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NEUTRINO OSCILLATIONS OF THE SECOND CLASS*

V. Barger and J. P. Leveille†
Physics Department, University of Wisconsin
Madison, Wisconsin 53706

P. Langacker‡
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

S. Pakvasa
Physics Department, University of Hawaii
Honolulu, Hawaii 96822

ABSTRACT

We consider a second class of neutrino oscillations which can arise when both Majorana and Dirac neutrino mass terms are present in the Lagrangian. These oscillations mix neutrino members of weak current doublets with singlets of the same chirality. A depletion of a neutrino beam would result, with apparent non-conservation of probability. Possible relevance to current oscillation experiments is discussed.

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† Address after September 1, 1980: The Randall Laboratory, University of Michigan, Ann Arbor, Michigan 48109.
‡ On leave from the University of Pennsylvania, Philadelphia, PA 19174.

The possibility that neutrinos have mass has received experimental and theoretical support in recent months. The experimental evidence[1,2] is not yet conclusive, but is tantalizing nevertheless. On the theoretical side many grand unified theories require non-vanishing neutrino masses. In the following we shall assume that neutrinos are massive. For convenience, we shall work in an $SU(2)\times U(1)$ framework with the usual lepton assignment:

$$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L \quad \begin{matrix}\eta_{eL}\\ e^+_L\end{matrix} \quad ; \quad \begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}_L \quad \begin{matrix}\eta_{\mu L}\\ \mu^+_L\end{matrix} \quad ; \quad \begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}_L \quad \begin{matrix}\eta_{\tau L}\\ \tau^+_L\end{matrix} \quad ; \quad \ldots \tag{1}$$

The number of families is arbitrary. We assume that the mass matrix is already diagonal in the sector with electric charge $Q=\pm 1$. For convenience we have chosen to use left-handed singlet fields which are conjugate fields of the more familiar right-handed singlets, e.g., $e^-_R = C\gamma^o(e^+_L)^\dagger$ and $\eta^c_{eR} = C\gamma^o(\eta_L)^\dagger$ with $C = i\gamma^2\gamma^o$. In Eq. (1) we have also assumed only one neutral singlet per family although more could be added. As defined above, ν_{eL} is the field associated with the electron neutrino produced in the inverse β-decay reaction $e^-p \to \nu_{eL}n$, whereas $\nu^c_{eR} \equiv C\gamma^o(\nu_{eL})^\dagger$ is the field of the electron anti-neutrino created in β-decay $n \to pe^-\nu^c_{eR}$. Note that the singlet fields η_{iL}, $i=e,\mu,\tau,\ldots$ are not coupled to the $W^\pm$ and Z bosons since they are electrically neutral isospin singlets. Because the singlets interact with other fermions via Higgs bosons only, they are effectively decoupled from light fermions.

Two choices are possible for the mass terms in the Lagrangian.[3,4] With the usual Higgs doublet only, a Dirac-type mass term can be constructed

$$\mathscr{L}_D = -\sum_{i,j=e,\mu,\tau}\left(d_{ij}\,\overline{\nu_{iL}}\,\eta^c_{jR} + \text{h.c.}\right) \quad . \qquad (2)$$

In Eq. (2), we have omitted by fiat possible bare mass terms of the form $\bar{\eta}_{iL}\eta^c_{jR}$. Such terms can be dismissed by imposing a discrete symmetry on the Lagrangian:

$$\begin{array}{c}\left[\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L,\begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}_L,\begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}_L\right] \to e^{+i\alpha}\left[\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L,\begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}_L,\begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}_L\right] \\ \left[e^+_L,\eta_{eL},\mu^+_L,\eta_{\mu L},\tau^+_L,\eta_{\tau L}\right] \to e^{-i\alpha}\left[e^+_L,\eta_{eL},\mu^+_L,\eta_{\mu L},\tau^+_L,\eta_{\tau L}\right]\end{array} \quad . \qquad (3)$$

Equation (2) is invariant under the transformation Eq. (3), while Majorana mass terms are not: $\overline{\eta_{iL}}\eta^c_{jR} \to e^{2i\alpha}\overline{\eta_{iL}}\eta^c_{jR}$. The invariance of the Lagrangian Eq. (2) under Eq. (3) leads to lepton number conservation, with all doublet fields chosen to carry lepton number $\ell = +1$, all left-handed singlets $\ell = -1$. Majorana mass terms change lepton number by two units. With the discrete symmetry of Eq. (3), electron, muon and τ numbers are not separately conserved.

