SLAC-PUB-2585
August 1980
(T/E)

MINIMAL SUBCONSTITUENT MODELS OF QUARKS AND LEPTONS*

R. Casalbuoni**, G. Domokos and S. Kovesi-Domokos

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

and

Department of Physics, Johns Hopkins University
Baltimore, Maryland 21218

ABSTRACT

We analyze the structure of composite quarks and leptons in the framework of chiral subconstituent models of minimal internal symmetry. The model of Harari and Shupe contains the correct number of composite fields but it cannot sustain an exact color symmetry. An extension of the Harari-Shupe model is proposed, with both color and electromagnetic properties being realized linearly on the subconstituents. This model contains two generations of quarks and leptons; all models of this type contain additional quark-like states or lepton-like states of higher spin. Problems associated with the implementation of the approximate weak-electromagnetic symmetry and with the statistics of subconstituents are discussed.

(Submitted to Physical Review D)

---

*Work supported in part by the Department of Energy under contracts DE-AC03-76SF00515 (SLAC) and DE-AS-01-76ER03285 (Johns Hopkins).

**On leave of absence from and address after September 1, 1980: INFN, Sezione di Firenze, Florence, Italy.

1. Introduction

Subconstituent models of quarks and leptons have received increasing attention.[1] There are several reasons why such models are of interest. First, and foremost, it is hard to believe that several generations of quarks and leptons -- presumably of increasing masses, but of the same internal symmetry pattern -- would all form "fundamental" constituents of matter. Second, there is at least some hope that the nagging problems associated with a large number of elementary Higgs bosons in "standard" unified theories may be resolved in the framework of such models through a dynamical breakdown of some symmetry groups, and thus, Higgs fields would no longer have to be regarded as fundamental dynamical variables.[2]

Roughly speaking, two broad classes of subconstituent models can be distinguished among those proposed so far. Models of the first class ("conservative models") incorporate all known symmetries, including approximate ones, like weak isospin, at the level of the fundamental dynamical variables (subconstituents). Such models are generalizations of standard gauge theories and their symmetry structure has been analyzed in detail.[3] Their main drawback is that the symmetry group at the level of subconstituents is, by necessity, a rather large one; consequently, one is plagued by a plethora of "exotic" (so far unobserved) quark- and lepton-like composite fields. Moreover, it seems to be difficult, if not impossible, to effectively decouple the unwanted states from the low-lying quarks and leptons by giving the former sufficiently high masses. By contrast, proponents of models of the second class ("radical models") argue in essence that only exact symmetries should be carried by the subconstituents; all approximate symmetries (perhaps even weak isospin) should emerge as classification groups only at the level of the composite fields. Consequently, all of the heavy gauge bosons of standard unified models should be regarded as bound states. The prototype of such models is the one proposed independently by Harari[4] and Shupe[5]: these authors insist that only the gauge group of electromagnetism is carried as a local symmetry by the subconstituents; all other gauge bosons, including the gluons of $SU(3)_{color}$ are composite. These models are intuitively appealing, for they minimize the number of subconstituents necessary for the description of the known quarks and leptons; however, little is known about the structure of a theory within which such ideas can be realized.

In this paper we carry out a kinematical analysis of subquark models of minimal symmetry: obviously, such an analysis must precede any dynamical calculation.

In carrying out the analysis, we assume that some basic algebraic properties of quantum field theory continue to hold at the subconstituent level. Specifically, we assume that the quantum fields, $\psi_A(x)$ say, generate an infinite dimensional algebra. All elements of this algebra can be generated by means of a binary product[6] which is associative, i.e.,

$$\left(\psi_A(x)\,\psi_B(y)\right)\psi_C(z) = \psi_A(x)\left(\psi_B(y)\,\psi_C(z)\right)$$

