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THE EFFECTIVE WEAK HAMILTONIAN BEYOND THE LEADING-LOGARITHM APPROXIMATION*

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ABSTRACT

The effective nonleptonic weak Hamiltonian is examined beyond the leading-logarithm approximation. In the $\Delta S = 1$, $\Delta C = 1$ part of the Hamiltonian no significant contribution is found. In the $\Delta S = 1$, $\Delta C = 0$ sector coefficients of the "penguin" operators depend strongly on added corrections. The momentum subtraction scheme has been used in the calculation. The independence of the result on the renormalization procedure, as well as on the choice of the renormalization point μ is discussed.

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1. Introduction

Considerable progress has been made in the understanding of non-leptonic weak decays by using the effective Hamiltonian [1,2], in which strong interaction effects have been incorporated by means of the renormalization-group (RG) analysis [3]. Theoretical calculations for a wide range of processes have been reasonably successful for both $\Delta S = 1$ [2,4-7] and $\Delta S = 0$ sectors [8,9]. However, it seems that the explanation of charm-changing decays of the D-meson is still missing [10], and it is not as yet quite clear what is the source of this discrepancy. One possible explanation is that the dynamical scheme in which the effective Hamiltonian serves as an input is oversimplified for this case. The other possibility is that there is some additional mechanism that could affect coefficients of various operators in the Hamiltonian.

The main purpose of this paper is to investigate quantum-chromodynamic (QCD) corrections to weak Hamiltonian beyond the leading logarithmic (LL) order, taking into account the power-corrections too. The complete analysis via RG becomes extremely complicated, but some feeling of the importance of power-corrections can be reached even by the g^2-order calculation. Note that useful approximations such as the Appelquist-Carazzone theorem [11] and the R-rule [12] become inadequate beyond the LL. The method used in the paper is based on the comparison of matrix elements calculated to the lowest order in G_F and g^2 by the direct perturbative calculation, with the matrix elements calculated using the related effective Hamiltonian.

In section 2 $\Delta S = 1$, $\Delta C = 1$ part of the effective Hamiltonian is considered in order to see how power-corrections of the type $(m_q/\mu)^\alpha$, where m_q is the mass of some quark, influence the coefficients of $\mathcal{O}_{20}$ and $\mathcal{O}_{84}$ operators (see (A2) for the definition). Section 3 is devoted to study of operators with mixed left-right chiral structure (penguin terms [2]) in $\Delta S = 1$, $\Delta C = 0$ sector of the effective Hamiltonian. One finds that power-corrections in this case can considerably enhance the order g^2 coefficient of the penguin-like operator.

The underlying theoretical model used in sections 2 and 3 is the standard weak, four-quark model with strong interactions included with the help of QCD. Applying the momentum subtraction scheme [13], relevant Green's functions of the theory have been renormalized at a symmetric point in the momentum space. The physical results, however, must be independent of the selection of the renormalization procedure as well as of the choice of the renormalization point. The meaning of this statement for the case when an effective Hamiltonian serves as an input in the calculation of some real nonleptonic process is discussed in the concluding section. A few details of the calculation are given in the appendix.

2. Effective Hamiltonian for $\Delta S = 1$, $\Delta C = 1$ Transitions

The relevant interaction-part of the standard [14,15] $SU(3) \times SU(2)_L \times U(1)_R$ Lagrangian for $\Delta S = 1$, $\Delta C = 1$ transitions is

$$\mathcal{L}_{int} = \frac{1}{2} g \sum_{quarks} \bar{\psi} \gamma_\mu \lambda^a \psi G^\mu_a + \left\{ \frac{h}{2\sqrt{2}} \left(\cos\theta \left[\bar{u}d + \bar{c}s \right]_{V-A} + \sin\theta \left[\bar{u}s - \bar{c}d \right]_{V-A} \right) W^+ + h.c. \right\}. \tag{2.1}$$

A renormalizable gauge for weak interaction part is used, with a W-boson propagator $-ig^{\mu\nu}/(k^2 - M^2)$. In principle the term with Higgsghost particles should be included in (2.1), but since in all cases considered their contribution is suppressed by an additional factor $(m_q/M)^2$ compared to the W-boson contribution, one can drop these particles from analyses.

One first calculates the matrix element for $c+d \to s+u$ "scattering" in the lowest order in G_F and zeroth order in g. The only possible contribution is shown on fig. 1a, and gives

$$\mathcal{M}_{1a} = i\,\frac{h^2}{8}\cos^2\theta\;\bar{s}\gamma_\mu(1-\gamma_5)c\;\bar{u}\gamma^\mu(1-\gamma_5)d\;\frac{1}{q^2-M^2}$$

$$= -i\,\frac{G}{\sqrt{2}}\cos^2\theta\Big[\big(\bar{s}c\big)_{V-A}\big(\bar{u}d\big)_{V-A} + \mathcal{O}(q^2/M^2)\Big]. \qquad (2.2)$$

The same matrix element could be calculated by help of the standard current × current Hamiltonian

$$H^{CC} = \frac{G}{\sqrt{2}}\cos^2\theta\;\bar{s}\gamma_\mu(1-\gamma_5)c\;\bar{u}\gamma^\mu(1-\gamma_5)d \qquad . \qquad (2.3)$$

Its contribution is displayed on fig. 1b. The heavy dot represents the local operator on the RHS of the expression (2.3). The related matrix element is

$$\mathcal{M}_{1b} = -i\,\frac{G}{\sqrt{2}}\cos^2\theta\;\big(\bar{s}c\big)_{V-A}\big(\bar{u}d\big)_{V-A} \qquad . \qquad (2.4)$$

Matrix elements (2.2) and (2.4) are identical when $\mathcal{O}(q^2/M^2)$ terms are neglected. As expected, the perturbative calculation gives the same result as the calculation using an effective Hamiltonian.

One proceeds now in the same way to calculate matrix elements to the order g^2. The diagrams needed in the perturbative calculation are shown on fig. 2. The dashed lines represent gluon exchanges.

