\Phi\Phi^\dagger\Phi \tag{5.4}$$

Let us first classically analyze the scalar sector at zero temperature. For $\lambda - h > 0$ this potential is bounded from below. The tachyonic mass term creates an instability of the symmetric phase, leading to spontaneous symmetry breaking. Because h is chosen positive, the potential can be minimized for $\langle\pi_\mu\rangle^2 = \langle\phi_\mu\rangle^2 \neq 0$ and for $\langle\pi_\mu\rangle\langle\phi_\mu\rangle = 0$. Of the infinite number of minima, we can choose the frame for which $\langle\sigma\rangle \neq 0$ and $\langle\phi^3\rangle = -\langle\sigma\rangle$ corresponding to both spontaneous chiral SU(2) symmetry breaking and spontaneous isospin breaking. Because the magnitudes of $\langle\sigma\rangle$ and $\langle\phi^3\rangle$ are equal, the u quark remains massless:

$$f\bar{\psi}_L\langle\Phi\rangle\psi_R + f\bar{\psi}_R\langle\Phi^\dagger\rangle\psi_L = f\bar{\psi}\left(\langle\sigma\rangle + \langle\phi^3\rangle\tau^3\right)\psi = (\bar{u}\ \bar{d})\begin{pmatrix}0 & 0\\ 0 & m_d\end{pmatrix}\begin{pmatrix}u\\ d\end{pmatrix} \tag{5.5}$$

where $m_d = 2f\langle\sigma\rangle$. Therefore there remains an unbroken chiral U(1) symmetry

$$\begin{pmatrix}u\\ d\end{pmatrix} \rightarrow \begin{pmatrix}e^{i\alpha\gamma_5}\ u\\ d\end{pmatrix} = e^{i\frac{1+\tau_3}{2}\alpha\gamma_5}\,\psi\ . \tag{5.6}$$

There are three pseudoscalar pion-like massless particles associated with the spontaneous chiral SU(2) symmetry breaking that gives the same mass to the u and d quarks, and two massless charged scalar particles associated with the spontaneous isospin breaking that splits the masses of the u and d quarks.

Now let us consider the complete model in order to compute quantum corrections to this potential. The Euclidean functional integral for this theory is

$$\int \mathcal{D}\Phi\ \mathcal{D}\Phi^\dagger\ e^{-W(\Phi)} \tag{5.7a}$$

where

$$e^{-W(\Phi)} = \exp\left\{-\int\left[\frac{1}{4}\,\mathrm{tr}\left|\partial_\mu\Phi\right|^2 + V(\Phi)\right]\right\}\int \mathcal{D}A\; e^{-S(A)}\; e^{i\theta\nu(A)}$$

$$\times\ \det\left[\not{D}(A) + f\left(\Phi\,\frac{1+\gamma_5}{2} + \Phi^\dagger\,\frac{1-\gamma_5}{2}\right)\right] \qquad (5.7b)$$

The semiclassical effective potential can be obtained from the semiclassical effective quantum action, $W(\Phi) - \mathrm{tr}\,\ell n\,\mathcal{D}^{-1}(\Phi)$, where $\mathcal{D}^{-1}(\Phi)$ is the inverse scalar propagator in a background scalar field. $W(\Phi)$ is calculated by approximating the QCD functional integral semiclassically, expanding about the approximate minima of $S(A)$. This gives

$$e^{-W(\Phi)} \simeq \frac{\det\left(-\partial_\mu^2\right)}{\sqrt{\det\ \mathcal{D}^{-1}_{\mu\nu}(0)}}\ \det\left(\not{\partial} + \Phi\,\frac{1+\gamma_5}{2} + \Phi^\dagger\,\frac{1-\gamma_5}{2}\right)$$

$$\times\ \left(1 + \frac{1}{Z(0)}\int d\begin{pmatrix}\text{collective}\\ \text{coordinates}\end{pmatrix}\frac{e^{-\frac{8\pi^2}{g^2}}\ \det - D_\mu^2(A)}{\sqrt{\det'\ \mathcal{D}^{-1}_{\mu\nu}(A)}}\right.$$

$$\times\ \left\{e^{i\theta}\det\left[\not{D}(A) + \Phi\,\frac{1+\gamma_5}{2} + \Phi^\dagger\,\frac{1-\gamma_5}{2}\right] + e^{-i\theta}\det\left[\not{D}(\bar{A}) + \Phi\,\frac{1+\gamma_5}{2}\right.\right.$$

$$\left.\left.\left. + \Phi^\dagger\,\frac{1-\gamma_5}{2}\right]\right\} + \ldots\right)\ \exp\left\{-\int\left[\frac{1}{4}\,\mathrm{tr}\left|\partial_\mu\Phi\right|^2 + V(\Phi)\right]\right\}, \qquad (5.8)$$

where $Z(0)$ is the first term on the right-hand side. These expressions are to be evaluated with finite temperature boundary conditions. The constant $\Phi$ dependence of the fermion determinants in background instanton fields is [from Eq. (3.23)],

$$f^2\left(e^{-i\theta}\det\Phi + e^{+i\theta}\det\Phi^\dagger\right)\left[1 + \frac{f^2}{2}\,\mathrm{tr}\,\Phi^\dagger\Phi\right.$$

$$\left.\times\ \left(\mathrm{tr}\,G(A)\,G(A) - \mathrm{tr}\,G(0)\,G(0)\right) + \ldots\right] \qquad (5.9)$$

In the dilute instanton gas limit, these single instanton terms exponentiate; the scalar potential $V(\Phi)$ is corrected from these instanton effects by the terms,

$$V(\Phi) + f^2\kappa^2\left(e^{-i\theta}\det\Phi + e^{+i\theta}\det\Phi^\dagger\right)\left(1 + \frac{\epsilon f^2}{2\kappa^2}\,\mathrm{tr}\,\Phi^\dagger\Phi\right) \qquad (5.10)$$

where

$$\kappa^2 \equiv \int\frac{d\rho}{\rho^3}\,\lambda(\rho)\quad , \qquad (5.11)$$

and where

$$\epsilon \equiv \int\frac{d\rho}{\rho^3}\,\lambda(\rho)\,\left(\mathrm{tr}\,G(A)\,G(A) - \mathrm{tr}\,G(0)\,G(0)\right)\quad . \qquad (5.12)$$

The effects of finite temperature boundary conditions on the instanton contribution, as were previously discussed, essentially just cut off the instanton density with the factor exp $\{-\mathrm{const.}\ \rho^2/\beta^2\}$. The temperature dependence of

$$Z(0) = \frac{\det\left(-\partial_\mu^2\right)\det\left[\not{\partial} + f\left(\Phi\,\frac{1+\gamma_5}{2} + \Phi^\dagger\,\frac{1-\gamma_5}{2}\right)\right]}{\sqrt{\det'\ \mathscr{D}_{\mu\nu}^{-1}(0)}\quad\det\ \mathscr{D}^{-1}(\Phi)} \qquad (5.13)$$

gives exp $\{-\beta$ (free energy of quark, gluon and scalar gases)$\}$. The leading high temperature terms in the free energy go like [36,37]

$$-\ \mathrm{const.}\ T^4 + \mathrm{const.}\ T^2\ \mathrm{tr}\,\Phi^\dagger\Phi \qquad (5.14)$$

The temperature independent terms from the quark and scalar determinants will renormalize tr $\Phi^\dagger\Phi$ and tr $\Phi^\dagger\Phi\Phi^\dagger\Phi$, as well as give terms of all higher powers of these invariants. Because of the explicit mass scale, though, all powers higher than quartic will be neglected.

Therefore, up to quartic terms, the $\Phi$ dependence of the potential is of the form

$$V(\Phi) = \left(\text{const. } T^2 - \mu^2\right) \frac{1}{4} \text{tr}\, \Phi^\dagger\Phi + \frac{\lambda}{16}\left(\text{tr}\, \Phi^\dagger\Phi\right)^2 - \frac{h}{16} \text{tr}\, \Phi^\dagger\Phi\Phi^\dagger\Phi$$
$$- \frac{\kappa^2}{2} f^2 \left(e^{i\theta} \det \Phi + e^{-i\theta} \det \Phi^\dagger\right)\left(1 + \frac{\epsilon f^2}{2\kappa^2} \text{tr}\, \Phi^\dagger\Phi\right) . \qquad (5.15)$$

To analyze the symmetry realizations from this potential, let us first briefly consider $\theta = 0$; we will return to the $\theta$ dependence. Then the most important modification of the original potential comes from the mass terms, which are now

$$\frac{1}{2}\left(\text{const. } T^2 - \mu^2 - \kappa^2(T)\right)\pi^2 + \frac{1}{2}\left(\text{const. } T^2 - \mu^2 + \kappa^2(T)\right)\phi^2 \qquad (5.16)$$

From these terms we see that the instanton contribution increases the tachyonic mass term for the $\sigma$ and $\vec{\pi}$ fields, increasing the tendency for spontaneous chiral symmetry breaking, while on the other hand it is trying with the thermal fluctuations to stabilize the mass term for the $\eta$ and $\vec{\phi}$ fields.

To study the symmetry realizations for $\theta \neq 0$, we can in general choose a basis for $\Phi$ so it is diagonal,

$$\Phi = \begin{pmatrix} m_u e^{-i\phi_u} & 0 \\ 0 & m_d e^{-i\phi_d} \end{pmatrix} \qquad (5.17)$$

Then the potential becomes

$$V = \frac{1}{2}\left(\text{const. } T^2 - \mu^2\right)\left(m_u^2 + m_d^2\right) + \frac{\lambda}{4}\left(m_u^2 + m_d^2\right)^2 - \frac{h}{4}\left(m_u^4 + m_d^4\right)$$

$$- \frac{\kappa^2}{2} f^2 m_u m_d \cos\left(\theta - \phi_u - \phi_d\right)\left[1 - \frac{\varepsilon}{\kappa^2} f^2 \left(m_u^2 + m_d^2\right)\right] \qquad (5.18)$$

For a range of parameters and temperatures, this potential has a global minimum for $m_u \neq m_d$, and $\phi_u + \phi_d = \theta$. The latter equation is the condition for no CP violation; it is a consequence of the chiral U(1) symmetry of the scalar sector as explained by Peccei and Quinn [39]. The elimination of strong CP violation is accompanied by an axion [40], but is corresponds to a state most naturally identified with $\pi^0$ rather than $\eta$.[15] Spontaneous chiral symmetry breaking would create massless $\pi^0$ and $\eta$ states; spontaneous isospin breaking then mixes these states. The massless state $\pi^0 - \eta$ could be identified as the neutral partner of $\pi^\pm$, while the massive state $\pi^0 + \eta$ could be identified with the "isosinglet" state. The instanton effect gives mass to the ordinary $\eta$ state, and so the neutral "isotriplet" particle acquires a small mass; the "isosinglet" pseudoscalar mass is slightly shifted. Also, the u quark which had been massless before including instanton effects now acquires a small mass.

$$m_u \simeq \frac{\kappa f}{\sqrt{h}\, m_d} \qquad (5.19)$$

Of course there is still a charged pair of massless pseudoscalars and scalars, as well as two neutral scalars, one the partner of the massless pair, and the other the $\sigma$.

As the temperature is lowered further $\kappa^2$ rapidly increases; there could then occur a phase transition to a phase with restored isospin symmetry.[16] The massless charged scalars will now acquire the same mass as their neutral partner; the quark masses will also become equal. Furthermore, the massive $\eta$ will become light — it will correspond to the axion [34], and there will again be an isotriplet of massless pseudoscalar $\vec{\pi}$'s

As the temperature is lowered still further, corrections to the dilute gas approximation become important; these will be discussed in Sections VI and VII. Already, from this very simple model, we see that QCD effects tend to restore the isospin symmetry and spontaneously break the chiral symmetry of this scalar model. This is an indication that the dynamics of QCD will choose this combination of symmetry realizations as the temperature goes to zero.

B.

In this model of spontaneous CP violation we choose the potential in Eq. (5.1) to be invariant under $SU_L(2) \times SU_R(2)$ [as well as $U_B(1)$], and of the form,

$$V(\pi,\phi) = -\frac{\mu_1^2}{2}\pi^2 - \frac{\mu_2^2}{2}\phi^2 + \frac{\lambda}{4}\left(\pi^2+\phi^2\right)^2 - \frac{h_1}{4}\left(\pi^2-\phi^2\right)^2 - h_2\left(\pi\cdot\phi\right)^2 - \frac{h}{2}\pi^2\phi^2 \quad . \tag{5.20}$$

This potential has a minimum for spontaneous symmetry breaking such that $\langle\pi_\mu\rangle$ and $\langle\phi_\mu\rangle$ are both nonzero (from the h term), are parallel (from the $h_2$ term), and are unequal in magnitude (from the $h_1$ term). We choose the frame so that $\langle\phi\rangle \neq 0$ and $\langle\eta\rangle \neq 0$. This potential therefore implies the spontaneous breaking of chiral SU(2) symmetry and P and CP; because there is no chiral U(1) symmetry of this potential, this CP violation cannot be rotated away. The spontaneous CP violation shows up as a phase in the quark mass term,

$$\bar{\psi}_L\langle\Phi\rangle\psi_R + \bar{\psi}_R\langle\Phi^\dagger\rangle\psi_L = \bar{\psi}_L(\langle\sigma\rangle + i\langle\eta\rangle)\psi_R + \bar{\psi}_R(\langle\sigma\rangle - i\langle\eta\rangle)\psi_L$$

$$= \sqrt{\langle\sigma\rangle^2 + \langle\eta\rangle^2}\left(e^{i\delta}\,\bar{\psi}_L\psi_R + e^{-i\delta}\,\bar{\psi}_R\psi_L\right) , \qquad (5.21)$$

where $\delta = \tan^{-1}(\langle\eta\rangle/\langle\sigma\rangle)$. Transforming this phase out of the quark mass term by a chiral U(1) rotation makes it show up in the scalar self-couplings and in $\theta\,\mathrm{tr}\,F_{\mu\nu}\tilde{F}^{\mu\nu}$, thus this weak interaction CP violation induces strong interaction CP violation.

