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STRUCTURE FUNCTIONS AND HIGH TWIST CONTRIBUTIONS IN PERTURBATIVE QUANTUM CHROMODYNAMICS*

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On page 5:

line 8: for "... experiment[14]" replace "... experiment[13]"

line 12: for "... analysis[15]" replace "... analysis[14]"

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ABSTRACT

Perturbative QCD predictions are presented for the x near 1 behavior of hadronic structure functions. The available energy xW^2 is shown to control structure function evolution. In the case of meson structure functions, the $x \sim 1$ behavior is dominated by a high twist contribution to the longitudinal structure function $F_L \sim Cx^2/Q^2$ which can be rigorously computed and normalized.

One of the most important areas of study of perturbative quantum chromodynamics is the behavior of the hadronic wavefunctions at short distances or at far off-shell kinematics. This behavior can be tested not only in exclusive reactions such as form factors at large momentum transfer but also in deep inelastic scattering reactions at the edge of phase space. In this talk we will review the QCD predictions for the behavior of the hadronic structure functions $F_i(x,Q)$ in the end-point $x_{Bj} \sim 1$ region.[1] The endpoint region is particularly interesting because one must understand in detail (a) the contributions of exclusive channels, (b) the effect of high twist terms (power-law scale-breaking contributions) which can become dominant at large x, and (c) the essential role of the available energy W in controlling the logarithmic evolution of the structure functions. Note that as $x \sim 1$, essentially all of the hadron's momentum must be carried by one quark (or gluon), and thus each propagator which transfers this momentum becomes far-off shell: $k^2 \sim -(\vec{k}_\perp^2 + \mathcal{M}^2)/(1-x) \to -\infty$ [see Fig. 1]. Accordingly, if the spectator mass $\mathcal{M}$ is finite the leading power-law behavior in (1-x) is determined by the minimum number of gluon exchanges required to stop the hadronic spectators, and only the valence Fock states, $|qqq\rangle$ for baryons, and $|q\bar{q}\rangle$ for mesons, contribute to the leading power behavior. If one simply computes the connected tree graphs, as in Fig. 2, one finds that the results depend in an essential way on the quark's helicity:

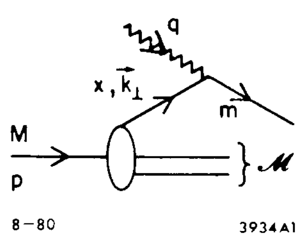

Fig. 1. Kinematics for inelastic structure functions.

$$G_{q/p}(x) \sim \begin{cases} (1-x)^3 & \text{parallel quark/nucleon helicity} \\ (1-x)^5 & \text{anti-parallel quark/nucleon helicity} \end{cases}$$

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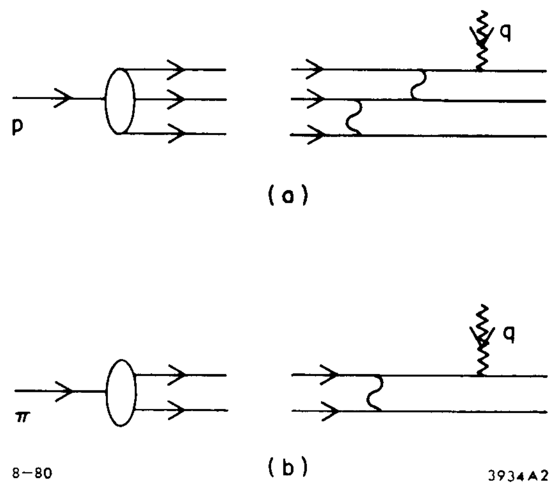

Fig. 2. Perturbative QCD tree diagrams for computing the $x \sim 1$ power behavior of baryon and meson structure functions.

and $G_{q/\pi} \sim (1-x)^2$. If the nucleon wavefunctions satisfy the standard SU(6) spin-flavor symmetry, then the above results imply[2]

$$\frac{G_{u/p}}{G_{d/p}} \underset{x\to 1}{=} 5 \quad .$$

Let us now consider how these results for the power-law behavior emerge within the complete perturbative structure of QCD. Including corrections from gluon radiation, vertex and self-energy corrections, and continued iteration of the gluon-exchange kernel, one finds for the nucleon's quark distribution[1]

$$G_{q/p}(x,Q) \underset{x\to 1}{=} (1-x)^3\, \alpha_s^4(k_x^2) \left| \sum_{j=0}^{\infty} b_j \left(\log \frac{k_x^2}{\Lambda^2} \right)^{-\gamma_j^N} \right|^2 P_q(x,Q)$$
$$\times \left[1 + \mathcal{O}\left(\alpha_s(k_x^2), 1/Q \right) \right] \quad . \qquad (1)$$

