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OBSERVABILITY OF HEAVY HIGGS BOSONS *

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ABSTRACT

It is argued that, in certain popular heavy color scenarios in which the Higgs sector of the standard $SU_2 \times U_1$ electro-weak theory is generated dynamically, the physical Higgs particle, of zeroth order mass m_H, has the pole of its propagator at

$$m_H^2 + \frac{(1-i)}{\sqrt{2}} \frac{m_H}{M_W} \frac{\sqrt{\alpha}}{\sin\theta_W} \frac{1}{\left(\frac{4\pi}{3}\right)^{2/3} R_{HC}^2}$$

when $m_H\left(\pi\alpha/(2\sin^2\theta_W)\right)^{\frac{1}{2}}/M_W > \frac{1}{2}$. Here, R_{HC} is the effective radius of the heavy color Wigner-Seitz vacuum cells introduced by K. Johnson. For conventional values of the parameters, a Higgs particle with $m_H = 0.383$ TeV has a width of 0.663 TeV!

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I. Introduction

One of the outstanding issues concerning the standard Salam-Ward-Glashow-Weinberg $SU_2 \times U_1$ model[1] of the electro-weak interaction is the detailed dynamical mechanism represented by the Higgs sector of this model. In an effort to understand this mechanism, several authors[2] have taken the view that the Higgs sector is generated by a more fundamental heavy color interaction (QHCD) [to be distinguished from the conventional color interaction of quantum chromodynamics (QCD)]. Thus, whereas the QCD theory strongly interacts at squared momentum transfers $\lesssim 1\ (\mathrm{GeV}/c)^2$, the QHCD theory strongly interacts at squared momentum transfers $\lesssim 1\ (\mathrm{TeV}/c)^2$. Here, we wish to consider the experimental accessibility of the physical Higgs particle in such a dynamical scheme when its zeroth order mass[3] m_H is large enough that the physical Higgs particle interacts with itself strongly.

More precisely, the Lagrangian for the standard $SU_2 \times U_1$ model is by now well known.[1] After spontaneous symmetry breaking, we adopt the unitary gauge for the model. Then, under the assumption that the physical Higgs particle ϕ_0 is strongly interacting, the relevant Higgs Lagrangian is

$$\mathscr{L}_H = \frac{1}{2}\left(\partial_\mu\phi_0\partial^\mu\phi_0 - 2M_1^2\phi_0^2\right) - h\left(\frac{1}{4}\phi_0^4 + \sqrt{2}\lambda\phi_0^3 - \lambda^4\right) \qquad (1)$$

where, to make contact with Ref. 1, we have

$$\begin{gathered} M_W = \lambda g/\sqrt{2}\ , \quad M_Z = \lambda\left(g^2 + g'^2\right)^{\frac{1}{2}}/\sqrt{2}\ , \\ e = g\sin\theta_W \equiv gg'/\sqrt{g^2 + g'^2}\ , \text{ and} \\ \lambda^2 = M_1^2/(2h) \quad . \end{gathered} \qquad (2)$$

Here, g and g' are the SU_2 and U_1 gauge couplings, respectively, and e is the electron's charge, M_W and M_Z are the charged and neutral weak gauge boson masses, respectively, and θ_W is Weinberg's angle. The zeroth order mass of the physical Higgs particle is clearly given by

$$m_H^2 = 2M_1^2 \quad . \tag{3}$$

Since we will take $2h^{\frac{1}{2}} > 1$, the remaining fermion and boson interactions in the model amount to small corrections to any conclusions which we arrive at by studying (1). While these corrections are interesting in themselves, they do not concern us here. [In other words, all the interaction terms in the complete $SU_2 \times U_1$ Lagrangian which are not shown in (1) have small couplings involving powers of g and g'.] The regime $2h^{\frac{1}{2}} > 1$ requires

$$\left(\frac{\pi\alpha m_H^2}{2\sin^2\theta_W M_W^2}\right)^{\frac{1}{2}} > 1/2 \tag{4}$$

or

$$m_H > 2.26 M_W \tag{5}$$

for $\sin^2\theta_W = 0.236$. Thus, we consider a large Higgs mass.

To study (1) in this strong coupling regime, we employ methods related to those we introduced in Ref. 4. As has been shown by Bender et al.,[5] and others,[6-11] none of the known properties of calculable systems has been omitted by the methods in Ref. 4. This gives us additional confidence in the development presented herein.

The methods in Ref. 4 are not generally familiar, however. Thus, we shall begin our formal presentation by adapting the relevant aspects of the results in Ref. 4 to the analysis of (1). This is done in the next section, Sec. II. Our objective is to make the arguments given in this paper self-contained.

The remainder of our discussion is then organized in the following way. In Sec. III, we use the results presented in Sec. II together with the ideas of Johnson[12] on vacuum Wigner-Seitz cells to compute the relationship between the width of the heavy Higgs particle and the scale Λ_{HC} of the QHCD theory. Section IV is composed of some concluding remarks. The appendix contains some technical details.

