SLAC-PUB-2584
August 1980
(M)

SUPPRESSION OF SUPERHEAVY MAGNETIC MONOPOLES IN GRAND UNIFIED THEORIES*

So-Young Pi
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

The superheavy magnetic monopoles predicted by grand unified theories would not be produced in significant numbers if electromagnetic gauge invariance is spontaneously broken when the temperature T is greater than $T_c \gtrsim 1$ TeV.

Grand unified theories predict the existence of superheavy magnetic monopoles.[1-7] These monopoles are of the type discovered by 't Hooft and Polyakov.[6] They exist if a semi-simple group is broken down to a subgroup which contains U_1 factor. The monopole mass M_m is of order M_X/α, where $\alpha = g^2/4\pi$, g is a gauge coupling and M_X is a typical mass of a gauge boson associated with a broken generator. For example, in Georgi-Glashow model $M_X \simeq 10^{14}$ GeV and $M_m \simeq 10^{16}$ GeV.

The problem of monopole production and their subsequent annihilation, in the context of a second order or weakly first order phase transition, was analyzed by Zeldovich and Khlopov[1] and by Preskill.[2] In preskill's analysis, it was found that relic monopoles would exceed present bounds by roughly 14 orders of magnitude. Since it seems difficult to modify the estimated annihilation rate, one must find a way which suppresses the production of these monopoles.

One interesting solution to this problem, suggested by Preskill,[2] Einhorn et al.,[3] and Guth and Tye[4] is that the phase transition at which the U_1 factor occurs is strongly first order. The problem of monopole production in a strongly first order phase transition was treated in detail by Guth and Tye.

In this talk I will describe an alternative scenario for the suppression of monopoles, developed in collaboration with P. Langacker,[7] in which the universe undergoes two or more phase transitions (which can be second order)

$$G \xrightarrow[T_1]{} H_1 \xrightarrow[T_2]{} H_2 \;\cdots\; \xrightarrow[T_n]{} H_n \xrightarrow[T_c]{} SU_3^c \times U_1^{EM} \quad , \tag{1}$$

where U_1^{EM} is not a subgroup of H_n. The critical temperature at which U_1^{EM} appears is $T_c \gtrsim 1$ TeV. For example, in SU_5 model,

$$SU_5 \xrightarrow[T_1 \lesssim M_X]{} SU_3^c \xrightarrow[T_c \gtrsim 1\ \text{TeV}]{} SU_3^c \times U_1^{EM} \quad . \tag{2}$$

Since $T_c <<< M_m \simeq 10^{16}$ GeV no monopoles will be produced.

We consider a model which at $T=0$ is the standard SU_5 model with symmetry breaking

$$SU_5 \rightarrow SU_3^c \times SU_2 \times U_1 \rightarrow SU_3^c \times U_1^{EM} \tag{3}$$

* Work supported by the Department of Energy, contract DE-AC03-76SF00515.
(Invited talk presented at the XXth International Conference on High Energy Physics, University of Wisconsin, Madison, July 17-23, 1980.)

by an adjoint Higgs representation and three five Higgs representations. (It turns out that this is the minimum number of five Higgs representations required for our purpose.) The Higgs potential at $T=0$ is

$$V = V_\Phi + V_{\Phi\phi} + V_\phi \tag{4a}$$

$$V_\Phi = -\frac{1}{2}m^2\,\mathrm{Tr}\Phi^2 + \frac{1}{4}a(\mathrm{Tr}\,\Phi^2)^2 + \frac{1}{2}b\,\mathrm{Tr}\,\Phi^4 \tag{4b}$$

$$V_{\Phi\phi} = \sum_{i=1}^{3}\left[\alpha_i\,\phi_i^\dagger\phi_i\,\mathrm{Tr}\Phi^2 + \beta_i\,\phi_{ia}^\dagger\,\Phi_{ab}^2\,\phi_{ib}\right] \tag{4c}$$

$$V_\phi = \sum_{i=1}^{3}\left[-\mu_i^2\,\phi_i^\dagger\,\phi_i + \lambda_i(\phi_i^\dagger\phi_i)^2\right] + \sum_{i<j}\left[\sigma_{ij}(\phi_i^\dagger\phi_i)(\phi_j^\dagger\phi_j) + \rho_{ij}(\phi_i^\dagger\phi_j)(\phi_j^\dagger\phi_i) + \eta_{ij}(\phi_i^\dagger\phi_j)^2 + \eta_{ij}^*(\phi_j^\dagger\phi_i)^2\right] \tag{4d}$$

where Φ is an adjoint Higgs representation and ϕ_i are five Higgs representations. We have imposed discrete symmetries $\Phi\to-\Phi$ and $\phi_i\to-\phi_i$ for simplicity.