At very high temperature this model is in its symmetric phase, and as the temperature is lowered it undergoes a phase transition to a phase with spontaneous chiral SU(2) and P and CP symmetry breaking. As the temperature is lowered further instanton effects sharply turn on. The effective mass term becomes

$$\frac{1}{2}\left(\text{const.}\ f^2T^2 - \mu_1^2 - \kappa^2\right)\pi^2 + \frac{1}{2}\left(\text{const.}\ f^2T^2 - \mu_2^2 + \kappa^2\right)\phi^2 \qquad (5.22)$$

Once again the instanton effects reinforce the tendency of the scalar theory to spontaneously break chiral SU(2) symmetry, but they also tend to restore the CP invariance (they tend to reverse the tachyonic sign of $\mu_2^2$).

This effect should be contrasted with the Pecci-Quinn [39] effect where a chiral U(1) symmetry of the weak interactions prevents weak interaction CP violation from inducing strong interaction CP violation.

Here there is no weak interaction chiral U(1) symmetry; the dynamics of the strong interactions tends to restore weak interaction CP violation. If in the real world weak interaction CP violation is spontaneous, the tendency of the strong interactions to restore this symmetry may be a clue to why CP violation is so small. As a consequence we would expect that at high energy, CP violating effects get larger.

We have seen that the effect of QCD instanton corrections on weak interaction symmetry realizations has been to enhance the tendency for spontaneous chiral SU(2) symmetry breaking, and suppress the tendency for spontaneous isospin and CP breaking. Further, these same instanton effects give mass to the weak interaction chiral U(1) massless particle (axion) [34]. Therefore, they have the tendency to create the same combination of symmetry realizations in the weak interactions that are presumed to occur in the strong interactions; that is, QCD wants to lock the weak interaction symmetry realizations to its own. When we later include the instanton chirality correlations, these tendencies will be further enhanced.

The reason for the restoration of CP symmetry has the same origin as the reason for the restoration of isospin — the opposite sign of the instanton induced mass terms for $\sigma^2 + \vec{\pi}^2$ and $\eta^2 + \vec{\phi}^2$. Chiral SU(2) symmetry links $\eta$ and $\vec{\phi}$ just as it links $\sigma$ and $\vec{\pi}$, therefore the realizations of isospin and CP are linked simply by chiral SU(2) symmetry. It is the instanton induced (spontaneous) chiral U(1) breaking that produces this interrelation of symmetry realizations in which the tendency for spontaneous chiral SU(2) symmetry breaking is linked to the tendency for isospin and CP symmetries to be manifest. The reason the instanton effect tends to produce this combination of symmetry realizations is completely a consequence of the chirality selection rules.

The relative minus sign between $\sigma^2 + \vec{\pi}^2$ and $\eta^2 + \vec{\phi}^2$ is a reflection of instanton induced attractive interactions between quarks in $\bar{\psi}\psi$ and $i\bar{\psi}\gamma_5\vec{\tau}\psi$ channels versus repulsive interactions in the $i\overline{\psi\tau}_5\psi$ and $\bar{\psi}\vec{\tau}\psi$ channels. The positive mass contribution to the $\eta$, for example, is due to the instanton induced repulsive interaction between quarks in the flavor singlet pseudoscalar channel. This repulsion must be contrasted, though, with the confinement dielectric effects, the onset of which are associated with the large dipole moments of the same instantons. There will certainly be confinement independent of flavor channel; the difference between the $\vec{\pi}$ and $\eta$, though, is that $\vec{\pi}$ can be a vacuum (phase oscillation) state before (at a higher temperature) becoming a confined state as well, while the $\eta$ is not a vacuum state.

Instantons also induce interactions between quarks and antiquarks in color octet channels, as seen from Eq. (4.15). In general we should consider models with color octet scalar fields, $\Theta$, which have $U_L(2) \times U_R(2)$ quantum numbers. Expectation values for these fields would simultaneously spontaneously break both the color gauge symmetry and the flavor symmetry. However, the instanton induced forces between quarks in these channels are much weaker than in the color singlet channels, so spontaneous symmetry breaking of this kind will not be considered.

On the other hand, spontaneous color gauge symmetry breaking could in principle occur, and without directly also breaking the flavor symmetry. This would require colored scalar glueball states. (It is difficult to see how this could happen, though, since gluon exchange gives repulsive forces between gluons in color octet scalar channels. These colored glueball channels would couple to colored scalars, $G^i$, but not directly to quarks. Induced nonlocal couplings with quarks would exist, though, and these would induce mixing terms in the effective potential between these colored scalars and the $\Phi$'s (see Fig. 7). This kind of mixing implies that in principle the realization of the color gauge symmetry and the flavor symmetry are interdependent.

## VI. EFFECTIVE FIELD THEORY FOR CHIRALITY CORRELATIONS

In this section we will include the effects of chirality correlations between instantons and anti-instantons. It will be shown that the contribution of these correlations can be approximated by the effective quantum field theory [2] of the 't Hooft Lagrangian [1,15].

At moderately high temperature the semiclassical approximation is assumed to be valid; the functional integral will therefore be dominated by configurations close to the minima of the classical gauge field action [7, 41]. While the exact minima of the Euclidean gauge field action are multi- (anti-) instanton configurations with all integer values of $\nu(A)$, the volume in field configuration space for these minima is very small [7]. On the other hand, there are many more configurations close to the minima than there are minima. The configurations close to the minima with large volume in field configuration space correspond to a plasma of well separated instantons and anti-instantons with $\nu(A) = \pm 1$. The instantons do not interact with one another classically in the sense that the action for an exact multi-instanton configuration equals the sum of actions of individual instantons [42]. Instantons and anti-instantons do interact classically, though, like 4-D color magnetic dipoles [7]. (Actually, the action for a configuration with a sum of instantons does not equal the sum of actions for separate instantons, but this interaction is weak relative to that between instantons and anti-instantons [43].) The instantons and anti-instantons also have additional quantum mechanical interactions, the most important of which arise because of the chirality selection rules for massless quarks; they interact by the exchange of chirality in all possible ways consistent with these selection rules [2].

The semiclassical computation of the QCD free energy is equivalent to computing the corrections to the perturbation theory free energy due to the external field from a plasma of thermal instantons and anti-instantons. The effect of finite temperature can be implemented by imposing (anti) periodic boundary conditions on the Euclidean theory; we will first consider the contribution to the Euclidean functional integral without the periodic boundary conditions, and later mention the effect of these boundary conditions. In the following discussion we will also neglect the dipolar interactions compared to the chirality correlations, and will later show how to include these corrections.

The QCD Euclidean functional integral is approximated by

$$Z \simeq \sum_{\substack{\text{configurations}\\ \text{close to minima}}} \int d\begin{pmatrix}\text{collective}\\ \text{coordinates}\end{pmatrix} \frac{e^{-S(A)}\, e^{i\theta\nu(A)} \det{-D_\mu^2}(A)}{\sqrt{\det' \ \mathscr{D}_{\mu\nu}^{-1}(A)}} \ \det \not{D}(A) \tag{6.1}$$

For A the field of a gas of instantons and anti-instantons, the evaluation of these determinants is analogous to the evaluation of the determinant of the Hamiltonian of a molecule. The eigenvalue problem is approximated by perturbatively expanding the molecular wavefunctions about a basis of atomic wavefunctions. In the lowest approximation, corresponding to large instanton separations, the determinants factorize into a product of determinants for separate instantons and anti-instantons. The collective coordinates are then those for separate instantons and anti-instanton This contribution to the functional integral vanishes, however; the operator $\not{D}(A_i)$, for $A_i$ an instanton or anti-instanton, has a zero eigenvalue due to the violation of the chirality selection rules. (When we previously considered QCD coupled to external fields, the external fields absorbed the required chirality changes. The approximation for QCD previously considered in our discussion of QCD coupled to weak interaction models was just this lowest order approximation.)

The correlations between instantons and anti-instantons necessary for consistency with the charality selection rules arise from the first order correction to the zero eigenvalues. Degenerate perturbation theory, in the basis of the zero-mode eigenfunctions of the Dirac operator in separate instantons and anti-instantons, is used to calculate the corrected eigenvalues:

$$\det\left[\int_x \psi_0^\dagger\left(X_i - x\right)\,\not{D}(A)\,\psi_0\left(x - X_j\right) - \varepsilon\delta_{ij}\right] = 0 \tag{6.2}$$

is an Nth order equation for $\varepsilon$, the N roots being the corrected eigenvalues. We need only the product of these eigenvalues, though, and this product is just

$$\det \left[ \int_x \psi_0^\dagger\left(X_i - x\right) \not{D}(A) \; \psi_0\left(x - X_j\right) \right] \equiv \det H\left(X_i, X_j\right) \tag{6.3}$$

The lowest nonvanishing contribution to the QCD Euclidean functional integral from this gas of instantons and anti-instantons, then takes the form of a grand partition function:

$$Z \simeq \sum_{N_\pm = 0}^{\infty} \frac{1}{N_+! N_-!} \int \prod_{i=0}^{N_+ + N_-} \left(d^4 X_i \, d\rho_i \, d\Omega_i \, \lambda(\rho_i)\right) e^{i\theta\left(N_+ - N_-\right)} \det H \ , \tag{6.4}$$

The matrix elements $H(X_i, X_j)$ are evaluated with the explicit zero-mode eigenfunctions of the Dirac operator in a singular gauge instanton or anti-instanton. These matrix elements are nonzero only if $\psi_0(x-X_i)$ and $\psi_0(x-X_j)$ have opposite chirality [2]; that is, if one is from a background instanton and the other from an anti-instanton. This is because

$$\int_x \psi_0^\dagger\left(X_i - x\right) \not{D}(A) \; \gamma_5^2 \psi_0\left(x - X_j\right) = - \int_x \left[\gamma_5 \psi_0\left(x - X_i\right)\right]^\dagger \not{D}(A) \left[\gamma_5 \psi_0\left(x - X_j\right)\right]. \tag{6.5}$$

The $H(X_i, X_j)$ describe the space-time dependence of the exchange of chirality between instantons and anti-instantons. For $(X_i - X_j)^2 \gg \rho_i \rho_j$, this space-time correlation approaches that of a free massless fermion propagator. This can be seen by approximately evaluating the matrix elements [2].

$$\int_x \psi_o^\dagger(X_i-x)\ \not{D}(A)\ \psi_0(x-X_j)$$

$$= \int_x \psi_0^\dagger(X_i-x)\left[\not{\partial} - \not{A}(x-X_1) - \ldots - \not{A}\left(x-X_{N_+ +N_-}\right)\right]\psi_0(x-X_j)$$

$$= \int_x \psi_0^\dagger(X_i-x)\left\{\left[-\not{\partial} - \not{A}(x-X_i)\right] + \left[\not{\partial} - \not{A}(x-X_j)\right]\right.$$

$$\left. + \not{\partial} - \sum_{k\neq i,j} \not{A}(x-X_k)\right\}\psi_0(x-X_j)$$

$$= \int_x \psi_0^\dagger(X_i-x)\left[\not{\partial} - \sum_{k\neq i,j} \not{A}(x-X_k)\right]\psi_0(x-X_j) \tag{6.6}$$

The first term in the last expression can be rewritten

$$\begin{aligned}&\int_{x,x'} \psi_o^\dagger(X_i-x)\ \delta^4(x-x')\ \not{\partial}\ \psi_0(x'-X_j)\\ &= \int_{x,x'} \psi_0^\dagger(X_i-x)\ G^{-1}(x-x')\ \psi_0(x'-X_j)\end{aligned} \tag{6.7}$$

and with Eq. (4.7) gives the stated result. The second term in the last expression of Eq. (6.6) is a small contribution of the overlap of three distantly separated wavefunctions; we will ignore it in the following discussion, but will discuss its effects in the next section.

We will now show that the space-time correlations between the instantons and anti-instantons can be described by a fermionic quantum field theory. The determinant of $H(X_i,X_j)$ is expanded in a cycle expansion

$$\det H(X_i,X_j) = \sum_{\mu_i=1}^{N} \varepsilon_{\mu_1 \ldots \mu_N} H\left(X_1,X_{\mu_1}\right) \cdots H\left(X_N,X_{\mu_N}\right) , \tag{6.8}$$

where $N' = N_+ + N_-$. This is the sum of all possible closed loops, with appropriate exchange minus signs. There is actually a product of determinants for each quark flavor since

$$\det\left(\int \psi^\dagger \not{D} \psi\right) = \det \begin{bmatrix} \int u^\dagger \not{D} u & 0 \\ 0 & \int d^\dagger \not{D} d \end{bmatrix} = \det\left(\int u^\dagger \not{D} u\right) \det\left(\int d^\dagger \not{D} d\right). \quad (6.9)$$

Some examples of terms for $N_+ = 3$, $N_- = 3$, and 2 flavors follow: the simplest term occurs when, for each flavor, $\mu_1 = 2$, $\mu_2 = 3$, ... $\mu_N = 1$, as illustrated graphically[17] in Fig. 8(a). Another kind of product of the two largest cycles comes from a different ordering of the vertices for the two flavors, as in Fig. 8(b). A term with the same structure as the first we considered in Fig. 8(a) comes from a product of smaller cycles, Fig. 8(c). Disconnected graphs are also generated, Fig. 8(d), as well as products of cycles of different sizes, Fig. 8(e). Other permutations give all possible combinations. Each vertex $X_i$ is multiplied by a factor $\lambda(\rho_i)e^{\pm i\theta}$, and integrations are performed over $\rho_i$, $\Omega_i$ and $X_i$.