Further, <u>all</u> continuous transformations of this algebra (whether or not they are symmetries of the theory) are generated by the usual commutators. (We do not consider transformations -- "supersymmetries" -- which change the symmetry character of an element of the algebra.) The commutator has the usual properties, *viz*. $[A,BC] = [A,B]C + B[A,C]$ and the Jacobi identity. (In technical terms, this means that all derivations of the algebra of the fields are inner.[7]) The Lie algebra obtained by considering every possible commutator is obviously infinite dimensional; it generates the automorphism group of the algebra of fields. For practical purposes, we select some convenient finite dimensional subgroups; henceforth called classification groups, useful in enumerating composite operators and their quantum numbers. All composite states of immediate interest ("leptons" and "quarks") are triple products of the "elementary" subconstituent fields. Motivated by the analyses of Shaw, Silverman and Slansky[8] and of Brodsky and Drell[9], we approximate the leptons and quarks by triple products <u>taken at the same spacetime point</u>: this should be a very good approximation at least for the first generation of quarks and leptons. (In any known quantum field theory such local products are singular and they have to be appropriately defined; however, this fact does not affect the algebraic properties we are concerned with.)

The yardstick of success of subconstituent models at present is whether or not they can reproduce the fields present in the SU(5) or SO(10) groups of grand unified theories.[10] In order to achieve this, we represent subconstituents by two independent Weyl spinors (or equivalently, by a single Dirac spinor). Throughout this paper, composite fields present in (16) of the spin-covering of SO(10) are called "physical;" others are "exotic." In the next two Sections we analyze the original Harari-Shupe model and one of its possible minimal extensions: the extension is obtained by assuming that $SU(3)_c$ and the gauge group of electromagnetism are exact symmetries and hence they are realized linearly on the subconstituents. The results are summarized and discussed in Section 4.

2. The Harari-Shupe Model

There are two types of subconstituents, $s^a_A(x)$ and $\bar{s}^a_A(x)$, where $A = 1,2$ is a Weyl spinor index and $a = 0,1$ labels a reducible representation of the electromagnetic gauge group:

$$
\begin{aligned}
[Q,\ s^a_A(x)] &= \frac{1}{3} a\, s^a_A(x) \\
[Q,\ \bar{s}^a_A(x)] &= -\frac{1}{3} a\, \bar{s}^a_A(x)
\end{aligned}
\tag{2.1}
$$

Q being the electric charge operator. The left-handed fields s^a_A and $\bar{s}^a_A$ can be embedded into the fundamental representation of a classification group $U(8) \supset U(4) \otimes SU(2)_\chi$, where $SU(2)_\chi$ stands for the chiral "rotations" of the left-handed fields induced by Lorentz transformations. However, in view of the fact that the "physical" leptons and quarks are constructed out of triple products of the form $(s\,s\,s)$ and $(\bar{s}\,\bar{s}\,\bar{s})$, it is more convenient to choose a classification group which keeps the subconstituents s and $\bar{s}$ separate from each other. Consequently, we choose a subgroup of U(8) as an effective classification group in the form of a direct product. The direct factors act on the fields s and $\bar{s}$, respectively. A convenient choice (in an obvious notation) is

$$
U(4)_s \otimes U(4)_{\bar{s}} \subset U(8)
$$

This corresponds to the embedding $U(2) \otimes SU(2)_\chi \subset U(4)$, the internal group being $U(2)_s \otimes U(2)_{\bar{s}}$. The Lie algebra of both direct factors here is spanned by a basis I_K, U in the usual way. The electric charge operator is in a "diagonal" U(2) subalgebra, viz.

$$
Q = \frac{1}{3}(U_s + I_{3s}) - \frac{1}{3}(U_{\bar{s}} + I_{3\bar{s}})
$$

The success of the Harari-Shupe model depends on the fact that the triple products $(s^a_A\, s^b_B\, s^c_C)$ and $(\bar{s}^a_A\, \bar{s}^b_B\, \bar{s}^c_C)$ must span locally eight different states of (chiral) spin 1/2. Obviously if spin is ignored, the <u>maximal</u> number of internal states is $2^3 = 8$. However, this implies that the model is viable only if the fields s and $\bar{s}$ either do not obey Fermi statistics or the model is non-minimal, i.e., it contains some "sub-color" group.[2,3] Indeed, Fermi statistics implies linear relationships between binary products, <u>viz</u>.

