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CHIRAL DYNAMICS INCLUDING VECTOR MESONS APPLIED TO THE DECAYS OF THE τ LEPTON*

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ABSTRACT

The decays of the τ-lepton up to four pions are investigated within the framework of phenomenological Lagrangians. These Lagrangians are invariant under the chiral group SU(2) × SU(2) which is nonlinearly realized on the pion field alone. The vector mesons ρ and A1 are introduced as gauge bosons of the chiral group. Due to the nonlinear realization, processes with different numbers of pions are interrelated. Our results are compared to the existing data.

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INTRODUCTION

Current algebra[1] which is the basis for chiral dynamics[2] relates processes with a different number of pions. As is well known, the predictions of current algebra are most easily obtained by the method of phenomenological Lagrangians in connection with nonlinear realizations of the chiral group.

The whole chiral group SU(2) × SU(2) can be nonlinearly realized on the pion field alone. This was the starting point of a previous work,[4] where the consequences of pion dynamics in τ-decays as derived from such a Lagrangian were discussed.

If we try to extend pion dynamics beyond the 'soft region' we have to deal with the resonances. Calculating total decay rates for the τ-lepton in Ref. 4 the ρ meson was included by hand via a form-factor wherever a two pion state with appropriate quantum numbers appeared. Within the framework of phenomenological Lagrangians the vector and axial vector mesons can however be introduced from the very beginning by gauging the pion Lagrangian. It is this dynamics that is used here to calculate τ-decays with up to four pions in the final state. The effect of the nonlinearity is that a set of processes given all processes with additional pions are predicted. In our case we can fix the parameters of the Lagrangian with τ-decays into two and three pions. The pion decay constant f_π which also enters can be thought of as determined e.g. in pion decay.

The paper is organized as follows: We first recall the general formulae for τ-decays. The second chapter contains a discussion of the interaction Lagrangian and the matrix elements for the decays under consideration. In the third chapter we present our results on the decay spectra and rates. The conclusions and two appendices follow. Appendix A contains the derivation of the Lagrangian and Appendix B the explicit expressions for the Lagrangian and the currents as used in this paper.

1. τ Lepton Decays

Heavy lepton decays have been of interest under various points of view.[3] Let us first write down the T-matrix element for τ decays into hadrons which according to standard weak interaction theory[5] is given by the product of the matrix elements of the leptonic and the hadronic currents.

$$T = \frac{G}{\sqrt{2}} \cdot \cos\theta_c \cdot \bar{u}(p')\gamma^\mu(1 - \gamma_s)u(p)J^h_\mu(q_1 \ldots q_n) \tag{1.1}$$

Using these matrix elements the decay rates $\tau \to \nu_\tau$ + n pions are calculated to be:

$$\Gamma_n = \frac{1}{2\cdot m_\tau} \int dLips_{n+1}(p;p',q_1 \ldots q_n) \frac{1}{2} \sum_{spins} |T|^2 \tag{1.2}$$

where $dLips_{n+1}$ denotes the invariant phase space integral[6] over the neutrino momentum p' and the n pion momenta $q_1 \ldots q_n$ · p is the momentum of the decaying τ lepton, m_τ its mass.

The integration over the pion momenta in (1.2) can be carried out independently and due to current conservation the result can be written in the form:

$$\int dLips_n(Q;q_1 \ldots q_n) J^h_\mu J^h_\nu = \frac{a_n}{6\pi} (Q_\mu Q_\nu - g_{\mu\nu}Q^2) \tag{1.3}$$

where Q denotes the total hadron momentum. $Q = q_1 + q_2 + \ldots + q_n$. After integrating over the ν_τ momentum p' the final result can be expressed as follows, with $y = \frac{Q^2}{m_\tau^2}$

$$\gamma_n = \frac{\Gamma_n}{\Gamma_L} = \int_o^1 dy(1 - y)^2(1 + 2y) \cdot a_n \tag{1.4}$$

where we have used the pure leptonic decay rate $\Gamma_L = \frac{G^2 m_\tau^5}{192\pi^3}$ for normalization.[7] Formula (1.4) reduces our problem to calculating the various functions a_n:

$$a_n = - \frac{2\pi}{Q^2} \int dLips_n(Q;q_1 \ldots q_n)(J_\mu^h)^2 \tag{1.5}$$

2. Chiral Dynamics With Vector Meson and Pion Interactions

From chiral gauge invariance the following Lagrangian for the interaction of massless pions and the vector mesons has been derived in Appendix A (A25, A28)

$$\mathscr{L} = L_{YM} + L_1 + \kappa \cdot f \cdot \left\{ (\vec{\rho}\,'_{\mu\nu}(\vec{p}^{\mu} \times \vec{p}^{\nu}) + (2\alpha - \tfrac{1}{2}) \cdot f \cdot (\vec{p}_\mu \times \vec{p}_\nu)^2 \right\} + L_M \tag{2.1}$$

Where L_{YM} is the standard SU(2) × SU(2) Yang Mills Lagrangian for the gauge bosons. L_M breaks the gauge invariance (but not global SU(2) × SU(2)) and makes the gauge bosons massive. $L_1 = \frac{f^2}{2} \eta^2 (\vec{p}_\mu)^2$ contains the kinetic term for the pion field. The rest are interaction terms between pions and vector mesons. The explicit form of (2.1) we are concerned with here is given in Appendix B (B1).

The Lagrangian (2.1) yields the vector current:

$$\vec{V}_\mu = \frac{m_\rho^2}{f} \cdot \vec{\rho}_\mu \tag{2.2}$$

which is proportional to the field of the ρ meson and the axial vector current:

$$\vec{A}_\mu = -\frac{m_\rho^2}{f}\vec{a}_\mu = -\frac{m_\rho^2}{f}\hat{\vec{a}}_\mu + f_\pi \partial_\mu \vec{\pi} \tag{2.3}$$

where $\hat{a}_\mu$ is the field of the physical A_1 meson. These currents are conserved due to the equations of motion corresponding to (2.1).

The matrix elements of the hadronic currents can be calculated from (2.2) and (2.3) in presence of the interactions given by (2.1). For any given process we have to sum <u>all</u> the tree diagrams. This guarantees a result that is independent of the parametrization[10] as well as current conservation. That means our current matrix elements are given by the tree diagrams of the perturbation expansion of:

$$\langle f|\left((\vec{V}_\mu + \vec{A}_\mu)e^{i\int L_I dx}\right)_t|0\rangle \tag{2.4}$$

L_I is the interaction part of the Lagrangian (2.1).

In the case n=1, that is $\tau^+ \to \nu_\tau \pi^+$ we get from the axial current (2.3) with our normalization:

$$J_\mu^1(Q) = -i\sqrt{2}\, f_\pi \cdot Q_\mu \tag{2.5}$$

In the case n=2 for the decay $\tau^+ \to \nu_\tau \pi^+ \pi^o$ we have:

$$J_\mu^2(q^+,q_o;Q) = \sqrt{2}\,\frac{m^2}{(q^+ + q^o)^2 - m^2 + i\varepsilon}(q^+ - q^o)_\mu\left(1 + \frac{\kappa - \frac{1}{2}}{2m^2}Q^2\right) \tag{2.6}$$

In the case n=3 there are two processes

$$\tau^+ \to \nu_\tau \pi^+ \pi^o \pi^o \text{ and } \tau^+ \to \nu_\tau \pi^+ \pi^+ \pi^-$$

with the same amplitude

$$J_\mu^3\left((q^+ q_1^o), q_2^o; Q\right) = J_\mu^3\left((q^- q_1^+), q_2^+; Q\right)$$

And in the case n=4 we have two different amplitudes for the processes

$$\tau^+ \to \nu_\tau \pi^+ \pi^o \pi^o \pi^o \text{ and } \tau^+ \to \nu_\tau \pi^+ \pi^- \pi^+ \pi^o$$

The amplitudes for the decays into three and four pions turn out to be rather lengthy and we have given the explicit expressions in Appendix B (B3 - B5).

3. Decay Spectra and Decay Rates

We now compute the decay spectra and decay rates resulting from the amplitudes presented in the last chapter. Our results are compared to the experimental data in Figs. 1-3 and in a table. The decay rate for the decay $\tau^+ \to \nu_\tau \pi^+$ is the usual one

$$\gamma_1(\pi^+) = \frac{3}{2} \left(\frac{4\pi f_\pi}{m_\tau}\right)^2 \cdot \cos\theta_c \tag{3.1}$$

In calculating the two pion rates we first replace the ρ meson propagator by an appropriate form factor.

$$\frac{m^2}{(q^+ + q^o)^2 - m^2 + i\varepsilon} \to \frac{m^2 - im\Gamma_\rho}{(q^+ + q^o)^2 - m^2 + im\Gamma_\rho} \tag{3.2}$$

We then study the dependence of the decay spectrum on the parameter κ and find that the data are represented very well if we choose $\kappa = \frac{1}{2}$ (Fig. 1). This choice also simplifies the matrix elements for the three and four pion final states. If we had used the narrow width approximation for the form factor (3.2) that is the replacement:

$$\frac{m^2(m^2 + \Gamma_\rho^{\ 2})}{(Q^2 - m^2) + m^2\Gamma_\rho^{\ 2}} \to \frac{\pi \cdot m^3}{\Gamma_\rho} \delta(Q^2 - m^2) \tag{3.3}$$

the result for the decay rate would have been 7% higher which is within the experimental error. We take this as justification for the use of the narrow width approximation whenever it seems appropriate.

The 3π matrix elements (B3) for $\kappa = \frac{1}{2}$ give rise to the decay spectra shown in Figs. 2 and 3. The narrow width approximation has been used only in the calculation of the interference term between the two ρ's in the amplitude.