II. Path-Space Approach to Strongly Coupled Higgs Fields

Our primary purpose in this section is to "review" the relevant aspects of the Feynman path-space theory of strongly coupled fields which we introduced in Ref. 4. We wish to use that theory to study the width of the Higgs particle in the $SU_2 \times U_1$ model in the regime delineated by (4) and (5). For this reason, our "review" will be effected by using the Lagrangian $\mathscr{L}_H$ itself in this latter regime as our vehicle of illustration.

More specifically, we wish to consider the large h limit of this Lagrangian $\mathscr{L}_H$ in (1). Toward this end, we study the generating functional for the connected Green's functions for $\mathscr{L}_H$,

$$iZ(J) = \ell n \int \mathscr{D}\phi_0 \exp\left\{i\int d^4x \left[\mathscr{L}_H(\phi_0) + J\phi_0\right]\right\} \quad , \tag{6}$$

where J(x) is an appropriate external source. The relevant physics in the large h limit of $\mathscr{L}_H$ can be obtained by computing Z(J) in this limit.

To compute Z(J) for large h, we will use the following functional identities:

$$\exp\left\{-i\int d^4x b\phi_0^4\right\} = \int \mathscr{D}\phi_0 \ \exp\left\{i\int d^4x\left(\sigma^2 + 2\sqrt{b}\sigma\phi_0^2\right)\right\} \tag{7}$$

$$\exp\left\{i\int d^4x F(\phi_0)\right\} = \int \mathscr{D}\kappa \ \mathscr{D}\rho \ \exp\left\{i\int d^4x\left(\rho(\kappa - \phi_0) + F(\kappa)\right)\right\} \ , \tag{8}$$

where $b > 0$ and $F(x)$ is an arbitrary function of its argument x. To prove (7), simply observe that

$$\sigma^2 + 2\sqrt{b}\sigma\phi_0^2 = \left(\sigma + \sqrt{b}\phi_0^2\right)^2 - b\phi_0^4 \quad . \tag{9}$$

Thus, the shift

$$\sigma \to \sigma - \sqrt{b}\phi_0^2 \tag{10}$$

leaves $\mathscr{D}\sigma$ unchanged but produces, on the RHS of (7),

$$\int \mathscr{D}\sigma \ \exp\left\{i\int d^4x \ \left(\sigma^2 - b\phi_0^4\right)\right\} \quad . \tag{11}$$

(Here, RHS denotes "the right-hand side.") The normalization of $\mathscr{D}\sigma$ is to be chosen such that

$$\int \mathscr{D}\sigma \ \exp\left\{i\int d^4x\sigma^2\right\} = 1 \quad . \tag{12}$$

Similarly, to obtain (8), recall that for an appropriate normalization of $\mathscr{D}\rho$,

$$\int \mathscr{D}\rho \ \exp\left\{i\int d^4x\rho(\kappa - \phi_0)\right\} = \delta(\kappa - \phi_0) \tag{13}$$

where $\delta(\kappa - \phi_0)$ is the delta functional of $\kappa - \phi_0$. In more detail, using a countable uniform covering $\mathscr{O} = \{U_j : j = 1,2,3,\ldots\}$ of space-time with sets U_j such that $\bar{x}_j$ is the geometric center of U_j and such that the measure of $U_j \cap U_i$ is zero for $i \neq j$ and the measure of each U_j is $\Delta x \downarrow 0$, then

$$\delta(\kappa - \phi_0) = \prod_{j=1}^{\infty} C_1 \int_{-\infty}^{\infty} \frac{d\rho_j}{2\pi} \exp\left\{i\rho_j\left(\kappa(\bar{x}_j) - \phi_0(\bar{x}_j)\right)\Delta x\right\} \tag{14}$$

for an appropriate choice of the constant C_1. The RHS of (8) is then the same as

$$\prod_{j=1}^{\infty} C_2 \int_{-\infty}^{\infty} d\kappa_j \int_{-\infty}^{\infty} C_1 \frac{d\rho_j}{2\pi} \exp\left\{i\rho_j\left(\kappa_j - \phi_0(\bar{x}_j)\right)\Delta x + F\left(\phi_0(\bar{x}_j)\right)\Delta x\right\} \quad (15)$$

for an appropriate normalization constant C_2. We obtain (8) whenever the constants C_1 and C_2 satisfy

$$C_1 C_2/\Delta x = 1 \quad . \quad (16)$$

We will always choose C_1 and C_2 in accordance with (16).

The strategy to be used to obtain the large h limit of iZ(J) is as follows: first, we shall use (7) to represent the quartic part of $\mathscr{L}_H$ as

$$\exp\left\{-i\int d^4x \, \frac{h}{4} \, \phi_0^4(x)\right\} = \int \mathscr{D}\sigma \, \exp\left\{-i\int d^4x\left(\sigma^2 + \sqrt{h}\sigma\phi_0^2\right)\right\} \quad . \quad (17)$$

Then we shall use (8) three times.