$$\begin{aligned} \left\{ s^a_A,\ s^b_B \right\} &= 0 \\ \left[s^a_A,\ s^b_B\, s^c_C \right] &= 0 \end{aligned} \tag{2.2}$$

hence the number of independent triple products is less than eight. In particular, composite lepton fields of the form $s^a_A\, s^a_B\, s^a_C$ vanish identically. In order to extract the full "quark" and "lepton" contents of the triple products, we proceed to reduce out the Kronecker product $4 \otimes 4 \otimes 4$ of the classification group U(4). We have in a straightforward notation,

$$\begin{gathered} \Box \otimes \Box \otimes \Box = \begin{smallmatrix}\Box\\\Box\\\Box\end{smallmatrix} \oplus \begin{smallmatrix}\Box\Box\\\Box\ \ \end{smallmatrix} \oplus \begin{smallmatrix}\Box\Box\\\Box\ \ \end{smallmatrix} \oplus \Box\Box\Box \\ (4) \times (4) \times (4) = (4^*) + (20) + (20) + (20) \end{gathered} \tag{2.3}$$

The irreducible representations on the r.h.s. of (2.3), in turn, are decomposed according to $U(2) \otimes SU(2)_X$; the result is as follows:

$$\begin{aligned} \begin{smallmatrix}\Box\\\Box\\\Box\end{smallmatrix} &= \underline{(2,2)} \\ \begin{smallmatrix}\Box\Box\\\Box\ \ \end{smallmatrix} &= \underline{(4,2)} \oplus (2,4) \oplus \underline{(2,2)} \\ \Box\Box\Box &= (4,4) \oplus \underline{(2,2)} \end{aligned} \tag{2.4}$$

The representations of spin 1/2 are underlined. It is important to observe that the spin 3/2 fields cannot be effectively decoupled[2] (sent to a higher mass scale) by breaking the U(2) classification group alone; for an effective decoupling, the forces binding the subconstituents have to have a strong spin dependence. The <u>total</u> number of fields of spin 1/2 is 16 instead of 8. This fact is the consequence of including spin (which was left out of consideration in Refs. 4,5); in considering all representations with spin 1/2, all possible anti-symmetric pairings of spin indices are counted separately. It is easy to see that within an associative algebra of the fields, there are no algebraic constraints which would remove the composite fields of spin 3/2 while retaining all physical fields. In essence, this is a consequence of the fact that all algebraic constraints have to be imposed at the level of the binary products. Since the subcomponent fields transform as (2,2) of $U(2) \otimes SU(2)_{\chi}$, the binary product transforms as

$$(2,2) \otimes (2,2) = (3,3) \oplus (1,3) \oplus (3,1) \oplus (1,1) \qquad (2.5)$$

Binary constraints consistent with associativity can always be formulated in a form which puts at least one of the representations occurring in (2.5) equal to zero. However, on doing so, one is never left with the correct charge count in the ternary products.

It is worth noting that the fields of spin 1/2 have to be collected from representations of different symmetries: hence a conventional scheme[2] with subcolored Fermi fields (assuming that quarks and leptons are singlets of a subcolor group) does not work.

The basic trouble with this model is, however, that -- despite the right number of composite fields being present -- the Lie algebra of the color group, $SU(3)_c$, cannot be realized as an inner derivation. In principle, one could think of two distinct ways of implementing $SU(3)_c$:

i) The subconstituents are color singlets; $SU(3)_c$ acts only on the composite quarks. (This is the way envisioned by Harari and Shupe.)

ii) The generators of $SU(3)_c$ act on the subconstituents in a non-linear way such that the composite quarks transform linearly under this action. (A linear action on the composite quarks is necessary in order to obtain eight gluons as collective excitations.) Consider the first alternative. This means that there exist eight color generators, C_i $(1 \le i \le 8)$, all of them homogeneous polynomials of the subconstituent fields and their conjugates, such that

$$\left[C_i,\ s^a_A\right] = 0 \tag{2.6a}$$

$$\left[C_i,\ s^a_A s^a_B s^b_C\right] \neq 0 \tag{2.6b}$$

However, by assumption, the algebra of fields is associative and the C_i are in the derivation algebra of the fields, which implies

