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THE SU(2) × U(1) × U'(1) MODELS WHICH ARE SLIGHTLY DIFFERENT FROM THE WEINBERG-SALAM MODEL*

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ABSTRACT

We discuss SU(2) × U(1) × U'(1) models by a uniform formula which is convenient for their comparison with the standard Weinberg-Salam model. As examples, we give three interesting models which are based on different grand-unification models. In one model, U'(1) does not contribute to the electromagnetic interaction; in the other two, both U(1) and U'(1) do contribute to the electromagnetic interaction. Also, the first two models can approach the standard Weinberg-Salam model as close as one wants; but the third model has limitations on it.

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The standard Weinberg-Salam model[1] of the electro-weak interactions seems consistent with the experiments. However, as the neutral current experiments are not so accurate, the intermediate bosons of the $SU(2) \times U(1)$ gauge have not yet been found, and the Higgs sector may change the model in a very subtle way, other possibilities have still not been ruled out -- especially those slightly different from the standard model.[2] In such a situation, when we study the low energy electro-weak phenomenology (EWP) of grand unification models, e.g. the SO(10) model,[3] the E_6 model, and other models unifying both vertical interactions and horizontal interactions, there is no reason now to demand that it has to have the same low energy EWP as dictated by the Weinberg-Salam (W-S) model (except the minimal grand unification SU(5) model[4] which has a large desert between 10^2 GeV to 10^{14} GeV).

What is the possible low energy EWP beyond the W-S model? This problem has been discussed continually.[6-10] The next simplest model beyond $SU(2) \times U(1)$ is $SU(2) \times U(1) \times U'(1)$ models. Many papers exist on $SU(2) \times U(1) \times U'(1)$ models,[7-10] while some of them discuss the case where the U'(1) does not contribute to the electromagnetic interaction, others discuss where U(1) and U'(1) do contribute to the electromagnetic interaction. Also, most papers discuss only "safe" models which can approach the standard Weinberg-Salam model as close as one wants so it will never be ruled out except when the Weinberg-Salam model is ruled out. Here in this paper we will discuss all $SU(2) \times U(1) \times U'(1)$ models in a uniform formula which is very convenient for comparing them with the Weinberg-

Salam model. Also, this formula can fit both safe and unsafe models.

The characteristic of the standard $SU(2)_L \times U(1)$ model (with Higgs doublets only),[5] is that one parameter, the Weinberg angle, defines three things at the tree level: (1) it defines the ratio of the electromagnetic coupling constant and the weak charged gauge coupling constant

$$\sin^2\theta_W = e^2/g_L^2 = g'^2/\left(g_L^2 + g'^2\right) \qquad ; \qquad (1)$$

(2) it defines the mass ratio of the charged gauge boson and the neutral gauge boson

$$\sin^2\theta_W = 1 - m_W^2/m_Z^2 \qquad ; \qquad (2)$$

and (3) it defines the form of the low energy neutral current interaction

$$H^{nc} = \frac{iG_F}{2\sqrt{2}} \rho J^o(1) J^o(2), \quad \left(\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W}\right) \qquad (3)$$

where

$$J^o(a) = 4\left[\bar{\psi}_L T_3 \psi_L - \sin^2\theta_W \bar{\psi} Q \psi\right]_a \quad . \qquad (4)$$

Here $G_F/\sqrt{2} = g_L^2/8m_W^2$, a the flavor index, $T_3 = \tau_3/2$, Q the number of the electric charge of the particle a. Hereafter, for easy writing we will omit all γ matrices and space time indices.

At the tree approximation, these three definitions of the Weinberg angle, Eqs. (1)-(3), are equivalent and $\rho = 1$ in the W-S model. However, all the present experimental data are from the low energy phenomena, especially the neutral current interactions which are relevant to Eq. (4) only. So the consistency of the three definitions has not really been tested yet. The measured average value of $\sin^2\theta_W$ is[2]

$$\sin^2\theta_W = 0.23 \pm 0.02 \quad . \tag{5}$$

We notice three points from the observation of these measurements: (1) The average has been taken over all different experiments. This procedure is meaningful only when all the relevant neutral current interactions are governed by Eq. (4) with the same $\sin^2\theta_W$. (2) The typical experimental uncertainty of the value of $\sin^2\theta_W$ is about 1/10 (in some experiments, the uncertainties are even worse than 1/4). (3) Most of the experimental results are model dependent, for example, Eq. (4) is used to fix part of the experimental parameter, e.g., g_A, and the parton model is used to deal with the hadrons.