The first use of (8) is to represent the free part of $\mathscr{L}_H$ and the source term $J\phi_0$ as

$$\exp\left\{i\int d^4x \left(\frac{1}{2}\left(\partial^\mu\phi_0\partial_\mu\phi_0 - 2M_1^2\phi_0^2\right) + J\phi_0\right)\right\}$$
$$= \int \mathscr{D}\kappa\mathscr{D}\rho \, \exp\left\{i\int d^4x \left(\frac{1}{2}\left(\partial_\mu\kappa\partial^\mu\kappa - 2M_1^2\kappa^2\right) + J\kappa + \rho(\kappa - \phi_0)\right)\right\} \quad . \quad (18)$$

Here, in the notation of (8),

$$F(\phi_0) = \frac{1}{2}\left(\partial_\mu\phi_0\partial^\mu\phi_0 - 2M_1^2\phi_0^2\right) + J\phi_0 \quad . \quad (19)$$

The second use of (8) is to represent the cubic part of $\mathscr{L}_H$ as

$$\exp\left\{-i\int d^4x\sqrt{2}h\lambda\phi_0^3\right\} = \int \mathscr{D}\eta\mathscr{D}\nu \, \exp\left\{i\int d^4x\left(\nu(\eta - \phi_0) - \sqrt{2}h\lambda\eta^3\right)\right\} \quad . \quad (20)$$

Here, in the notation of (8),

$$F(\phi_0) = -\sqrt{2}h\lambda\phi_0^3 \quad . \quad (21)$$

The final use of (8) is to express the RHS of (20) as

$$\int \mathscr{D}\eta \mathscr{D}\nu \exp\left\{i\int d^4x\left(\nu(\eta - \phi_0) - \sqrt{2}h\lambda\eta^3\right)\right\} = \int \mathscr{D}\eta\mathscr{D}\nu\mathscr{D}\eta'\mathscr{D}\nu' \exp\left\{i\int d^4x\left(\nu(\eta - \phi_0) + \nu'(\eta - \eta') - \sqrt{2}h\lambda\eta^2\eta'\right)\right\} . \tag{22}$$

In the notation of (8), in (22)

$$F(\eta) = -\sqrt{2}h\lambda\eta^3 \quad . \tag{23}$$

These representations (17), (18), and (22) allow us to isolate the small parts of $\mathscr{L}_H$ for h large.

More specifically, if we introduce the results (17), (18), and (22) into (6) we have

$$iZ(J) = \ell n\int \mathscr{D}\phi_0\mathscr{D}\kappa\mathscr{D}\rho\mathscr{D}\sigma\mathscr{D}\eta\mathscr{D}\nu\mathscr{D}\eta'\mathscr{D}\nu' \exp\left\{i\int d^4x \left[\frac{1}{2}\left(\partial_\mu\kappa\partial^\mu\kappa - 2M_1^2\kappa^2\right) + J\kappa + \sigma^2 + \sqrt{h}\sigma\phi_0^2 + \rho(\kappa - \phi_0) - \sqrt{2}h\lambda\eta^2\eta' + \nu'(\eta - \eta') + \nu(\eta - \phi_0) + h\lambda^4\right]\right\} . \tag{24}$$

The shifts

$$\phi_0 \to \phi_0 + (\rho + \nu)/(2\sqrt{h}\sigma) \tag{25}$$

$$\eta \to \eta + \nu'/(2\sqrt{2}h\lambda\eta') \tag{26}$$

then give

$$iZ(J) = \ell n\int \mathscr{D}\phi_0\mathscr{D}\kappa\mathscr{D}\rho\mathscr{D}\sigma\mathscr{D}\eta\mathscr{D}\nu\mathscr{D}\eta'\mathscr{D}\nu' \exp\left\{i\int d^4x \left[\frac{1}{2}\left(\partial_\mu\kappa\partial^\mu\kappa - 2M_1^2\kappa^2\right) + J\kappa + \sigma^2 + \sqrt{h}\sigma\phi_0^2 + \rho\kappa - \sqrt{2}h\lambda\eta^2\eta' - \nu'\eta' + \nu\eta + \frac{\nu'^2}{4\sqrt{2}h\lambda\eta'} + \frac{\nu\nu'}{2\sqrt{2}h\lambda\eta} - \frac{(\rho + \nu)^2}{4\sqrt{h}\sigma} + h\lambda^4\right]\right\} . \tag{27}$$

Thus, to leading order in 1/(4h) one has

$$iZ(J) = \ell n \int \mathscr{D}\phi_0 \mathscr{D}\kappa \mathscr{D}\rho \mathscr{D}\sigma \mathscr{D}\eta \mathscr{D}\nu \mathscr{D}\eta' \mathscr{D}\nu' \exp\left\{ i \int d^4x \left[\frac{1}{2}\left(\partial^\mu\kappa\partial_\mu\kappa - 2M_1^2\kappa^2\right) + J\kappa + \sigma^2 + \sqrt{h}\sigma\phi_0^2 + \rho\kappa - \sqrt{2}h\lambda\eta^2\eta' - \nu'\eta' + \nu\eta - (\rho+\nu)^2/(4\sqrt{h}\sigma) + h\lambda^4 \right] \right\} . \tag{28}$$

In other words, we shall work to lowest order in the interactions

$$\frac{\nu'^2}{4\sqrt{2}h\lambda\eta'} = \frac{g\nu'^2}{8hM_W\eta'} \quad \text{and} \quad \frac{\nu\nu'}{2\sqrt{2}h\lambda\eta} = \frac{g\nu\nu'}{4hM_W\eta} , \tag{29}$$

the small parts of the super-renormalizable $\eta^2\eta'$ term in (24).