$$\left[C_i,\ s^a_A s^a_B s^b_C\right] = \left[C_i,\ s^a_A\right] s^a_B s^b_C + s^a_A \left[C_i,\ s^a_B\right] s^b_C + s^a_A s^a_B \left[C_i,\ s^b_C\right] \neq 0 \ ;$$

this contradicts (2.6a). In order to exclude alternative ii), we recall that in order to support a nonlinear realization of the Lie algebra of $SU(3)_c$, the quark fields, as a vector space, have to be isomorphic to tangent vectors of a space symmetric under SU(3). All such spaces are cosets of SU(3) with respect to one of its subgroups.[11] The maximal subspace admitted by the Harari-Shupe model is isomorphic to $SU(3)/SU(2) \otimes U(1)$. However, the group $SU(2) \otimes U(1)$ is realized linearly on this space and its representations can be read off from Eq. (2.4). We note that there are no composite fields (of any charge) invariant under the action of $SU(2) \otimes U(1)$. Hence, in physical terms, a nonlinearly realized Lie algebra SU(3) cannot be identified with $SU(3)_c$, for it does not leave the leptons invariant. This proves our assertion.

3. An Extended Harari-Shupe Model

In order to alleviate the difficulty with representing color, one can consider a minimal extension of the Harari-Shupe model including all exact symmetries at the subconstituent level. To this end, we consider chiral subconstituent fields s_A^α and $\bar{s}_A^\alpha$, $(0 \le \alpha \le 3)$, with internal group $U(4) \otimes U(4)$, where the s_A^i $(\bar{s}_A^i)$, $(1 \le i \le 3)$ are triplets (antitriplets) under $SU(3)_c \subset U(4)$, respectively, while s_A^o and $\bar{s}_A^o$ are singlets. This assignment is a simple generalization of the Pati-Salam four-color scheme[12] at the subconstituent level. By a straightforward generalization of the Harari-Shupe model, we discover a unique electric charge assignment, viz.

$$\begin{gathered} \left[Q,\ s_A^o\right] = \left[Q,\ \bar{s}_A^o\right] = 0 \\ \left[Q,\ s_A^i\right] = \frac{1}{3}\, s_A^i \quad , \quad \left[Q,\ \bar{s}_A^i\right] = -\frac{1}{3}\, \bar{s}_A^i \end{gathered} \tag{3.1}$$

The charge operator obviously commutes with $SU(3)_c$. In addition, one can define (just as in the original Harari-Shupe model) a conserved global quantum number, B-L; the conventional assignments being ±1/3 for s^i and s^o, respectively. The fields $\bar{s}$ carry quantum numbers B-L of opposite sign. No separate baryon and lepton numbers can be defined in these models.[3] The complete classification group is U(16). However, as explained in the previous Section, we utilize the classification group $U(8)_s \otimes U(8)_{\bar{s}} \subset U(16)$. The quark and lepton contents of the triple products (s s s) and ($\bar{s}\,\bar{s}\,\bar{s}$) are extracted by reducing out $(8) \otimes (8) \otimes (8)$ with respect to $U(4) \otimes SU(2)_X \subset U(8)$.

The result of the reduction process is the following:

$$\begin{aligned} (8) \otimes (8) \otimes (8) = {} & (20_S,\ 4) \oplus \underline{(20_M,\ 2)} \oplus (4^*,\ 4) \oplus \underline{(20_M,\ 2)} \\ & + (20_S,\ 2)^2 \oplus (20_M,\ 4)^2 \oplus \underline{(4^*,\ 2)}^2 \oplus (20_M,\ 2)^2 \ , \end{aligned} \tag{3.2}$$

where the two 20-dimensional representations of U(4), corresponding to the Young patterns (3,0,0) and (2,1,0), are distinguished by subscripts S and M, respectively. The representation (4*) of U(4) corresponds to the Young pattern (1,1,1). As before, representations with spin 1/2 are underlined. In order to distinguish quarks from leptons, the representations of U(4) are further reduced out with respect to $SU(3)_c$. This gives:

$$\begin{aligned}(20)_S &= (10)_1 \oplus (6)_{2/3} \oplus (3)_{1/3} \oplus (1)_0 \,, \\ (20)_M &= (8)_1 \oplus (6)_{2/3} \oplus (3^*)_{2/3} \oplus (3)_{1/3} \,, \\ (4^*) &= (1)_1 \oplus (3^*)_{2/3} \,,\end{aligned} \tag{3.3}$$