The width Γ_A of the A_1 meson has been introduced in the same way as for the ρ in (3.2). We now try to determine the parameter α. It is clear from Fig. 5c, the dominant diagram where α enters in the 3π matrix element, that the variation of α tends to have a similar effect as a variation of Γ_A. This ambiguity cannot be resolved with present data. We therefore choose to let all that variation be due to Γ_A and set $\alpha = 0$. That again simplifies then the 4π amplitudes. In comparing our spectra with the data we have to be careful since in[15] the data are given for the selected mode $\tau \to \rho\pi$ whereas in[18] all three pion final states are taken. It turns out however that for all practical purposes the spectra for the two cases are equal due to interference of the diagrams 5b, 5b', 5c with 5d. In Figs. 2 and 3 we also compare this result to the previous one[4] where there was no A_1 meson. (Dashed-dotted line in Figs. 2, 3.)

The decay spectra resulting from the 4 pion matrix elements (B4,5) are shown in Fig. 4. It turns out that the major contribution to the decay spectrum arises from the graphs 5g and 5i which have an A_1 in it. The rest is small and is indicated in Fig. 4 with a dashed-dotted line. The spectra for (B4) and (B5) are practically the same. The contribution of diagram 5j which is in the amplitude (B5) only is negligible since it has a treshold factor $\left(1 - \frac{4m^2}{Q^2}\right)^{3/2}$ that starts at the very end of the available phase space.

CONCLUSIONS

We have calculated spectra and rates for τ-decays up to four pions in the final state. The vector mesons ρ and A_1 enter the dynamics via gauge couplings to the pions.

Usual weak interaction theory predicts the decay rate into one pion. The two pion sector clearly shows the ρ-meson and prediction and experimental data agree very well if the parameter κ is chosen to be $\kappa \approx \frac{1}{2}$. The situation in the three pion sector is less clear. The experimental rates have very large errors and even if adjusted to the area of the predicted spectra one cannot find compelling evidence for the A_1-meson. Interference among the various amplitudes makes the mode $\tau \to \nu\rho\pi$ the real dominant one. Varying the parameter α that appears in the three pion sector has roughly the same effect as varying the A_1 width. We therefore chose $\alpha = 0$ in all further calculations. The four pion sector is similar to the three pion sector in the sense that the dominant decay amplitude is $\tau \to \nu A_1 \pi$. If therefore the τ-decay spectrum and rate into four pions would have been measured there would be clear evidence in favor or against an A_1, since the contribution including an A_1 is at least three times bigger than in a model without an A_1.[4]

Our calculation accounts for approximately 78% of all decays depending on the normalization to $\Gamma(\tau \to \nu e\bar{\nu})$ which we take to be 17%. In addition 5% are expected for the Cabibbo suppressed decays, leaving about 17% , a few percent of which can be attributed to decays into more than four pions.

The question could be asked whether the remaining 10-15% are due to some other mechanisms like second class currents. But we have to keep in mind, however, that a change in the leptonic width from 17% to 20% would leave no room for such other effects.

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APPENDIX A

The formalism of nonlinear realizations and phenomenological Lagrangians has been worked out in great detail.[8,9,10] We give here a short derivation of the formulas necessary. The group of interest is SU(2) × SU(2) with the general element b × d; b and d are elements of an SU(2) group. The diagonal elements U × U form a subgroup, the isospin group. An arbitrary element can be decomposed into a product of a chiral element $V \times V^{-1}$ and an element of the isospin subgroup:

$$b \times d = (V \times V^{-1})(U \times U) \tag{A1}$$

This equation can be solved for V and U and we obtain:

$$V^2 = b \cdot d^{-1}; \quad U = V \cdot d = V^{-1} \cdot b \tag{A2}$$

To find a nonlinear realization of the chiral transformations we have to study the multiplication law of the group $g^o \cdot g = g'$ in the decomposition (A1). The effect of an isospin transformation g^o on g is:

$$(U_0 \times U_0)(V \times V^{-1})(U \times U) = (V' \times V'^{-1})(U' \times U') \tag{A3}$$

Using (A2) we obtain:

$$V' = U_0 V U_0^{-1}, \quad U' = U_0 U \tag{A4}$$

If g^o is a chiral element we find

$$(V_0 \times V_0^{-1})(V \times V^{-1})(U \times U) = (V \times V^{-1})(U' \times U') \tag{A5}$$

where

$$V'^2 = V_0 V^2 V_0 \text{ and } U' = \tilde{U} U \tag{A6}$$

We have used the abbreviation

$$\tilde{U} = V' V_0^{-1} V^{-1} = V'^{-1} V_0 V \tag{A7}$$

which should indicate that $\tilde{U}$ is an element of an SU(2) group.

To obtain an explicit form of a nonlinear transformation law we have to parametrize the SU(2) group elements. We choose the exponential parametrization, the final result of the computation of S-matrix elements is independent of the choice of the parametrization.[11]

$$V = e^{i\vec{\xi}\frac{\vec{\tau}}{2}} \qquad \vec{\tau} \text{ are the Pauli matrices} \tag{A8}$$

from (A4) and (A6) follows

$$\vec{\xi}'\vec{\tau} = U_0\vec{\xi}\,\vec{\tau}\,U_0^{-1} \tag{A9}$$

$$e^{i\vec{\xi}'\vec{\tau}} = V_0\, e^{i\vec{\xi}\,\vec{\tau}}\, V_0 \tag{A10}$$

which gives under infinitesimal transformations

$$U_0 = 1 + i\vec{\alpha}\frac{\vec{\tau}}{2}, \quad V_0 = 1 + i\vec{\beta}\frac{\vec{\tau}}{2}$$

the usual isospin transformations

$$\delta^{iso}\vec{\xi} = -(\vec{\alpha}\times\vec{\xi}) \tag{A11}$$

and the nonlinear chiral transformations

$$\delta^{chi}\vec{\xi} = \vec{\beta} + \left[\vec{\xi}\times(\vec{\xi}\times\vec{\beta})\right]\left\{\frac{1}{|\vec{\xi}|^2} - \frac{\cot|\vec{\xi}|}{|\vec{\xi}|}\right\} \tag{A12}$$

With the help of the nonlinear transforming $\vec{\xi}$ it is possible to associate with any linear representation of the isospin group

$$\psi' = \mathcal{D}(U_0)\,\psi \tag{A13}$$

a nonlinear realization of the whole chiral group and any nonlinear realization of the full group which transforms linearly under isospin transformations can be parametrized that it transforms in the same way as ψ.[11]

For chiral transformations (A13) is extended to

$$\psi' = \mathcal{D}(\tilde{U})\psi \qquad \text{with } \tilde{U} \text{ given in (A7)} \tag{A14}$$

In order to construct Lagrangians we now apply this to the nonlinearly transforming object $\partial_\mu\vec{\xi}$ and want our isospin and chiral transformations to be space-time dependent: $\vec{\alpha} = \vec{\alpha}(x)$; $\vec{\beta} = \vec{\beta}(x)$. From (A9) we get the isospin transformations of $V\partial_\mu V^{-1}$ and $V^{-1}\partial_\mu V$ (both expressions if expanded start $\sim\partial_\mu\vec{\xi}$)

$$\begin{aligned} V'\partial_\mu V'^{-1} &= U_0\left\{V\,(\partial_\mu + U_0^{-1}\partial_\mu U_0)\;V^{-1}\right\}U_0^{-1} + U_0\partial_\mu U_0^{-1} \\ V'^{-1}\partial_\mu V' &= U_0\left\{V^{-1}\,(\partial_\mu + U_0^{-1}\,\partial_\mu U_0)\,V\right\}U_0^{-1} + U_0\partial_\mu U_0^{-1} \end{aligned} \tag{A15}$$

And from (A10) and (A7) for chiral transformations

$$\begin{aligned} V'\partial_\mu V'^{-1} &= \tilde{U}\left\{V(\partial_\mu + V_0\partial_\mu V_0^{-1})\;V^{-1}\right\}\tilde{U}^{-1} + \tilde{U}\partial_\mu\tilde{U}^{-1} \\ V'^{-1}\partial_\mu V' &= \tilde{U}\left\{V^{-1}(\partial_\mu + V_0^{-1}\partial_\mu V_0)V\right\}\tilde{U}^{-1} + \tilde{U}\partial_\mu\tilde{U}^{-1} \end{aligned} \tag{A16}$$

This shows that by introducing as usual covariant derivatives

$$V\left(\partial_\mu - if(\vec{\rho}_\mu + \vec{a}_\mu)\frac{\vec{\tau}}{2}\right)V^{-1}$$

and

$$V^{-1}\left(\partial_\mu - if(\vec{\rho}_\mu - \vec{a}_\mu)\frac{\vec{\tau}}{2}\right)V$$

these quantities transform like connections from which follows that their difference transforms like a tensor and their sum again like a connection.