In a $SU(2)_L \times U(1) \times U'(1)$ model, only the first definition, Eq. (1), remains unchanged, the other two definitions, Eqs. (2) and (3), change even at the tree level, especially the low energy behavior of the neutral current interaction, viz., Eq. (3). Obviously, if an $SU(2)_L \times U(1) \times U'(1)$ model is really the underlying physics of low energy EWP, some corrections up to 1/6 to the standard W-S model, which may have escaped present experimental observation, are not impossible. What are these corrections like is interesting for the phenomenological analysis of more accurate experiments in the near future. The connection of these corrections to the underlying unification models is an interesting problem from the theoretical standpoint.

The method we use here to deal with any $SU(2)\times U(1)\times U'(1)$ model is first by making a suitable notation in the U(1) and U'(1) group-space to make two new abelian gauge groups $U(1)_y \times U(1)_{y'}$ where $U(1)_{y'}$ does not contribute to the electromagnetic interaction and second, by discussing the mixing between the $U(1)_{y'}$ gauge boson and the $SU(2)\times U(1)_y$ massive gauge boson to get the new formula for the neutral current interactions.

Let us call the operator of the second U(1) and Y', which has nothing to do with the electric charge operator, and the first $U(1)_y$ as Y as in the W-S model, then the electric charge is

$$Q = I_3 + Y/2 \quad . \tag{6}$$

Now we may have a mixing between the $SU(2)\times U(1)_Y$ neutral massive gauge boson and the $U(1)_{Y'}$ gauge boson (but, of course, no mixing between the photon field corresponding to the operator Eq. (6) and the $U(1)_{Y'}$ field). Thus the low energy neutral current interaction Hamiltonian becomes exactly

$$\begin{aligned} H^{nc} = \frac{iG_F}{2\sqrt{2}} \Bigg[& \frac{\cos^2\theta_2}{\lambda_1} J^o(1)J^o(2) + \eta \frac{\sin\theta_2\cos\theta_2}{\lambda_1} \big(J^o(1)J'(2) + J^o(2)J'(1)\big) \\ & + \eta^2 \frac{\sin^2\theta_2}{\lambda_1} J'(1)J'(2) + \frac{\sin^2\theta_2}{\lambda_2} J^o(1)J^o(2) \\ & + \eta \frac{\sin\theta_2\cos\theta_2}{\lambda_2} \big(J^o(1)J'(2) + J^o(2)J'(1)\big) + \eta^2 \frac{\cos^2\theta_2}{\lambda_2} J'(1)J'(2) \Bigg] \end{aligned} \tag{7}$$

where J^o = Eq. (4); J' is the current connected with the extra $U(1)_{Y'}$ symmetry; θ_2 is the mixing angle between the two neutral gauge bosons; λ_1 and λ_2 are the diagonalized mass squares of the two neutral gauge bosons with $\lambda_1 < \lambda_2$ and normed by $m_W^2/\cos^2\theta_W$,

$$\eta = g''/\sqrt{g_L^2 + g'^2} \qquad ; \qquad (8)$$

and g' and g" are the coupling constants of $U(1)_Y$ and $U(1)_{Y'}$, respectively. We call Eq. (7) the small deviation form. Obviously, when the last five terms are negligible, Eq. (7) becomes Eq. (3), the W-S model.

The concrete form of J' and the values of θ_2, λ_1 and λ_2 are model dependent, in other words, they depend on the meaning of the extra $U(1)_{Y'}$ and the structure of the Higgs sector, which in turn may connect with a grand-unification model and its hierachy pattern. Let us discuss some interesting examples.