where the subscripts indicate the eigenvalues of the electric charge. Altogether, therefore, this model contains two families of the physical quarks and leptons, $((1)_1 \oplus (1)_0 \oplus (3^*)_{2/3} \oplus (3)_{1/3})^2$, all of them coming from the mixed representation, (2,1,0,0,0,0,0) of U(8). These composite fields correspond to $(e^+_L, \nu_{eL}, u_L, \bar{d}_L)^2$; the assignment of u_L to (3*) of $SU(3)_c$ is purely a matter of convention. In addition, however, there are other, unwanted representations of $SU(3)_c$ present, as it can be read off from Eqs. (3.2) and (3.3); it is not possible to eliminate those by means of algebraic constraints within the framework of an associative algebra of the fields, while retaining all the physical composite fields. As in the original Harari-Shupe model, the duplication of the states in (2,1,0,0,0,0,0) of U(8) is a consequence of two independent spin coupling schemes being present, viz.

$$\begin{aligned}\big((2) \otimes (2)\big) \otimes 2 &= \big((3) \oplus (1)\big) \otimes (2) \\ &= \big((2) \oplus (4)\big) \oplus (2)\end{aligned} \tag{3.4}$$

In principle, there exists the possibility of splitting the masses of the two families by means of a spin dependence of the binding forces; however, so far, we have not found a simple and convincing mechanism for doing so. Again, similarly to the Harari-Shupe model, all the "physical" states come from a representation of mixed symmetry, hence a conventional subcolored Fermi algebra of the subconstituents does not provide us with a viable model.

Given the subconstituent fields, the most general Lagrangian invariant under a local $U(4) \otimes U(4)$ symmetry is given by

$$\mathscr{L} = \frac{i}{2}\left[s_A^{\alpha *} D_{\alpha\beta}^{\dot{A}B} s_B^{\beta} + \bar{s}_A^{\alpha *} D_{\alpha\beta}^{\dot{A}B} \bar{s}_B^{\beta} - (c.c.)\right] + \mathscr{L}_G \tag{3.5}$$

where $\mathscr{L}_G$ is the kinetic term of the gauge fields and D stands for the standard covariant derivative.[13] The symmetry U(4) has to be badly broken: indeed, on writing $U(4) \simeq SU(4) \otimes U(1)$ we can further decompose SU(4) into representations of $SU(3)_c \otimes U(1)$ as follows:

$$(15) = (3) \oplus (3^*) \oplus (8) \oplus (1) \ . \tag{3.6}$$

The SU(4) singlet and color triplet gauge bosons are responsible for proton decay. Indeed, the elementary processes contributing to this decay are:

$uu \to e^+\bar{d}$ (color triplet boson in the direct channel and exchange of a SU(4) singlet)

$ud \to e^+\bar{u}$ (double exchange of color triplets)

All these bosons can, in principle, be made superheavy if some bilinears develop a zero mass condensate[2]: the propagators required to have zero mass poles of sufficient strength are the following

$$\left\langle \left(\bar{s}_A^{\alpha} s^{\alpha A} \bar{s}_B^{\beta *} s^{B\beta *}\right)_+ \right\rangle \doteq F \tag{3.7a}$$

$$\left\langle \left(\bar{s}_A^{i} s^{oA} \bar{s}_A^{j *} s^{oA*}\right)_+ \right\rangle = \delta^{ij} G^{(1)} + G^{(8)i}_{\ \ j} \tag{3.7b}$$

Whether or not the composite propagators can develop poles at zero mass is a dynamical question which is beyond the scope of the present analysis; however, a severe kinematic constraint imposed by (3.7) is that the <u>color octet part</u> of (3.7b) <u>must not contain a zero mass pole</u>, for we want to leave $SU(3)_c$ intact.