The gauge fields $\vec{\rho}_\mu$ and $\vec{a}_\mu$ transform as usual under infinitesimal gauge transformations:

$$\begin{aligned} \delta^{iso}\vec{\rho}_\mu &= -(\vec{\alpha}\times\vec{\rho}_\mu) + \frac{1}{f}\partial_\mu\vec{\alpha} \\ \delta^{iso}\vec{a}_\mu &= -(\vec{\alpha}\times\vec{a}_\mu) \\ \delta^{chi}\vec{\rho}_\mu &= (\vec{\beta}\times\vec{a}_\mu) \\ \delta^{chi}\vec{a}_\mu &= (\vec{\beta}\times\vec{\rho}_\mu) - \frac{1}{f}\partial_\mu\vec{\beta} \end{aligned} \tag{A17}$$

We define the tensor $\vec{p}_\mu$:

$$-i\frac{\vec{\tau}}{2}\vec{p}_\mu = \frac{1}{2f}\left\{V\left(\partial_\mu - if(\vec{\rho}_\mu + \vec{a}_\mu)\frac{\vec{\tau}}{2}\right)V^{-1} - V^{-1}\left(\partial_\mu - if(\vec{\rho}_\mu - \vec{a}_\mu)\frac{\vec{\tau}}{2}\right)V\right\} \tag{A18}$$

and the connection $\vec{v}_\mu$:

$$-i\frac{\vec{\tau}}{2}\vec{v}_\mu = \frac{1}{2f}\left\{V\left(\partial_\mu - if(\vec{\rho}_\mu + \vec{a}_\mu)\frac{\vec{\tau}}{2}\right)V^{-1} + V^{-1}\left(\partial_\mu - if(\vec{\rho}_\mu - \vec{a}_\mu)\frac{\vec{\tau}}{2}\right)V\right\} \tag{A19}$$

or explicitly

$$\begin{aligned}\vec{p}_\mu = \frac{1}{f}\,\partial_\mu\vec{\xi} + \frac{1}{f}\left(\vec{\xi}\times(\vec{\xi}\times\partial_\mu\vec{\xi})\right)\frac{|\vec{\xi}| - \sin|\vec{\xi}|}{|\vec{\xi}|^3} + \vec{a}_\mu + \vec{\xi}\times(\vec{\xi}\times\vec{a}_\mu)\,\frac{1-\cos|\vec{\xi}|}{|\vec{\xi}|^2} \\ - (\vec{\xi}\times\vec{\rho}_\mu)\,\frac{\sin|\vec{\xi}|}{|\vec{\xi}|}\end{aligned} \tag{A20}$$

$$\vec{v}_\mu = \frac{1}{f}\,(\vec{\xi}\times\partial_\mu\vec{\xi})\,\frac{\cos|\vec{\xi}| - 1}{|\vec{\xi}|^2} + \vec{\rho}_\mu - (\vec{\xi}\times\vec{a}_\mu)\,\frac{\sin|\vec{\xi}|}{|\vec{\xi}|} + \vec{\xi}\times(\vec{\xi}\times\vec{\rho}_\mu)\,\frac{1-\cos|\vec{\xi}|}{|\vec{\xi}|^2} \tag{A21}$$

From $\vec{v}_\mu$ we can construct the tensor

$$\vec{v}_{\mu\nu} = \partial_\mu\vec{v}_\nu - \partial_\nu\vec{v}_\mu + f(\vec{v}_\mu\times\vec{v}_\nu)$$

and define

$$\vec{\rho}\,'_{\mu\nu} = \vec{v}_{\mu\nu} + f(\vec{p}_\mu\times\vec{p}_\nu)$$

which can be expressed more easily in the original fields $\vec{\xi}$, $\vec{a}_\mu$, $\vec{\rho}_\mu$

$$\vec{\rho}\,'_{\mu\nu} = \vec{\rho}_{\mu\nu} + \frac{1-\cos|\vec{\xi}|}{|\vec{\xi}|^2}\,\vec{\xi}\times(\vec{\xi}\times\vec{\rho}_{\mu\nu}) - \frac{\sin|\vec{\xi}|}{|\vec{\xi}|}\,(\vec{\xi}\times\vec{a}_{\mu\nu}) \tag{A22}$$

where

$$\vec{\rho}_{\mu\nu} = \partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu + f(\vec{\rho}_\mu\times\vec{\rho}_\nu) + f(\vec{a}_\mu\times\vec{a}_\nu)$$

and

$$\vec{a}_{\mu\nu} = \partial_\mu\vec{a}_\nu - \partial_\nu\vec{a}_\mu + f(\vec{\rho}_\mu\times\vec{a}_\nu) + f(\vec{a}_\mu\times\vec{\rho}_\nu)$$

We just remark that $\vec{\rho}\,'_{\mu\nu}$ can also be obtained in the following way

$$\vec{\rho}\,'_{\mu\nu}\frac{\vec{\tau}}{2} = \frac{1}{2}\left\{V(\vec{\rho}_{\mu\nu}\frac{\vec{\tau}}{2} + \vec{a}_{\mu\nu}\frac{\vec{\tau}}{2})V^{-1} + V^{-1}(\vec{\rho}_{\mu\nu}\frac{\vec{\tau}}{2} - \vec{a}_{\mu\nu}\frac{\vec{\tau}}{2})V\right\} \tag{A23}$$

and that there is another tensor

$$\vec{a}'_{\mu\nu}\frac{\vec{\tau}}{2} = \frac{1}{2}\left\{V(\vec{\rho}_{\mu\nu}\frac{\vec{\tau}}{2} + \vec{a}_{\mu\nu}\frac{\vec{\tau}}{2})V^{-1} - V^{-1}(\vec{\rho}_{\mu\nu}\frac{\vec{\tau}}{2} - \vec{a}_{\mu\nu}\frac{\vec{\tau}}{2})V\right\}$$

$$\vec{a}'_{\mu\nu} = \vec{a}_{\mu\nu} + \frac{1-\cos|\vec{\xi}|}{|\vec{\xi}|^2}\vec{\xi}\times(\vec{\xi}\times\vec{a}_{\mu\nu}) - \frac{\sin|\vec{\xi}|}{|\vec{\xi}|}(\vec{\xi}\times\vec{\rho}_{\mu\nu}) \tag{A24}$$

We can now write down the most general gauge invariant Lagrangian relevant for our processes. (The squares of (A23) and (A24) are not relevant up to four pions.)

$$\mathscr{L} = L_{YM} + L_1 + \kappa \cdot f\left\{\vec{\rho}'_{\mu\nu}\cdot(\vec{p}^{\mu}\times\vec{p}^{\nu}) + (2\alpha - \frac{1}{2})f(\vec{p}_{\mu}\times\vec{p}_{\nu})^2\right\} \tag{A25}$$

$$L_{YM} = -\frac{1}{4}(\partial_{\mu}\phi^a_{\nu} - \partial_{\nu}\phi^a_{\mu} + f\cdot c^{abc}\phi_{\mu b}\phi_{\nu c})^2 \tag{A26}$$

where $\phi^{1,2,3}_{\mu} = \rho^{1,2,3}_{\mu}$ and $\phi^{4,5,6}_{\mu} = a^{1,2,3}_{\mu}$ and c^{abc} are the structure constants of SU(2) × SU(2) $c^{abc} = \varepsilon^{abc}$ if {a,b,c} = {1,2,3} etc.

$$L_1 = \frac{f^2}{2}\eta^2(\vec{p}_{\mu})^2 \tag{A27}$$

In order to apply this Lagrangian to physical processes we have to break the gauge invariance since our vector mesons should be massive. We do this by adding a mass term L_M to our Lagrangian (A25) that preserves global SU(2) × SU(2) symmetry.

$$L_M = \frac{1}{2}m^2(\vec{\rho}^{\,2}_{\mu} + \vec{a}^{\,2}_{\mu}) \tag{A28}$$

The next step is to diagonalize L_1 since there appears a term $\sim\vec{a}_{\mu}\partial^{\mu}\vec{\xi}$. We do this by redefining our field $\vec{a}_{\mu}$

$$\vec{a}_{\mu} = \hat{\vec{a}}_{\mu} - \frac{\eta^2 f}{[\eta^2 f^2 + m^2]}\partial_{\mu}\vec{\xi} \tag{A29}$$

which is already the most general possibility since additional terms would cancel out in the S-matrix.[10]

We require that vector meson dominance holds for the ρ meson. This means that in (A21) there should not appear a term $(\vec{\xi} \times \partial_\mu \vec{\xi})$ with the same quantum numbers in addition to $\vec{\rho}_\mu$ since $\vec{v}_\mu$ is the quantity which enters universally via the covariant derivative in all interactions especially in the nucleon system. This can be done by choosing $\eta^2 f^2 = m^2$ which also fixes the ratio of the masses of the vector mesons.

$$\frac{m_\rho^2}{m_{\hat{a}}^2} =: \frac{m^2}{\bar{m}^2} = \frac{1}{2} \qquad \text{(A30)}$$

The vector current resulting from our Lagrangian is

$$\vec{V}_\mu = \frac{m^2}{f} \cdot \vec{\rho}_\mu \qquad \text{(A31)}$$

and the axial vector current is

$$\vec{A}_\mu = - \frac{m^2}{f} \vec{a}_\mu = - \frac{m^2}{f} \vec{\hat{a}}_\mu + \frac{\eta^2}{2} \partial_\mu \vec{\xi} \qquad \text{(A32)}$$

From L_1 after shifting $\vec{a}_\mu$ (A29) we can identify the physical pion field $\frac{\eta}{\sqrt{2}} \vec{\xi} = \vec{\pi}$ and from (A32) applied to pion decay we conclude that $\frac{\eta}{\sqrt{2}} = f_\pi$ the pion decay constant (≈ 92 MeV).