In the case where lepton number is conserved, Eq. (2) is the most general mass Lagrangian, and the mass eigenstates are three (Dirac) four-component neutrinos, in the case of three families. The weak interaction eigenstates ν_{eL}, $\nu_{\mu L}$, $\nu_{\tau L}$ are linear superpositions of these Dirac mass eigenstates. Through the usual formalism, one is led to oscillation among the three flavors, $\nu_{eL} \leftrightarrow \nu_{\mu L} \leftrightarrow \nu_{\tau L}$. Similarly, one has oscillations $\eta_{eL} \leftrightarrow \eta_{\mu L} \leftrightarrow \eta_{\tau L}$ which are undetectable, however, since the singlets are not coupled to the gauge bosons. We call these oscillations among flavors (without chirality change) first class oscillations.

If we do not impose lepton number conservation, then Eq. (2) is not the most general Lagrangian mass term. Including all allowed Majorana couplings, we obtain

$$\mathscr{L} = -\frac{1}{2} \sum_{i,j=e,\mu,\tau} a_{ij}\overline{\nu_{iL}}\nu_{jR}^{c} + d_{ij}\left(\overline{\nu_{iL}}\eta_{jR}^{c} + \overline{\eta_{jL}}\nu_{iL}^{c}\right) + s_{ij}\overline{\eta_{iL}}\eta_{jR}^{c} + \text{h.c.} \tag{4}$$

Here we have used the identity $\overline{\nu_L}\eta_R^c = \overline{\eta_L}\nu_R^c$ to reduce the numbers of independent constants. Within an $SU(2)\times U(1)$ context additional Higgs fields would be needed to generate the extra terms: $a_{ij} \neq 0$ requires a Higgs triplet whereas a singlet Higgs or simply bare mass terms will allow non-vanishing s_{ij}'s.

Diagonalization[3,4] of Eq. (3) reveals that the mass eigenstates are 6 Majorana (i.e., self-conjugate) neutrinos in the case of 3 families. The weak eigenstates ν_{iL} and η_{iL}, $i = e,\mu,\tau$, are linear superpositions of these six states. Besides the first class oscillations $\nu_{iL} \leftrightarrow \nu_{jL}$, $\eta_{iL} \leftrightarrow \eta_{jL}$, one can now have lepton-number changing oscillations involving singlet-to-doublet transitions, $\nu_{iL} \leftrightarrow \eta_{jL}$. We call these oscillations of the second class. We note that these oscillations do not flip chirality. Chirality flip oscillations are suppressed by powers of m_ν/E_ν and are negligible.

We consider in detail the consequences of having both Majorana and Dirac neutrino mass terms in a single family. Defining the doublets $\omega_L^\alpha \equiv (\nu_L,\eta_L)$, $\omega_R^{\alpha c} \equiv (\nu_R^c, \eta_R^c)$, the Lagrangian mass term from Eq. (3) can be cast in the form

$$\mathscr{L}_{\text{mass}} = -\frac{1}{2}\,\bar{\omega}_L^{\alpha} M^{\alpha\beta}\,\omega_R^{\beta c} + \text{h.c.} \tag{5}$$