Effecting the integral over $\mathscr{D}\nu'$ in (28) simply produces the delta functional $\delta(\eta')$, so that the subsequent integration over $\mathscr{D}\eta'$ simply sets $\eta' = 0$. But, for $\eta' = 0$, the integration over $\mathscr{D}\eta$ in (28) produces the delta functional $\delta(\nu)$; the subsequent integration over $\mathscr{D}\nu$ simply sets $\nu = 0$. Thus, (28) is the same as

$$iZ(J) = \ell n \int \mathscr{D}\phi_0 \mathscr{D}\kappa \mathscr{D}\rho \mathscr{D}\sigma \exp\left\{ i \int d^4x \left[\frac{1}{2}\left(\partial_\mu\kappa\partial^\mu\kappa - 2M_1^2\kappa^2\right) + J\kappa + \sigma^2 + \sqrt{h}\sigma\phi_0^2 + \rho\kappa - \rho^2/(4\sqrt{h}\sigma) + h\lambda^4 \right] \right\} . \tag{30}$$

To leading order in 1/(4h), the super-renormalizable interaction in (1) is negligible.

Analyzing (30) in more detail, we have

$$iZ(J) = \ell n\left\{\int \mathcal{D}\phi_0 \mathcal{D}\kappa \mathcal{D}\rho \mathcal{D}\sigma \ \exp\left\{i\int d^4x \left[\frac{1}{2}\left(\partial_\mu\kappa\partial^\mu\kappa - 2M_1^2\kappa^2\right) + h\lambda^4 \right.\right.\right.$$

$$\left.\left.\left. + J\kappa + \sigma^2 + \sqrt{h}\sigma\phi_0^2 + \rho\kappa\right]\right\} \sum_{n=0}^{\infty} \frac{i^n}{n!} \prod_{j=1}^{n} \int d^4x_j \frac{\left(-\rho^2(x_j)\right)}{4\sqrt{h}\sigma(x_j)}\right\}$$

$$= \ell n \int \mathcal{D}\phi_0 \mathcal{D}\kappa \mathcal{D}\rho \mathcal{D}\sigma \quad \sum_{n=0}^{\infty} \frac{i^n}{n!} \prod_{j=1}^{n} \int d^4x_j \frac{\left(-\rho^2(x_j)\right)}{4\sqrt{h}} \int_{-\infty}^{\infty} \frac{d\beta_j}{\beta_j + i\varepsilon}$$

$$\int_{-\infty}^{\infty} \frac{d\alpha_j}{2\pi} \exp\left\{i \sum_{j=1}^{n} \alpha_j\left(\beta_j - \sigma(x_j)\right) + i\int d^4x \right.$$

$$\left.\left[\frac{1}{2}\left(\partial_\mu\kappa\partial^\mu\kappa - 2M_1^2\kappa^2\right) + J\kappa + \sigma^2 + \sqrt{h}\sigma\phi_0^2 + \rho\kappa + h\lambda^4\right]\right\} \quad , \tag{31}$$

where $\varepsilon \downarrow 0$ and we define $\prod_{j=1}^{0} a_j = 1$ and $\sum_{j=1}^{0} a_j = 0$ for all expressions a_j, $j = 1,2,3,\ldots.$ The integrations over $\mathcal{D}\phi_0$ and $\mathcal{D}\sigma$ may now be done by observing that

$$\frac{\int \mathcal{D}\phi_0 \mathcal{D}\sigma \ \exp\left\{i\left\{\int d^4x \left[\sigma^2 + \sqrt{h}\sigma\phi_0^2\right] - \sum_{j=1}^{n} \alpha_j \sigma(x_j)\right\}\right\}}{\int \mathcal{D}\phi_0 \mathcal{D}\sigma \ \exp\left\{i\left\{\int d^4x \left[\sigma^2 + \sqrt{h}\sigma\phi_0^2\right]\right\}\right\}} \tag{32}$$

$$= \frac{\left(\prod_{\ell=1}^{\infty} \int_{-\infty}^{\infty} d\phi_{0\ell} \int_{-\infty}^{\infty} d\sigma_\ell\right) \exp\left\{i\left\{\Delta x \sum_{\ell=1}^{\infty} \left(\sigma_\ell^2 + \sqrt{h}\sigma_\ell \phi_{0\ell}^2\right) - \sum_{j=1}^{n} \alpha_j \sigma_j\right\}\right\}}{\left(\prod_{\ell=1}^{\infty} \int_{-\infty}^{\infty} d\phi_{0\ell} \int_{-\infty}^{\infty} d\sigma_\ell\right) \exp\left\{i\left\{\Delta x \sum_{\ell=1}^{\infty} \left(\sigma_\ell^2 + \sqrt{h}\sigma_\ell \phi_{0\ell}^2\right)\right\}\right\}} \quad .$$