This grand partition function therefore corresponds to an infinite number of graphs;[18] these graphs are exactly all the graphs of an effective quantum field theory. Therefore expression (6.4) for Z is approximated by

$$\int \mathscr{D}u \mathscr{D}\bar{u} \mathscr{D}d \mathscr{D}\bar{d} \quad \exp -\int \left[ (\bar{u}\ \bar{d}) \not{\partial} \begin{pmatrix} u \\ d \end{pmatrix} + \mathscr{L}_{eff} \right] \quad (6.10)$$

where

$$\mathscr{L}_{eff}(X) = \int \frac{d\rho}{\rho^5}\lambda(\rho) \int d\Omega \left\{ e^{i\theta} \; \mathscr{L}_+(\Omega,X,\rho) + e^{-i\theta} \; \mathscr{L}_-(\Omega,X,\rho) \right\} \qquad (6.11a)$$

where

$$\mathscr{L}_\pm = \int_{\substack{x,x' \\ y,y'}} \left[ u(x')G^{-1}(x'-x) \; u_0(x-X)\bar{u}_0(X-y) \quad G^{-1}(y-y')u(y') \right]$$

$$\times \int_{\substack{z,z' \\ w,w'}} \left[ \bar{d}(z')G^{-1}(z'-z) \; d_0(z-X)d_0^\dagger(X-w) \quad G^{-1}(w-w')d(w') \right] \qquad (6.11b)$$

The only difference between the instanton contribution, $\mathscr{L}_+$, and anti-instanton contribution, $\mathscr{L}_-$, is the chirality of the constant spinors $\chi$ in the zero-mode wavefunctions, Eq. (4.6). Performing the integration over all global color gauge transformations gives contractions of the color indices of the $\chi$ spinors in different ways; the color contractions of these spinors give, from Eq. (4.9) chirality projection operators, consequently implying contractions of Dirac indices. The result is

$$\mathscr{L}_{eff} = \int \rho d\rho \lambda(\rho) \Bigg[ e^{i\theta} \Bigg( \Bigg\{ \left[ (\bar{u}\ \bar{d}) \, I \, \frac{1-\gamma_5}{2} \begin{pmatrix} u \\ d \end{pmatrix} \right]^2 - \left[ (\bar{u}\ \bar{d}) \, I\vec{\tau} \, \frac{1-\gamma_5}{2} \begin{pmatrix} u \\ d \end{pmatrix} \right]^2 \Bigg\}$$

$$- \frac{3}{16} \Bigg\{ \left[ (\bar{u}\ \bar{d}) \, I\lambda^a \, \frac{1-\gamma_5}{2} \begin{pmatrix} u \\ d \end{pmatrix} \right]^2 - \left[ (\bar{u}\ \bar{d}) \, I\lambda^a\vec{\tau} \, \frac{1-\gamma_5}{2} \begin{pmatrix} u \\ d \end{pmatrix} \right]^2 \Bigg\} \Bigg)$$

$$+ e^{-i\theta} \left( \frac{1-\gamma_5}{2} \rightarrow \frac{1+\gamma_5}{2} \right) \Bigg] , \qquad (6.12)$$

where $\vec{\tau}$ are the ordinary $2 \times 2$ flavor isospin matrices, $\lambda^a$ are the color matrices, and where the implied space-time structure in, for example, the color matrix current

$$(\bar{u}\ \bar{d})\ I\ \lambda^a\ \frac{1+\gamma_5}{2}\begin{pmatrix} u \\ d \end{pmatrix}$$

is

$$\int_{x,x'} (\bar{u}\ \bar{d})(x)\ I(x-X,x'-X;\ )\ \lambda^a \begin{pmatrix} u \\ d \end{pmatrix}(x') \quad , \tag{6.13}$$

where

$$I(x-X,x'-X;\rho) \equiv \int_P e^{iP\cdot(x-X)}\ h(\rho P) \int_{P'} e^{iP'\cdot(X-x')}\ h(\rho P') \quad , \tag{6.14a}$$

and where $h(\rho P)$ is the Fourier transform of the space-time dependence of the singular gauge zero mode Dirac eigenfunction, Eq. (4.7), with a factor of $1/\not{P}$ taken out [17],

$$h(\rho P) = \sqrt{\frac{2}{\pi^2}} \int_0^\infty ds\ \frac{s^2}{\left(1+s^2\right)^{3/2}}\ J_2(\rho P s) \ . \tag{6.14b}$$

The complicated looking expression for $\mathscr{L}_{eff}$ is simply the analytic realization of $e^{i\theta}$ times a $\Delta Q_5 = -4$ operator that couples quarks (without regard for different color contractions), like

$$\bar{u}_R u_L \bar{d}_R d_L - \bar{u}_R d_L \bar{d}_R u_L \quad , \tag{6.15}$$

and $e^{-i\theta}$ times a $\Delta Q_5 = +4$ term which couples quarks like

$$\bar{u}_L u_R \bar{d}_L d_R - \bar{u}_L d_R \bar{d}_L u_R \quad . \tag{6.16}$$

Note that if we perform a chiral U(1) rotation of the quark fields by $\alpha$, $\psi \to e^{-i\alpha\gamma_5}\psi$, then $\theta \to \theta - 4\alpha$, consistent with the transformation law implied by the chiral U(1) Ward identity. This expression has a $\theta$

periodicity of $2\pi$ as expected from the fact that the contributing configurations had locally $\nu(A) = \pm 1$.

While the current structure in Eq. (6.12) for $\mathscr{L}_{eff}$ simply illustrates the chirality properties, for purposes of studying possible spontaneous symmetry breaking it is best to reexpress the current structure as

$$\mathscr{L}_{eff} = \int \rho d\rho \; \lambda(\rho) \left\{ \left[ e^{i\theta} \det \Phi(\psi,\bar{\psi}) + e^{-i\theta} \det \Phi^{\dagger}(\psi,\bar{\psi}) \right] - \frac{3}{16} \left[ e^{i\theta} \det \Theta^{a}(\psi,\bar{\psi}) + e^{-i\theta} \det \Theta^{a\dagger}(\psi,\bar{\psi}) \right] \right\} \quad (6.17)$$

where the $2 \times 2$ flavor matrices $\Phi$ and $\Theta$ are defined

$$\Phi(\psi,\bar{\psi}) \equiv \begin{bmatrix} \bar{u} I \frac{1-\gamma_5}{2} u & \bar{u} I \frac{1-\gamma_5}{2} d \\ \bar{d} I \frac{1-\gamma_5}{2} u & \bar{d} I \frac{1-\gamma_5}{2} d \end{bmatrix} \quad (6.18)$$

and

$$\Theta^{a}(\psi,\bar{\psi}) \equiv \begin{bmatrix} \bar{u} I \lambda^{a} \frac{1-\gamma_5}{2} u & \bar{u} I \lambda^{a} \frac{1-\gamma_5}{2} d \\ \bar{d} I \lambda^{a} \frac{1-\gamma_5}{2} u & \bar{d} I \lambda^{a} \frac{1-\gamma_5}{2} d \end{bmatrix} . \quad (6.19)$$

These composite fields can be decomposed exactly as in Eqs. (2.12) and (3.4) for the scalar fields, where now

$$\Phi(\psi,\bar{\psi}) = \left[ \left( \bar{\psi} I \psi \right) + i \left( i \bar{\psi} I \gamma_5 \vec{\tau} \psi \right) \cdot \vec{\tau} \right] + i \left[ \left( i \bar{\psi} I \gamma_5 \psi \right) - i \left( \bar{\psi} I \vec{\tau} \psi \right) \cdot \vec{\tau} \right] \quad (6.20)$$

and where

$$\Theta^a(\psi,\bar{\psi}) = \left[\left(\bar{\psi} I \lambda^a \psi\right) + i\left(i\bar{\psi} I \gamma_5 \vec{\tau} \lambda^a \psi \cdot \vec{\tau}\right)\right]$$
$$+ i\left[\left(i\bar{\psi} I \gamma_5 \lambda^a \psi\right) - i\left(\bar{\psi} I \vec{\tau} \lambda \ \psi \cdot \vec{\tau}\right)\right] \qquad (6.21)$$

This effective field theory of quarks represents the leading semiclassical approximation to QCD. The nonlocal quark dynamics results from chirality correlations of gluon topological fluctuations, and is not the result of semiclassically integrating out the gluon degrees of freedom keeping the quark fields fixed. The later effective nonlocal field theory of quark degrees of freedom would be an appropriate approximation for heavy quark dynamics [46].

Finite temperature boundary conditions on the original QCD functional integral have the effect on this effective field theory of including the modifications discussed previously for a thermal instanton contribution to finite temperature QCD: the coupling $\lambda(\rho)$ is exponentially suppressed for large $\rho$, and the form factors in I (from the zero-mode wavefunctions) must be corrected to be consistent with the antiperiodic boundary conditions. Also, the fermonic functional integral here must be over antiperiodic fields.

We must still approximate the effective field theory in order to obtain the instanton contribution to the QCD free energy in the temperature range for which $\bar{g}^2(\beta\mu)/8\pi^2 << 1$. From this free energy we can explore for phase structure in QCD in this temperature range; if the free energy becomes complex as the temperature is lowered to a critical value, this signals an instability, and thus a phase transition. In order to explore for phase structure in QCD, we can consider the phase structure of the effective field theory. The characteristic feature of the onset of a second-order phase transition is a buildup of long-range correlations in the associated order parameter. We can therefore obtain the behavior of the free energy near a phase transition by finding an approximation to the effective field theory that emphasizes long-range correlations of the relevant order parameter.

Now in the effective field theory the nonrenormalizable dimensions of the operator in $\mathscr{L}_{eff}$ implies the dominant importance of short distances, although the very short distance behavior is tempered here by the form factors that reflect the asymptotic freedom of QCD. Correspondingly, the dimensions of this operator suggests the unimportance of long distances. How, then could such an interaction lead to long-distance correlations: This can happen if composite order parameter fields can create massless bound states; these could form for strong enough attractive forces between quarks at moderately short distances [20]. The long-distance behavior of these composite states would then be that of a weakly coupled (IR free) effective renormalizable field theory [47,48].

We therefore first attempt to convert the effective field theory to a representation that emphasizes the interactions of composite fields closely related to the order parameters. Now at first sight it seems natural to introduce auxiliary fields $\Phi$ and $\Theta$ of the same form as the external fields, Eqs. (2.12) and (3.4). These auxiliary fields can be introduced so that their field equations equate them with the composite fields [49,50],

$$\Phi = \Phi\left(\psi,\bar{\psi}\right) \tag{6.22a}$$

$$\Theta = \Theta\left(\psi,\bar{\psi}\right) \tag{6.22b}$$

The functional integral would then become

$$Z = \int \mathscr{D}\Phi\, \mathscr{D}\Phi^\dagger\, \mathscr{D}\Theta\, \mathscr{D}\Theta^\dagger \; e^{-W(\Phi,\Theta)} \tag{6.23a}$$

where

$$\begin{aligned} e^{-W(\Phi,\Theta)} = & \int \mathscr{D}\psi\, \mathscr{D}\bar{\psi} \; \exp\left\{-\int \left[\bar{\psi}\partial\!\!\!/\psi + \mathscr{L}_{eff}(\psi,\bar{\psi})\right]\right\} \\ & \times \exp\left\{-\int \lambda \; \mathrm{tr}\left|\left(\sigma + i\vec{\pi}\cdot\vec{\tau}\right) + i\left(\eta - i\vec{\phi}\cdot\vec{\tau}\right) - \Phi\left(\psi,\bar{\psi}\right)\right|^2\right\} \\ & \times \exp\left\{-\frac{3}{16}\int \lambda \; \mathrm{tr}\left|\left(\xi^a + i\vec{\beta}^a\cdot\vec{\tau}\right) + i\left(\zeta^a - i\vec{\rho}^a\cdot\vec{\tau}\right) - \Theta\left(\psi,\bar{\psi}\right)\right|^2\right\} \end{aligned} \tag{6.23b}$$

Introducing the auxiliary fields in this way leads to certain simplifications. For example, for fields such as $\sigma$,

$$\begin{aligned} & \exp\left\{\int \frac{\lambda}{2}\left(\bar{\psi}I\psi\right)^2\right\} \int \mathscr{D}\sigma \; \exp\left\{-\int \frac{\lambda}{2}\left(\sigma - \bar{\psi}I\psi\right)^2\right\} \\ & = \int \mathscr{D}\sigma \; \exp\left\{-\int \lambda \left(\frac{\sigma^2}{2} - \bar{\psi}I\psi\sigma\right)\right\} \end{aligned} \tag{6.24}$$

where the four-quark field terms have cancelled. On the other hand, for composite fields such as $\eta$, for example, we have

$$\begin{aligned} & \exp\left\{-\int \frac{\lambda}{2}\left(i\bar{\psi}I\gamma_5\psi\right)^2\right\} \int \mathscr{D}\eta \; \exp\left\{-\int \frac{\lambda}{2}\left(\eta - i\bar{\psi}I\gamma_5\psi\right)^2\right\} \\ & = \exp\left\{-\int \lambda \left(i\bar{\psi}I\gamma_5\psi\right)^2\right\} \int \mathscr{D}\eta \; \exp\left\{-\int \lambda \left(\frac{\eta^2}{2} - i\bar{\psi}I\gamma_5\psi\eta\right)\right\} \end{aligned} \tag{6.25}$$

with no cancellation of the four-quark field terms. The mathematical reason why some terms cancel and others do not is relative minus signs. Physically, these signs represent attractive versus repulsive interactions between quarks in the different channels, as we have seen to some extent from our discussion in Section V. It is therefore best not to introduce auxiliary fields for those composite fields associated with channels which have repulsive forces between quarks. Also, even for the color octet channels with attractive forces, since these forces are much weaker than for the color singlet channels, it is not appropriate to introduce auxilliary fields. This is because there could be no phase transition driven by massless bound states in these channels until a much lower temperature (where this effective field theory is certainly not a valid approximation) than for the color singlet channels. We will therefore deal with the repulsive $\eta$ and $\phi$ channels and the color octet channels one way, and the attractive $\sigma$ and $\vec{\pi}$ channels another.