Let us consider the effective Hamiltonian to the order g^2 of the form

$$H_{eff} = \frac{G}{\sqrt{2}} \cos^2\theta \left[\bar{s}c\bar{u}d + \frac{g^2}{16\pi^2} \left(A\bar{s}c\bar{u}d + B\bar{s}d\bar{u}c \right) \right]_{(V-A)(V-A)} + \mathcal{O}(g^4) \quad , \tag{2.5}$$

where A and B are the quantities to be determined. g^0 part of (2.5) gives rise to diagrams on fig. 3, and g^2 part to diagrams on fig. 4. If (2.5) is a correct Hamiltonian, the relation

$$\mathcal{M}_{2a} + \mathcal{M}_{2b} = \mathcal{M}_{3a} + \mathcal{M}_{3b} + \mathcal{M}_4 \quad , \tag{2.6}$$

must be satisfied. It is easy to see that $\mathcal{M}_{2b} = \mathcal{M}_{3b}$ (to the order $\mathcal{O}(q^2/M^2)$), and instead of (2.6) one gets

$$\mathcal{M}_{2a} - \mathcal{M}_{3a} = \mathcal{M}_4 \quad . \tag{2.7}$$

In the appendix it is shown how the matrix elements in fig. 2a can be separated in the form

$$g^2 \frac{h^2}{8M^2} \left(F + \mathcal{O}(1/M^2) \right) \quad .$$

Only the first part of this expression is interesting; terms of order $\mathcal{O}(1/M^4)$ are either suppressed by the big mass in the denominator or are of a dimension higher then six, and will not be considered explicitly. The result is then

$$\mathcal{M}_{2a} = i \frac{G}{\sqrt{2}} \cos^2\theta \frac{g^2}{16\pi^2} \left[-\frac{9}{4} + \frac{3}{2} \ln M^2 \right.$$
$$\left. + \int_0^1 2y\,dy \int_0^1 dx \left(-\ln \phi_1\phi_2 + \frac{1}{4} \ln \phi_3\phi_4 \right) \right] \left(\bar{s}\lambda^a c \right)_{V-A} \left(\bar{u}\lambda^a d \right)_{V-A} \quad . \tag{2.8}$$

Functions ϕ_1 $(i = 2-4)$ are written in the appendix (A11), and here the function ϕ_1 corresponding to the first diagram on fig. 2a is listed:

$$\phi_1 = y^2 \left[p^2x^2 + r^2(1-x)^2 - 2prx(1-x) \right] + y \left[\left(m_c^2 - p^2 \right) x + \left(m_d^2 - r^2 \right)(1-x) \right] \quad . \tag{2.9}$$

The diagrams in fig. 3a are evidently divergent, and have to be renormalized by means of the subtraction in the symmetric point

$$p^2 = p'^2 = r^2 = r'^2 = -\mu^2$$
$$2pr = 2p'r' = -2rp' = -2rr' = -2pp' = -2r'p = \frac{2}{3}\mu^2 \quad . \qquad (2.10)$$

Then one gets finite, but μ^2-dependent contribution for the matrix element $\mathcal{M}_{3a}$:

$$\mathcal{M}_{3a} = i\frac{G}{\sqrt{2}} \cos^2\theta \frac{g^2}{16\pi^2}\int_0^1 2y\,dy\int_0^1 dx\left(-\ln\frac{\phi_1\phi_2}{\phi_{1S}\phi_{2S}} + \frac{1}{4}\ln\frac{\phi_3\phi_4}{\phi_{3S}\phi_{4S}}\right)$$
$$\times\left(\bar{s}\lambda^a c\right)_{V-A}\left(\bar{u}\lambda^a d\right)_{V-A} \quad . \qquad (2.11)$$

ϕ_{iS} denotes the function ϕ_i calculated in the symmetric point (2.10). For example

$$\phi_{1S} = \mu^2 y\left[-y\left(\frac{4}{3}x^2 - \frac{4}{3}x + 1\right) + 1 + \frac{m_d^2}{\mu^2}(1-x) + \frac{m_c^2}{\mu^2}x\right] \quad . \qquad (2.12)$$

In fact, there are some additional contributions to (2.11), typically of the form

$$Gg^2\frac{mm'}{\mu^2}\bar{s}(1\pm\gamma_5)\lambda^a c\;\bar{u}(1\pm\gamma_5)\lambda^a d \quad ,$$

or

$$Gg^2\frac{mm'}{\mu^2}\bar{s}\sigma_{\mu\nu}(1\pm\gamma_5)\lambda^a c\;\bar{u}\sigma^{\mu\nu}(1\pm\gamma_5)\lambda^a d \quad ,$$

where m and m' are masses of some in-(out-)going quarks. In a more careful treatment such terms have to be included in (2.5) too, but for $\mu \sim m_c$ they are suppressed.

Using the known property of λ-matrices

$$\left(\lambda^a\right)_{ij}\left(\lambda^a\right)_{kl} = -\frac{2}{3}\delta_{ij}\delta_{kl} + 2\delta_{il}\delta_{jk} \quad ,$$

one gets

$$\mathcal{M}_{2a} - \mathcal{M}_{3a} = i \frac{G}{\sqrt{2}} \cos^2\theta \frac{g^2}{16\pi^2} \Big[-\frac{9}{4} + \frac{3}{2} \ln M^2 \tag{2.13}$$

$$+ \int_0^1 2y dy \int_0^1 dx \left(-\ln \phi_{1S}\phi_{2S} + \frac{1}{4} \ln \phi_{3S}\phi_{4S} \right) \Big] \left(-\frac{2}{3} \bar{s}c\bar{u}d + 2\bar{s}d\bar{u}c \right)_{(V-A)(V-A)} .$$

Since the contribution of fig. 4 is

$$\mathcal{M}_4 = -i \frac{G}{\sqrt{2}} \cos^2\theta \frac{g^2}{16\pi^2} \left[A\, \bar{s}c\bar{u}d + B\, \bar{s}d\bar{u}c \right]_{(V-A)(V-A)} , \tag{2.14}$$

the final result is*

$$A = -\frac{1}{3} B = \frac{2}{3} \left[-\frac{9}{4} + \frac{3}{2} \ln M^2 + \int\!\!\int \left(-\ln \phi_{1S}\phi_{2S} + \frac{1}{4} \ln \phi_{3S}\phi_{4S} \right) \right] . \tag{2.15}$$

It is not a great task to find the LL part of the quantity A explicitly. It depends on the relations between μ and quark masses. Two typical values are

$$\bar{A} = \ln \frac{M^2}{\mu^2} \left[+ \mathcal{O}\left(\frac{m_q^2}{\mu^2} \right) \right] \quad \text{for } \mu^2 > m_c^2 > m_s^2 \quad \ldots$$

$$\bar{A} = \ln \frac{M^2}{\mu^2} - \frac{1}{2} \ln \frac{m_c^2}{\mu^2} \left[+ \mathcal{O}\left(\frac{\mu^2}{m_c^2}, \frac{m_s^2}{\mu^2} \ldots \right) \right] \quad \text{for } m_c^2 > \mu^2 > m_s^2 \quad \ldots \tag{2.16}$$

and one recognizes the standard LL results (A3) and (A5).