Here, Δx is the size of the measure of each of the sets in a uniform covering of space-time by sets U_j with geometric centers $\{\bar{x}_j\}$ such that $\{x_j\}$ in (32) satisfy

$$\{x_j\} \subset \{\bar{x}_j\} \tag{33}$$

so that we can take $\sigma_j = \sigma(\bar{x}_j)$ and $\phi_{0j} = \phi_0(\bar{x}_j)$. The RHS of (32) is thus the same as

$$R_n = \prod_{j=1}^{n} \frac{\int_{-\infty}^{\infty} d\phi_{0j} \int_{-\infty}^{\infty} d\sigma_j \, \exp\left\{i\left\{\Delta x\left(\sigma_j^2 + \sqrt{h}\sigma_j\phi_{0j}^2\right) - \alpha_j\sigma_j\right\}\right\}}{\int_{-\infty}^{\infty} d\phi_{0j}\int_{-\infty}^{\infty} d\sigma_j \, \exp\left\{i\left\{\Delta x\left(\sigma_j^2 + \sqrt{h}\sigma_j\phi_{0j}^2\right)\right\}\right\}} \, . \tag{34}$$

For, all terms in the infinite products in the numerator and denominator of (32) cancel except those for the points $\{x_j\}$. Since the denominator of the LHS (left-hand side) of (32) is a constant independent of x_j, we have that (34) is equal to the integrations over $\mathscr{D}\phi_0$ and $\mathscr{D}\sigma$ in (31) up to an unimportant constant factor. Thus, for an appropriate normalization of $\mathscr{D}\phi_0$, we can write

$$\int \mathscr{D}\phi_0 \mathscr{D}\sigma \, \exp\left\{i\left\{\int d^4x\left[\sigma^2 + \sqrt{h}\sigma\phi_0^2\right] - \sum_{j=1}^{n} \alpha_j\sigma(x_j)\right\}\right\} = R_n \quad . \tag{35}$$

The quantity R_n is evaluated in the Appendix. Its value is given by

$$R_n = \prod_{j=1}^{n} \frac{2i^{3/4}}{\Gamma(1/4)} \sqrt{\frac{\pi}{\alpha_j}} \, (\Delta x)^{1/4} e^{-i\alpha_j^2/(4\Delta x)} \quad , \tag{36}$$

where $\Gamma(1/4)$ is the Euler gamma function of argument 1/4 and the branch of the radicals in (36) is

$$-\pi < \text{Arg } z \leq \pi \tag{37}$$

for complex numbers z. We therefore have the result

$$\begin{aligned} &\int \mathscr{D}\phi_0 \mathscr{D}\sigma \, \exp\left\{i\left\{\int d^4x\left[\sigma^2 + \sqrt{h}\sigma\phi_0^2\right] - \sum_{j=1}^{n} \alpha_j\sigma(x_j)\right\}\right\} \\ &\qquad = \prod_{j=1}^{n} \frac{2i^{3/4}}{\Gamma(1/4)} \sqrt{\frac{\pi}{\alpha_j}} \, (\Delta x)^{1/4} e^{-i\alpha_j^2/(4\Delta x)} \quad . \end{aligned} \tag{38}$$

Further, the attendant integration over $d\beta_j d\alpha_j$ in (31) can now be done. For, by elementary methods

$$\int_{-\infty}^{\infty} \frac{d\beta_j}{\beta_j + i\varepsilon} \int_{-\infty}^{\infty} \frac{d\alpha_j}{2\pi\sqrt{\alpha_j}} e^{i\left(\alpha_j\beta_j - \alpha_j^2/(4\Delta x)\right)} = \frac{-(\Delta x)^{1/4}\Gamma(1/4)}{\sqrt{2}i^{1/4}} \quad . \tag{39}$$

Hence, using (38) and (39), we see that, for appropriate normalizations of the functional integrals, (31) is the same as

$$\begin{aligned} iZ(J) &= \ell n \int \mathcal{D}\kappa\mathcal{D}\rho \sum_{n=0}^{\infty} \frac{i^n}{n!} \prod_{j=1}^{n} \int d^4x_j \, \frac{\rho^2(x_j)}{2\sqrt{2}h} \, \frac{(\Delta x)^{1/2}\sqrt{\pi}}{i^{-1/2}} \\ &\quad \exp\left\{i\int d^4x \left[\frac{1}{2}\left(\partial_\mu\kappa\partial^\mu\kappa - 2M_1^2\kappa^2\right) + J\kappa + \rho\kappa + h\lambda^4\right]\right\} \\ &= \ell n \int \mathcal{D}\kappa\mathcal{D}\rho \quad \exp\left\{i\int d^4x \left[\frac{1}{2}\left(\partial_\mu\kappa\partial^\mu\kappa - 2M_1^2\kappa^2\right) + J\kappa + \rho\kappa \right.\right. \\ &\quad \left.\left. + \rho^2(\Delta x)^{1/2}\sqrt{\pi}/(-8hi)^{1/2} + h\lambda^4\right]\right\} \quad . \end{aligned} \tag{40}$$

This is the desired large h limit of the Higgs theory represented by $\mathcal{L}_H$ in (1).