We introduce $\sigma$ and $\vec{\pi}$ auxiliary fields, and sources for the quark fields. With the help of these quark sources the functional integral for the effective field theory can be reexpressed.

$$Z = z\left(\frac{\delta}{\delta\eta}, \frac{\delta}{\delta\bar{\eta}}\right) e^{-W(\eta,\bar{\eta})} \tag{6.26}$$

where

$$e^{-W(\eta,\bar{\eta})} = \int \mathcal{D}\sigma\, \mathcal{D}\vec{\pi} \exp \quad - \int \frac{\lambda}{2} \left(\sigma^2 + \vec{\pi}^2\right) + \mathrm{tr}\,\ln \mathbb{G}^{-1}\left(\sigma,\vec{\pi}\right) + \bar{\eta}\mathbb{G}\left(\sigma,\vec{\pi}\right)\eta \tag{6.27}$$

where

$$\left\langle x \middle| \mathbb{G}^{-1}\left(\sigma,\vec{\pi}\right) \middle| x' \right\rangle \equiv \delta^4\left(x-x'\right)\not{\partial} + \int \rho d\rho \lambda(\rho) \int_X I\left(x-X, x'-X:\rho\right)$$

$$\times \left[\sigma\left(X,\rho\right) + i\gamma_5\vec{\pi}\left(X,\rho\right)\cdot\vec{\tau}\right] \quad , \tag{6.28}$$

and where

$$z\left(\frac{\delta}{\delta\eta}, \frac{\delta}{\delta\bar{\eta}}\right) \equiv \exp\left\{-\int \frac{\lambda}{2}\, \phi_\mu^2 \left(\frac{\delta}{\delta\eta}, \frac{\delta}{\delta\bar{\eta}}\right)\right\}$$

$$\times \exp\left\{-\int \frac{3\lambda}{64}\left[\det \Theta^a \left(\frac{\delta}{\delta\eta}, \frac{\delta}{\delta\bar{\eta}}\right) + \det \Theta^{a\dagger}\left(\frac{\delta}{\delta\eta}, \frac{\delta}{\delta\bar{\eta}}\right)\right]\right\} \tag{6.29}$$

where $\phi'_\mu$ and $\Theta$ are of the same form as the fields in Eqs. (6.20) and (6.21), but with $\psi$ replaced by $\delta/\delta\bar{\eta}$ and $\bar{\psi}$ replaced by $\delta/\delta\eta$.

The remaining functional integral is an order parameter field theory. The instanton contribution to the free energy of QCD in this moderately high temperature range is approximately equal to the free energy of this order parameter field theory. It is not unreasonable, because of universality, that QCD should be well approximated by an order parameter field theory in the neighborhood of a second-order phase transition. It is therefore suggestive that the semiclassical approximation to QCD reasonably well describes the onset of correlations approaching the spontaneous chiral SU(2) symmetry breaking phase transition.

For an effective infrared free-order parameter field theory, the dominant long-distance correlations arise from the semiclassical approximation. At the same time, for a phase transition to occur, the composite order parameter fields must propagate like massless particles, and this can only happen if there are strong enough attractive forces between quarks at short distances. The forces between quarks get stronger as the effective coupling $\lambda$ gets larger, and $\lambda$ gets larger very rapidly as the temperature is decreased. Thus a mixing of long- and short-distance effects, from the moderately short-distance behavior of the most infrared important graphs, is necessary to produce the correlations responsible for a phase transition. We will therefore approximate the functional integral in our effective field theory semiclassically, and treat the effects generated by z perturbatively.

Thus, in the lowest approximation we have

$$e^{-W} \simeq \frac{\exp\left\{-\,S_{eff}(\sigma,\vec{\pi})\right\}}{\sqrt{\det\,\Delta_\sigma^{-1}(\sigma,\vec{\pi})}\;\sqrt{\det\,\Delta_\pi^{-1}(\sigma,\vec{\pi})}} \qquad (6.30)$$

where (with the fermi sources turned off),

$$S_{eff}(\sigma,\vec{\pi}) \;\equiv\; \frac{\lambda}{2}\left(\sigma^2+\vec{\pi}^2\right) - \mathrm{tr}\,\ln\,\mathbb{G}^{-1}(\sigma,\vec{\pi}) \qquad (6.31)$$

and where the inverse $\sigma$ and $\vec{\pi}$ propagators are defined by

$$\Delta_\sigma^{-1}(X,\rho;X',\rho') \;\equiv\; \frac{\delta^2\,S_{eff}}{\delta\sigma(X,\rho)\;\delta\sigma(X',\rho')} \qquad (6.32a)$$

and

$$\overset{-1}{\Delta_\pi^{ab}}(X,\rho;X',\rho') \qquad \frac{\delta^2\,S_{eff}}{\delta\pi^a(X,\rho)\;\delta\pi^b(X',\rho')} \qquad (6.32b)$$

These functions are to be evaluated at the minimum of $S_{eff}$, and thus at the solution to the equations

$$\frac{\delta\,S_{eff}}{\delta\sigma} = 0 \quad , \qquad \frac{\delta\,S_{eff}}{\delta\pi^a} = 0 \quad . \qquad (6.33)$$

By a chiral SU(2) rotation the direction of the minimum can be chosen in the $\sigma$ direction, and so the minimum condition implies

$$\sigma(X,\rho) \;=\; \int_{x,x'} \mathrm{tr}\;I(x-X,x'-X;\rho)\;\langle x'|\,\mathbb{G}(\sigma)\,|x\rangle \qquad (6.34)$$

which is independent of X by translation invariance of the ground state. On expressing the integral in terms of Fourier transforms, this equation becomes

$$\sigma(\rho) = \text{tr} \int_P h^2(\rho P) \frac{1}{\not{P} + M(P)} \tag{6.35}$$

where

$$M(P) \equiv \int \rho' d\rho' \; \lambda(\rho') \; h^2(\rho' P) \; \sigma(\rho') \tag{6.36}$$

This equation for $\sigma(\rho)$ can be reexpressed in the form

$$M(Q) = 8 \int_P \rho d\rho \; \lambda(\rho) \frac{h^2(\rho Q) \; h^2(\rho P)}{P^2 + M^2(P)} \; M(P) \tag{6.37}$$

Essentially this equation, but without the effects of the finite temperature boundary conditions, has been discussed by several authors [7,16,17,51].

In the temperature range for which the instanton density is small, this equation (probably) has only a trivial solution. To this approximation, then, the semiclassical correction to the QCD free energy from a gas of gluons and quarks is $\frac{1}{2}$ tr $\ell n \; \Delta^{-1}$, the free energy of a gas of composite $\sigma$ and $\vec{\pi}$ particles. (Even for couplings too weak to form a massless $\vec{\pi}$, there are weakly bound $\sigma$ and $\vec{\pi}$ resonances.[19]

With scalar fields coupled to the quarks, as in the models previously considered, the effective field theory must be modified. While it seems the scalars should just couple to the quarks in the effective field theory like $\bar{\psi}\{\not{\partial} + \Phi\,[(1+\gamma_5)/2] + \Phi^\dagger\,[(1-\gamma_5)/2]\}\psi$, this is only approximately correct. In the presence of the external scalar fields there are two new kinds of graphs in the effective field theory, Fig. 9, to this order of approximation. In one a quark can propagate from an instanton, interact with a scalar field and be reabsorbed by the same instanton. In the other a quark can be exchanged between two instantons or two anti-instantons, interacting with a scalar field in between. Instanton graphs with two or more scalar field insertions on a quark line should not be included to this order of approximation (see Fig. 10). (As was shown in Fig. 6, these graphs should have quarks propagating in background instanton fields between scatterings.)

In the presence of external scalar fields, we can consider corrections to the free energy $W(\Phi)$ of Section V. Now the minimum equations (6.33) are nontrivial. Including the first-order effects generated by the operator z, we now have

$$W(\Phi) \simeq -\frac{\kappa^2}{4}\left(\mathrm{Re}\det\Phi - \mathrm{tr}\,\Phi^\dagger\Phi\right) + \int \frac{\lambda}{2}\left(\sigma^2(\Phi) + \vec{\pi}^2(\Phi)\right) \tag{6.38}$$

$$- \mathrm{tr}\,\ln \mathbb{G}^{-1}\left(\sigma(\Phi),\vec{\pi}(\Phi),\Phi\right) + \frac{1}{2}\,\mathrm{tr}\,\ln \Delta^{-1}\left(\sigma(\Phi),\vec{\pi}(\Phi)\right) + \text{const. } T^2\ \mathrm{tr}\,\Phi^\dagger\Phi$$

where

$$\mathbb{G}^{-1}(\sigma,\vec{\pi},\Phi) \equiv \not{\partial} + \left[\lambda I\left(\sigma + i\vec{\pi}\cdot\vec{\tau}\right) + \Phi\right]\frac{1+\gamma_5}{2} + \left[\lambda I\left(\sigma - i\vec{\pi}\cdot\vec{\tau}\right) + \Phi^\dagger\right]\frac{1-\gamma_5}{2} \tag{6.39}$$

If the second and third terms are approximated perturbatively in $\lambda$, in first-order they will produce a term $(-\kappa^2/4)(\mathrm{Re}\det\Phi + \mathrm{tr}\,\Phi^\dagger\Phi)$ which, combined with the first term, reproduces our previous result.

Without making this approximation, though, the third and fourth terms of Eq. (6.38) can be interpreted two ways. First, they represent the free energy of free quarks and composite mesons but for which the quarks have acquired a dynamical mass due to the external scalar fields. (See Fig. 11.) Alternatively, the second and third terms, when expanded in powers of $\Phi$ and $\Phi^{\dagger}$ represent the lowest approximation to the sum of n-point Green's functions for composite $\sigma$ and $\vec{\pi}$ mesons. (See Fig. 12a.) The fourth term of Eq. (6.38) contributes meson radiative corrections to the external meson propagators and to the n-point vertex. (See Fig. 12b.)

## VII. EFFECTIVE FIELD THEORY FOR SEMICLASSICAL QCD

We have shown how, at moderately high temperature, the semiclassical approximation to QCD is approximately described by a finite temperature effective chiral SU(2) $\sigma$-model like field theory. As the temperature is lowered, the contribution of larger instanton-scale sizes becomes important as $\bar{g}^2/8\pi^2$ gets larger. The larger instantons have a higher density, and so interactions between them, apart from those required by the chirality selection rules, become more important. We therefore discuss some of the corrections to this picture.

The classical gauge field configurations we have expanded about correspond to the sum of fields from separated instantons and anti-instantons. The instanton field can be interpreted as the vector potential for a 4-D color magnetic dipole [7], for $(x-X)^2 >> \rho^2$,

$$A_\mu(x) \simeq M_{\mu\nu} \frac{(x-X)_\nu}{(x-X)^4} \quad , \tag{7.1}$$

with the dipole moment

$$M_{\mu\nu} = 2\rho^2 \, \bar{\eta}^a_{\mu\nu} \, \Omega \frac{\lambda^a}{2} \, \Omega^\dagger \quad . \tag{7.2}$$

The dipole moment for an anti-instanton has $\bar{\eta}^a_{\mu\nu}$ replaced by $\eta^a_{\mu\nu}$. The corrections we consider correspond to the interaction of these dipoles, and to the propagation of the quarks in these dipole fields. These effects are corrections as the instanton density increases. It will turn out that it is natural to also consider higher-loop corrections to the semiclassical approximation at the same time. These are effects associated with gluons propagating in background instanton fields.

We first consider the interaction of these dipoles. Because the gauge field configurations of separated instantons and anti-instantons are not exact minima of the classical gauge field action, the difference between the action for these configurations and the sum of actions for separate instantons and anti-instantons represents an interaction-action. The dominant interaction-action corresponds to the 4-D Abelian magnetic field energy of the superposition of fields minus the field energy of separate dipoles; for $\mathcal{F}_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$

$$V\left(X_i - X_j\right) \simeq 2\,\frac{1}{2g^2} \int_x \operatorname{tr}\ \mathcal{F}_{\mu\nu}\left(X_i - x\right)\ \mathcal{F}_{\mu\nu}\left(x - X_j\right) \tag{7.3}$$

For an instanton and anti-instanton, this takes the form

$$V\left(X_i - X_j\right) \simeq \left(\frac{2\pi}{g}\right)^2 \operatorname{tr}\left(\frac{M^i_{\lambda\mu}\left(X_i - X_j\right)_\mu\ \bar{M}^j_{\lambda\nu}\left(X_i - X_j\right)_\nu}{\left(X_i - X_j\right)^6}\right) \tag{7.4}$$

which is the result of Callan, Dashen and Gross [7]. For two instantons or two anti-instantons, though, this interaction is smaller by an extra power of $\rho^2/(X_i - X_j)^2$, as shown by Bernard [43]. The interactions between pairs of instantons or pairs of anti-instantons will be neglected in the following; there are other interaction terms of the same order (see below).