APPENDIX B

In Appendix B we give the explicit expressions for the Lagrangian and the matrix elements relevant to τ-decays up to four pions. In our notation we supress the vector character of the fields; ρ, π, $\hat{a}$ is meant to be $\vec{\rho}$, $\vec{\pi}$, $\vec{\hat{a}}$. The Lagrangian (A25) reduces to:

$$\begin{aligned}
\mathscr{L}^{\text{relevant}} = & \frac{1}{2}(\partial_\mu \pi)^2 - \frac{1}{4}(\partial_\mu \rho_\nu - \partial_\nu \rho_\mu)^2 - \frac{1}{4}(\partial_\mu \hat{a}_\nu - \partial_\nu \hat{a}_\mu)^2 + \frac{1}{2} m^2 \rho^2 + \frac{1}{2}\bar{m}^2 \hat{a}^2 \\
& + \frac{1}{12} \cdot \frac{1}{f_\pi^2} (\pi \times \partial_\mu \pi)^2 \\
& + \frac{1}{3} \frac{f}{f_\pi} \Big(\pi \times (\pi \times \partial_\mu \pi)\Big) \hat{a}^\mu \\
& - 2f^2 \cdot f_\pi \cdot (\pi \times \rho_\mu) \hat{a}^\mu \\
& + f \cdot (\pi \times \partial_\mu \pi) \rho^\mu \\
& + f^2 \cdot (\pi \times \rho_\mu)^2 \\
& + \left[\kappa (2\alpha + \frac{1}{2}) - \frac{1}{4}\right] \frac{1}{16 f^2 f_\pi^4} (\partial_\mu \pi \times \partial_\nu \pi)^2 \\
& + \frac{\kappa}{4f \cdot f_\pi^3} (\partial_\mu \hat{a}_\nu - \partial_\nu \hat{a}_\mu) \Big(\pi \times (\partial^\mu \pi \times \partial^\nu \pi)\Big) \\
& + \frac{1}{4 \cdot f_\pi} (\partial_\mu \hat{a}_\nu - \partial_\nu \hat{a}_\mu) \Big((\rho^\mu \times \partial^\nu \pi) + (\partial^\mu \pi \times \rho^\nu)\Big) \\
& + \left[\kappa (2\alpha - \frac{1}{2}) + \frac{1}{4}\right] \frac{1}{4f \cdot f_\pi^3} (\partial_\mu \pi \times \partial_\nu \pi) \Big((\hat{a}^\mu \times \partial^\nu \pi) + (\partial^\mu \pi \times \hat{a}^\nu)\Big) \\
& - \frac{1}{2} f (\partial_\mu \rho_\nu - \partial_\nu \rho_\mu)(\rho^\mu \times \rho^\nu) \\
& + \left[\kappa + \frac{1}{2}\right] \frac{1}{2f_\pi} (\partial_\mu \rho_\nu - \partial_\nu \rho_\mu) \Big((\hat{a}^\mu \times \partial^\nu \pi) + (\partial^\mu \pi \times \hat{a}^\nu)\Big) \\
& + \frac{\kappa}{6f \cdot f_\pi^4} \Big(\pi (\partial_\mu \rho_\nu - \partial_\nu \rho_\mu)\Big) \Big(\pi (\partial^\mu \pi \times \partial^\nu \pi)\Big)
\end{aligned}$$

$$+ \frac{\kappa}{2f_\pi^2} \Big((\partial_\mu \rho_\nu - \partial_\nu \rho_\mu)\pi\Big)\Big((\rho^\mu \partial^\nu \pi) - (\partial^\mu \pi \rho^\nu)\Big)$$

$$+ \left[\kappa - \frac{1}{2}\right] \cdot \frac{1}{4f\ f_\pi^2} (\partial_\mu \rho_\nu - \partial_\nu \rho_\mu)(\partial^\mu \pi \times \partial^\nu \pi)$$

$$- \frac{\kappa}{12f \cdot f_\pi^4} (\partial_\mu \rho_\nu - \partial_\nu \rho_\mu)(\partial^\mu \pi \times \partial^\nu \pi)|\pi|^2$$

$$+ \frac{\kappa}{2f_\pi^2} (\partial_\mu \rho_\nu - \partial_\nu \rho_\mu)\Big(\rho^\nu (\pi \partial^\mu \pi) - \rho^\mu (\pi \partial^\nu \pi)\Big)$$

$$+ \left[\kappa - \frac{1}{2}\right] \cdot \frac{1}{4f_\pi^2} (\rho_\mu \times \rho_\nu)(\partial^\mu \pi \times \partial^\nu \pi)$$

$$- \frac{1}{8f_\pi^2} (\rho_\mu \times \partial_\nu \pi)\Big((\rho^\mu \times \partial^\nu \pi) - (\rho^\nu \times \partial^\mu \pi)\Big)$$

$$- \left[\kappa(2\alpha - \frac{1}{2})\right] \frac{1}{4f \cdot f_\pi^4} (\partial_\mu \pi \times \partial_\nu \pi)\Big(\rho^\mu (\pi \partial^\nu \pi) - \rho^\nu (\pi \partial^\mu \pi)\Big)$$

$$+ \kappa \cdot \alpha \cdot \frac{1}{2f \cdot f_\pi^4} \Big(\pi(\partial_\mu \pi \times \partial_\nu \pi)\Big)\Big((\rho^\mu \partial^\nu \pi) - (\rho^\nu \partial^\mu \pi)\Big) \qquad \text{(B1)}$$

The matrix element for the decay into two pions is:

$$J_\mu^2(q^+, q^o; Q) = \sqrt{2} \cdot m^2 \cdot (q^+ - q^o)_\mu \cdot P(q^+ q^o) \left\{ 1 + \frac{\kappa - \frac{1}{2}}{2m^2} Q^2 \right\} \qquad \text{(B2)}$$

where q^+, q^o, q^- are the pion momenta and Q is their sum.

$$P(q^+, q^o) = \frac{1}{(q^+ + q^o)^2 - m^2 + i\varepsilon}$$

The corresponding diagram is shown in Fig. 5a.

The matrix element for the decay into 3 pions is:

$$
\begin{aligned}
&J^3_\mu\Big((q^+ q^o_1),\ q^o_2;\ Q\Big) = J^3_\mu\Big((q^- q^+_1),\ q^+_2;\ Q\Big) \\
&= \frac{i\sqrt{2}}{f_\pi}\Bigg\{ -\frac{1}{2}\Big((q^-_\mu - \frac{Q_\mu}{Q^2}(Qq^-)\Big)(\kappa\alpha + \frac{3}{4}\kappa + \frac{1}{8}) \\
&\quad + \frac{\bar{m}^2}{Q^2 - \bar{m}^2 + i\varepsilon}\Bigg\{ \frac{1}{2}\Big(q^-_\mu - \frac{Q_\mu}{Q^2}(Qq^-)\Big)\Big(-\Big[\kappa\alpha + \frac{3}{4}\kappa + \frac{5}{8}\Big] + \frac{1}{m^2}(Qq^-)(\kappa\alpha - \frac{\kappa}{4} + \frac{1}{8})\Big) \\
&\qquad + \Big(q_{1\mu} - \frac{Q_\mu}{Q^2}(Qq_1)\Big)(Qq_2)\frac{1}{m^2}(\kappa\alpha - \frac{\kappa}{4} + \frac{1}{8}) \\
&\qquad + \frac{1}{2}m^2\Big[1 + \frac{\kappa - \frac{1}{2}}{m^2}(q^- q_1)\Big] P(q^- q_1)\Bigg\{ (q^- - q_1)_\mu - \frac{Q_\mu}{Q^2}\Big(Q(q^- - q_1)\Big) \\
&\qquad\quad - \frac{2(\kappa + \frac{1}{2})}{m^2}\Big(Q(q^- - q_1)\Big)\Big(q_{2\mu} - \frac{Q_\mu}{Q^2}(Qq_2)\Big) \\
&\qquad\quad + \frac{(\kappa + 1)}{m^2}\left\{ \begin{array}{l} q_{2\mu} Q^2 - Q_\mu (Qq_2) \\ + 2q_{1\mu}(Qq_2) - 2q_{2\mu}(Qq_1) \end{array} \right\} \\
&\quad + \text{the same where } 1 \leftrightarrow 2
\end{aligned}
\tag{B3}
$$

The diagrams for the amplitude (B2) are shown in Figure 5b-d.

The matrix element for the decay into $\pi^+\pi_1^o\pi_2^o\pi_3^o$ is:

$$\frac{\sqrt{2}}{f_\pi^2}\frac{1}{Q^2-m^2+i\varepsilon}\times$$

$$\Bigg\{\frac{m^2}{3}(4q_\mu^+-Q_\mu)+\sum_{i\neq j\neq \ell=1}^{3}\Big\{(2q_i-Q)_\mu\frac{m^2+(\kappa-\frac{1}{2})(Qq_i)}{(Q-q_i)^2+i\varepsilon}$$

$$+q_{i\mu}(\kappa-\frac{1}{2})\Big\}\cdot q_\ell(q^+-q_j)\cdot\Big\{\frac{1}{3}+(q^+q_j)\frac{\kappa(2\alpha+\frac{1}{2})-\frac{1}{4}}{2m^2}\Big\}$$

$$+\frac{1}{2}\Big[\kappa(2\alpha+\frac{1}{2})-\frac{1}{4}\Big]\Big\{q_\mu^+(Qq^+)-\sum_{\ell=1}^{3}q_{\ell\mu}(q^+q_\ell)\Big\}$$

$$+\sum_{\ell=1}^{3}\Big\{q_{\ell\mu}(Qq^+)-q_\mu^+(Qq_\ell)\Big\}\Big\{\frac{1}{2}\Big[\kappa(2\alpha+\frac{1}{2})-\frac{1}{4}\Big]-\frac{2}{3}(\kappa+1)\Big\}$$

$$+\frac{1}{2}\frac{\Big[\kappa(2\alpha-\frac{1}{2})+\frac{1}{4}\Big]}{m^2}\sum_{\ell=1}^{3}\Big\{Q_\mu\Big(Q(q^+-q_\ell)\Big)-(q^+-q_\ell)_\mu Q^2\Big\}(q_\ell q^+)$$