The powers of α_s and $(1-x)$ reflect the behavior of the hard scattering amplitude at the off-shell value $k_x^2 = (\langle k_\perp^2 \rangle + \mathcal{M}^2)/(1-x)$ where $\langle k_\perp^2 \rangle$ is set by the spectator transverse momentum integrations. The anomalous dimensions γ_j^N are the anomalous dimensions of the nucleon's valence Fock state wavefunction at short distances. Their contribution to $G_{q/p}(x,Q)$ are due to the evolution of the wavefunction integrated up to the transverse momentum scale $\vec{\ell}_\perp^2 < k_x^2$ as in the corresponding exclusive channel analyses.[1] The last factor $P_q(x,Q)$ represents the target-independent evolution of the structure function due to gluon emission from the struck quark: $(C_F = 4/3)$

$$P_q(x,Q) \sim (1-x)^{4C_F \xi(Q)} \qquad (2)$$

where

$$\xi(Q) = \int_{Q_0^2}^{Q^2} \frac{dj_\perp^2}{j_\perp^2}\, \alpha_s(j_\perp^2) \sim \log\left(\frac{\log Q^2/\Lambda^2}{\log Q_0^2/\Lambda^2} \right) \quad . \qquad (3)$$

The lower limit Q_0^2 of the gluon's transverse momentum integration is set by the mean value of the spectator quark's transverse momenta and masses. This hadronic scale sets the starting point for structure functions evolution.[3] Equation (1) then gives the light-cone momentum distribution for parallel-helicity quarks with x near 1 at the transverse momentum scale Q.

It should be emphasized that the actual momentum scale probed by various deep inelastic inclusive reactions depends in detail on the process under consideration; the actual upper limit of the transverse momentum integration is set by kinematics. For example, if we consider the contribution of Fig. 3 to the deep inelastic structure functions, the propagator (or energy denominator) associated with the top loop reduces to the usual Bjorken structure $2q\cdot p - Q^2/x + i\varepsilon$ only if $k_\perp^2 \ll (1-y)Q^2 \leq (1-x)Q^2$ where $k_\perp$ is the quark's transverse momentum and $y \geq x$ is the light-cone variable indicated in the figure. The remaining structure factorizes into a form which defines $G_{q/p}(x/y, k_\perp)$. Thus the actual relation between the structure function and the momentum distribution for $x \sim 1$ is[1,4]

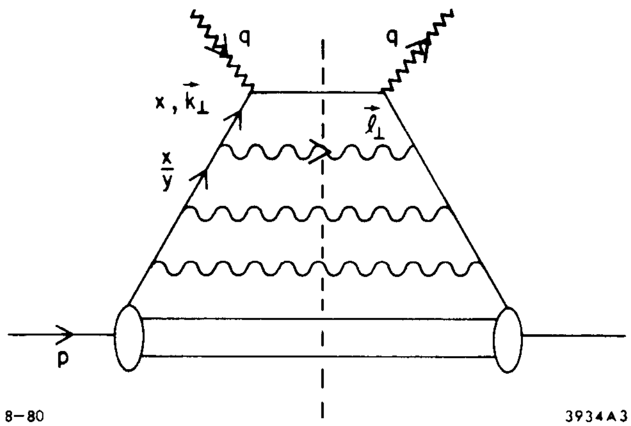

Fig. 3. Perturbative QCD diagrams for structure function evolution.

$$F_2(x,Q) = \sum_i e_i^2\, x_{Bj}\left[G_{q_i/p}(x_{Bj},Q) + \delta G_{q_i/p}(x_{Bj},Q)\right] \tag{4}$$

where

$$\delta G_{q/p}(x,Q) = -2C_F \int_0^1 dy\, \frac{1+y^2}{1-y} \int_{(1-y)Q^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2}\, \frac{\alpha_s(k_\perp^2/y)}{4\pi}$$
$$\times \left\{G_{q/p}(x/y,k_\perp)\, \frac{\theta(y>x)}{y} - G_{q/p}(x,k_\perp)\right\} \tag{5}$$

corrects for the fact that the top loop is integrated to $k_\perp^2 < (1-y)Q^2$ ($\leq x_{Bj}W^2$) not Q^2. [The argument of α_s is also crucial here.] Other inclusive reactions have to be individually examined: in the case of the Drell-Yan process $q_a\bar{q}_b \to \mu^+\mu^-$, the structure functions evolve to $(1-y_a)Q^2$ and $(1-y_b)Q^2$ not $Q^2 = (p_+ + p_-)^2$.