For $0\le T \ll M_X$ we need only to consider the $SU_2\times U_1$ part of the model. (We assume that SU_3^c is never broken.) Therefore let us first consider $SU_2\times U_1$ part of the model with Higgs potential V_ϕ, Eq. (4d), in which ϕ_i are SU_2 doublets.

At $T=0$ we choose the parameters in the potential such that the vacuum expectation values (VEV) of the Higgs fields are $\langle\phi_1(0)\rangle = (0\ v_1)^T/\sqrt{2}$ and $\langle\phi_2(0)\rangle = \langle\phi_3(0)\rangle = 0$. $SU_2\times U_1$ symmetry is broken down to U_1^{EM}. We also require the parameters satisfy the sufficient conditions for V_ϕ to be bounded below.[7] We also take $\rho_{ij} > 2|\eta_{ij}|$ so that when two fields ϕ_i and ϕ_j develop VEV they want to be orthogonal, i.e., $\phi_i = (0\ v_i)^T/\sqrt{2}$ and $\phi_j = (v_j\ 0)^T/\sqrt{2}$. We want U_1^{EM} to be unbroken at $T=0$ but broken for $T>T_c$.

At high temperatures, we have to calculate the finite temperature effective potential to study the symmetry behavior of the system.[8-10] For sufficiently high T, the ensemble averages $\langle\phi_i(T)\rangle$ can be obtained by minimizing the effective potential[9,10]

$$V_\phi(T) = V_\phi(0) + \sum_{i=1}^{3}\frac{1}{2}T^2 F_i\,\phi_i^\dagger\,\phi_i \quad . \tag{5}$$

The functions F_i are given by[7]

$$F_i = (3g^2+g'^2)/8 + \lambda_i + \sum_{j\ne i}\left[\frac{\sigma_{ij}}{3} + \frac{\rho_{ij}}{6}\right] + \text{Yukawa terms} \ . \tag{6}$$

For small fermion masses Yukawa terms are negligible. The effective mass terms at high temperature will be $M_i^2(T) = \mu_i^2 - \frac{1}{2}F_iT^2$. Therefore, if $F_i>0$ then for $T^2 \gtrsim 2\mu_i^2/F_1$ $SU_2\times U_1$ will be restored. However, if $F_i<0$ the symmetry will stay broken[11,12] or may be further broken down to a lower symmetry[9] at high temperature. We choose parameters so that $F_{1,2}<0$. This turns out to require $F_3>0$ so that for sufficiently high T, we may have a phase transition to a phase where $SU_2\times U_1$ is completely broken.

We have found a range of parameters such that at high temperature $\langle\phi_1(T)\rangle = (0\ v_1(T))^T/\sqrt{2}$, $\langle\phi_2(T)\rangle = (v_2(T)\ 0)^T/\sqrt{2}$, $\langle\phi_3(T)\rangle = 0$ is (at least) a local minimum of $V_\phi(T)$.[7] These parameters satify

$$\begin{aligned}&\lambda_1 \simeq \lambda_2 \simeq \lambda >> g^4\ ,\ |\rho_{ij}|\\ &-\sigma_{13} \simeq -\sigma_{23} \simeq \sigma > 3\lambda + \sigma_{12} + 3X\\ &|\mu_2^2| > \mu_1^2\ ,\ 2 > \sigma_{12}/\lambda > -1\ ,\ \lambda_3 > \sigma^2/\lambda\quad .\end{aligned} \tag{7}$$

where $X = (3g^2 + g'^2)/8 \simeq 0.16$. The condition $\lambda << g^4$ allows us to neglect radiative corrections to V_ϕ. For a typical set of numbers, choose $\lambda \simeq -\sigma_{12} \simeq g^2 \simeq 0.4$, $\sigma \gtrsim 1.3$, $\lambda_3 \gtrsim 4.1$. We see that there is a range of parameters which satisfy the above conditions, but a rather large value for λ_3 is required. The second order phase transition occurs at T_c such that $v_2(T_c) = 0$. T_c is given by

$$T_c = A\mu_1/\sqrt{\lambda_1} = (246\ \text{GeV})A \tag{8}$$

where A is a function of the parameters in the potential and is typically of order unity, but can be made much larger or smaller by adjusting parameters. We will assume $T_c \gtrsim 1$ TeV. We have therefore demonstrated the existence of a phase transition in which $SU_2 \times U_1$ is broken to U_1^{EM} at $T = 0$ and $SU_2 \times U_1$ is completely broken for $T > T_c$.