$$+\frac{\kappa\Big[\kappa(2\alpha-\frac{1}{2})+\frac{1}{4}\Big]}{2m^2}\sum_{i\neq\ell=1}^{3}(q_\ell q^+)\Big\{q_{i\mu}\Big(Q(q^+-q_\ell)-(q^+-q_\ell)_\mu(Qq_i)\Big)\Big\}$$

$$+\Big[\frac{\kappa}{2}-\frac{1}{4}\Big]m^2\sum_{i\neq\ell=1}^{3}P(q^+q_\ell)\Big\{(q^+-q_\ell)_\mu(Qq_i)-q_{i\mu}\Big(Q(q^+-q_\ell)\Big)\Big\}$$

$$+\Big[\frac{\kappa}{2}+\frac{1}{4}\Big]\sum_{i\neq j\neq\ell=1}^{3}P(q^+q_\ell)\Big\{q_j(q^+-q_\ell)\Big\}\Big\{q_{j\mu}\Big(Q(Q-q_i)-(Q-q_i)_\mu(Qq_j)\Big)\Big\}$$

$$+\frac{1}{2}\Big[\kappa+\frac{1}{2}\Big]^2\sum_{i\neq j\neq\ell=1}^{3}P(q^+q_\ell)[q_j(q^+-q_\ell)]\Big\{q_{i\mu}(Q-q_j)^2-(Q-2q_j)_\mu(Qq_i)\Big\}$$

$$+\frac{m^2}{4}\sum_{i\neq j\neq\ell=1}^{3}P(q^+q_\ell)\Big\{(q^++q_\ell)_\mu\Big(Q(q^+-q_\ell)\Big)-(q^+-q\ell)_\mu\Big(Q(q^++q_\ell)\Big)\Big\}$$

$$+\frac{m^2}{2}\left(\kappa-\frac{1}{2}\right)\sum_{i\neq j\neq \ell=1}^{3} P(q^+q_\ell)\left\{(q^+ + q_\ell)_\mu\Big(q_j(q^+ - q_\ell)\Big) - (q^+ - q_\ell)_\mu\Big(q_j(q^+ + q_\ell)\Big)\right\}$$

$$+ m^4 \sum_{\ell=1}^{3} (q^+ - q_\ell)_\mu P(q^+q_\ell)$$

$$+\sum_{i\neq j\neq \ell=1}^{3} m^4\left\{(2q_i - Q)_\mu \frac{\left[1+\frac{\kappa-\frac{1}{2}}{m^2}(Qq_i)\right]}{(Q-q_i)^2} + q_{i\mu}\frac{(\kappa-\frac{1}{2})}{m^2}\right\}q_j(q^+ - q_\ell)\left\{1+\frac{\kappa-\frac{1}{2}}{m^2}(q^+q_\ell)\right\}P(q^+q_\ell)$$

$$+\frac{1}{2}\sum_{i\neq j\neq \ell=1}^{3}\left\{Q^2 A_\mu\Big((q_\ell q^+),\ q_j,\ (Q-q_i)\Big) - Q_\mu\left[Q^\sigma A_\sigma\Big((q_\ell q^+),\ q_j,\ (Q-q_i)\Big)\right]\right\}$$

$$+\kappa\sum_{i\neq j\neq \ell=1}^{3}\left\{(Qq_i)\ A_\mu\Big((q_\ell q^+),\ q_j,\ (Q-q_i)\Big) - q_{i\mu}\left[Q^\sigma A_\sigma\Big((q_\ell q^+),\ q_j,\ (Q-q_i)\Big)\right]\right\}\Bigg\} \qquad \text{(B4)}$$

where

$$A_\mu\Big((q_\ell q^+),\ q_j,\ (Q-q_i)\Big)$$

$$= \frac{1}{(Q-q_i)^2 - \bar{m}^2 + i\varepsilon}\Bigg\{ -\frac{1}{2}\left[\kappa\left(2\alpha+\frac{1}{2}\right)+\frac{3}{4}\right]\left\{q^+_\mu (Q-q_i)^2 - (Q-q_i)_\mu\Big((Q-q_i)q^+\Big)\right\}$$

$$+\frac{\left[\kappa\left(2\alpha-\frac{1}{2}\right)+\frac{1}{4}\right]}{2m^2}(q^+q_\ell)\left\{(q^+ - q_\ell)_\mu (Q-q_i)^2 - (Q-q_i)_\mu\Big((Q-q_i)(q^+ - q_\ell)\Big)\right\}$$

$$+\frac{1}{2}P(q^+q_\ell)\Bigg\{m^2\left\{(q^+ - q_\ell)_\mu (Q-q_i)^2 - (Q-q_i)_\mu\Big((Q-q_i)(q^+ - q_\ell)\Big)\right\}$$

$$-[2\kappa+1](Q-q_i)(q^+ - q_\ell)\left\{q_{j\mu}(Q-q_i)^2 - (Q-q_i)_\mu\Big((Q-q_i)q$$

$$+ 2m^2(\kappa+1)\left\{q_{j\mu}(Q-q_i)(q^+ - q_\ell) - (q^+ - q_\ell)_\mu\Big((Q-q_i)q_j\Big)\right\}$$

The corresponding graphs are shown in Figure 5e-i.

The matrix element for $\tau^+ \to \nu_\tau \pi^+ \pi^- \pi^+ \pi^o$ is given by:

$$\frac{\sqrt{2}}{f_\pi^{\,2}} \frac{1}{Q^2 - m^2 + i\varepsilon} \times$$

$$\left(m^2 \left\{ \frac{1}{3} q_\mu^o - \frac{1}{6} (q_{1\mu}^+ + q_{2\mu}^+) \right\} \right.$$

$$\left. + \frac{1}{2} \left[2\kappa\alpha + \frac{\kappa}{2} - \frac{1}{4} \right] \left\{ q_\mu^- (q^- q_1^+) - q_\mu^- (q^- q^o) + q_\mu^o (q^o q^-) - q_{1\mu}^+ (q_1^+ q^-) \right\} \right.$$

$$+ \left\{ (2q_o - Q)_\mu \frac{\left[m^2 + (\kappa - \frac{1}{2})(Qq_o) \right]}{(Q - q_o)^2} + q_\mu^o (\kappa - \frac{1}{2}) \right\} \cdot \left(q_1^+ (q^- - q_2^+) \right) \left(\frac{1}{3} + \frac{\left[2\kappa\alpha + \frac{\kappa}{2} - \frac{1}{4} \right]}{m^2} (q_2^+ q^-) \right)$$

$$+ \left\{ (2q_2^+ - Q)_\mu \frac{\left[m^2 + (\kappa - \frac{1}{2})(Qq_2^+) \right]}{(Q - q_2^+)^2} + q_{2\mu}^+ (\kappa - \frac{1}{2}) \right\} \cdot \left(q^- (q^o - q_1^+) \right) \left(\frac{1}{3} + \frac{\left[2\kappa\alpha + \frac{\kappa}{2} - \frac{1}{4} \right]}{m^2} (q_1^+ q^o) \right)$$

$$+ \left\{ (2q_1^+ - Q)_\mu \frac{\left[m^2 + (\kappa - \frac{1}{2})(Qq_1^+) \right]}{(Q - q_1^+)^2} + q_{1\mu}^+ (\kappa - \frac{1}{2}) \right\} \cdot \left(q_2^+ (q^o - q^-) \right) \left(\frac{1}{3} + \frac{\left[2\kappa\alpha + \frac{\kappa}{2} - \frac{1}{4} \right]}{m^2} (q^- q^o) \right)$$

$$+ \left\{ q_\mu^- (Qq_o) - q_\mu^o (Qq^-) + q_{1\mu}^+ (Qq^-) - q_\mu^- (Qq_1^+) \right\} \left[\kappa\alpha + \frac{\kappa}{4} - \frac{1}{8} + \frac{1}{3} (\kappa - \frac{1}{2}) \right]$$

$$+ \left\{ q_\mu^o (Qq_1^+) - q_{1\mu}^+ (Qq^o) \right\} \left[\kappa + \frac{1}{2} \right]$$

$$+ \frac{1}{4} \frac{\left[2\kappa\alpha - \frac{\kappa}{2} + \frac{1}{4} \right]}{m^2} \left\{ \begin{array}{l} (q^- q_2^+) \left[Q_\mu \left(Q(q^- - q_2^+) \right) - (q^- - q_2^+)_\mu Q^2 \right] \\ + (q^o q_1^+) \left[Q_\mu \left(Q(q^o - q_1^+) \right) - (q^o - q_1^+)_\mu Q^2 \right] \\ + (q^o q^-) \left[Q_\mu \left(Q(q^o - q^-) \right) - (q^o - q^-)_\mu Q^2 \right] \end{array} \right.$$

$$+ \frac{\kappa \left[2\kappa\alpha - \frac{\kappa}{2} + \frac{1}{4} \right]}{2m^2} \left\{ \begin{array}{l} (q^- q_2^+) \left[q_\mu^o \left(Q(q^- - q_2^+) \right) - (q^- - q_2^+)_\mu (Qq^o) \right] \\ + (q^o q_1^+) \left[q_{2\mu}^+ \left(Q(q^o - q_1^+) \right) - (q^o - q_1^+)_\mu (Qq_2^+) \right] \\ + (q^o q^-) \left[q_{1\mu}^+ \left(Q(q^o - q^-) \right) - (q^o - q^-)_\mu (Qq_1) \right] \end{array} \right.$$

$$+ \frac{(\kappa - \frac{1}{2})}{2} m^2 \left\{ \left\{ (q^- - q_2^+)_\mu (Qq^o) - q_\mu^o \Big(Q(q^- - q_2^+) \Big) \right\} P(q^- q_2^+) \right.$$

$$+ \left\{ (q^o - q_1^+)_\mu (Qq_2^+) - q_{2\mu}^+ \Big(Q(q^o - q_1^+) \Big) \right\} P(q^o q_1^+)$$