The QCD functional integral is approximated by the grand partition function describing this 4-D instanton dipole plasma with chirality correlations; from Eq. (6.1) with

$$S\left(\sum_i A_i\right) \simeq \sum_i S\left(A_i\right) + \sum_{i \neq j} V\left(X_i - X_j\right) \quad , \tag{7.5}$$

we have

$$\begin{aligned} Z \simeq \sum_{N_\pm = 0}^{\infty} \frac{1}{N_+!} \frac{1}{N_-!} \int \prod_{i=1}^{N_+ + N_-} d^4X_i \, d\Omega_i \, \frac{d\rho_i}{\rho_i^5} \, \lambda\left(\rho_i\right) \\ \times \; e^{i\theta(N_+ - N_-)} \exp\left\{- \sum_{i \neq j} V\left(X_i - X_j\right)\right\} \det H\left(X_i, X_j\right) \end{aligned} \tag{7.6}$$

The effective coupling $\lambda(\rho_i)$ again arises from exp $\{-S(A_i)\}$ times the factorized nonzero-mode determinants and zero-mode Jacobians. We should also include the correction to the factorized nonzero-mode determinants and zero-mode Jacobians that renormalize V. However, this correction remains to be done; the only corrections to factorization have been computed for exact multi-instanton configurations [43,52] which give higher order instanton interactions that we neglect in the following discussion. These interaction terms are $\mathcal{O}[\rho^2/(X_i - X_j)^6]$ just as the dipolar interactions between pairs of instantons or pairs of anti-instantons. Also, H is again the matrix of zero-mode matrix elements of the Dirac operator in the background field from the instanton gas, Eq. (6.3). When these matrix elements were previously considered the terms

$$\int_x \psi_0^\dagger\left(X_i - x\right) \not{A}\left(x - X_k\right) \psi_0\left(x - X_j\right) \tag{7.7}$$

were neglected. Since the zero-mode wavefunctions are effective quark propagators, these terms simply represent the lowest approximation to quarks porpagating between $X_i$ and $X_j$ in the external dipole field of an instanton at $X_k$. They will be included below.

This grand partition function can again be represented by an effective quantum field theory. Without the quarks, Jevicki [18] has given the generalization, appropriate to a dipole plasma, of Polyakov's [41] effective field theory for the monopole plasma in 2+1 dimensional compact QED. We will generalize Jevicki's effective field theory to include the chirality correlations between instantons and anti-instantons due to the massless quarks. Defining the dipole field

$$M_{\mu\nu}(x) \equiv \sum_{i=1}^{N_+} M^i_{\mu\nu}\ \delta^4\left(x-X_i\right) \tag{7.8}$$

and the corresponding field $\bar{M}_{\mu\nu}(x)$, which is the sum over all anti-instantons with the $\bar{\eta}^a_{\mu\nu}$ in $M_{\mu\nu}$ replaced by $\eta^a_{\mu\nu}$, we consider the functional integral formula,

$$\int \mathcal{D}A \exp\left\{-\frac{1}{g^2}\int \mathrm{tr}\, A_\mu\left(-\partial^2\right) A_\mu - \frac{4\pi^2}{g^2}\int \mathrm{tr}\left(M_{\mu\nu}+\bar{M}_{\mu\nu}\right)\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)\right\}$$

$$\simeq \frac{1}{\sqrt{\det\left(-\partial^2\ \delta_{\mu\nu}\right)}} \exp\left\{-\frac{2\pi^4}{g^2}\int \mathrm{tr}\, M_{\lambda\mu}\ \partial_\mu\ \frac{1}{-\partial^2}\ \overleftarrow{\partial}_\nu\ \bar{M}_{\nu\lambda}\right\}$$

$$= \frac{1}{\sqrt{\det\left(-\partial^2\ \delta_{\mu\nu}\right)}} \exp\left\{-\frac{4\pi^2}{g^2}\sum_{i\neq j} \mathrm{tr}\, M^i_{\lambda\mu}\ \frac{\left(X_i-X_j\right)_\mu \left(X_i-X_j\right)_\nu}{\left(X_i-X_j\right)^6}\ \bar{M}^j_{\lambda\nu}\right\} \tag{7.9}$$

This formula is derived by completing the square in the Gaussian functional integral. It is only approximate since we neglect the dipolar interaction between pairs of instantons or pairs of anti-instantons. The functional integral is over a color matrix of vector potentials, $A_\mu = A_\mu^a\,(\lambda^a/2)$ and can be made to look more electromagnetic-like with a gauge constraint.

$$\int \mathscr{D}A\;\delta\left(\partial_\mu A_\mu\right)\;\exp\left\{-\frac{1}{2g^2}\int \mathrm{tr}\;\mathscr{F}^2_{\mu\nu}(A)\;-\;\frac{4\pi^2}{g^2}\int \mathrm{tr}\left(M_{\mu\nu}+\bar{M}_{\mu\nu}\right)\;\mathscr{F}_{\mu\nu}(A)\right\}\;. \tag{7.10}$$

This represents the interaction of a color matrix of Euclidean electromagnetic fields with 4-D color magnetic dipole fields. Inserting this relation into the grand partition function, Eq. (7.6), gives

$$\int \mathscr{D}A\;\delta\left(\partial_\mu A_\mu\right)\;\exp\left(-\frac{1}{2g^2}\int \mathrm{tr}\;\mathscr{F}^2_{\mu\nu}(A)\right)\left\{\sum_{N_\pm=0}^{\infty}\frac{1}{N_+!}\;\frac{1}{N_-!}\right.$$

$$\times\int\prod_{i=1}^{N_+}\left[d^4X_i\;\frac{d\rho_i}{\rho_i^5}\;d\Omega_i\;\lambda\left(\rho_i\right)\;e^{i\theta}\;\exp\left(\frac{4\pi^2}{g^2}\;\mathrm{tr}\;M_{\mu\nu}\left(\rho_i,\Omega_i\right)\;\mathscr{F}_{\mu\nu}\left(X_i\right)\right)\right]$$

$$\times\int\prod_{j=1}^{N_-}\left[d^4X_j\;\frac{d\rho_j}{\rho_j^5}\;d\Omega_j\;\lambda\left(\rho_j\right)\;e^{-i\theta}\;\exp\left(\frac{4\pi^2}{g^2}\;\mathrm{tr}\;\bar{M}_{\mu\nu}\left(\rho_j,\Omega_j\right)\;\mathscr{F}_{\mu\nu}\left(X_j\right)\right)\right]$$

$$\left.\times\;\det\;H\left(X_i,\rho_i,\Omega_i;X_j,\rho_j,\Omega_j\right)\right\} \tag{7.11}$$

where we have made explicit the $\rho$ and $\Omega$ dependence of the dipole moments. The expression in curly brackets is the same as Eq. (6.4), but with modified couplings

$$\lambda\, e^{i\theta} \longrightarrow e^{i\theta} \exp\left\{ \frac{4\pi^2}{g^2}\ \mathrm{tr}\ M_{\mu\nu}\ \mathscr{F}_{\mu\nu} \right\} \tag{7.12}$$

and analogously for the anti-instanton term. Therefore the effective Lagrangian, Eq. (6.11), is modified to

$$\mathscr{L}_{\mathrm{eff}}(X) = \int \rho d\rho\ \lambda(\rho) \int d\Omega \left\{ e^{i\theta} \exp\left( \frac{4\pi^2}{g^2}\ \mathrm{tr}\ M_{\mu\nu}(\rho,\Omega)\ \mathscr{F}_{\mu\nu}(X) \right) \right.$$

$$\left. \times\ \mathscr{L}_{+}(\Omega,X,\rho) + e^{-i\theta} \exp\left( \frac{4\pi^2}{g^2}\ \mathrm{tr}\ \bar{M}_{\mu\nu}(\rho,\Omega)\ \mathscr{F}_{\mu\nu}(X) \right) \mathscr{L}_{-}(\Omega,X,\rho) \right\} \tag{7.13}$$

where $\mathscr{L}_{\pm}$ are the same expressions as Eq. (6.11b).

The grand partition function describing the plasma of instantons and anti-instantons interacting through both dipolar interactions and chirality correlations is therefore described by the field theory

$$\int \mathscr{D}A \exp\left\{ -\frac{1}{2g^2} \int \mathrm{tr}\ \mathscr{F}^2_{\mu\nu}(A) \right\}\ \delta\left(\partial_\mu A_\mu\right) e^{-W(A)} \tag{7.14}$$

where

$$e^{-W(A)} \equiv \int \mathscr{D}\psi\, \mathscr{D}\bar{\psi}\ \exp\left\{ -\int \left[ \bar{\psi}\not{D}(A)\psi + \mathscr{L}_{\mathrm{eff}}\left(\psi,\bar{\psi},\ \mathscr{F}_{\mu\nu}\right) \right] \right\} \tag{7.15}$$

W(A) is the free energy of an instanton plasma interacting only through chirality correlations but in an external field. (In this expression we have included a gauge interaction with the fermions which we have not yet explained; we will do so later.)

In order to begin to discuss the physical content of this effective field theory we must consider the gauge averaging integration in $\mathscr{L}_{eff}$. The gauge averaging gives contractions of color indices in all possible color singlet ways. In general it will mix terms from the instanton's dipole moment interaction with the interaction of fermions. There will be terms, though, for which the gauge averaging associated with the quark interactions and the dipole interactions factorize; these we will consider first, and then consider the terms for which they mix.

One of the factorized terms in $\mathscr{L}_{eff}$ is proportional to

$$\int d\Omega \left[ \exp\left\{ \frac{4\pi^2}{g^2} \operatorname{tr} M_{\mu\nu}(\Omega) \ \mathscr{F}_{\mu\nu} \right\} e^{i\theta} \det \Phi\left(\psi,\bar{\psi}\right) \right.$$
$$\left. + \exp\left\{ \frac{4\pi^2}{g^2} \operatorname{tr} \bar{M}_{\mu\nu}(\Omega) \ \mathscr{F}_{\mu\nu} \right\} e^{-i\theta} \det \Phi^\dagger\left(\psi,\bar{\psi}\right) \right] \tag{7.16}$$

Expanding the exponentials to perform the gauge averaging, just as for the theory without quarks, gives the leading term

$$\left[ 1 + \frac{8\pi^2}{g^4} \frac{\pi^2\rho^4}{4} \operatorname{tr}\left( \frac{\mathscr{F}_{\mu\nu} - \tilde{\mathscr{F}}_{\mu\nu}}{2} \right)^2 \right] e^{i\theta} \det \Phi\left(\psi,\bar{\psi}\right)$$
$$+ \left[ 1 + \frac{8\pi^2}{g^4} \frac{\pi^2\rho^4}{4} \operatorname{tr}\left( \frac{\mathscr{F}_{\mu\nu} + \tilde{\mathscr{F}}_{\mu\nu}}{2} \right)^2 \right] e^{-i\theta} \det \Phi^\dagger\left(\psi,\bar{\psi}\right)$$
$$= \left[ 1 + \frac{1}{2g^2} \operatorname{tr}\left(\mathscr{F}^2_{\mu\nu}\right) \frac{8\pi^4\rho^4}{g^2} \right] \operatorname{Re} \ e^{i\theta} \det \Phi - \frac{i}{16\pi^2} \operatorname{tr}\left( \mathscr{F}_{\mu\nu} \ \tilde{\mathscr{F}}_{\mu\nu} \right)$$
$$\times \frac{(2\pi)^6}{g^4} \rho^4 \operatorname{Im} \ e^{i\theta} \det \Phi \tag{7.17}$$

The first term has the effect of increasing the instanton density, that is

$$\lambda(\rho) \to \lambda(\rho) \left[ 1 + \frac{8\pi^2}{g^2} \frac{\pi^2 \rho^4}{2g^2} \; \mathrm{tr}\, \mathscr{F}^2_{\mu\nu} \right] \tag{7.18}$$

This is the modification of the instanton density in an external field found by Callan, Dashen and Gross [7]. The external field is now, however, a quantum field that must be integrated. Alternatively[20] this term can (formally) be seen to modify the dielectric properties of the vector field,

$$\frac{1}{2g^2} \int \mathrm{tr}\, \mathscr{F}^2_{\mu\nu} \left[ 1 - \pi^2 \int \rho d\rho \, \frac{8\pi^2}{g^2} \lambda(\rho)\, \mathrm{Re} \;\; e^{i\theta} \det \Phi\left(\psi,\bar{\psi}\right) \right] \tag{7.19}$$

The fermions modify the Callan, Dashen and Gross susceptibility [7] [apart from the usual coupling renormalization effects in $\lambda(\rho)$] space-time dependent $\det \Phi(\psi,\bar{\psi})$ terms which must be integrated over the fermion fields. Other quark contributions to this susceptibility arise from additional interaction vertices generated by the gauge averaging, which we now schematically discuss.