What we wish to do with (40) is to use it to compute the decay characteristics for the respective Higgs particle. To these characteristics we turn in the next section.

III. Heavy Higgs Particle Decay

To relate our result (40) to the Higgs particle decay properties at strong coupling, the one unknown and undefined parameter in (40), Δx, must be determined. It is at this stage of our analysis that the heavy color origin of this Higgs particle plays an essential role.

More precisely, the determination of the parameter Δx in (40) requires some discussion. In particular, we reiterate that Δx is the size of the measures of the sets in a uniform covering $\mathcal{O}$ of space-time which we used to effect the functional integrals for the effective Higgs theory (1). If the Higgs field ϕ_0 were fundamental, we could consistently take $\Delta x \downarrow 0$. However, according to Ref. 2, ϕ_0 is a composite of heavy color fields. The dynamics of these heavy color fields is believed to be the same as the QCD theory, except that the characteristic scale of masses is 3×10^3 times larger than that of QCD. With this last remark in mind, we may therefore adopt a bag[12,13,14,15] point of view in treating the compositeness of ϕ_0 as it relates to Δx. We find it convenient to use the M.I.T. version[12,13] of the bag approach to hadron dynamics.

More specifically, we follow the recent formulation of Johnson[12] of the M.I.T. bag and view the QHCD vacuum as composed of Wigner-Seitz cells on the surfaces of which the M.I.T. bag heavy color confining conditions are satisfied. The constituents of ϕ_0 are confined in such a Wigner-Seitz cell of effective radius R_{HC}. Outside of the cell (or box), ϕ_0 looks fundamental -- inside, it looks composite. Further, inside of this cell, QHCD is perturbative. While this perturbative regime is interesting in itself, here, we work to zeroth order in the respective small perturbative effects.

To make contact with Δx in (40), we proceed as follows. We suppose that the sets in $\mathcal{O}$ are chosen so that if $U_j \varepsilon \mathcal{O}$ and $\bar{x}_j = (\bar{t}_j, \vec{\bar{x}}_j)$ is at the center of U_j then

$$U_j = \left[\bar{t}_j - \frac{1}{2}(\Delta x)^{1/4},\ \bar{t}_j + \frac{1}{2}(\Delta x)^{1/4}\right] \times V(\vec{\bar{x}}_j) \qquad (41)$$

where $V(\vec{\bar{x}}_j)$ is the respective Wigner-Seitz cell with $\vec{\bar{x}}_j$ as its geometrical center point and with volume $(\Delta x)^{3/4}$. When[16]

$$(\Delta x)^{3/4} = \frac{4}{3}\pi R_{HC}^3 \quad , \qquad (42)$$

we may identify the Wigner-Seitz cells $\{V(\vec{x}_j)\}$ with the respective QHCD vacuum Wigner-Seitz cells of Johnson. Since we are working to zeroth order in the perturbative effects within these cells, we ignore all dynamics within a single such cell. Thus, we may not resolve ϕ_0 beyond the value specified for Δx in (42); for, in doing the functional integrals in (7)-(40), we have taken, for example,

$$\int d^4x\phi_0^4(x) \to \Delta x \sum_{\{\bar{x}_j\}} \phi_0^4(\bar{x}_j) \tag{43}$$

so that using Δx smaller than $(4\pi R_{HC}^3/3)^{4/3}$ would imply that ϕ_0 looks fundamental inside of the respective Wigner-Seitz cells and would involve dynamics inside such cells -- both of which are excluded here. Thus, (43) represents the appropriate lower limiting value of Δx, i.e., $\Delta x = (4\pi R_{HC}^3/3)^{4/3}$.