The complete function A together with its LL part is plotted on fig. 5. Evidently, the inclusion of the power-corrections makes no significant change to the $\Delta S = 1$, $\Delta C = 1$ Hamiltonian, as long as μ is chosen near the mass of the charmed quark.

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*Due to the fact that $A = -B/3$ in (2.15), the known [1] relation $C_{84} = (C_{20})^{-1/2}$ is preserved even beyond the LL approximation.

3. The Influence of the Power-Corrections to the Penguin Terms

It has been argued [2,4-7] that operators with mixed left-right chirality in $\Delta S = 1$, $\Delta C = 0$ effective Hamiltonian might play an important role in understanding of nonleptonic weak processes. However, due to the GIM mechanism [15] such terms arise in a LL approximation only for $\mu < m_c$ [2]. There is no reason to believe that this result will survive once the power-corrections are taken into account.

In this section the coefficient of the operator $\mathscr{P}$ is considered.

$$\mathscr{P} = \frac{2}{g}\,\bar{s}\gamma^{\mu}(1-\gamma_5)\,\lambda^a d\, D^{\nu}_{ab}\, G^{b}_{\nu\mu} \quad . \tag{3.1}$$

By the equation of motion, $\mathscr{P}$ could be related (see, e.g., refs. [12,16]) to the standard penguin operator. D^{ν}_{ab} in (3.1) is the covariant derivative acting on the gluon energy-momentum tensor.

One first considers the $g^2 G_F$ contribution of the ordinary perturbative calculation to the matrix element of the form (3.1). The related diagrams are presented on fig. 6. It will be enough to calculate diagrams 6a and 6b with only one external gluon leg.* It is easy to see that diagrams 6c give contribution of the form

$$\bar{s}\nabla\nabla\nabla d \quad , \tag{3.2}$$

(where ∇ denotes the covariant derivative of the fermion field), and can also be dropped from consideration. The matrix element of the diagram on fig. 6a is

$$\mathscr{M}_{6a} = \sin\theta\cos\theta\,\frac{h^2}{8}\,\frac{g}{2}\,\lambda^a \times \int \frac{d^n k}{(2\pi)^n}\,\gamma_{\mu}(1-\gamma_5)\,\frac{1}{\hat{k}-\hat{q}-m_u}\,\gamma_{\alpha}\,\frac{1}{\hat{k}-m_u}\,\gamma^{\mu}(1-\gamma_5)\,\frac{1}{(k-p)^2-M^2} \tag{3.3}$$

*The calculation of diagrams 6d and 7c with more gluon legs confirms the result (3.8).

The expression (3.3) is divergent, but when the analogous contribution from fig. 6b is subtracted (due to $-\sin\theta\cos\theta$ coupling), the resulting integral is finite:

$$\mathcal{M}_{6(a+b)} = i \sin\theta\cos\theta \frac{G}{\sqrt{2}} \frac{g}{4\pi^2} \lambda^a\left(q^2\gamma^\alpha-\hat{q}q^\alpha\right)(1-\gamma_5)$$

$$\times\left[\int_0^1 dx x(1-x)\, \ln \frac{m_c^2-q^2x(1-x)}{m_u^2-q^2x(1-x)} + \mathcal{O}(1/M^2)\right]$$

$$+\left(\text{contribution of the form (3.2)}\right) \quad . \qquad (3.4)$$

Let us suppose that the relevant $\Delta S = 1$, $\Delta C = 0$ effective Hamiltonian has the form

$$H_{eff} = \sin\theta\cos\theta \frac{G}{\sqrt{2}} \Big[\left(\bar{s}u\bar{u}d - \bar{s}c\bar{c}d\right)_{(V-A)(V-A)} + g^2 D \cdot \mathcal{P}$$

$$+ g^2 \left(\text{lin. comb. of four-quark operators}\right) + \mathcal{O}(g^4)\Big] \quad . \qquad (3.5)$$

Diagrams that could be built from the g^0 part of (3.5) are presented on fig. 7, and the contribution of the operator $\mathcal{P}$ to the required matrix element is indicated on fig. 8.

The diagram 7a is divergent, and has to be subtracted at a symmetric point $p^2 = p'^2 = -\mu^2$. The resulting finite, μ-dependent matrix element is then

$$\mathcal{M}_{7a} = i \sin\theta\cos\theta \frac{G}{\sqrt{2}} \frac{g}{4\pi^2} \lambda^a\left(q^2\gamma^\alpha-q^\alpha\hat{q}\right)(1-\gamma_5)$$

$$\times\int_0^1 dx x(1-x)\, \ln \frac{m_u^2+\mu^2x(1-x)}{m_u^2-q^2x(1-x)} + (\text{other uninteresting terms}) . \qquad (3.6)$$

By replacing m_u by m_c in (3.6), and changing the sign, one gets the matrix element $\mathcal{M}_{7b}$.