(A) Suppose that above a very high energy scale M', ($M' >>> M_W$), the EW interactions are described by $SU(2)_L \times U(1)_Y \times SU(2)_H$ where the $SU(2)_H$ is the horizontal gauge group. The $SU(2)_H$ is broken down to $U(1)_H$ at the energy scale M'. Thus we get the $SU(2)_L \times U(1)_Y \times U(1)_H$ model below the energy scale M', which then is broken down to $U(1)_{em}$. Now we study only the last symmetry breaking. We choose the effective Higgs sector as follows:

$$\begin{array}{llll} \langle\phi\rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix} & \text{with} & T_L = \tfrac{1}{2} \ , \ Y = 1 \ , \ H = 1 & \\ & & & . \qquad (9) \\ \langle\chi\rangle = v_2 & \text{with} & T_L = 0 \ , \ Y = 0 \ , \ H = 1 & \end{array}$$

Then we get (when $\alpha^2 = v_2^2/v_1^2 >> 1$)

$$\begin{array}{lll} \lambda_1 \simeq 1 - \dfrac{2+\eta^2}{\eta^2\alpha^2} & , & \lambda_2^2 \simeq \eta^2\alpha^2 \\ \cos\theta_2 \simeq 1 & , & \sin\theta_2 = \dfrac{2+\eta^2}{\eta^3\alpha^2} \ , \qquad (10) \end{array}$$

and the low energy neutral current interaction Hamiltonian to the first order of $\frac{1}{\alpha^2}$ is:

$$H^{nc} \simeq \frac{iG_F}{2\sqrt{2}} \left[\left(1 + \frac{2+\eta^2}{\eta^2\alpha^2}\right) J^o(1)J^o(2) + \frac{2+\eta^2}{\eta^2\alpha^2} \left(J^o(1)J'(2) + J^o(2)J'(1)\right) + \frac{1}{\alpha^2} J'(1)J'(2) \right] \tag{11}$$

with

$$J' = 2\bar{\psi} H \psi \quad , \tag{12}$$

where the operator H/2 is the third component of the $SU(2)_H$ group in the Fermion representation. H has to be the same number for all low mass Fermions to suppress the strangeness changed neutral current. According to the present experimental status, $\alpha > 5$ is already safe enough.

The additional $U(1)_{Y'}$ coming from an SU(N) (N > 5) or GL(n,c) has been discussed by Ref. 8.

(B) The SO(10) group can be decomposed down to

$$SU(2)_L \times SU(2)_R \times SU(3)_C \times U(1)_B \subset SO(10) \quad . \tag{13}$$

The $U(1)_B$ means B-L U(1) gauge symmetry, where B is the baryon number and L is the lepton number and we will call the B-L simply by B. We notice that at the grand-unification level, the ratio of the coupling constants is

$$g_L^2 : g_R^2 : g_B^2 = 1 : 1 : 3/2 \quad . \tag{14}$$

We suppose that at the energy scale $M \sim 10^{14}$ GeV, the SO(10) symmetry is broken down to the subgroup Eq. (13). Because of the different behaviors of the running coupling constants of the nonabelian gauge group $SU(2)_L \times SU(2)_R \simeq SO(4)$ and the $U(1)_B$ gauge group, the ratio changes as the energy goes down. Say, at a still very high energy scale M' we reach

$$g_L^2 : g_R^2 : g_B^2 = 1.8 : 1.8 : 1 \tag{15}$$

where the $SU(2)_R$ is broken down to $U(1)_R$ and at the energy scale about 1 TeV we get a model[9]

$$SU(2)_L \times U(1)_R \times U(1)_B$$

with, say

$$g_L^2 : g_R^2 : g_B^2 = 2.2 : 1.8 : 1 \qquad . \tag{16}$$

In this model we have Y in Eq. (5) as

$$Y = 2T_{3R} + B \qquad . \tag{17}$$

We choose the effective Higgs sector as follows

$$\begin{aligned} \langle\phi\rangle &= \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \quad \text{with} \quad T_L = \tfrac{1}{2} \,,\; T_{3R} = \tfrac{1}{2} \,,\; B = 0 \\ \langle\chi\rangle &= v_2 \quad \text{with} \quad T_L = 0 \,,\; T_{3R} = \tfrac{1}{2} \,,\; B = -1 \end{aligned} \qquad . \tag{18}$$