The linear terms in the expansion of the exponential of the dipole interaction in $\mathscr{L}_{eff}$, Eq. (7.13), for example,

$$\int d\Omega \; \mathrm{tr}\, M_{\mu\nu}(\Omega,\rho) \quad \mathscr{F}_{\mu\nu}(x) \quad \mathscr{L}_+(\Omega,X,\rho) \quad , \tag{7.20}$$

will lead to a vertex involving a color nonsinglet quark current interacting with the field $\mathscr{F}_{\mu\nu}$ through the dipole moment (and analogously for the anti-instanton terms). This vertex is depicted graphically in Fig. 13(a). The quadratic term in the expansion of the exponential requires the gauge averaging integral,

$$\int d\Omega \left( \mathrm{tr}\, M_{\mu\nu}(\Omega,\rho) \quad \mathscr{F}_{\mu\nu}(x) \right)^2 \quad \mathscr{L}_+(\Omega,X,\rho) \tag{7.21}$$

There will result a factorized term, previously discussed, as well as new vertices coupling two dipole moments with two quark currents, depicted in Fig. 13(b). The physics of these new vertices is exemplified by treating $\mathscr{L}_{eff}$ perturbatively in Eq. (7.15) for W(A). In second-order (1 instanton and 1 anti-instanton contribution) we have the graphs of Fig. 14. All of these graphs are proportional to tr $\mathscr{F}^2_{\mu\nu}$, and therefore give a contribution to the susceptibility.

If we further integrate over $A_\mu$ in Eq. (7.14) perturbatively, we can check that this field theory generates all the effects of the semi-classical approximation to QCD included in Eqs. (7.6) and (7.7). The graphs in Fig. 15 show that the field theory generates instantons and anti-instantons interacting through both dipolar interactions and quark exchange, with quarks propagating in the background dipole fields of the instantons, and interacting through gluon exchange. The graph in Fig. 15(c) will actually be canceled by a corresponding graph with the quark interacting with the dipole field of the other (anti) instanton; this is because of the zero-modes (see Eq. (6.6)). Quarks do propagate, though, in the background dipole fields of instantons that are not their sources. Besides the semiclassical effects of Eq. (7.6), this effective field theory generates additional corrections not yet discussed. First, there are multiple insertions of the background instanton fields on quark propagation, as depicted in Fig. 16. These multiple insertions on the quark lines sum to give nonzero-mode quark propagators in background instanton fields. These effects can be derived either from the degenerate perturbation theory expansion of the determinant of the Dirac operator in a background field from an instanton gas, as in Section VI, but for the nonzero-modes, or by expanding exp $\{\mathrm{tr}\ \ell n\ \not{D}(A)\}$ in powers of A, as was done by Mottola [44] and Levine and Yaffe [45]. Finally, this effective field theory also generates gluon exchange between quarks. Now in higher order in the semiclassical approximation (in $\hbar$), gluon corrections are generated, but these gluons are propagating in background instanton fields. This is depicted in Fig. 17. Our effective field theory generates only the lowest approximation to the graph in Fig. 17(a), depicted in Fig. 17(b). However, the graph in Fig. 17(a) could be generated by the full non-Abelian version of our vector field theory!

Therefore, semiclassically integrating out the instanton gauge degrees of freedom in QCD should reproduce another non-Abelian effective gauge field theory, but with more complicated quark interactions. This is what one might expect from renormalization group ideas. This effective field theory is

$$Z = \int \mathcal{D}A\,\mathcal{D}\psi\,\mathcal{D}\bar{\psi}\ \exp\left\{-\frac{1}{2g^2}\int \mathrm{tr}\ F^2_{\mu\nu}(A) + \int \bar{\psi}\not{D}(A)\psi + \int \mathcal{L}_{\mathrm{eff}}\left(\psi,\bar{\psi},\ F_{\mu\nu}\right)\right\} \tag{7.22}$$

where

$$\mathcal{L}_{\mathrm{eff}} \simeq \int \rho\, d\rho\ \lambda(\rho) \int d\Omega \left[ e^{i\theta} \exp\left\{\left(\frac{2\pi}{g}\right)^2 \mathrm{tr}\ M_{\mu\nu}(\rho,\Omega)\ F_{\mu\nu}(A)\right\} \times \mathcal{L}_+(\Omega,\rho,X) + e^{-i\theta} \exp\left\{\left(\frac{2\pi}{g}\right)^2 \mathrm{tr}\ \bar{M}_{\mu\nu}(\rho,\Omega)\ F_{\mu\nu}(A)\ \mathcal{L}_-(\Omega,\rho,X)\right\}\right] \tag{7.23}$$

The boundary conditions on this field theory, besides those of finite temperature, must exclude integration over instanton degrees of freedom, and require the constraint of singular gauge. The perturbative analysis of this field theory then reproduces the instanton effects.

## VIII. SUMMARY

In this paper we have considered QCD at finite temperature in order to begin to study phase structure. Since temperature serves to define an energy scale, the high temperature behavior of the theory is calculable because of the asymptotic freedom. As the temperature is lowered, nonperturbative effects must be included, not only because the effective coupling is getting large, but because some quantities in QCD are dominated by nonperturbative effects even for perturbatively weak coupling. Instanton contributions are the nonperturbative effects we have studied here; these are the weak coupling effects that seen to be responsible for the onset of the rapid transition from weak to strong coupling behavior in the theory. Their effects can be qualitatively compared and contrasted with the perturbative effects. Perturbative effects have $U_L(2) \times U_R(2)$ symmetry. They are weak for short distance scales and get stronger slowly as the scale increases. They produce equally attractive forces in all color singlet channels. Instanton effects, on the other hand, effectively have $SU_L(2) \times SU_R(2) \times U_B(1)$ symmetry. They are exponentially small at very short distances and correspondingly turn on suddenly at relatively weak coupling. They effect the vacuum in two ways. First, they contribute to the dielectric function of the QCD vacuum (just as perturbative effects do) which leads to attractive forces in all color singlet channels. However, the chirality correlations that also follow from the instantons give rise to additional attractive forces between massless quarks in the $\sigma$ and $\vec{\pi}$ channels, but repulsive forces in the $\eta$ and $\vec{\phi}$ channels. The forces between quarks due to these chirality correlations in the color octet channel also depend on flavor, but they are much weaker than the forces in the color singlet channels.

In the high temperature phase, the theory can be probed with external fields. Its response to these fields offers an indication of the kinds of symmetry realizations to expect from the theory at low temperature. At high temperature, but approaching the critical temperature, the QCD response to external scalar field theory probes indicates an interrelation of symmetry realizations. The chirality selection rules associated with color gauge field configurations with nontrivial topological fluctuations, that prevent the chiral U(1) phase oscillation, lead to operators with $\Delta Q_5 = \pm 4$ and with $SU_L(2) \times SU_R(2) \times U_B(1)$ symmetry that tend to induce spontaneous chiral SU(2) symmetry breaking and restore isospin and CP symmetry in these models. These configurations with instanton and anti-instanton fluctuations, which are near minima of the classical action and therefore dominate semiclassically, furthermore give large contributions to the dielectric susceptibility, and thus are also important for the onset of confinement.

Probing the response of QCD to external scalar fields may give a reasonable indication of the symmetry realizations that will result when the temperature is lowered, but is of course no replacement for a computation of the free energy as the temperature approaches the critical temperature for phase transitions. In this direction we have considered the contribution of an instanton plasma to the QCD free energy. The usual high temperature plasma of quarks and gluons feels a background field from correlated topological field fluctuations. At very high temperature these topological fluctuations give no contribution to the free energy due to the violation of chirality selection rules. This is manifest in the vanishing of the determinant of the Dirac operator in the extreme dilute limit. The leading contribution from these instanton fluctuations comes from corrections to the zero-eigenvalues of the Dirac operator that preserve consistency with the chirality selection rules. The grand partition function for this instanton plasma was shown to be equal to the functional integral for a fermiinic field theory, the field theory of the finite temperature version of the 't Hooft effective Lagrangian. This field theory was transformed to an order parameter field theory, the dominant approximation to which adds to the quark and gluon gas contribution to the QCD free energy that of a gas of excitations with $\sigma$ and $\vec{\pi}$ quantum numbers.

At still lower temperatures these instanton and anti-instanton gauge field fluctuations are correlated both due to chirality selection rules and due to 4-D magnetic dipole-dipole interactions. The grand partition function for this complicated statistical mechanics model was shown to be equivalent to the functional integral for an effective gauge field theory,

$$\int \mathscr{D}A\, \mathscr{D}\psi\, \mathscr{D}\bar{\psi} \quad \exp\left\{ - \left[ S_{QCD} + \int \mathscr{L}_{eff}\left(\psi, \bar{\psi}, F_{\mu\nu}\right) \right] \right\} \qquad (8.1)$$

where the functional integral over $A_\mu$ does not include instanton configurations; their contribution is contained in the graphs of this effective field theory. One of the important instanton effects arises from the term in $\mathscr{L}_{eff}$ proportional to tr $F^2_{\mu\nu}$. This term gives a temperature dependent coupling renormalization, $g^2 \rightarrow g^2\mu$. We have shown how to compute the effect of quarks on the Callan, Dashen and Gross susceptibility. (This evaluation will be considered elsewhere.)

Because the statistical mechanics system that arises from the semiclassical approximation to QCD is so physical, a 4-D color magnetic dipole plasma with quarks interacting through chirality correlations and propagating in the dipole fields, we expect an understanding of its properties is possible. Instanton interactions with anti-instantons will align the dipoles, and quarks propagating in this vacuum will feel attractive forces in color singlet channels. Combined with the effects of the chirality correlations, this additional attractive interaction, associated with the onset of confinement, may perhaps be enough to induce the spontaneous chiral SU(2) symmetry breaking phase transition.

ACKNOWLEDGMENTS

This work was begun while I was a member of the Institute for Advanced Study; I wish to thank Roger Dashen and Steve Adler for extending to me the hospitality of the Institute, and for their encouragement. I especially wish to thank Roger Dashen for his many insights on which this work is based. It is also a pleasure to thank Helen Quinn for many productive discussions, improving my understanding of many of the topics discussed, and also for her encouragement.

I would also like to thank John Collins, Predrag Cvitanovic, David Gross, Alan Guth, Pierre Sikivie and Larry Yaffe for important conversations improving my understanding of particular aspects of this work. I would further like to thank Varouz Baluni, Walter Dittrich, Gerry Guralnik, David Horn, Ken Johnson, Stu Kasdan, Emil Mottola, Rob Ore and Malcolm Perry for general discussions. Finally, I would like to thank the members of the theory group at Los Alamos for their hospitality during two visits while this work was in progress.

FOOTNOTES

1 $\langle\bar{\psi}\psi\rangle$ spontaneously breaks both chiral SU(2) and chiral U(1) symmetry.

2 A coset G/H is a set of elements of G that are considered to be equivalent if they differ only by multiplication (from the right) by an element of the subgroup H. A particular spontaneous symmetry breaking vacuum is actually a set of vacuua that differ only by a transformation by an element of the unbroken subgroup. The set of such vacuum cosets is a coset-space.

3 Particles fall into representations of the vacuum symmetry. For chiral SU(N) symmetry breaking for N ≥ 3, there are nontrivial representations of the discrete subgroup of vacuum global symmetries combined with discrete space-time symmetries. As Dashen has shown [21], this allows the possibility of parity doubling.

4 Weak interaction CP violating perturbations, however, can change $\theta$.

5 The chiral U(2) transformation $\begin{pmatrix} u \\ d \end{pmatrix} \to \begin{pmatrix} e^{i(\phi_u/2)\gamma_5} & 0 \\ 0 & e^{i(\phi_d/2)\gamma_5} \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix}$

takes $\left\langle \begin{pmatrix} \bar{u}\{(1+\gamma_5)/2\}u & 0 \\ 0 & \bar{d}\{(1+\gamma_5)/2\}d \end{pmatrix} \right\rangle$ with $\theta = 0$ into

$\left\langle \begin{pmatrix} \bar{u}\, e^{i\phi_u}\{(1+\gamma_5)/2\}u & 0 \\ 0 & \bar{d}\, e^{i\phi_d}\{(1+\gamma_5)/2\}d \end{pmatrix} \right\rangle$ ; with $\theta = \phi_u + \phi_d$, this

vacuum is CP and isospin invariant and spontaneously breaks chiral U(2) symmetry. I thank Sidney Coleman for a discussion of this point.

6 This is because while $\langle\bar{\psi}\psi\rangle$ is even under both P and C, $\langle i\bar{\psi}\gamma_5\psi\rangle$ is odd under P and even under C; spontaneous CP violation arises from interference effects. This is to be compared to usual weak interaction (explicit) parity violation: $\bar{\psi}\gamma_\mu\psi$ is even under P and odd under C, while $\bar{\psi}\gamma_\mu\gamma_5\psi$ is odd under P and even under C. Interference effects are thus odd under both P and C and thus even under CP.

7 Questions associated with which vacuum state is picked out by a mass perturbation requires additional considerations. Dashen's theorem [25] states that the correct vacuum state is the one that minimizes the energy of the symmetry breaking perturbation. For a real diagonal mass matrix, $\Phi = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}$, the vacuum state in footnote five with $m_u \sin\phi_u = m_d \sin\phi_d$ and $\theta = \phi_u + \phi_d$ minimizes the perturbation. For $\theta \neq 0$, there is now CP violation due to a mismatch between the conserved CP of the chiral perturbation, and the conserved CP of the spontaneous chiral symmetry breaking vacuum. For $\theta = \pi$, however, there is CP invariance except when $m_u = m_d$. In that case there are two CP conjugate degenerate solutions of the minimum equations. This is an example of Dashen's mechanism [25] for spontaneous CP violation. (In this particular case with two flavors, it also happens that $m^2 = 0$ to first-order in $m_u = m_d \neq 0$.) For further discussion of these points see, for example, Refs. [26]. These remarks imply there are subtleties involved in taking the limit $\Phi \to 0$ in Eq. (2.25).

8 I thank Alan Guth for a crucial discussion on this subject.

[9] This 't Hooft from (singular gauge) can be obtained from the BPST [13] form by a local gauge transformation. See, for example, [32].

[10] For a discussion of these collective coordinates in singular gauge, see for example [33]; at finite temperature see [14].