The contribution of the operator $\mathcal{P}$ from (3.5) is

$$\mathcal{M}_8 = i \sin\theta\cos\theta \frac{G}{\sqrt{2}} \frac{g}{4\pi^2} \lambda^a\left(q^2\gamma^\alpha-q^\alpha\hat{q}\right)(1-\gamma_5)\, 8\pi^2 D \quad . \qquad (3.7)$$

Because of the equality $\mathcal{M}_{6(a+b)} = \mathcal{M}_{7(a+b)} + \mathcal{M}_8$, the unknown function D is given by

$$8\pi^2 D = \int_0^1 dx x(1-x)\ \ell n\ \frac{m_c^2+\mu^2 x(1-x)}{m_u^2+\mu^2 x(1-x)} \quad . \tag{3.8}$$

The LL part of the integral in (3.8) can be easily found:

$$8\pi^2\bar{D} = \text{zero}\left[+\mathcal{O}\left(\frac{m_c^2}{\mu^2}, \frac{m_u^2}{\mu^2}\right)\right] \quad \text{for } \mu^2 > m_c^2 > m_u^2 \tag{3.9a}$$

$$8\pi^2\bar{D} = \frac{1}{6}\ \ell n\ \frac{m_c^2}{\mu^2}\left[+\mathcal{O}\left(\frac{\mu^2}{m_c^2}, \frac{m_u^2}{\mu^2}\right)\right] \text{for } m_c^2 > \mu^2 > m_u^2 \tag{3.9b}$$

By the equation of motion (3.9b) leads to the well known [2] left-right part of the $\Delta S = 1$, $\Delta C = 0$ weak Hamiltonian:

$$H_{eff}^{L-R} = \sin\theta\cos\theta\ \frac{G}{\sqrt{2}}\ \frac{g^2}{4\pi^2}\left(\frac{1}{12}\ \ell n\ \frac{m_c^2}{\mu^2}\right)\mathcal{P}$$

$$\longrightarrow \sin\theta\cos\theta\ \frac{G}{\sqrt{2}}\ \frac{g^2}{4\pi^2}\left(-\frac{1}{12}\ \ell n\ \frac{m_c^2}{\mu^2}\right)\bar{s}\gamma_\mu(1-\gamma_5)\lambda^a d \sum_{quarks} \bar{\psi}\gamma^\mu\lambda^a\psi \tag{3.10}$$

The function $8\pi^2 D$ (3.8) is plotted on fig. 9. It is calculated under the assumption that values of mass-parameters m_c and m_u are nearly constant for considered values of μ. The numbers between the curves denote ratios of the function D (3.8) and its LL part $\bar{D}$ (3.9). Even the low-μ ratio shows the importance of power-corrections to the LL result, and for values $\mu \gtrsim m_c$, (3.8) is almost completely dependent on the power-correction terms.

4. Discussion and Conclusion

The main aim of this paper was to learn how the power-corrections of the form $(m_q/\mu)^\alpha$ influence a standard LL part of the effective Hamiltonian. It was shown that in $\Delta S = 1$, $\Delta C = 1$ part of the Hamiltonian there was no significant contribution* to the lowest order part of the coefficients C_{20} and C_{84}.

The result in $\Delta S = 1$, $\Delta C = 0$ part of Hamiltonian deserves much more attention. For the values of $\mu \sim 0.5$ GeV power-corrections to the coefficients of penguin-like operators are as important as the LL contribution. As μ increases the old result [2] becomes overshadowed by corrections. Since the corrections have the same sign as LL part (see fig. 9), the coefficients get considerably bigger.

This result is not in a contradiction with the present theory of hyperon decays. It has been shown [4] that the calculation could reproduce s-wave amplitudes correctly while giving p-wave amplitudes somewhat too small if the penguin coefficients are taken to be just such as given by the leading-log analysis. The result is improved [5] when these coefficients are increased by hand. The same is true [2] for kaon decays into two pions: the theoretical predictions are in better agreement with experiment if penguin coefficients are larger than given by the LL analysis. Some changes are expected to appear in the analysis of CP-violation parameters [6] too.

While the GIM mechanism [15] "kills" penguin terms for a certain choice of μ in the LL approximation, terms with mixed chirality survive in the effective Hamiltonian even for $\mu > m_c$ in the calculation beyond the leading log. This new effect might play an important role in analyses of the $\Delta S = 1$, $\Delta C = 0$ decays of charm particles, such as $F \to D\pi$, etc.

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*The known disagreement between the theory and experiment for charm-changing decays of the D-meson [10] becomes slightly worse when power-corrections are taken into account.

However, as far as one is able to handle only the lowest order strong corrections, no final answer can be given. An analysis via RG equation, which helps when LL result is considered, becomes extremely involved if power-corrections are included. The RG functions β, γ and δ, that are μ-independent in LL approximation, become complicated functions of the renormalization point. Furthermore, the boundary condition on the solution of the RG equation, $C(\mu, g^2 \to 0) = C_{free}$, happens to be insufficient and one has to know how fast C goes to C_{free}, i.e., derivatives of C with respect to g^2 must be known in the $g^2 \to 0$ limit.

It is shown that coefficients in the effective Hamiltonian have considerably richer structure when the complete analysis rather than LL approximation is performed. However, results (2.15) and (3.8) apparently depend on the renormalization scheme as well as on the choice of μ, and a comment of this fact has to be made.

Let us forget for a moment that quarks are hadronized in real particles, and consider the world in which free quarks interact. Furthermore, for the sake of simplicity, imagine that quarks are massless. Their behaviour is then governed by the Lagrangian of the form (2.1), where g is an unknown parameter of the theory.* In order to find it one has (i) to choose some appropriate experiment, (ii) to make a selection of the renormalization scheme as well as of the renormalization point, and then to calculate the S-matrix, and (iii) to relate the experimentally measured value to the parameter g. (It is the property of asymptotically free theories [17] that no matter what renormalization scheme one chooses, the value of the parameter g determined from an experiment through steps

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*The weak interaction part of (2.1) is forgotten in the further analysis.

(i)-(iii) will decrease as μ in step (ii) grows). From now on, g acquires a fixed value, and in order to have a scheme- and μ- independent results one is forced to analyze any other experiment with just this value of g and just the same selection of the renormalization as used in the step (ii).

In the real world the situation would be analogous* if the mechanism of hadronization were known, and under the assumption that strong interactions could be treated perturbatively. However, that is not the case, and hadronic matrix elements of an effective Hamiltonian are usually determined [4,5,8] by using quark-model wave functions to describe baryon and meson states. Thus the insensitivity of the result on a renormalization prescription in a sense given above is spoiled completely. One can just hope that matrix elements calculated for a specific choice of μ have the same behaviour as elements that would be calculated by a correct calculation (if such could be performed). A common approach [1,2,18], i.e., choosing the value of the average mass of quarks involved in a process for μ, works surprisingly well.

Let us conclude with the following remark. It seems that, at least in principle, the weak effective Hamiltonian could be calculated quite accurately. The part of corrections not considered in previous works is discussed in this paper. Another problem that exceeds the intentions of this work deserves further attention: how and why the present scheme of calculation of matrix elements of the weak Hamiltonian, although apparently incapable of feeling the problems of the renormalization gives still good results.

