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ON THE SHAPE OF HADRON STRUCTURE FUNCTIONS*

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ABSTRACT

The hypothesis that, in the leading twist approximation and to all orders of perturbative QCD, there exists a momentum scale Q_0^2 at which hadrons are pure valence quark (or antiquark) bound states gives good results for nucleon, pion and kaon structure functions.

Perturbative QCD tells us how hadron structure functions, e.g., $F_2^{eN}(x,Q^2)$, evolve with Q^2 for high enough Q^2. But what about their shapes as functions of x? Our aim here is to investigate the possibility of predicting those shapes. To do so we consider the experimental structure functions in a Q^2 region where higher than twist two effects ($1/Q^2$ terms) are negligible ($Q^2 \gtrsim$ 1-10 GeV^2) and extrapolate them at low Q^2 using perturbative QCD. We assume that this procedure leads to a momentum scale Q_0^2 at which the extrapolated hadron pictures correspond to pure valence quark or antiquark bound states with no glue and no sea[1-4] (qqq state for the nucleon and $q\bar{q}$ states for pion and kaon). Of course those perturbative QCD extrapolations of hadron structure functions have nothing to do with what is actually measured at the scale Q_0^2 because of the presence of very large higher twist effects at this scale. Our model consists in using perturbative QCD in the leading twist approximation with the following boundary conditions $xG(x,Q_0^2) = xq_s(x,Q_0^2) = 0$ (G and q_s stand respectively for gluon and light quark sea distributions). The valence boundary distributions $xq_v(x,Q_0^2)$ are the bound state distributions computed in the leading twist approximation ($1/Q_0^2$ terms neglected) as given, e.g., in the meson case, by the diagram of Fig. 1. Note that we never have to precise the exact value of Q_0^2.

We are able to predict the kaon structure functions in terms of the pion ones by using data on pion structure functions[5] together with the following formula[3]

$$M_{u_v^{K^+}}(n,Q^2) = M_{u_v^{\pi^+}}(n,Q^2) \times \frac{M_{u_v^{K^+}}(n,Q_0^2)}{M_{u_v^{\pi^+}}(n,Q_0^2)} \qquad (1)$$

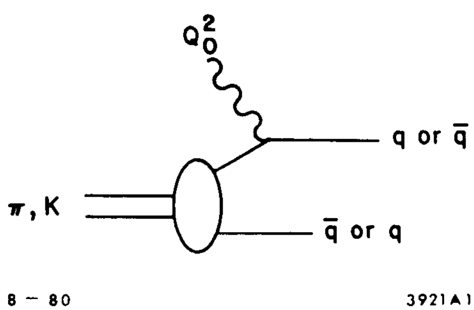

Fig. 1. Meson as a $q\bar{q}$ bound state.

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valid to all orders of perturbative QCD in the leading twist approximation $\left(M_q(n,Q^2) = \int_0^1 x^{n-1}\, q(x,Q^2)\, dx\right)$. In principle the distributions $q_V(x,Q_0^2)$ should be given by the QCD bound state solution, but so far this problem has not been solved. We are therefore forced to make approximations.

(1) With a non relativistic approximation the distributions $q_V(x,Q_0^2)$ obtained are peaked at $x_0 = \mu/(m+\mu)$ with a width of order $1/RM$. μ, m and M are respectively the masses of the struck quark (or antiquark), the recoiling partons and the hadronic target, R^2 is the mean square radius of the hadron. For nucleon, pion and kaon x_0 equals respectively 0.33, 0.5 and 0.38 (0.38 corresponds to u_V in K^+ when using $\mu = m_u = 336$ MeV and $m = m_s = 540$ MeV) and $1/RM = 0.26$, 2.4 and 0.80. The resulting distributions $q_V(x,Q_0^2)$ (see Fig. 6 in Ref. 4) are quite different from those experimentally measured at $Q^2 \sim 20$ GeV^2. This means that a large amount of perturbative QCD corrections are needed to obtain $q_V(x,Q^2 = 20\ \text{GeV}^2)$. This is a feasible possibility which is in fact realized when using the leading log approximation for the perturbative QCD corrections.[2,4] The shapes of the distributions $q_V(x,Q_0^2)$ are quite different from the experimental ones but the two following main features are not modified by the perturbative QCD corrections and are in fact experimentally observed: (a) the nucleon distribution is much more peaked and concentrated at small x than the pion one; and (b) the kaon distribution drops faster than the pion one at large x.