[11] This form for the 't Hooft term, without the color octet scalar fields, is also given by Mottola [34].

[12] I thank John Collins for an important discussion on this subject.

[13] A careful analysis [14] shows that only $A_0$ acquires a mass; that is, there is only electric screening to this order.

[14] The model given by the Lagrangian, Eq. (5.1), and potential, Eq. (5.4), but with the opposite sign for h, was considered by Mottola [34] to ellucidate many features of the chiral U(1) problem.

[15] I thank Helen Quinn for an important discussion of this point.

[16] Thermodynamically, this is a very interesting situation; the lower temperature phase has more symmetry than the higher temperature phase. This is like the melting of crystalline $He^3$ as the temperature is lowered further. The superfluid $He^3$ has a lower entropy (more order) than the crystalized phase.

[17] I thank Larry Yaffe for showing me this elegant graphical representation for the product of determinants.

18 See also [44] and [45].

19 I thank Fred Cooper and Dick Haymaker for stressing this point to me.

20 The terms in Eq. (7.17), with $\mathscr{F}_{\mu\nu}$ replaced by the full $F_{\mu\nu}$ (see below), tr $F_{\mu\nu}F_{\mu\nu}$Re $e^{i\theta}$ det $\Phi$ and tr $F_{\mu\nu}\tilde{F}_{\mu\nu}$Im $e^{i\theta}$ det $\Phi$, can also be interpreted as mixing meson pairs and glueballs.

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# FIGURE CAPTIONS

Fig. 1. Graphical expansion of $\mathrm{tr}\,\ell n\,[i\not{D}(A) - \Phi(1+\gamma_5)/2 - \Phi^\dagger(1-\gamma_5)/2]$. The heavy solid line represents a quark propagating in a background color gauge field. The external dashed lines with crosses represent external scalar fields.

Fig. 2. Two possible ways the chirality selection rules can be satisfied for any configuration with $\nu(A) = 1$. The shaded circles represent a region of localized field strength in Euclidean space-time; the dashed lines with crosses at their ends represent sources that absorb the massless quarks.

Fig. 3. A Euclidean space-time vacuum event consistent with the chirality selection rules. A region of space-time with $\nu(A) = +1$ creates quark pairs that are absorbed in a region with $\nu(A) = -1$.

Fig. 4. Graphical representation of Eq. (3.7). The heavy solid lines represent quark propagators in the same background color gauge field configuration.

Fig. 5. Graphical representation of Eq. (4.15). The solid circle represents an instanton or anti-instanton, the thin solid line represents a zero-mode wavefunction, and the heavy solid line represents a nonzero-mode quark propagator in the background field of the same instanton or anti-instanton; the dashed lines with crosses at the ends represent external scalar fields.

Fig. 6. Graphical example of a correction to Eq. (4.15) due to external (nonconstant) scalar field induced mixing of zero- and nonzero-mode quark propagators.

Fig. 7. Mixing of colored and flavored scalars.

Fig. 8. Examples of graphs arising from the product of cycle expansions of the zero-mode determinants. Each line between $X_i$ and $X_j$ represents the matrix element $H(X_i,X_j)$.

Fig. 9. New graphs due to external scalar fields, depicted as a cross on the quark lines. In the second graph, the quark line connects two instantons or two anti-instantons.

Fig. 10. Examples of graphs with scalar insertions that should not be included to this order of approximation.

Fig. 11. (a) Quark vacuum graph in which quarks have a dynamical mass.
(b) Composite meson vacuum graph in which the constituent quarks have a dynamical mass.

Fig. 12. (a) n-point Green's function for interacting composite mesons. Dashed lines represent composite mesons, and the crosses represent external scalar fields.
(b) Examples of meson radiative corrections to Fig. 12(a).

Fig. 13. Examples of new vertices implied by the chirality selection rules and dipole moments of instantons. The wavy lines with crosses at their ends represent external $\mathcal{F}_{\mu\nu}$ fields.

Fig. 14. Graphs (a), (b) and (c) represent the interaction with an external field $\mathcal{F}_{\mu\nu}$, represented by a wavy line with a cross; Graph (d) represents the interaction with an external field $A_\mu$, represented by a curly line with a cross.

Fig. 15. Graphs (a) and (b) represent dipole-dipole interactions between an instanton and anti-instanton, as well as quark exchange. Graph (c) represents a quark propagating in the dipole field of an instanton. Graph (d) represents gluon exchange between quarks.

Fig. 16. Multiple insertions of instanton fields on quark propagation.

Fig. 17. (a) Gluon (curly lines) interaction between quarks, with the gluon in a background instanton field; (b) is lowest approximation to (a).