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*Note that the true Lagrangian contains additional parameters, e.g., masses of quarks.

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Appendix

A. Effective Hamiltonian in Leading-Log Approximation

(i) $\Delta S = 1$, $\Delta C = 1$ Part

The effective $\Delta S = 1$, $\Delta C = 1$ Hamiltonian for the standard model is usually written in the form [18]

$$H_{eff} = \frac{G}{2\sqrt{2}} \cos^2\theta \left(C_{84}\, \mathcal{O}_{84} + C_{20}\, \mathcal{O}_{20} \right) \quad , \tag{A1}$$

where

$$\mathcal{O}^{84}_{20} = \left(\bar{u}d\bar{s}c \pm \bar{u}c\bar{s}d \right)_{(V-A)(V-A)} \quad . \tag{A2}$$

In (A1) Cabibbo-suppressed terms are omitted. The coefficients C_{20} and C_{84} in the LL approximation are for $\mu > m_c$ given by [1,18,19]

$$\begin{aligned} C_{20} &= \left(1 + \frac{25}{3} \frac{g^2}{16\pi^2} \ln \frac{M^2}{\mu^2} \right)^{0.48} = 1 + \frac{g^2}{4\pi^2} \ln \frac{M^2}{\mu^2} + \mathcal{O}(g^4) \\ C_{84} &= \left(C_{20} \right)^{-1/2} = 1 - \frac{g^2}{8\pi^2} \ln \frac{M^2}{\mu^2} + \mathcal{O}(g^4) \quad . \end{aligned} \tag{A3}$$

For $\mu \leq m_c$, the coefficients are [12]

$$\begin{aligned} C_{20} &= (\kappa_1)^{2/9} (\kappa_2)^{12/25} \quad ; \qquad C_{84} = \left(C_{20} \right)^{-1/2} \\ \kappa_1 &= 1 + 9 \frac{g^2}{16\pi^2} \ln \frac{m_c^2}{\mu^2} \quad ; \qquad \kappa_2 = 1 + \frac{25}{3} \frac{g^2/\kappa_1}{16\pi^2} \ln \frac{M^2}{m_c^2} \quad . \end{aligned} \tag{A4}$$

Thus one gets

$$C_{20} = 1 + \frac{g^2}{4\pi^2} \left(\ln \frac{M^2}{\mu^2} - \frac{1}{2} \ln \frac{m_c^2}{\mu^2} \right) + \mathcal{O}(g^4) \quad , \tag{A5}$$

and a similar expression for C_{84}.

(ii) $\Delta S = 1$, $\Delta C = 0$ Part

The well known [2] LL $\Delta S = 1$, $\Delta C = 0$ Hamiltonian contains, besides the usual four-quark operators, also the operator $\mathcal{P}$ (3.1) in the region $\mu \leq m_c$. The coefficient of the operator is calculated by help of the RG equation, and is given approximately by

$$C_P = \sin\theta\cos\theta \frac{G}{2\sqrt{2}} \left\{ (\kappa_2)^{12/25}\left[-0.48(\kappa_1)^{0.42} - 0.01(\kappa_1)^{-0.30} - 0.03(\kappa_1)^{0.80} + 0.51(\kappa_1)^{0.50}\right] + (\kappa_2)^{-6/25}\left[0.02(\kappa_1)^{0.42} - 0.06(\kappa_1)^{-0.30} - 0.01(\kappa_1)^{0.80} + 0.04(\kappa_1)^{0.50}\right]\right\} \quad , \qquad (A6)$$

where κ_1 and κ_2 are defined in (A4). The expression (A6) can be written as

$$C_P = \sin\theta\cos\theta \frac{G}{\sqrt{2}} \left[\frac{1}{12} \frac{g^2}{4\pi^2} \ln \frac{m_c^2}{\mu^2} + \mathcal{O}(g^4)\right] \quad ,$$

and one recognizes the result (3.10) obtained by the perturbative calculation.

B. Calculation of Diagrams on Fig. 2

When the finite part of some weak interaction amplitude is calculated, the terms of order $1/M^4$ (where M is the mass of a weak boson) give usually no significant contribution, and the analysis is much simpler if it is possible to isolate the dominant $1/M^2$ part. Here the method of such a separation is sketched in the calculation of the Feynman integral related to the first diagram on fig. 2a.

If the momentum of gluon is denoted by k, the matrix element is ($c^2 \equiv \cos^2\theta$)

$$M'_{2a} = c^2 \frac{h^2}{8} \frac{g^2}{4} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2\left[(k+p)^2-m_c^2\right]\left[(k-r)^2-m_d^2\right]\left[(k+q)^2-M^2\right]} \times \left\{\bar{s}\gamma_\mu(1-\gamma_5)(\hat{k}+\hat{p}+m_c)\gamma_\alpha\lambda_a c \times \bar{u}\gamma^\mu(1-\gamma_5)(\hat{r}-\hat{k}+m_d)\gamma^\alpha\lambda^a d\right\} \quad . \qquad (A7)$$

The straight expansion of W-boson propagator around M^2 is not allowed, since the higher order terms become more and more divergent, and another method has to be found (see Appendix D of ref. [20]). Let us first write the denominator by help of the Feynman parametrization as

$$\int_0^1 2y\,dy \int_0^1 dx \frac{1}{\left[k^2-2kc-d\right]^3\left[(k+q)^2-M^2\right]}$$

$$c_\mu = -xyp_\mu + (1-x)yr_\mu \tag{A8}$$

$$d = xy\left(m_c^2-p^2\right) + y(1-x)\left(m_d^2-r^2\right) \quad .$$

After the translation $k_\mu \to k_\mu + c_\mu$ is done, one can expand W-propagator around k^2-M^2. Thus

$$M'_{2a} \to c^2 \frac{h^2}{8}\frac{g^2}{4}\int \frac{d^4k}{(2\pi)^4} \iint \frac{\left\{k_\mu \to k_\mu + c_\mu\right\}}{(k^2-\phi_1)^3(k^2-M^2)}$$

$$\times\left[1 - \frac{2(q+c)k-(q+c)^2}{k^2-M^2} + \mathcal{O}\left(1/(k^2-M^2)^2\right)\right] \quad , \tag{A9}$$

where $\phi_1 = c^2 + d$ is defined in (2.9), and $q = p-p' = r'-r$. The simple analysis shows that only the first term in the square bracket in (A9) contributes to the order $\mathcal{O}(1/M^2)$, and all the other terms are of the order $\mathcal{O}(1/M^4)$. The final result is

$$M'_{2a} = i\,c^2 \frac{h^2}{8M^2}\frac{g^2}{16\pi^2}\int_0^1 2y\,dy \int_0^1 dx\left[-\frac{3}{2} - \ln\frac{\phi_1}{M^2}\right]$$

$$\times (\bar{s}\lambda^a c)_{V-A}(\bar{u}\lambda^a d)_{V-A} + \mathcal{O}(1/M^4) \quad . \tag{A10}$$

The terms proportional to $(m_c m_d/\phi_1)$ and those of dimensions higher than six have not been written explicitly in (A10).