The actual evolution of structure functions in deep inelastic lepton scattering is thus controlled by the available energy $x_{Bj}W^2$, and is more moderate at $x_{Bj} \sim 1$ than would be expected from lowest order expectations. Analytic forms for the (1-x) behavior are readily computed.[1] The most important features are the following: (1) The $\delta G/G$ correction to leading order in α_s reproduces the critical $2C_F(\alpha_s(Q^2)/4\pi)\log^2 n$ terms in the structure function moments as calculated using the operator product expansion and renormalization group. In our analysis a series of terms of all orders in $(\alpha_s \log^2(1/1-x))^p$ or $(\alpha_s \log^2 n)^p$ arises simply from the fact that the natural evolution parameter for the structure functions $F_i(x,Q)$ and moments $\mathcal{M}_n(x,Q)$ is controlled by $(1-y)Q^2 < (1-x)Q^2$ and not Q^2; the basic momentum distributions $G(x,Q)$ do not contain the anomalous double-log terms and have a straightforward perturbative evolution. (2) The extended evolution equations based on Eqs. (4) and (5) have a number of phenomenological advantages. After taking into account the appropriate evolution limits, each deep inelastic process can be related to the basic distributions $G_q(x,Q)$, avoiding large kinematic corrections. The scale parameter Λ_n which has been introduced to eliminate the strong n-dependence of the higher order corrections to the moments is unnecessary. The fact that xW^2 controls the evolution suggest its use in structure function parameterizations and studies of moment factorization in fragmentation processes. A study of the application of this method to photon structure functions is in progress. (3) The exclusive-inclusive connection fails in QCD.[1,5] At fixed but large W^2, $F_{2N}(x,Q)$ falls as $(1-x)^{3+\delta}$ where $\delta > 0$, whereas, modulo logarithmic factors, exclusive channels in QCD give contributions $\sim(1-x)^3$ from the Q^{-4} scaling of the leading nucleon form factors. Thus exclusive channels will eventually dominate the leading twist contributions to inclusive cross sections at fixed W^2, $Q^2 \to \infty$.

A complete treatment of the hadron structure functions must take into account higher twist contributions. Although such contributions are suppressed by powers of $1/Q^2$, they can have fewer powers of $(1-x)$ and, accordingly, may be phenomenologically important in the large x domain.[6] In the case of nucleons, the $\ell + qq \to \ell' + qq$ subprocess (in which the lepton recoils against two quarks) leads to a structure function contribution $\sim(1-x)/Q^4$ since only one quark spectator is required. A large longitudinal structure function is also expected.[6,7,8] Although complete calculations of such terms have not been done, the presence of such terms can reduce the amount of logarithm scale-violation required from the leading twist contributions in phenomenological fits.[6,9]

The analysis of meson structure functions at $x \sim 1$ is similar to that of the baryon, with two striking differences: (1) The controlling power behavior of the leading twist contribution is $(1-x)^2$ from perturbative QCD.[2,10] The extra factor of $(1-x)$ -- compared to what would have been expected from spectator counting -- can be attributed to the mismatch between the quark spin and that of the meson. (2) The longitudinal meson structure function has an anomalous non-scaling component[7,11] which is finite at $x \to 1$: $F_L(x,Q) \sim Cx^2/Q^2$. This high twist term, which comes from the lepton scattering off an instantaneous fermion-line in light-cone perturbation theory, can be rigorously computed and normalized in perturbative QCD. The crucial fact is that the wavefunction evolution and spectator transverse momentum integrations in Fig. 4 can be written directly in terms of a corresponding calculation of the meson form factor. The result for the pion structure function to leading order in $\alpha_s(k_x^2)$ and $\alpha_s(Q^2)$ is[11,12]

$$F_L^{\pi}(x,Q) = \frac{2x^2}{Q^2} C_F \int_{m^2/(1-x)}^{Q^2} dk^2 \; \alpha_s(k^2)\, F_{\pi}(k^2) \qquad (6)$$

which numerically is $F_L \sim x^2/Q^2$ (GeV^2 units).

The dominance of the longitudinal structure functions in the fixed W limit for mesons is an essential prediction of perturbative

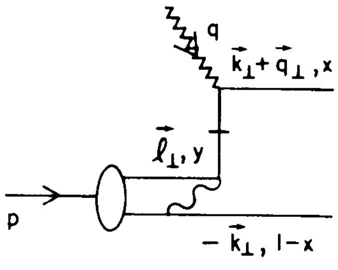

Fig. 4. Perturbative contribution to the meson longitudinal structure function $F_L \sim C/Q^2$.

QCD. Perhaps the most dramatic consequence is in the Drell-Yan process $\pi p \to \ell^+\ell^- X$; one predicts[7] that for fixed pair mass Q, the angular distribution of the ℓ^+ (in the pair rest frame) will change from the conventional $(1+\cos^2\theta_+)$ distribution to $\sin^2(\theta_+)$ for pairs produced at large x_L. A recent analysis of the Chicago-Illinois-Princeton experiment[13] at FNAL appears to confirm the QCD high twist prediction with about the expected normalization. Striking evidence for the effect has also been seen in a Gargamelle analysis[14] of the quark framentation functions in $\nu p \to \pi^+\mu^- X$. The results yield a quark fragmentation distribution into positive charged hadrons which is consistent with the predicted form: $dN^+/dzdy \sim B(1-z)^2 + (C/Q^2)(1-y)$ where the (1-y) behavior corresponds to a longitudinal structure function. It is also crucial to check that the $e^+e^- \to MX$ cross section becomes purely longitudinal $(\sin^2\theta)$ at large z at moderate Q^2. The implications of this high twist contribution for meson production at large p_T will be discussed elsewhere.[12]

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