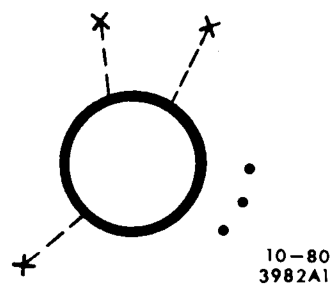


Fig. 1

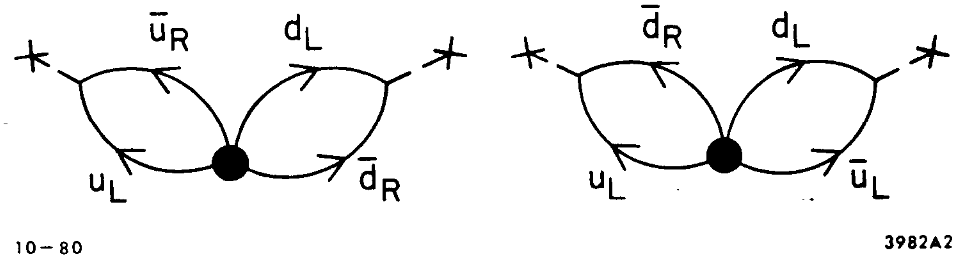


Fig. 2

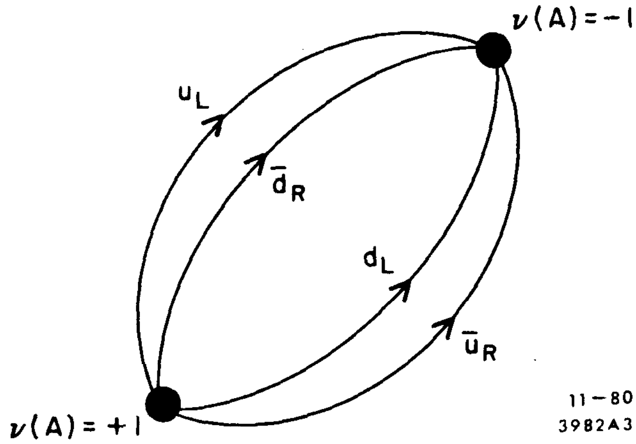


Fig. 3

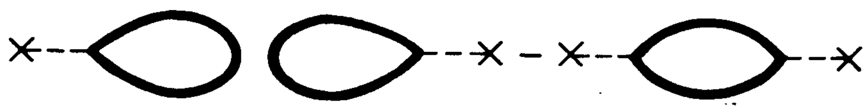


Fig. 4

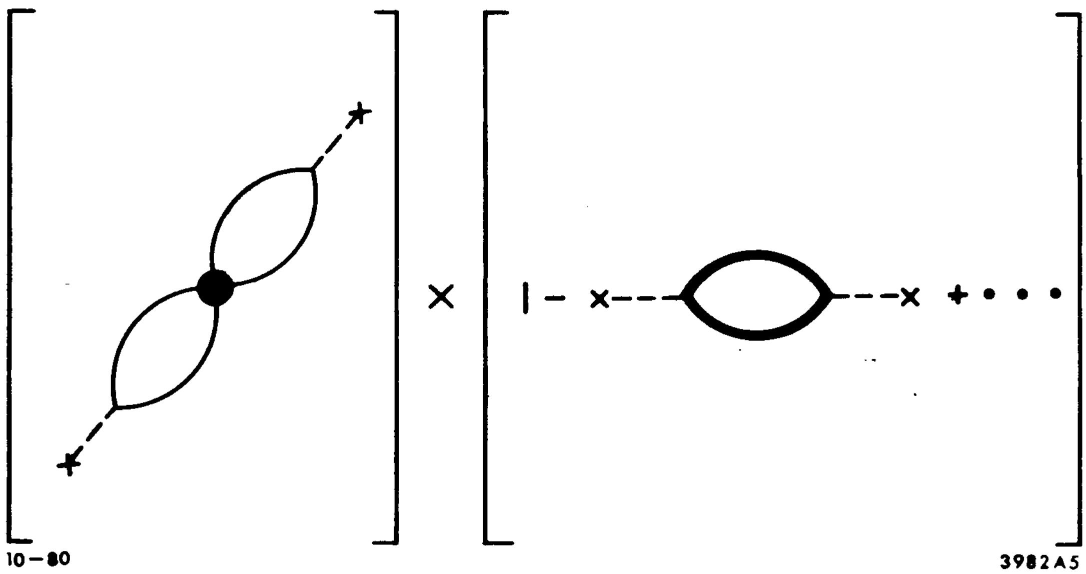


Fig. 5

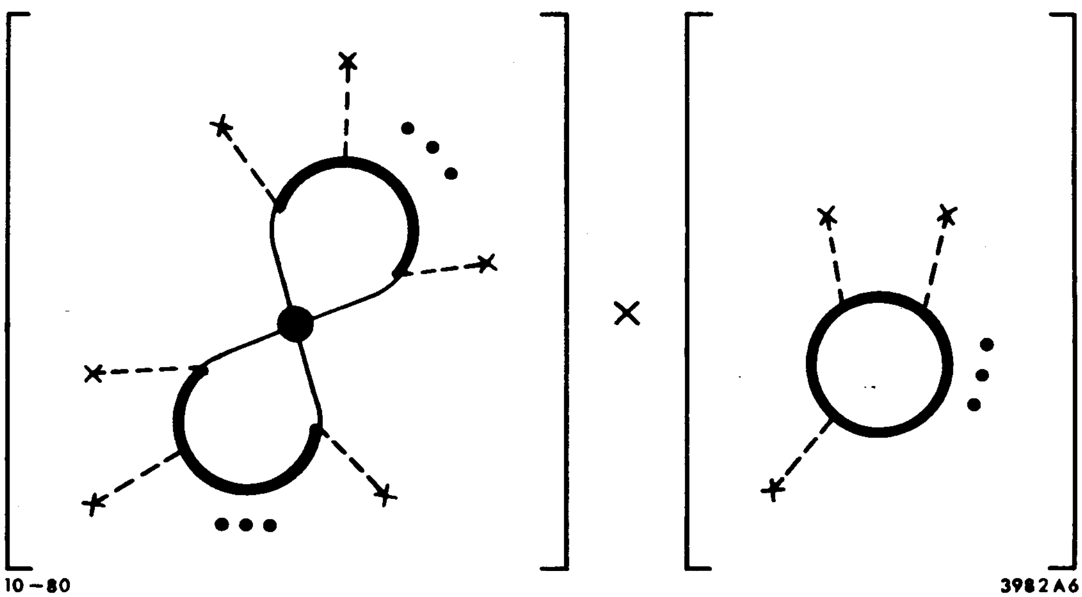


Fig. 6

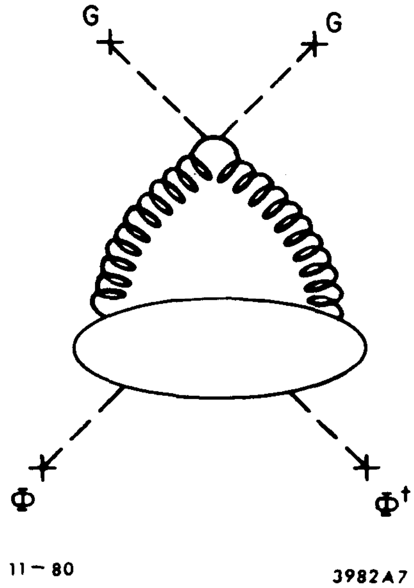


Fig. 7

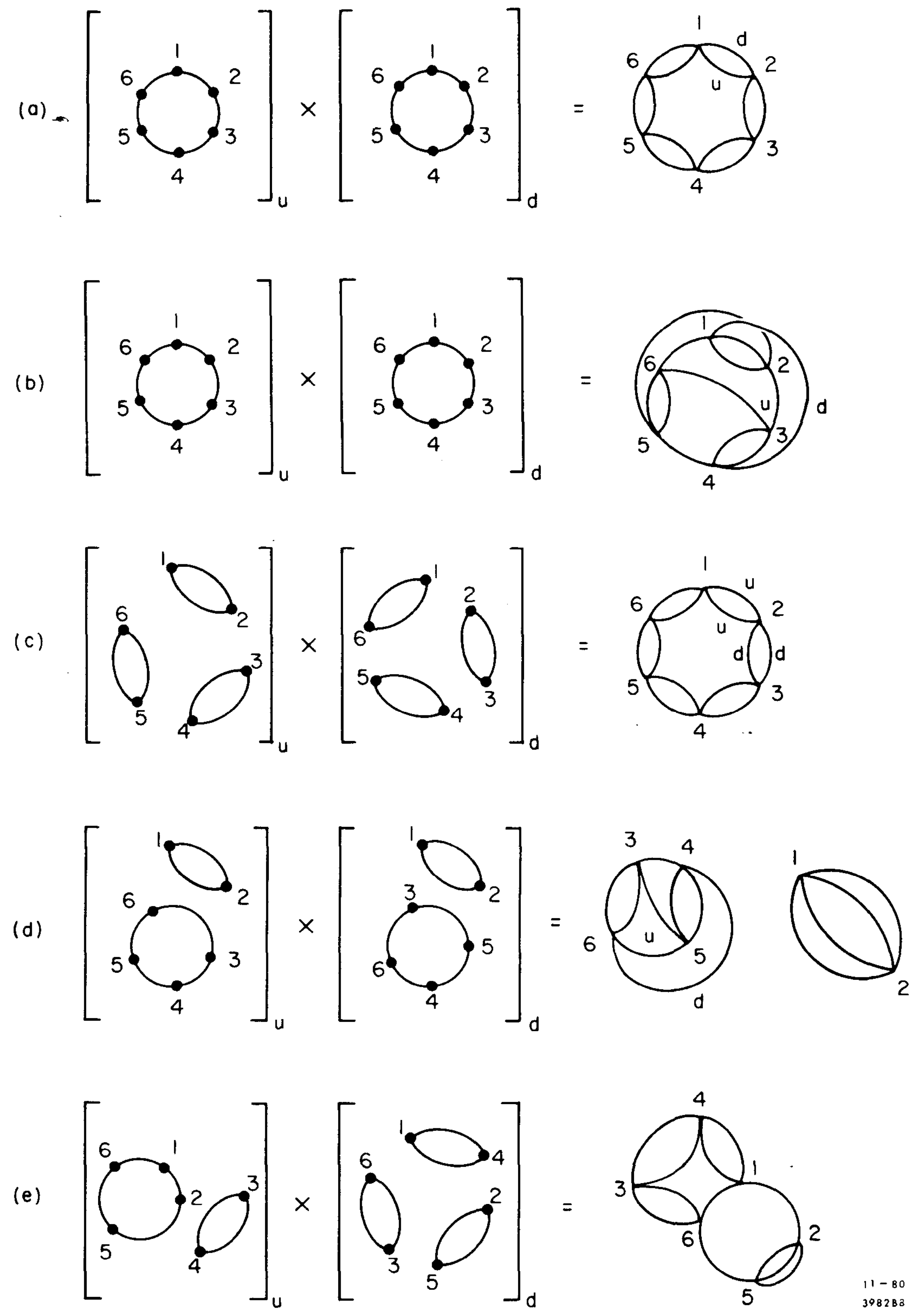


Fig. 8

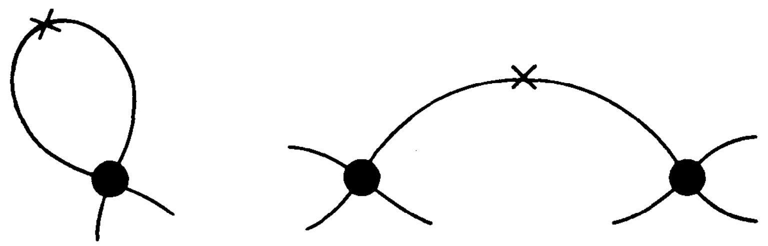

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Fig. 9

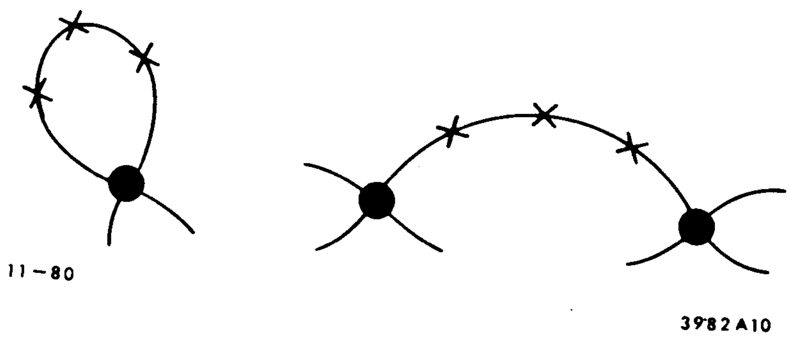



Fig. 10

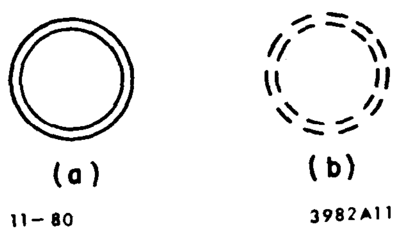
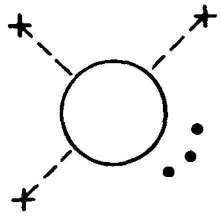
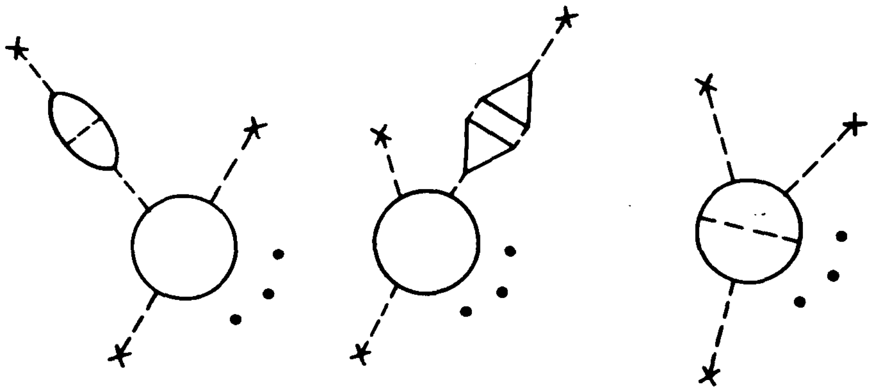



Fig. 11



(a)



(b)

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Fig. 12

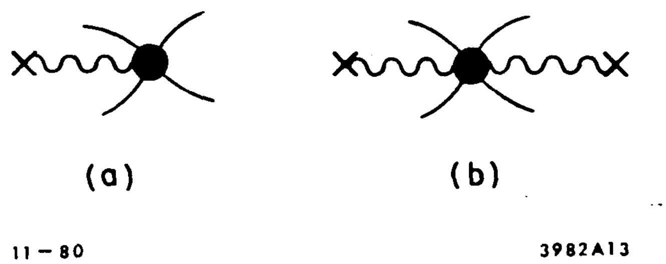


Fig. 13

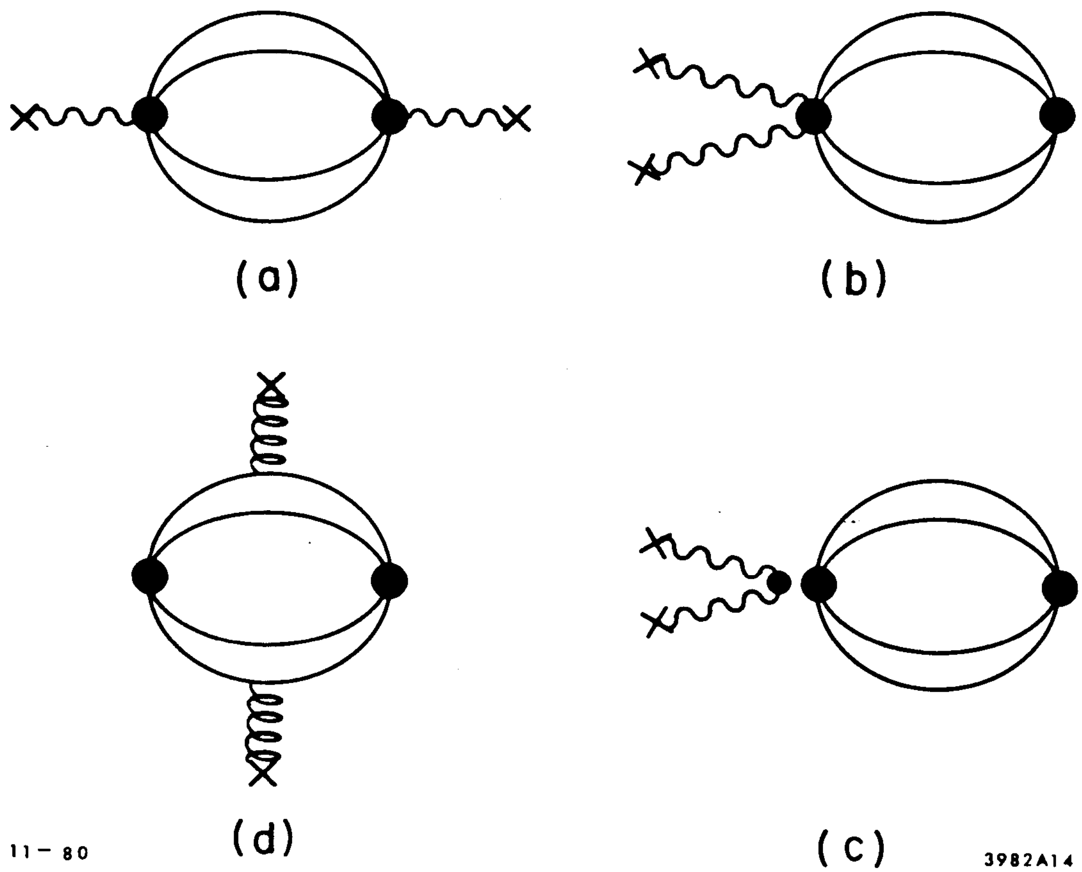


Fig. 14

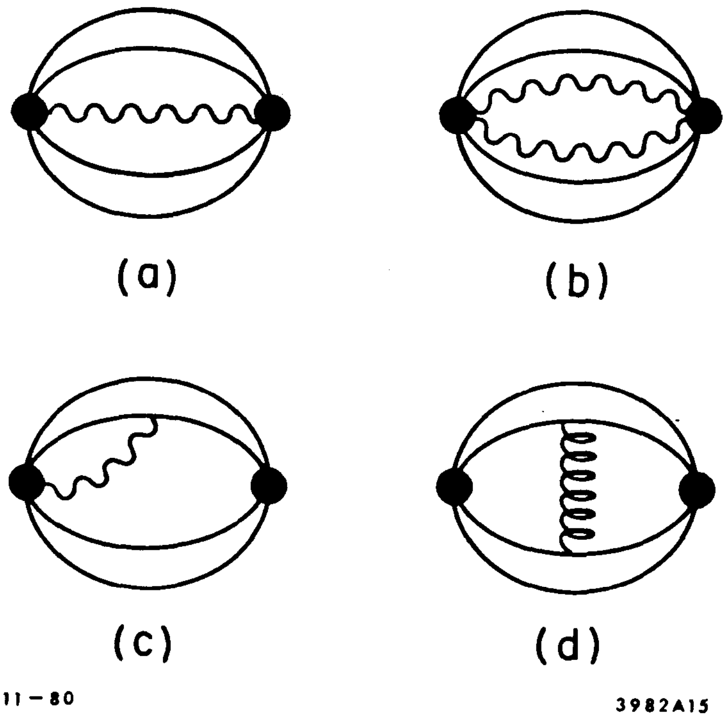


Fig. 15

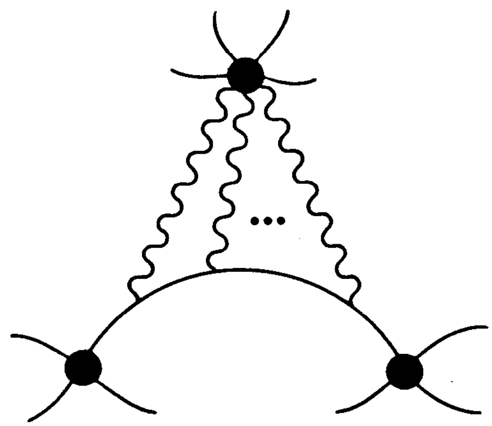


Fig. 16

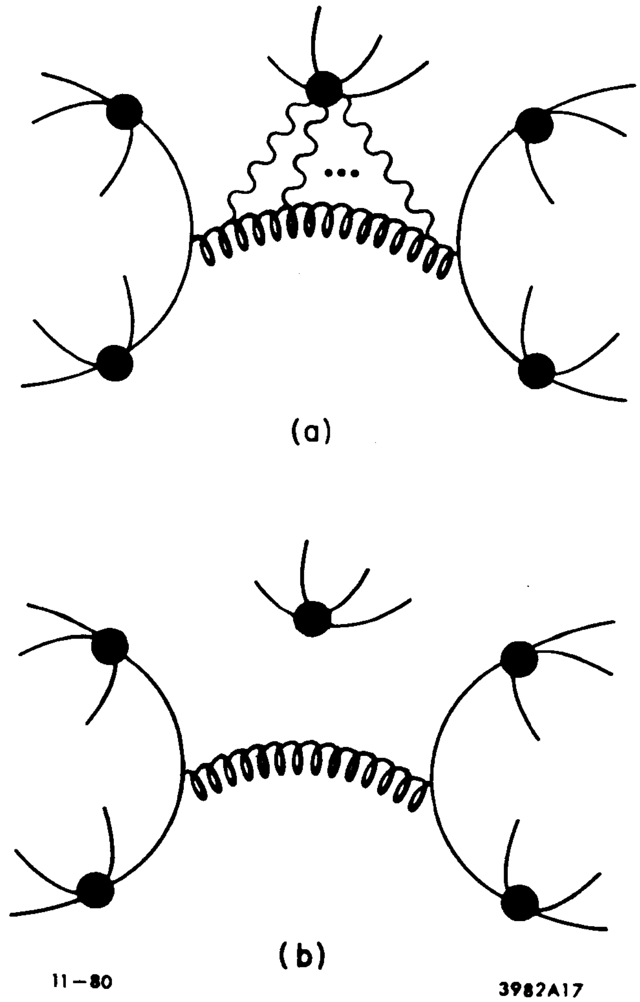


Fig. 17