$$\left. + \left\{ (q^o - q^-)_\mu (Qq_1^+) - q_{1\mu}^+ \Big(Q(q^o - q^-) \Big) \right\} P(q^o q^-) \right.$$

$$+ \frac{1}{2} (\kappa + \frac{1}{2}) \left\{ \Big(q_1^+ (q^- - q_2^+) \Big) \left\{ q_1^\mu \Big(Q(Q - q^o) \Big) - (Q - q^o)_\mu (Qq_1^+) \right\} P(q^- q_2^+) \right.$$

$$+ \Big(q^- (q^o - q_1^+) \Big) \left\{ q_\mu^- \Big(Q(Q - q_2^+) \Big) - (Q - q_2^+)_\mu (Qq^-) \right\} P(q^o q_1^+)$$

$$\left. + \Big(q_2^+ (q^o - q^-) \Big) \left\{ q_{2\mu}^+ \Big(Q(Q - q_1^+) \Big) - (Q - q_1^+)_\mu (Qq_2^+) \right\} P(q^o q^-) \right.$$

$$+ \frac{1}{2} (\kappa + \frac{1}{2})^2 \left\{ \Big(q_1^+ (q^- - q_2^+) \Big) \left\{ q_\mu^o (Q - q_1^+)^2 - (Q - 2q_1^+)_\mu (Qq^o) \right\} P(q^- q_2^+) \right.$$

$$+ \Big(q^- (q^o - q_1^+) \Big) \left\{ q_{2\mu}^+ (Q - q^-)^2 - (Q - 2q^-)_\mu (Qq_2^+) \right\} P(q^o q_1^+)$$

$$\left. + \Big(q_2^+ (q^o - q^-) \Big) \left\{ q_{1\mu}^+ (Q - q_2^+)^2 - (Q - 2q_2^+)_\mu (Qq_1^+) \right\} P(q^o q^-) \right.$$

$$+ \frac{m^2}{4} \left\{ \left\{ (q^- + q_2^+)_\mu \Big(Q(q^- - q_2^+) \Big) - (q^- - q_2^+)_\mu \Big(Q(q_2^+ + q^-) \Big) \right\} P(q^- q_2^+) \right.$$

$$+ \left\{ (q^o + q_1^+)_\mu \Big(Q(q^o - q_1^+) \Big) - (q^o - q_1^+)_\mu \Big(Q(q_1^+ + q^o) \Big) \right\} P(q^o q_1^+)$$

$$\left. + \left\{ (q^o + q^-)_\mu \Big(Q(q^o - q^-) \Big) - (q^o - q^-)_\mu \Big(Q(q^o + q^-) \Big) \right\} P(q^o q^-) \right.$$

$$+ \frac{m^2}{2} (\kappa - \frac{1}{2}) \left\{ \left\{ q_{1\mu}^+ \Big(q^o (q^- - q_2^+) \Big) - q_\mu^o \Big(q_1^+ (q^- - q_2^+) \Big) + (q_1^+ + q_2^+)_\mu \Big(q_1^+ (q^- - q_2^+) \Big) \right. \right.$$

$$\left. - (q^- - q_2^+)_\mu \Big(q_1^+ (q_2^+ + q^-) \Big) \right\} P(q^- q_2^+)$$

$$+ \left\{ q_\mu^- \Big(q_2^+ (q^o - q_1^+) \Big) - q_{2\mu}^+ \Big(q^- (q^o - q_1^+) \Big) + (q_1^+ + q^-)_\mu \Big(q^- (q^o - q_1^+) \Big) \right.$$

$$\left. - (q^o - q_1^+)_\mu \Big(q^- (q_1^+ + q^o) \Big) \right\} P(q^o q_1^+)$$

$$+ \left\{ q_{2\mu}^+ \Big(q_1^+ (q^o - q^-) \Big) - q_{1\mu}^+ \Big(q_2^+ (q^o - q^-) \Big) + (q^- + q_2^+)_\mu \Big(q_2^+ (q^o - q^-) \Big) \right.$$

$$\left. \left. - (q^o - q^-)_\mu \Big(q_2^+ (q^- + q^o) \Big) \right\} P(q^o q^-) \right.$$

$$+ \frac{m^4}{2} \Big\{ (q^- - q_2^+)_\mu \, P(q^- q_2^+) + (q^o - q_1^+)_\mu \, P(q^o q_1^+) + (q^o - q^-)_\mu \, P(q^o q^-)$$

$$+ m^4 \left\{ \left\{ (2q^o - Q)_\mu \frac{\left[1 + \frac{(\kappa - \frac{1}{2})}{m^2} (Qq^o) \right]}{(Q - q^o)^2} + q^o_\mu \frac{(\kappa - \frac{1}{2})}{m^2} \right\} \times \right.$$

$$\left[1 + \frac{(\kappa - \frac{1}{2})}{m^2} (q_2^+ q^-) \right] [q_1^+ (q^- - q_2^+)] \cdot P(q^- q_2^+)$$

$$+ \left\{ (2q_2^+ - Q)_\mu \frac{\left[1 + \frac{(\kappa - \frac{1}{2})}{m^2} (Qq_2^+) \right]}{(Q - q_2^+)^2} + q^+_{2\mu} \frac{(\kappa - \frac{1}{2})}{m^2} \right\} \times$$

$$\left[1 + \frac{(\kappa - \frac{1}{2})}{m^2} (q_1^+ q^o) \right] [q^- (q^o - q_1^+)] \quad P(q^o q_1^+)$$

$$+ \left\{ (2q_1^+ - Q)_\mu \frac{\left[1 + \frac{(\kappa - \frac{1}{2})}{m^2} (Qq_1^+) \right]}{(Q - q_1^+)^2} + q^+_{1\mu} \frac{(\kappa - \frac{1}{2})}{m^2} \right\} \times$$

$$\left. \left[1 + \frac{(\kappa - \frac{1}{2})}{m^2} (q^o q^-) \right] [q_2^+ (q^o - q^-)] \quad P(q^o q^-) \right.$$

$$+ \left\{ \frac{1}{2} Q^2 + \kappa (Qq^o) \right\} A_\mu \Big((q^- q_2^+), \, q_1^+, \, (Q - q^o) \Big) - \left(\frac{1}{2} Q_\mu + \kappa q^o_\mu \right) Q^\sigma A\sigma \Big((q^- q_2^+), \, q_1^+, \, (Q - q^o) \Big)$$

$$+ \left\{ \frac{1}{2} Q^2 + \kappa (Qq_2^+) \right\} A_\mu \Big((q^o q_1^+), \, q^-, \, (Q - q_2^+) \Big) - \left(\frac{1}{2} Q_\mu + \kappa q_{2\mu} \right) Q^\sigma A\sigma \Big((q^o q_1^+), \, q^-, \, (Q - q_2^+) \Big)$$

$$+ \left\{ \frac{1}{2} Q^2 + \kappa (Qq_1^+) \right\} A_\mu \Big((q^o q^-), \, q_2^+, \, (Q - q_1^+) \Big) - \left(\frac{1}{2} Q_\mu + \kappa q^+_{1\mu} \right) Q^\sigma A\sigma \Big((q^o q^-), \, q_2^+, \, (Q - q_1^+) \Big)$$

$$+ m^4 \left\{ (q^- - q_2^+)_\mu \Big(Q (q_1^+ - q^o) \Big) - \Big(Q (q^- - q_2^+) \Big) (q_1^+ - q^o)_\mu \right\} P(q^- q_2^+) P(q^o q_1^+)$$

$$+ \frac{m^4}{2} \left\{ (q_1^+ + q^o)_\mu (q^- - q_2^+)(q_1^+ - q^o) - (q^- + q_2^+)_\mu (q^- - q_2^+)(q_1^+ - q^o) \right\} P(q^- q_2^+) \; P(q^o q_1^+)$$

$$\left. + \text{ the same expressions where } 1 \leftrightarrow 2. \right\} \qquad \text{(B4)}$$