By a similar analysis one gets the leading contribution of other diagrams on fig. 2a. In Section 2, the following functions have been used in addition to ϕ_1:

$$\phi_2 = y^2\left[p'^2x^2 + r'^2(1-x)^2 - 2p'r'x(1-x)\right] + y\left[\left(m_s^2 - p'^2\right)x + \left(m_u^2 - r'^2\right)(1-x)\right]$$

$$\phi_3 = y^2\left[p^2x^2 + r'^2(1-x)^2 + 2pr'x(1-x)\right] + y\left[\left(m_c^2 - p^2\right)x + \left(m_u^2 - r'^2\right)(1-x)\right]$$

$$\phi_4 = y^2\left[p'^2x^2 + r^2(1-x)^2 + 2rp'x(1-x)\right] + y\left[\left(m_s^2 - p'^2\right)x + \left(m_d^2 - r^2\right)(1-x)\right] \quad . \tag{A11}$$

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Figure Captions

Fig. 5. The function A (solid line), and its leading-log part (dashed line), calculated from (2.15) and (2.16), using values $M = 80$ GeV, $m_c = 1.5$ GeV and $m_s = 0.15$ GeV.

Fig. 9. The function $8\pi^2 D$ (3.8), and its leading-log part (3.9). The calculation has been done with the fix value $m_c = 1.5$ GeV.

(Please note: figures 1,2,3,4,6,7 and 8 are described in the main text.)