Then we get the similar result as Eq. (11) for the low energy neutral current interaction Hamiltonian when $\alpha^2 = v_2^2/v_1^2\cos^4\varphi >> 1$ with

$$J' = 4\left(J_{3R} - tg^2\varphi\, \bar{\psi}\, \frac{B}{2}\, \psi\right) \qquad , \tag{19}$$

where $tg^2\varphi = g_B^2/g_R^2$. Equation (1) becomes

$$\sin^2\theta_W = g_B^2 g_R^2 \Big/ \left(g_B^2 g_R^2 + g_L^2 g_B^2 + g_L^2 g_R^2\right) \qquad , \tag{20}$$

and Eq. (8) becomes

$$\eta^2 = g_R^4 \Big/ \left(g_B^2 g_R^2 + g_L^2 g_B^2 + g_L^2 g_R^2\right) \qquad . \tag{21}$$

Substituting Eq. (16) into Eq. (20) we get

$$\sin^2\theta_W = 0.23 \qquad . \tag{22}$$

The numbers in Eqs. (15), (16) and (22) are given for definiteness; they have no definitely physical meaning. The reason is that since a lot of unknown things may happen at the energy area 10^{14} GeV down to 1 TeV, we do not want to go into the details of the hierarchy here. However, we would stress that these numbers are reasonable if the hierarchy of the SO(10) model is in the way mentioned above, which is very different from the standard hierarchy.[4]

The models (A) and (B) can approach the standard W-S model closer and closer by adjusting the parameter (only one) α bigger and bigger. We can indeed choose such kind of Higgs sector to build a model, which has more than one adjustable parameter, however, even if we adjust all the parameters, we still have to face an appreciable deviation from the W-S model. This is the model (C).

(C) In model (B), we choose different Higgs contents[10]

$$\begin{aligned} \langle\phi_1\rangle &= \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \quad \text{with} \quad T_L = \tfrac{1}{2}\ ,\ T_{3R} = \tfrac{1}{2}\ ,\ B = 0 \\ \langle\phi_2\rangle &= \begin{pmatrix} v_2 \\ 0 \end{pmatrix} \quad \text{with} \quad T_L = \tfrac{1}{2}\ ,\ T_{3R} = \tfrac{1}{2}\ ,\ B = -2 \end{aligned} \quad . \tag{23}$$

The neutral current Hamiltonian Eq. (7) has the following parameterization

$$\lambda_{\substack{1\\2}} = \frac{1+\eta^2(a^2\beta^2+\alpha^2)}{2}\left[1 \mp \sqrt{1 - \frac{4\eta^2\alpha^2\beta^2(1+a)^2}{[1+\eta^2(a^2\beta^2+\alpha^2)]^2}}\right] \tag{24}$$

$$tg\theta_2 = \frac{1-\lambda_1}{\eta(\alpha^2 - a\beta^2)} \tag{25}$$

with $a = 1+2tg^2\varphi$, $\alpha^2 = v_1^2/(v_1^2+v_2^2)$, and $\beta^2 = (1-\alpha^2)$. If we choose the parameterization as Eq. (16), then $\eta = 0.64$, $\eta^2 = 0.41$ and we get (when

$\alpha^2 = 1/25)$

$$\lambda_1 \simeq 0.73 \ , \qquad \lambda_2 = 2.0 \ , \qquad tg\theta_2 \simeq -0.20 \quad . \tag{26}$$

Then in Eq. (7) every term in the last five terms is smaller than 1/6 of the first term. However, this may not be the best parameterization.

There are two problems in this model: (1) The total strength of the neutral current interaction is almost two times bigger than what is in the standard model though this can be improved a little. (2) One use of the Higgs ϕ_1 is to give Fermions masses through the Yukawa couplings. Because its VEV decreases five times more than that in the W-S model, we have to increase the coupling constants five times to get the same Fermion masses. The Yukawa interactions of Higgs and Fermions have not yet been detected. Will this increase bring bad news for such a model, we have to wait and see.

Our conclusion is that when energy of the accelerators goes up, we may find one more Z boson, whereas in the presently reachable energy region, we may find some signature of the other Z boson. If that is true, the low mass Z boson will be lighter than what expected in the standard W-S model at the tree level. We notice that the mass of the Z boson will become heavier if we consider radiative corrections in the standard W-S model.[10] If the exotic Higgs multiplets[5] get VEV, it may cause the ratio m_z/m_w smaller than that in the standard model.[10] However, the form of the neutral current interaction Eq. (3) will not change except there is more than one neutral gauge boson.

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