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TABLE

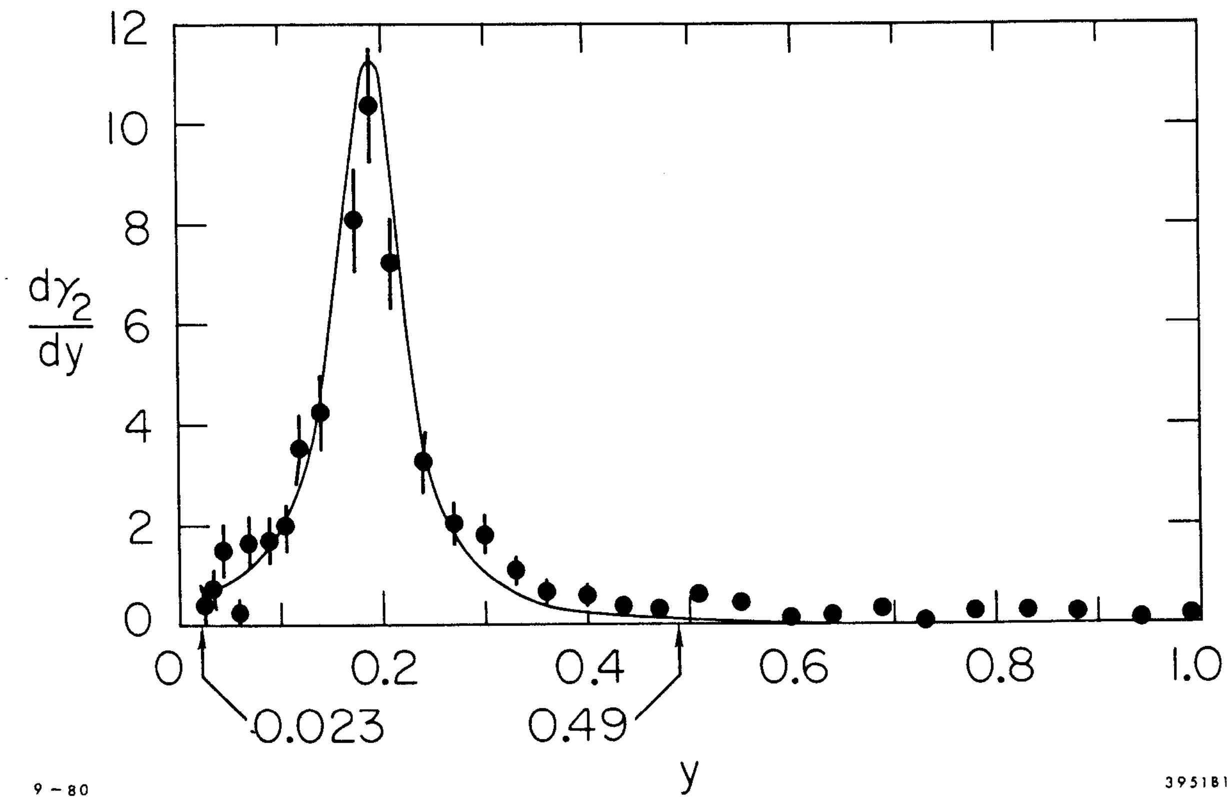
$$\text{Ratios } \gamma_n = \frac{\Gamma(\tau \to n \text{ pions} + \nu_\tau}{\Gamma(\tau \to {}^{\mu}_{(e)}\nu\nu)}$$

The parameters entering the theoretical calculations are: $f_\pi = 92$ MeV, $m_\tau = 1782$ MeV, $\Gamma_\rho = 155$ MeV, $m_\rho = 775$ MeV, $m_{A1} = \sqrt{2}\, m_\rho$, $\Gamma_{A1} = 250 \ldots 300$ MeV.[17] A review of the experimental situation is found in Ref. 12.

n	Mode	Theory	Experiment	total rate γ_n
1	$\pi^+\nu$	0.60	0.54 ± .18 Pluto[18]	
			SLAC-LBL[14]	.60
			0.50 ± 18 DELCO[14]	
2	$\rho^+\nu$	1.20	1.43 ± .53 DASP[13]	1.20
		1.17	1.21 ± .12 ± .20 SLAC-LBL[16]	
3	$\pi^+\pi^-\pi^+\nu$	0.21...0.25	0.34 ± .25 SLAC-LBL[18]	
	$\pi^+\pi^o\pi^o\nu$	0.21...0.25		0.42–0.50
	$\rho^o\pi^+\nu$	0.21...0.25	0.31 ± .10 Pluto[15]	
	$\rho^+\pi^o\nu$	0.21...0.25		
4	$\pi^+\pi^o\pi^o\pi^o$	0.14...0.16		0.28...0.32
	$\pi^+\pi^-\pi^+\pi^o$	0.14...0.16		

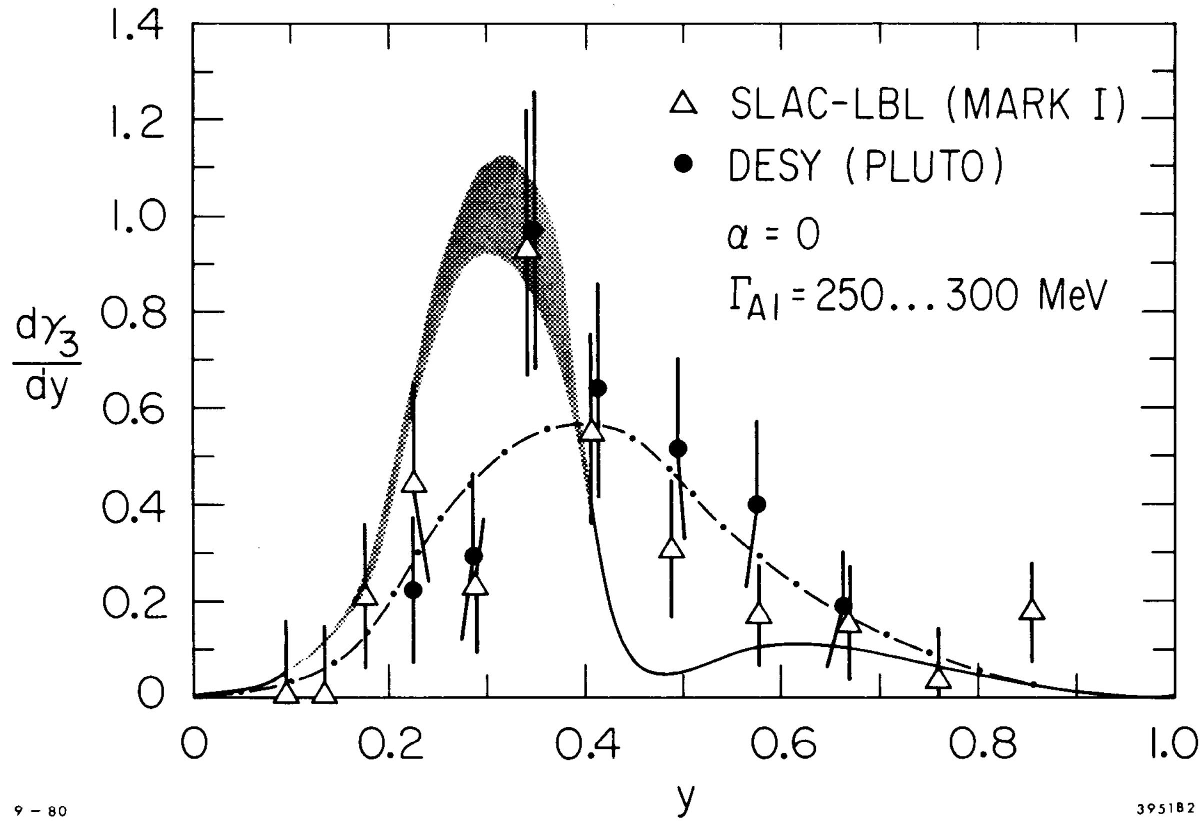
FIGURE CAPTIONS

1. Decay spectrum $\frac{d\Gamma_2}{dy}$ for $\tau^+ \to \nu_\tau \pi^+ \pi^o$. $y = \frac{Q^2}{m_\tau^2}$ the invariant mass of the pion system relative to the mass of the τ. The data are from Ref. 16. The curve is obtained from (2.6) for $\kappa = \frac{1}{2}$. 0.023 ... 0.49 is the interval from which the experimental rate is calculated.
2. Decay spectrum $\frac{d\Gamma_3}{dy}$ for $\tau^+ \to \nu_\tau \pi^+ \pi^o \pi^o$ and $\tau^+ \to \nu_\tau \pi^+ \pi^+ \pi^-$. The shaded area corresponds to the variation of Γ_{A1} in the range from 250 to 300 MeV. The dashed-dotted line indicates the spectrum without an A_1 meson as calculated in Ref. 4. The data are normalized to an area corresponding to $\Gamma_3 = 0.23$.
3. Same as in Fig. 2. But here Γ_{A1} is kept fixed at 300 MeV and the parameter α is varied from 0 to 0.9.
4. Decay spectrum $\frac{d\Gamma_4}{dy}$ for $\tau^+ \to \nu_\tau \pi^+ \pi^o \pi^o \pi^o$ and $\tau^+ \to \nu_\tau \pi^+ \pi^- \pi^+ \pi^o$. The shaded area corresponds to a variation of Γ_{A1} from 250 to 300 MeV. The dashed-dotted line is the contribution to the spectrum from graphs without an A_1 meson, $\alpha = 0$.
5. Shown are the graphs that contribute to the various processes: 5a to $\frac{d\Gamma_2}{dy}$; 5b - 5d to $\frac{d\Gamma_3}{dy}$; 5e - 5i to $\tau^+ \to \nu_\tau \pi^+ \pi^o \pi^o \pi^o$ and 5j together with 5e - 5i to $\tau^+ \to \nu_\tau \pi^+ \pi^- \pi^+ \pi^o$. The charge states of 5b - 5i have to be relabeled accordingly.