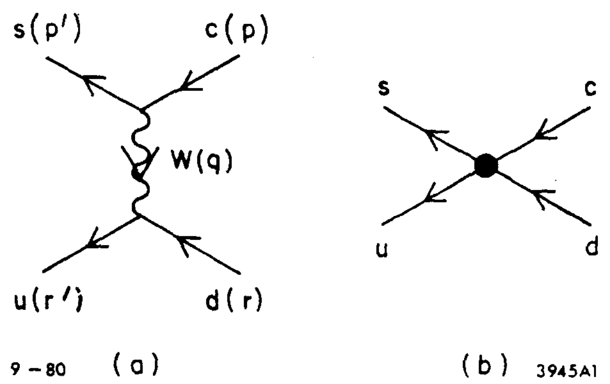

Fig. 1

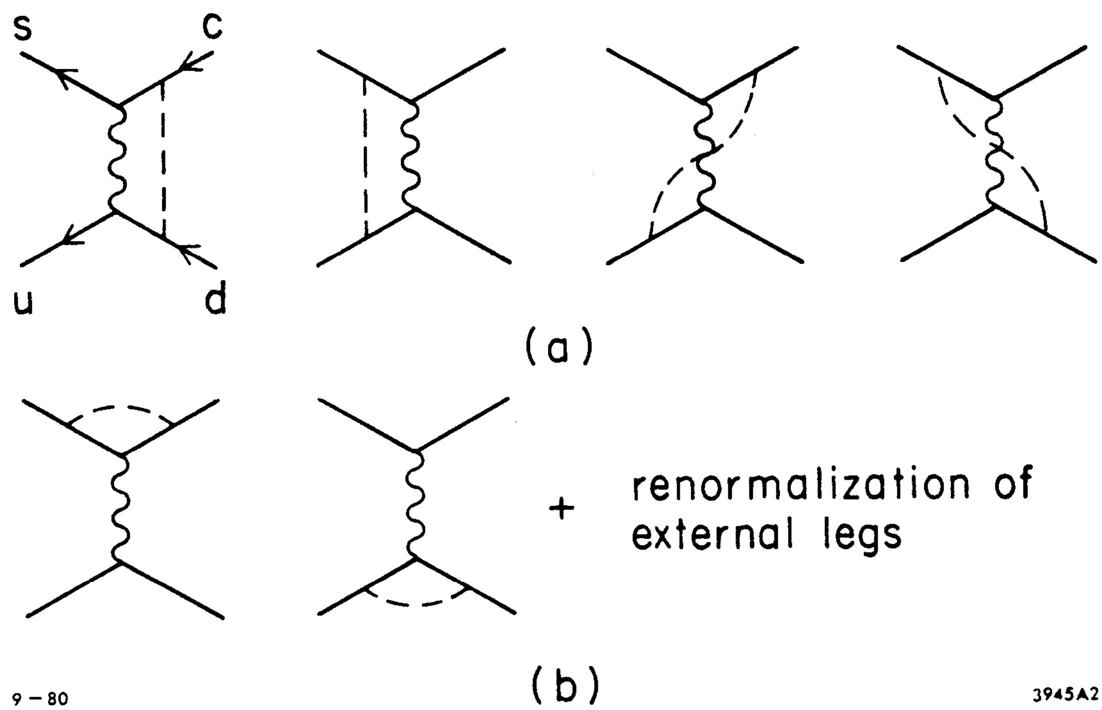

Fig. 2

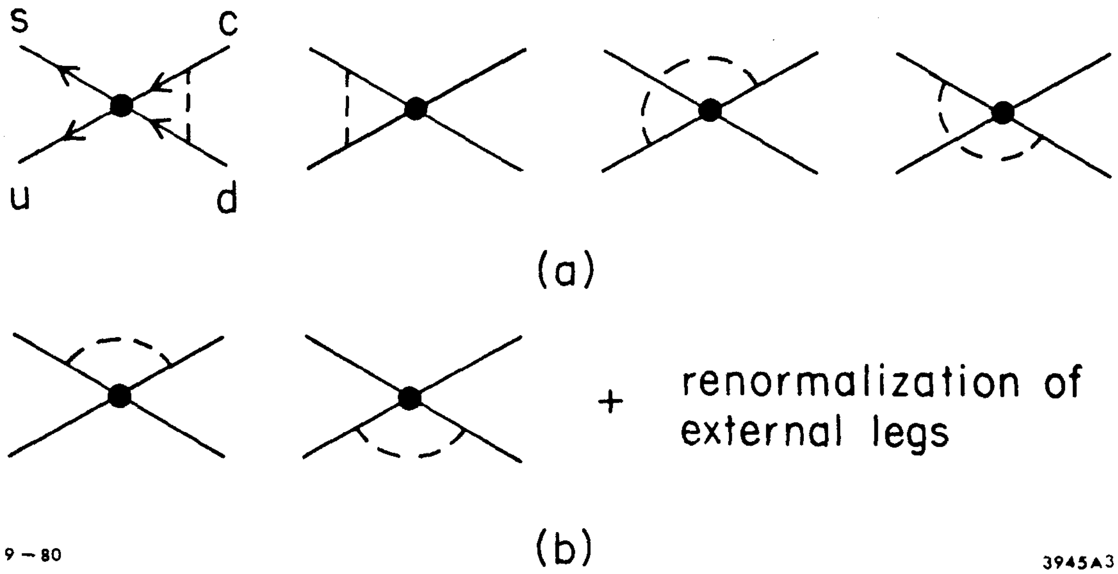

Fig. 3

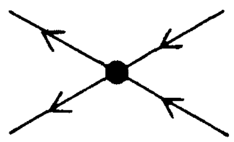

Fig. 4

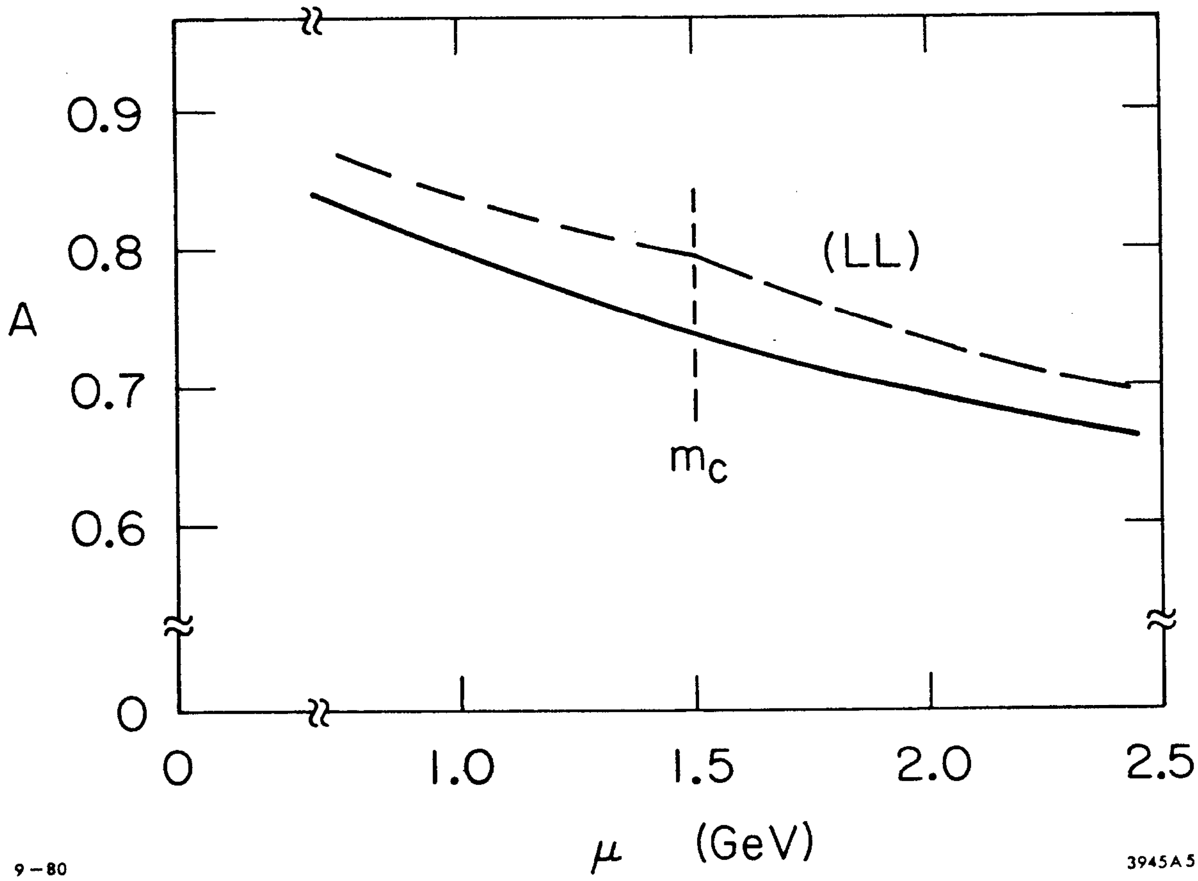

Fig. 5

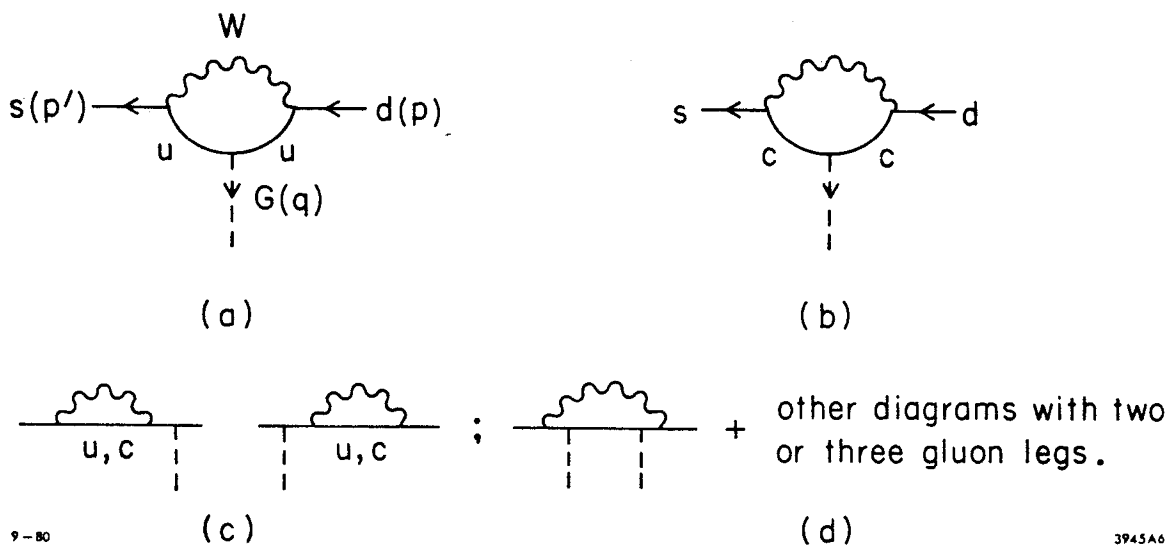
W
s(p′)
d(p)
u
u
G(q)
(a)
s
d
c
c
(b)
u, c
u, c
+ other diagrams with two or three gluon legs.
9-80
(c)
(d)
3945A6