With this last identification we have

$$\begin{aligned} iZ(J) &= \ln\int \mathcal{D}\kappa\mathcal{D}\rho\ \exp\left\{i\int d^4x\left[\frac{1}{2}\left(\partial_\mu\kappa\partial^\mu\kappa - 2M_1^2\kappa^2\right) + J\kappa\right.\right. \\ &\qquad \left.\left. + \rho\kappa + \rho^2\sqrt{\pi}\left(\frac{4\pi}{3}\right)^{2/3} R_{HC}^2/(-8hi)^{1/2} + h\lambda^4\right]\right\} \\ &= \ln\int\mathcal{D}\kappa\ \exp\left\{i\int d^4x\left[\frac{1}{2}\left(\partial_\mu\kappa\partial^\mu\kappa - \left(2M_1^2\right.\right.\right.\right. \\ &\qquad \left.\left.\left.\left. + \frac{(-2hi)^{1/2}}{\sqrt{\pi}}\frac{1}{\left(\frac{4\pi}{3}\right)^{2/3}R_{HC}^2}\right)\kappa^2\right) + J\kappa + h\lambda^4\right]\right\} \quad . \end{aligned} \tag{44}$$

From (44), we obtain our basic result: the pole of the composite Higgs field is at

$$2M_1^2 + \frac{(-2hi)^{1/2}}{\sqrt{\pi}} \frac{1}{\left(\frac{4\pi}{3}\right)^{2/3} R_{HC}^2} = m_H^2 + \frac{(1-i)}{\sqrt{2}} \frac{m_H}{M_W} \frac{\sqrt{\alpha}}{\sin\theta_W} \frac{1}{\left(\frac{4\pi}{3}\right)^{2/3} R_{HC}^2}$$

$$= m_H^2 + \frac{(1-i)}{\sqrt{2}} \frac{m_H}{M_W} \frac{\sqrt{\alpha}}{\sin\theta_W} \Lambda_{HC}^2 \quad , \tag{45}$$

where α is the fine structure constant and $\Lambda_{HC}^2 \equiv 1/\left((4\pi/3)^{2/3} R_{HC}^2\right)$. The corresponding decay width is

$$\Gamma_H = \sqrt{2}\left[\left(m_H^4 + \frac{\sqrt{2} m_H^3 \sqrt{\alpha} \Lambda_{HC}^2}{M_W \sin\theta_W} + \frac{m_H^2 \alpha \Lambda_{HC}^4}{M_W^2 \sin^2\theta_W}\right)^{1/2} - m_H^2 - \frac{m_H \sqrt{\alpha} \Lambda_{HC}^2}{\sqrt{2} M_W \sin\theta_W}\right]^{1/2} \quad . \tag{46}$$

From Refs. 12 and 13, the effective radius[17] of the appropriate Wigner-Seitz cells for QCD is $R_C \doteq 2.2/\text{GeV}$. From Ref. 2 the corresponding value of R_{HC} is $R_{HC} \doteq R_C/(3 \times 10^3) \doteq .73/\text{TeV}$. Thus, as expected,[2] $\Lambda_{HC} \cong 1$ TeV/c. With $\Lambda_{HC} = 1$ TeV/c and $\sin^2\theta_W = 0.236$, the decay widths of the physical Higgs field corresponding to $m_H = 5M_W = 383$ GeV, 50 M_W, and 500 M_W are easily seen to be 0.663 TeV, 1.34 TeV, and 1.59 TeV, respectively. In the limit $m_H \to \infty$, this contribution to the Higgs width equals 1.62 TeV.

Our message is quite clear: composite Higgs fields of the type considered here will be very broad affairs[18] -- quite a challenge for experimentalists!

IV. Discussion

We have found that the QHCD theory, taken together with the recent M.I.T. bag ideas of Johnson, permits one to compute the precise relationship between the heavy Higgs particle decay width Γ_H and its mass m_H in the standard $SU_2 \times U_1$ model. The question[19] which most naturally arises is "To what does this composite heavy Higgs particle decay?"

To answer this latter question, we simply recall that the degrees of freedom of the effective Higgs particle theory on scales smaller than the size of the Johnson-QHCD-vacuum-Wigner-Seitz cell were suppressed. Thus, the Higgs field instability is to these degrees of freedom. Since these degrees of freedom are confined, the decay energy of the Higgs particle will ultimately materialize as both ordinary hadrons and presumably new hadrons, assuming there are new absolutely stable heavy color, color singlet hadrons of appropriate mass generated by QHCD.

It should be understood that we do not claim to be the first to have pointed out that a heavy Higgs particle could have a large width of the same size as its mass.[2,18] Rather, what we have reported here is, to repeat somewhat, a calculation of the detailed relationship between the heavy Higgs mass m_H, the fine structure constant α, $\sin^2\theta_W$, and the heavy color scale Λ_{HC}. Such a calculation has not appeared elsewhere.[20]

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APPENDIX

Evaluation of R_n

We desire to evaluate the quantity

$$R_n = \prod_{j=1}^{n} \frac{\int_{-\infty}^{\infty} d\phi_{0j} \int_{-\infty}^{\infty} d\sigma_j \exp\left\{i\left\{\Delta x\left(\sigma_j^2 + \sqrt{h}\sigma_j\phi_{0j}^2\right) - \alpha_j\sigma_j\right\}\right\}}{\int_{-\infty}^{\infty} d\phi_{0j} \int_{-\infty}^{\infty} d\sigma_j \exp\left\{i\left\{\Delta x\left(\sigma_j^2 + \sqrt{h}\sigma_j\phi_{0j}^2\right)\right\}\right\}} \quad . \qquad (A.1)$$