The formal subconstituent currents corresponding to the gauge symmetry are of the form

$$\begin{aligned} H^{\alpha\beta}_{\dot{A}B} &= s^{\alpha *}_{A}\, s^{\beta}_{B} \\ J^{\alpha\beta}_{\dot{A}B} &= \bar{s}^{\alpha *}_{A}\, \bar{s}^{\beta}_{B} \end{aligned} \tag{3.8}$$

and their conjugates. The electromagnetic current is contained in an $SU(3)_c$ singlet combination of the currents (3.8), <u>viz</u>.

$$j_{\dot{A}B} = \frac{1}{3}\left(s^{i*}_{A}\, s^{i}_{B} - \bar{s}^{i*}_{A}\, \bar{s}^{i}_{B}\right) + (\mathrm{c.c.}) \tag{3.9}$$

In addition, the Lagrangian (3.5) possesses a <u>global</u> (generalized Gürsey-Pauli) symmetry, with the currents

$$\begin{aligned} K^{\alpha\beta}_{\dot{A}B} &= \bar{s}^{\alpha *}_{A}\, s^{\beta}_{B} \\ K^{\dagger\alpha\beta}_{\dot{A}B} &= s^{\alpha *}_{A}\, \bar{s}^{\beta}_{B} \quad . \end{aligned} \tag{3.10}$$

(There can be no elementary gauge fields associated with the currents (3.10), for that theory would not be free of anomalies.)

The main difficulty with this model arises on examining the question of weak interactions. Since the standard weak-electromagnetic group, $SU(2)_L \otimes U(1)$, is not an exact symmetry, no proof can be given of the absence of normal weak interactions (as we, for instance, proved that the model discussed in Section 2 does not contain $SU(3)_c$). Nevertheless, it is very plausible that this model either does not contain weak interactions or else, their existence depends on some delicate and so far completely unforeseen properties of the dynamics.

In order to see this, we first of all observe that the vector space spanned by the bilinears (3.8) and (3.10) does not contain elements with the quantum numbers corresponding to the generators of $SU(2)_L \otimes U(1)$. It can be easily demonstrated that any quantity carrying the quantum numbers of the charged generators must be at least trilinear in the currents (3.8) and (3.10). The minimal combination is given by the expressions:

$$
\begin{aligned}
T^{(-)}_{\dot{B}D} &= \varepsilon_{\dot{A}\dot{E}}\, \varepsilon_{CF}\, \varepsilon_{ijk} \Big\{ (\bar{s}^{i*}_{A}\; s^{o}_{C})\, (\bar{s}^{j*}_{B}\; s^{o}_{D})\, (\bar{s}^{k*}_{E}\; s^{o}_{F}) \\
&\quad + (\bar{s}^{o*}_{A}\; s^{i}_{C})\, (\bar{s}^{o*}_{B}\; s^{j}_{D})\, (\bar{s}^{k*}_{E}\; s^{o}_{F}) \Big\} + (\text{perm}) \qquad (3.11) \\
T^{(+)}_{\dot{B}D} &= T^{(-)*}_{B\dot{D}} \; .
\end{aligned}
$$

The electrically neutral generators can be decomposed into the orthogonal combinations, $\propto (T_3 \pm 1/2\ Y)$. One of those, the electromagnetic current, is exactly conserved and, hence, it must be represented by the bilinear expression (3.9) at the level of subconstituents. The combination "orthogonal" to this must again be trilinear in (3.8) and (3.10), since $U(4) \otimes U(4)$ does not contain enough diagonal operators to describe both $T_3 \pm 1/2\ Y$. We have:

$$
\begin{aligned}
Z^{(0)}_{\dot{B}D} &= \varepsilon_{\dot{A}\dot{E}}\, \varepsilon_{CF} \Big\{ (s^{o*}_{A}\; s^{o}_{C})\, (s^{o*}_{B}\; s^{o}_{D})\, (s^{o*}_{E}\; s^{o}_{F}) \\
&\quad + \frac{1}{6}\, (s^{o*}_{A}\; s^{o}_{C})\, (s^{i*}_{B}\; s^{p}_{D})\, (s^{j*}_{E}\; s^{q}_{F})\, \varepsilon_{ijk}\, \varepsilon_{pqk} \qquad (3.12) \\
&\quad - \frac{5}{6}\, (\bar{s}^{o*}_{A}\; \bar{s}^{o}_{C})\, (\bar{s}^{o*}_{B}\; \bar{s}^{o}_{D})\, (\bar{s}^{*i}_{E}\; \bar{s}^{j}_{F})\, \delta_{ij} \Big\} + (\text{perm})
\end{aligned}
$$

In the previous formulae, the symbol "(perm)" stands for terms which differ from those written out explicitly in permutations of the spinor indices contracted. Equations (3.11) and (3.12) express the fact (first noted by Harari, Ref. 4) that weak vector bosons must be composite states of (at least) three rishons and three antirishons.