with mass matrix

$$M = \begin{pmatrix} a & d \\ d & s \end{pmatrix} \quad . \tag{6}$$

The diagonalized mass matrix is $M_D = U_L^\dagger M U_R$ where U_L and U_R are unitary transformations of the ω_L and ω_R^c fields. Since M is symmetric, $U_R = U_L^* K^\dagger$ with K a unitary matrix. The relation of mass eigenstates ν_{iL} to ω_L^α is $\omega_L^\alpha = U_L^{\alpha i}\nu_{iL}$ (i = 1,2). The corresponding right-handed transformation is $\omega_R^{\alpha c} = C(\overline{\omega_L^\alpha})^T = U_R^{\alpha i} K_{ij}\nu_{jR}^c \equiv U_R^{\alpha i}\nu_{iR}^{\tilde{c}}$ where $\nu_{iR}^{\tilde{c}} \equiv K_{ij}\nu_{jR}^c = K_{ij}C(\overline{\nu_{jL}})^T$. The free Lagrangian for the neutral leptons is diagonal in the basis $\nu_i = \nu_{iL} + \nu_{iR}^{\tilde{c}}$. We find $\nu_i^{\tilde{c}} = \nu_i$, where $\nu_i^{\tilde{c}} \equiv K_{ij}C(\bar{\nu}_i)^T$. Hence, the ν_i are Majorana neutrino fields since they are self-conjugate. The combined Dirac and Majorana mass terms in the Lagrangian produce two Majorana eigenstates which in general have different masses m_1 and m_2. When $m_1 \neq m_2$, there is no conserved lepton number. The weak eigenstates ν_L and η_L are linear superpositions of the two Majorana mass eigenstates

$$\nu_L = \cos\alpha\ \nu_{1L} + \sin\alpha\ \nu_{2L} \quad , \quad \eta_L = -\sin\alpha\ \nu_{1L} + \cos\alpha\ \nu_{2L} \tag{7}$$

where $\cos\alpha = (U_L)^{11}$, $\sin\alpha = (U_L)^{12}$. The doublet member ν_L has the usual charged and neutral current couplings to gauge bosons. In the mass eigenstate basis the neutral current is non-diagonal, leading to the possibility of decays of the mass eigenstates by neutral currents, but with lifetimes which are much longer than the age of the universe.

Our primary considerations are for the logical possibility in which both m_1 and m_2 are small compared to the electron mass. This possibility has interesting implications for neutrino oscillations. Since the mass eigenstates propagate differently in time, second class oscillations

$\nu_{eL} \leftrightarrow \eta_{eL}$ which conserve chirality can occur. At a distance L from a source of ν_{eL}, the probability (for energy $E >> m_1, m_2$) of finding ν_{eL} is

$$P\left(\nu_{eL} \to \nu_{eL}\right) = 1 - \sin^2 2\alpha \sin^2(\tfrac{1}{2}\Delta) \qquad (8)$$

where the oscillation argument is $\tfrac{1}{2}\Delta = 1.27\delta m^2 L/E$, with $\delta m^2 = m_1^2 - m_2^2$ in eV^2 units and L/E in m/MeV units. The oscillations result in a depletion of an electron neutrino beam, or equivalently a deviation from a $1/r^2$ law for a point ν_{eL} source. Moreover, since η_{eL} is effectively non-interacting, probability conservation would appear to be violated by an amount $P(\nu_{eL} \to \eta_{eL}) = 1 - P(\nu_{eL} \to \nu_{eL})$, in contrast to first class oscillations where a depletion in $\nu_{eL} \to \nu_{eL}$ coincides with $\nu_{eL} \to \nu_{\mu L}, \nu_{\tau L}, \ldots$ transitions which are in principle observable.

In second class oscillations, both the charged current (CC) $\nu_{eL} p \to e^- X$ and neutral current (NC) $\nu_{eL} p \to \nu_{eL} X$ cross sections oscillate, $\sigma(L)/\sigma(L=0) = P(\nu_{eL} \to \nu_{eL}\,; L/E)$, and the ratio σ_{NC}/σ_{CC} is unaffected in the one-family case. This should be contrasted with first class oscillations where σ_{CC} and σ_{NC}/σ_{CC} oscillate, but σ_{NC} does not. Corresponding statements apply to ν_{eR} cross sections.

We now turn to possible phenomenological implications of second class oscillations for current experiments.

Solar: Lepton number violating oscillations have the capability of explaining the deficiency in the ratio of observed to expected solar neutrinos.[5] With first and second class oscillations among three families, the minimum probability for $\nu_e \to \nu_e$ transitions is 1/6.

Reactor: The cross sections for an initial ν_{eR}^c beam scattering on proton and deuteron targets indicate depletions[1] in $\sigma_{CC}(p)$, $\sigma_{CC}(d)$ and

$\sigma_{CC}(d)/\sigma'_{NC}(d)$ but not (at the $\approx$20% uncertainty level) in $\sigma_{NC}(d)$. To explain both the σ_{CC} and σ_{CC}/σ_{NC} results, first class oscillations are required with $\delta m^2 \simeq 1$ eV^2.