12
10
8
6
4
2
0
$\frac{d\gamma_2}{dy}$
0
0.2
0.4
0.6
0.8
1.0
0.023
0.49
y
9 – 80
3951B1

Fig. 1



1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
$\frac{d\gamma_3}{dy}$
SLAC-LBL (MARK I)
DESY (PLUTO)
$\alpha = 0$
$\Gamma_{A1} = 250 \ldots 300$ MeV
0
0.2
0.4
0.6
0.8
1.0
y
9 – 80
3951B2

Fig. 2

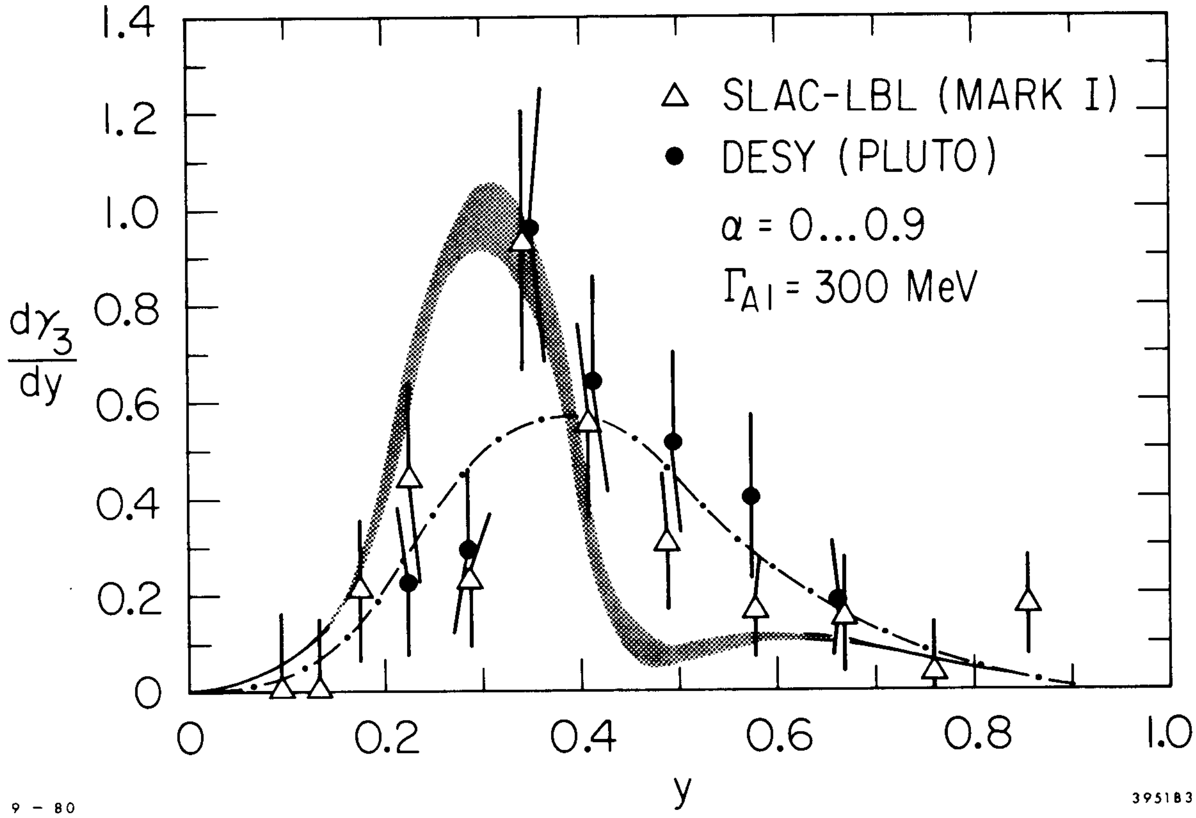

1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
$\frac{d\gamma_3}{dy}$
△ SLAC-LBL (MARK I)
● DESY (PLUTO)
α = 0...0.9
Γ_{A1} = 300 MeV
0
0.2
0.4
0.6
0.8
1.0
y
9 - 80
395183

Fig. 3

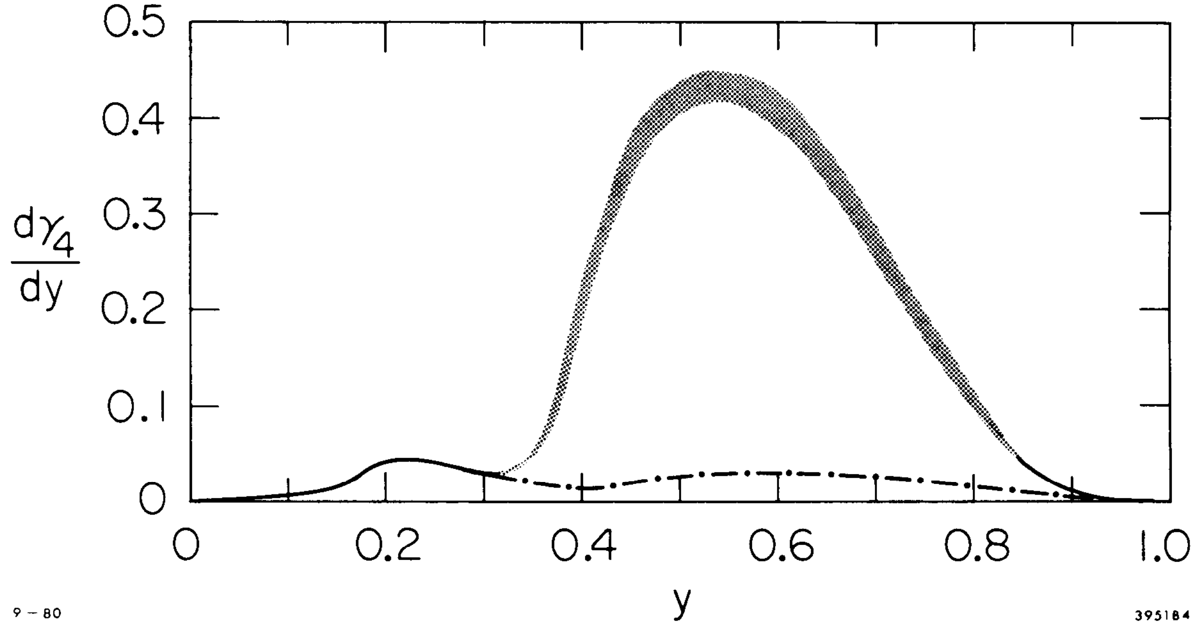


Fig. 4

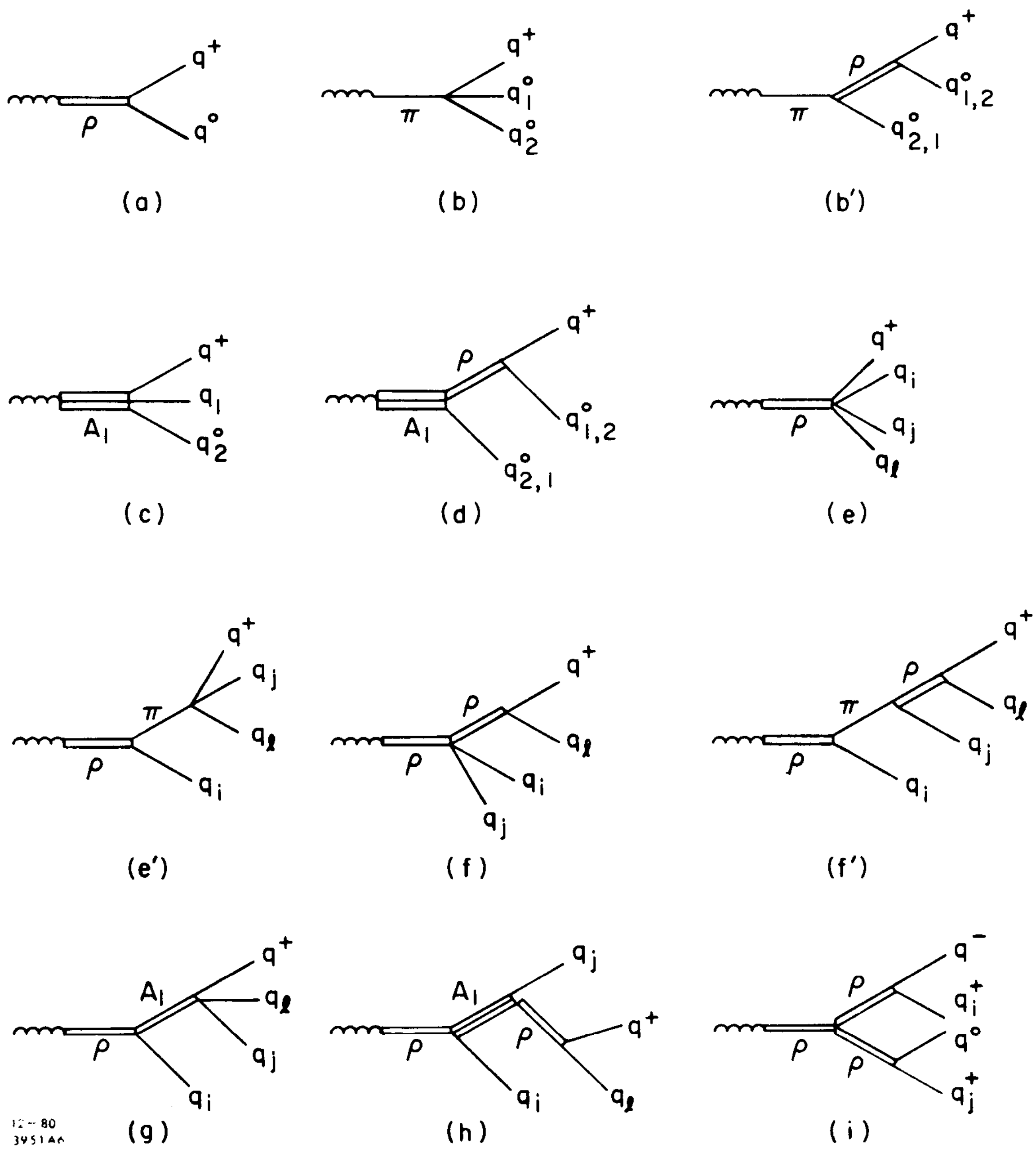

q+
ρ
q°
(a)
q+
π
q°1
q°2
(b)
q+
ρ
π
q°1,2
q°2,1
(b′)
q+
A1
q1
q°2
(c)
q+
ρ
A1
q°1,2
q°2,1
(d)
q+
ρ
qi
qj
qℓ
(e)
q+
π
qj
qℓ
ρ
qi
(e′)
q+
ρ
qℓ
qi
qj
(f)
q+
ρ
π
qℓ
qj
qi
(f′)
q+
A1
qℓ
ρ
qj
qi
12-80
3951A6
(g)
qj
A1
ρ
q+
qi
qℓ
(h)
q−
ρ
q+i
q°
ρ
q+j
(i)

Fig. 5