(2) Relativistic bound states: If we use a field theory with usual quark propagators, i.e., having poles at the quark constituent masses ($m_u \sim 336$ MeV), and massless gluon exchanges, we run into problems. The first problem is obviously that it is incompatible with confinement since it leads to free constituent quark states with the corresponding masses. The second problem has to do with structure functions. We get a nucleon distribution $u_V^N(x,Q_0^2)$ which is too narrowly peaked around $x_0 = 1/3$ (it does not reproduce $R_N = 0.8$ fm) and drops faster than the experimental one for $0.33 < x < 1$. We also obtain a pion distribution[6] $xu_V^{\pi^+}(x,Q_0^2) \sim Ax^3(1-x)^2$ which is steeper than the experimental one $(\sim 0.5\sqrt{x}\,(1-x))$ before being corrected by perturbative QCD. Therefore it is impossible for perturbative QCD, i.e., mainly gluon bremsstrahlung, to reproduce both the experimental nucleon and pion structure functions. One way out of these difficulties is to replace the first order perturbative pole of the quark propagator by a cut along the real axis.[3,7] A possibility is that the cut starts at m_0 which can be related to the current quark mass and that the pole at the constituent quark mass m is smeared out with a width γ^2 of the order of the strong interaction scale Λ^2. Explicitly if we write the quark propagator as

$$P(k) = \Pi(k^2)\not{k} + M(k^2) \qquad (2)$$

we can choose the absorptive part of $\Pi(k^2)$ as

$$\mathrm{Abs}\ \Pi(k^2) \sim \frac{\pi^{-1}\gamma^2\,\theta(k^2-m_0^2)}{(k^2-m^2)^2+\gamma^2} \tag{3}$$

with $\gamma^2 = \mathcal{O}(\Lambda^2)$. For the u and d quarks m_0 and m will be respectively of the order of 10 and 336 MeV whereas for the s quark they will be of the order of 150 and 540 MeV. Moreover we can choose the large k^2 behavior of our propagator to be given by perturbative QCD. Let us note that, since it has no more real pole, this propagator is now in agreement with confinement. Using this propagator, the nucleon, pion and kaon distributions $q_V(x,Q_0^2)$ obtained do not have the problem mentioned above. Note that their shapes are closer to the experimentally measured ones (corresponding to $Q^2 \sim 20\ \mathrm{GeV}^2$) than the non relativistic ones are. If, for u and d quarks we assume $m_0 = 0$, which is a good approximation, we find that $u_V^{\pi^+}(x,Q_0^2)$ is roughly proportional to $(1-x)$ for $0.45 < x < 1$. On the other hand using $m_s = 150$ MeV for the s quark we find that $u_V^{K^+}(x,Q_0^2)$ is roughly a decreasing linear function of x for $0.4 < x < 0.85$ and behaves like $(1-x)^2$ for $0.85 < x < 1$. Therefore we do not have any more the $(1-x)^2$ behavior for the pion structure function.[3] For the nucleon we find that $u_V^N(x,Q_0^2)$ behaves like $(1-x)^3$ when x goes to 1.