Beam dump: Charged and neutral current events are produced by prompt neutrinos created in the dump. Since the prompt neutrinos originate from decays of charmed particles, identical ν_e and ν_μ spectra and numbers are generated. The charged and neutral current interactions of the prompt neutrinos are measured in bubble chamber and counter experiments at CERN at a distance L $\simeq$ 800-900 m downstream.

In the bubble chamber experiment, the measured e/μ ratio[6] is $R(e/\mu) = 0.59^{+0.35}_{-0.21}$. Such deviations of the e/μ ratio from unity may indicate a $P(\nu_e \to \nu_e)$ depletion arising from oscillations.[2,7] For the CERN beam dump L/E $\simeq$ 0.01 m/MeV, so the mass scale of the oscillations would be $\delta m^2 \simeq 100$ eV^2. To discuss such oscillations we assume a prompt neutrino beam with equal parts of ν_{eL} and $\nu_{\mu L}$, neglecting any ν^c_{eR} and $\nu^c_{\mu R}$ contributions for simplicity.

For second class oscillations of the ν_e family alone, the e/μ ratio is given by

$$R(e/\mu) = \left[\langle P(\nu_e \to \nu_e)\sigma_{CC}\rangle\right] / \langle\sigma_{CC}\rangle \tag{9}$$

where σ_{CC} is the inclusive production cross section for e or μ and $\langle\ \rangle$ denotes a spectrum average. For first class oscillations $\nu_e \to \nu_e$, $\nu_e \to \nu_\tau$ (stringent experimental limits exist on $\nu_\mu \to \nu_e$ and $\nu_\mu \to \nu_\tau$ oscillations in this L/E range), the corresponding prediction is

$$R(e/\mu) = \frac{\langle P(\nu_e \to \nu_e)\sigma_{CC}\rangle + 0.17\langle P(\nu_e \to \nu_\tau)\sigma^\tau_{CC}\rangle}{\langle\sigma_{CC}\rangle + 0.17\langle P(\nu_e \to \nu_\tau)\sigma^\tau_{CC}\rangle} \tag{10}$$

where $\sigma_{CC}^{\tau}{}'$ is the inclusive τ cross section. For comparable mixing in the two classes, the predictions in Eqs. (9) and (10) are similar. One can discriminate experimentally between the classes of oscillations by ascertaining whether τ is produced and whether σ_{NC}/σ_{CC} changes.

The beam dump counter experiments measure the ratio $N(0\mu)/N(1\mu)$ of muonless to single muon events. With second class oscillations of the ν_e family the prediction is

$$N(0\mu)/N(1\mu) = \left[\left\langle\left(1+P(\nu_e\to\bar{\nu}_e)\right)\sigma_{NC}\right\rangle + \left\langle P(\nu_e\to\bar{\nu}_e)\sigma_{CC}\right\rangle\right]/\langle\sigma_{CC}\rangle \quad (11)$$

in the limit of perfect acceptance. The corresponding prediction for first class oscillations is

$$\frac{N(0\mu)}{N(1\mu)} = \frac{2\langle\sigma_{NC}\rangle + \langle P(\nu_e\to\nu_e)\sigma_{CC}\rangle + 0.83\langle P(\nu_e\to\nu_\tau)\sigma_{CC}^{\tau}\rangle}{\langle\sigma_{CC}\rangle + 0.17\langle P(\nu_e\to\nu_\tau)\sigma_{CC}^{\tau}\rangle} \quad . \qquad (12)$$

Taking comparable mixing in the two classes (and hence similar $R(e/\mu)$ predictions), the value of $N(0\mu)/N(1\mu)$ is significantly lower for second class oscillations. A detailed analysis with experimental cuts could thereby differentiate between first and second class oscillations in this L/E range on the basis of measured $R(e/\mu)$ and $N(0\mu)/N(1\mu)$ values. Still other alternatives are simultaneous first and second class oscillations or first class oscillations involving additional families.

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