Now I would like to describe how to embed our scheme to SU_5. We study the complete SU_5 potential, Eqs. (4a)-(4d). The conditions that V is bounded below and have symmetry breaking Eq. (3) at $T = 0$ are

$$\begin{aligned}&b > 0,\quad 15a + 7b > 0,\quad \beta_i < 0,\quad 5\alpha_i + 4\beta_i > 0\\ &\lambda_i > 0,\quad \sqrt{\lambda_i\lambda_j} + \sigma_{ij} > 0\ .\end{aligned} \tag{9}$$

For $T > M_X$ we have to consider the heavy particle contributions to V(T). The effective potential at high temperature, $T > M_X$, are given by

$$V(T) = V_\Phi(0) + V_\phi(0) + V_{\Phi\phi}(0) + \frac{1}{2}GT^2\mathrm{Tr}\Phi^2 + \sum_{i=1}^{3}\frac{1}{2}F_i'T^2\phi_i^\dagger\phi_i \tag{10}$$

where

$$G = \frac{1}{60}\left[13a + 94b + 75g^2 + \sum_{i=1}^{3}(50\alpha_i + 10\beta_i)\right]$$

$$F_i' = 2\lambda_i + \sum_{j\neq i}\left[\frac{5}{6}\sigma_{ij} + \frac{1}{6}\rho_{ij}\right] + \frac{6}{5}g^2 + \text{Yukawa terms}$$

G is always positive for the parameters satisfying Eq. (9). Therefore, at sufficiently high T, the effective mass of Φ, $-\frac{1}{2}m^2 + \frac{1}{2}GT^2 > 0$ so that VEV $\langle\Phi(T)\rangle$ will vanish. The parameters λ_i, σ_{ij} and ρ_{ij} have been already chosen as Eq. (7) for the phase transition at $T_c \gtrsim 1$ TeV. For those λ_i, σ_{ij} and ρ_{ij} and any α_i and β_i in Eq. (9) F_3 will be always positive. F_1 and F_2 may be positive or negative depending upon the values of α_i and β_i. It is very likely that SU_5 symmetry have been restored in the very early universe. In this case α_i and β_i should be chosen such that F_1 and F_2 are also positive.

It is a difficult problem to study the effective potential near $T \lesssim M_X$. There may be intermediate phases between SU_5 and SU_3^c phases (for example, $SU_3^c \times SU_2 \times U_1$):

$$SU_5 \underset{T_1}{\longrightarrow} \text{intermediate phases} \underset{T_n}{\longrightarrow} SU_3^c \underset{T_c}{\longrightarrow} SU_3^c \times U_1^{EM} \quad . \qquad (11)$$

There should be essentially no magnetic monopoles in our model. Any monopoles produced during intermediate phases at $T \lesssim M_X$ will become unstable once the SU_3^c phase is entered. They would presumably either decay or be confined in pairs which could subsequently annihilate. Stable monopoles of mass $M_m \approx 10^{16}$ GeV could, in principle, exist for $T < T_c$, but the number $r \simeq \exp(-M_m/T_c)$ expected from thermal fluctuations when $T \simeq T_c$ is extremely small.

For $T_n > T > T_c$ the U_1^{EM} is spontaneously broken. During this period the photon has a mass and electric charge is violated. Charge violating reactions are in equilibrium for $T \gtrsim T_c$.[7] If there is a net charge density in the present universe left over from fluctuations from equilibrium as $T > T_c$ it is far smaller[7] than the observational limit[13,14] from galaxies and cosmology.

REFERENCES

1. Ya. B. Zel'dovich and M. Y. Khlopov, Phys. Lett. 79B, 239 (1979).
2. J. P. Preskill, Phys. Rev. Lett. 43, 1365 (1979).
3. M. B. Einhorn, D. L. Stein and D. Toussaint, Phys. Rev. D21, 3295 (1980).
4. A. H. Guth and S.-H. H. Tye, Phys. Rev. Lett. 44, 631, 963 (1980).
5. H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974).
6. G. 't Hooft, Nucl. Phys. B79, 276 (1974); A. M. Polyakov, Pis'ma Eksp. Teor. Fiz. 20, 430 (1974) [JETP Lett. 20, 194 (1974)]. For an introduction, see S. Coleman in New Phenomena in Subnuclear Physics, Part A, ed. A. Zichichi (Plenum, N.Y., 1977), p. 297.
7. P. Langacker and S.-Y. Pi, Phys. Rev. Lett. 45, 1 (1980).
8. D. A. Kirzhnits and A. D. Linde, Phys. Lett. 42B, 471 (1972) and Ann. Phys. 101, 195 (1976).
9. S. Weinberg, Phys. Rev. D9, 3357 (1974).
10. L. Dolan and R. Jackiw, Phys. Rev. D9, 3320 (1974).
11. R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 42, 1651 (1979), Phys. Rev. D20, 3390 (1979), and CCNY preprint HEP-7916 (1979).
12. A. Zee, Phys. Rev. Lett. 44, 703 (1980).
13. R. A. Lyttleton and H. Bondi, Proc. R. Soc. Lond. A252, 313 (1959).
14. A. Barnes, Astron. J. 227, 1 (1979).