We observed earlier in this Section that the "physical" composite fields come from representations of mixed symmetry in the decomposition of trilinear products: this could be realized, for instance, by assuming an (associative) parafermi algebra satisfied by the subconstituents. No matter what the exact algebraic structure spanned by the subconstituents is, the set of currents given by (3.9), (3.11), (3.12) cannot be isomorphic to a local $SU(2)_L \otimes U(1)$. From a physical point of view this means that the Glashow-Salam-Weinberg unification of weak and electromagnetic interactions cannot be formulated in an algebraic language within such a minimal subconstituent model: in particular, the Weinberg angle has no simple algebraic expression of the usual type, viz.

$$\mathrm{Tr}\left(T^{(0)}\, T^{(0)}\right) = \sin^2\theta_W \,\mathrm{Tr}\,(QQ)$$

While we see no logical reason for excluding minimal subconstituent models on this ground alone, their failure for providing a simple picture of the unification of weak and electromagnetic interactions is somewhat discouraging.

4. Discussion

There are several features emerging from the previous analysis which seem to be of interest for further development of subconstituent models. Some of these are the following:

i) Spin (or, more precisely, the Lorentz group) presents an essential complication. In particular, the existence of several ways of contracting three subconstituents into spin 1/2 quarks and leptons increases the multiplicity of those states. This is a conclusion resting on rather firm group theoretical grounds. Mathematically (but not physically) this phenomenon is the same as observed by Nelson[14] who considered a Harari-Shupe model enlarged by an SU(2) "color" group. Whether or not the increased multiplicity of "physical" composite fields can be used for the description of several generations is an open question; it was already commented on in the previous Sections. The basic difficulty is, however, that leptons do not multiply at the same rate as quarks do, for in all subconstituent models considered so far they occur in representations of the highest weight of the internal classification group. As a consequence, one invariably ends up with some incomplete generations, having more quark-like states than leptons.

ii) In <u>all</u> subconstituent models proposed so far, there exist numerous exotic composite fields; some of those can be read off from the reduction schemes given in Sections 2 and 3, others, occurring in trilinears like $(s\, s\, \bar{s})$, $(\bar{s}\, \bar{s}\, s)$ etc. and in higher products have not been analyzed here. Although, for instance, Terazawa[1] insists that such exotics should be taken seriously, we know of no consistent symmetry breaking scheme which would remove the exotic fields (and only those!) from the realm of the known light quarks and leptons.

iii) The question of "statistics" of the subconstituents is still an open one. In the original Harari-Shupe model the physical composite fields have to be collected from triple products of different symmetries, whereas in the extended Harari-Shupe model they all come from a representation of mixed symmetry (which, however, also contains some exotic fields). This means that either the triple product is essentially non-local (but then where are the orbital excitations?) or that the subconstituents are not fermions; they may be parafermions of order two: that's why Nelson's model[14] seems to be better than the original Harari-Shupe model, cf. the celebrated paper by Han and Nambu[15] and references quoted there.

iv) It is possible that quantum theory, as we know it, has to be transcended at the level of subconstituents; in particular, the subconstituents may not obey an associative algebra. In essence, such a possibility has been advocated in a recent paper by Adler[16] and it has been considered (at the level of now-familiar quarks) by Gürsey and his collaborators.[17] The trouble with this approach is that non-associative algebras are easy to create[18] but difficult to live with.[19] All known not-associative algebras[20] possessing "nice" properties[7] are finite dimensional and therefore it is hard to see how field variables living on a locally Minkowskian continuum could span some "nice" non-associative algebras.