Fig. 6

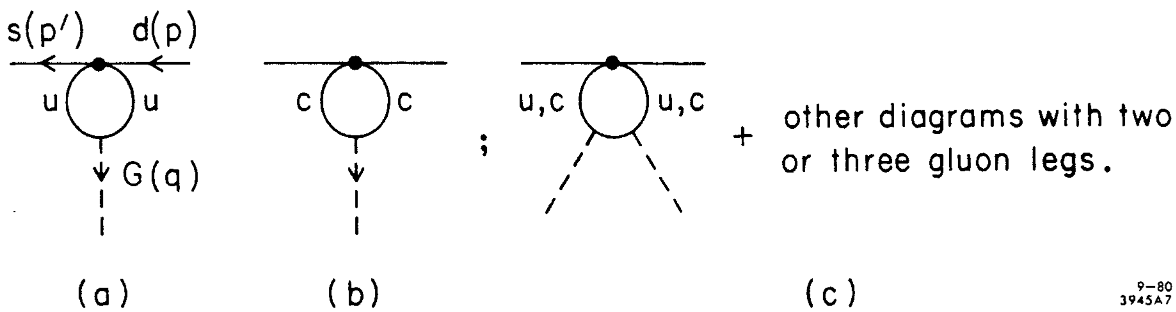
s(p′)
d(p)
u
u
G(q)
c
c
;
u,c
u,c
+
other diagrams with two
or three gluon legs.
(a)
(b)
(c)
9–80
3945A7

Fig. 7

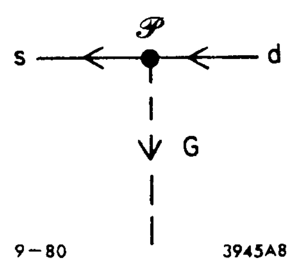

Fig. 8

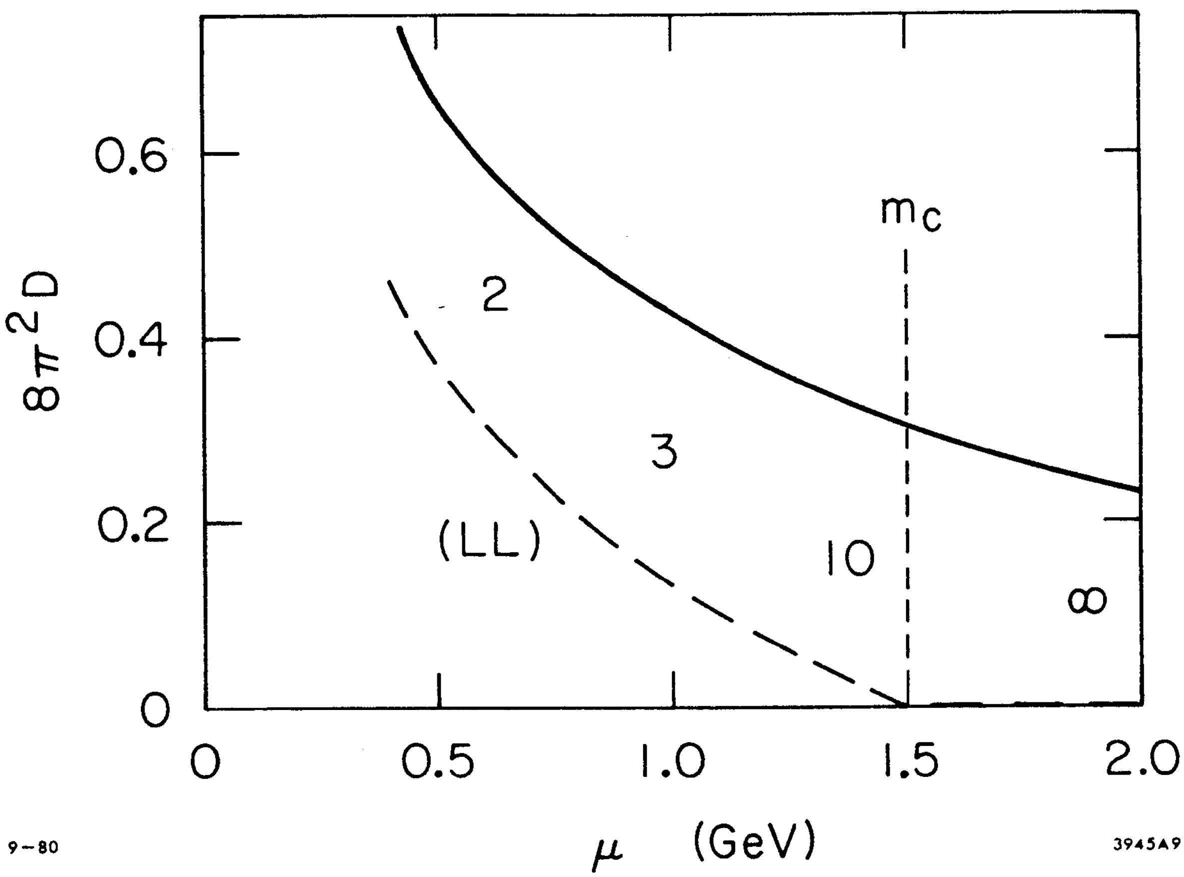

Fig. 9