Using a formula similar to (1) for nucleon and pion together with nucleon data[8] we can predict the pion structure functions. Agreement with pion data[5] is good both when using non relativistic calculations[4] or our relativistic model[3] for the input at Q_0^2. We can also predict the kaon structure functions using formula (1) together with pion data. Figure 2 shows our results for the ratio $[\bar{u}_V^{K^-}/\bar{u}_V^{\pi^-}](x,Q^2=20\ \mathrm{GeV}^2)$ together with the data points of the CERN-NA3 experiment.[9] The solid line corresponds to a non relativistic approximation[4] and the dashed one to a relativistic calculation which uses the following parameters $(m_0,m) = (0,336\ \mathrm{MeV})$ for the u and d quarks and $= (150\ \mathrm{MeV},\ 540\ \mathrm{MeV})$ for the s quark. The dashed line is very sensitive to those parameters. So most probably an adjustment of the parameters can give a better agreement with data. Note also that this curve has been obtained assuming no spin correlation between the struck quark and the recoiling one.

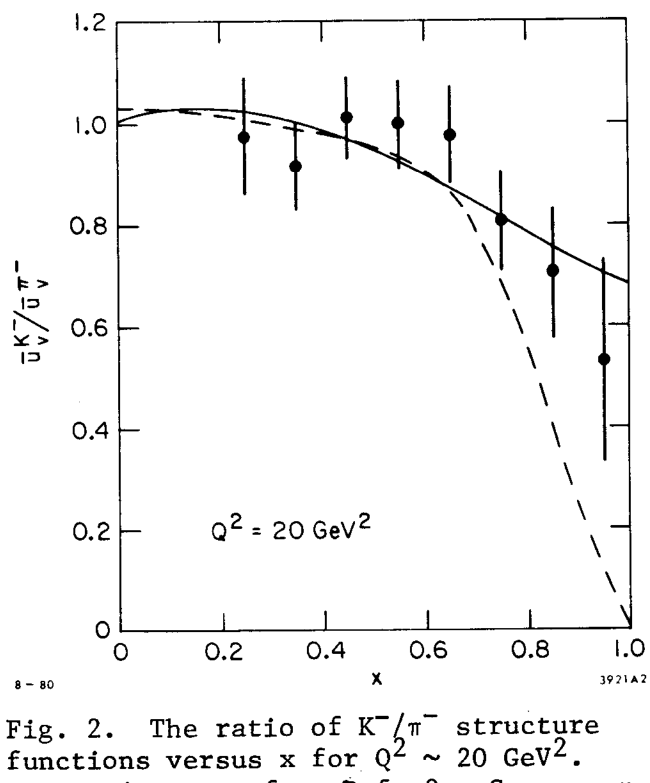

Fig. 2. The ratio of K^-/π^- structure functions versus x for $Q^2 \sim 20\ \mathrm{GeV}^2$. Data points are from Ref. 9. Curves are explained in the text.

It is possible to make predictions on a hadron structure function without referring to any other one if we use the leading log approximation of perturbative QCD.[1,2,4] In doing so we get the right partition of nucleon momentum carried between the valence, glue and sea.[1] Using the non relativistic input at Q_0^2, we obtain, in first approximation, good results for the valence distributions inside the nucleon, pion and kaon.[2,4] However the gluon and sea distributions obtained seem too steep near $x=0$.[2,10] This may indicate the need for higher orders of perturbative QCD. Note that if we suppress the effect of the 3-gluon coupling for $Q^2 \lesssim 1$ GeV^2 which corresponds to a freezing of the QCD running coupling constant $\alpha_s(Q^2)$ in this region[11] we obtain gluon and sea distributions which are in good agreement with the experimentally measured ones.[12]

We conclude that the idea of a hadron in which "real" gluons and sea quark-antiquark pairs all come from radiation processes (perturbative QCD) seems to work. This assumption used together with non relativistic or relativistic valence quark bound states gives good results when comparing nucleon, pion and kaon structure functions. In the relativistic case hadron structure functions can give information on the quark propagators in the non perturbative region. In particular the kaon structure functions can tell us about the s quark propagator.

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