ACKNOWLEDGEMENTS

This research has been completed during the authors' visit at the Stanford Linear Accelerator Center: we thank S. D. Drell for the hospitality offered to us at SLAC, and to H. Harari for some interesting conversations on subconstituent models. One of us (R. C.) thanks B. R. Judd, Department of Physics, Johns Hopkins University, for the hospitality offered to him during his visit there, where this investigation had begun. This research has been carried out with financial support from the U.S. Department of Energy under contract number DE-AS-01-76ER03285, and in part by contract DE-AC03-76SF00515 (SLAC).

REFERENCES

1. For recent reviews of subcomponent models, see H. Terazawa, Phys. Rev. D22, 184 (1980). R. Gatto, University of Geneva report, UGVA-DPT 1980/04-236; unpublished. C. S. Kalman, Concordia University report, CUQ/EPP 33 (1980); unpublished.

2. G. 't Hooft, Cargèse Summer Institute Lectures (1979); unpublished. S. Dimopoulos, S. Raby and L. Susskind, Stanford University preprint ITP-662 (1980); unpublished.

3. R. Casalbuoni and R. Gatto, Phys. Letters 93B, 47 (1980); see also R. Gatto, Ref. 1.

4. H. Harari, Phys. Letters 86B, 83 (1979).

5. M. A. Shupe, Phys. Letters 86B, 87 (1979).

6. For a recent attempt to generalize this assumption, see I. Bars and M. Günaydin, Yale University preprint YTP80-14 (1980); unpublished.

7. See e.g. R. D. Schafer, An Introduction to Nonassociative Algebras, Chapter 2, Academic Press, New York, 1966.

8. G. L. Shaw, D. Silverman and R. Slansky, Phys. Lett. 94B, 57 (1980).

9. S. J. Brodsky and S. D. Drell, SLAC preprint SLAC-PUB-2534 (1980); unpublished.

10. H. Georgi and S. L. Glashow, Phys. Rev. Letters 32, 236 (1974). For a recent review, see H. Georgi, Harvard University preprint, HUTP-80/A054 (1980); unpublished.

11. See e.g. S. Helgason, Differential Geometry and Symmetric Spaces, Chapter 4, Sec. 3. (Academic Press, New York, 1962).

12. J. Pati and A. Salam, Phys. Rev. D8, 1240 (1973).

13. It is conceivable that the gauge fields themselves are collective modes of the subconstituent fields, cf. H. Sugawara, in Proceedings of the International Conference on High Energy Physics, Tokyo, 1978, edited by S. Homma, M. Kawaguchi and H. Miyazawa (Physical Society of Japan, Tokyo, 1979). See also I. Bars and M. Günaydin, Ref. 6. The eventual compositeness of the gauge fields leaves the subsequent analysis unaffected.

14. C. A. Nelson, Phys. Letters 93B, 143 (1980).

15. Y. Nambu and M.-Y. Han, Phys. Rev. D10, 674 (1974).

16. S. L. Adler, Phys. Rev. D21, 2903 (1980).

17. For a review, see F. Gürsey, in Proceedings of the Second Johns Hopkins Workshop on Current Problems in High Energy Particle Theory, edited by G. Domokos and S. Kövesi-Domokos (Johns Hopkins University, Baltimore, MD 1978) and references quoted there.

18. Consider a vector space spanning the fundamental representation of a simple Lie group, G, of rank ≥ 2 and its dual, $V \oplus V^*$. Define binary products in $V \otimes V$, $V^* \otimes V^*$ and $V \otimes V^*$ by putting some of the irreducible representations occurring in the reduction of the tensor products equal to zero. As a rule, the resulting algebra is not-associative. (In particular, octonions and split octonions can be generated in this way.) For a detailed description, see G. Domokos and S. Kövesi-Domokos, Jour. Math. Phys. 19, 1477 (1978) and references quoted there.

19. G. Domokos and S. Kövesi-Domokos, Phys. Rev. D19, 2984 (1979).

20. We follow the terminology of Schafer, Ref. 7: an algebra is "non-associative" if we allow for the possibility that for some three elements $(ab)c \neq a(bc)$, whereas it is "not-associative" if there are at least three elements of the algebra for which this inequality holds.