We first note that the σ_j integrals are straightforward. For the numerator of (A.1) we have

$$\int_{-\infty}^{\infty} d\sigma_j \exp\left\{i\left\{\Delta x\left(\sigma_j^2 + \sqrt{h}\sigma_j\phi_{0j}^2\right) - \alpha_j\sigma_j\right\}\right\}$$

$$= \int_{-\infty}^{\infty} d\sigma_j \exp\left\{i\left\{\Delta x\left(\sigma_j + \frac{1}{2}\left(\sqrt{h}\phi_{0j}^2 - \alpha_j/\Delta x\right)\right)^2 - \frac{\Delta x}{4}\left(\sqrt{h}\phi_{0j}^2 - \alpha_j/\Delta x\right)^2\right\}\right\} \qquad (A.2)$$

$$= \sqrt{\frac{\pi}{-i\Delta x}} \exp\left\{\frac{-i\Delta x}{4}\left(h\phi_{0j}^4 - 2\sqrt{h}\phi_{0j}^2\alpha_j/\Delta x + \alpha_j^2/(\Delta x)^2\right)\right\} \quad .$$

For the σ_j integral in the denominator, simply set $\alpha_j = 0$ in (A.2). Thus, on introducing (A.2) into (A.1) we obtain

$$R_n = \prod_{j=1}^{n} \frac{\int_{-\infty}^{\infty} d\phi_{0j} \exp\left\{\frac{-i\Delta x}{4}\left(h\phi_{0j}^4 - 2\sqrt{h}\phi_{0j}^2\alpha_j/\Delta x + \alpha_j^2/(\Delta x)^2\right)\right\}}{\int_{-\infty}^{\infty} d\phi_{0j} \exp\left\{\frac{-i\Delta x}{4} h\phi_{0j}^4\right\}} \quad . \qquad (A.3)$$

To proceed, we next observe that

$$N(\alpha_j) \equiv \int_{-\infty}^{\infty} d\phi_{0j} \exp\left\{\frac{-i\Delta x}{4}\left(h\phi_{0j}^4 - 2\sqrt{h}\phi_{0j}^2\alpha_j/\Delta x\right)\right\}$$

$$= \frac{1}{2}\left(\frac{\alpha_j}{-\Delta x\sqrt{h}/2}\right)^{1/2} \exp\left\{i\alpha_j^2/(8\Delta x)\right\} K_{1/4}\left(i\alpha_j^2/\left((-)^2 8\Delta x\right)\right) \quad , \qquad (A.4)$$

where $K_{1/4}$ is the modified Bessel function of the second kind of order 1/4. Thus, the numerator integral in (A.3) is now known. To obtain the denominator integral in (A.3), we simply take the limit $\alpha_j \to 0$ of $N(\alpha_j)$ in (A.4). We find

$$\int_{-\infty}^{\infty} d\phi_{0j} \exp\left\{\frac{-i\Delta x}{4} \phi_{0j}^4\right\} = \frac{\Gamma(1/4)}{2i^{1/4}} \frac{1}{(\Delta x h/4)^{1/4}} \quad . \tag{A.5}$$

Here, we have used the result that

$$K_{1/4}(z) \xrightarrow[z\to 0]{} \Gamma(1/4) 2^{-3/4} z^{-1/4} \tag{A.6}$$

for $|\text{Arg } z| < \pi$. In (A.2), (A.4) and (A.5), the branch of the radicals is always such that

$$-\pi < \text{Arg } z \leq \pi \tag{A.7}$$

for the complex number z.

To complete the evaluation of R_n, we now take limit $\Delta x \downarrow 0$ in $N(\alpha_j)$. We find

$$N(\alpha_j) \xrightarrow[\Delta x \downarrow 0]{} \frac{1}{2}\left(\frac{-4\pi}{i\alpha_j \sqrt{h}/2}\right)^{1/2} = \left(\frac{-2\pi}{i\alpha_j \sqrt{h}}\right)^{1/2} \quad , \tag{A.8}$$

since

$$K_{1/4}(z) \xrightarrow[|z|\to\infty]{} \sqrt{\frac{\pi}{2z}}\, e^{-z} \quad . \tag{A.9}$$

On introducing (A.5) and (A.8) into (A.3), we find

$$\begin{aligned} R_n \xrightarrow[\Delta x \downarrow 0]{} & \prod_{j=1}^{n} \frac{\left(-\pi/(i\alpha_j \sqrt{h}/2)\right)^{1/2} \exp\left\{-i\alpha_j^2/(4\Delta x)\right\}}{\Gamma(1/4)/\left(2i^{1/4}(\Delta x h/4)^{1/4}\right)} \\ & = \prod_{j=1}^{n} \frac{2i^{3/4}\sqrt{\pi/\alpha_j}\,(\Delta x)^{1/4}}{\Gamma(1/4)} \exp\left\{-i\alpha_j^2/(4\Delta x)\right\} \quad . \end{aligned} \tag{A.10}$$

This is the desired result. It agrees with (36).

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