SLAC-PUB-2580
August 1980
(T)

HADRON WAVEFUNCTIONS AND STRUCTURE FUNCTIONS IN QCD*

Tao Huang†
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305
and
Institute of High Energy Physics, Beijing, China

ABSTRACT

We present[1] the theoretical and empirical constraints on the hadronic wavefunction and hadronic structure functions. In particular, we obtain a new type of low energy theorem for the pion wavefunction from the $\pi^0 \to \gamma\gamma$. Thus we can get the probability of finding the valence $|q\bar{q}\rangle$ state. All these constraints allow us to construct a possible model which describes hadronic wavefunctions, probability amplitudes, and distributions.

The underlying link between hadronic phenomena in quantum chromodynamics at large and small distance is the hadronic wavefunction. By studying the wavefunctions themselves, one could in principle understand not only the origin of the standard structure functions, but also the nature of multiparticle longitudinal and transverse momentum distribution and helicity dependence, as well as the effects of coherence. In this talk, we will discuss the theoretical and experimental constraints on the hadronic wavefunction and structure functions and construct a simple model to implement these constraints.

We define the states at equal $\tau = t + z$ on the light-cone using the light-cone gauge $A^+ = A^0 + A^3 = 0$. The amplitude to find n (on-mass-shell) quarks and gluons in a hadron with 4-momentum P directed along the Z direction and spin projection S_z is defined as $\psi_{S_z}^{(n)}(x_i, k_{\perp i}, s_i)$ [see Fig. 1] $[x_i \equiv (k_i^+/P^+)]$,[2] where by momentum conservation $\sum_{i=1}^n x_i = 1$ and $\sum_{i=1}^n k_{\perp i} = 0$. The s_i specify the spin projection of the constituents. The state is off the light-cone energy shell. For each fermion or antifermion constituent $\psi_{S_z}^{(n)}(x_i, k_{\perp i}, s_i)$ multiplies the spin factor $u(\vec{k}_i)/\sqrt{k_i^+}$ or $v(\vec{k}_i)/\sqrt{k_i^+}$. The wavefunction normalization condition is

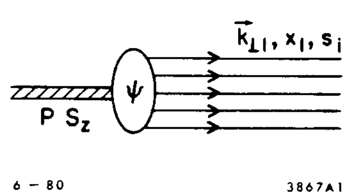

Fig. 1. The amplitude to find n(on-mass-shell) quarks and gluons in a hadron.

$$\sum_{(n)(s_i)} \int \left|\psi_{S_z}^{(n)}(x_i, k_{\perp i}, s_i)\right|^2 \left[d^2k_\perp\right]\left[dx\right] = 1 \quad . \qquad (1)$$

* Work supported in part by the Department of Energy, under contract DE-AC03-76SF00515, and by Academia Sinica of China.

† This work was done in collaboration with S. J. Brodsky (SLAC) and G. P. Lepage (Cornell University, LNS).

(Invited talk presented at the XXth International Conference on High Energy Physics, Madison, Wisconsin, July 17-23, 1980.)

We will discuss the following theoretical and experimental constraints on the wavefunctions and the structure functions.
(a) The predictions of perturbative QCD for the large transverse momentum tail of the Fock state $\psi(x_i, k_{\perp i})$. For the case of inclusive reactions, the standard quark and gluon structure functions, which control large momentum transfer inclusive reactions at the large scale Q^2 can be found

$$G_{a/H}\left(x_a, Q^2\right) \equiv d_a^{-1}\left(Q^2\right) \overline{\sum_{n, s_i, S_z}} \int^{k_{\perp a}^2 < Q^2} \left[d^2 k_\perp\right]\left[dx\right] \times \left|\psi_{S_z}^{(n)}\left(x_i, k_{\perp i}, s_i\right)\right|^2 \delta\left(x - x_a\right) \quad , \tag{2}$$

where $d_a^{-1}(Q^2)$ is due to the wavefunction renormalization of the constituent a. Note that only terms which fall-off $|\psi|^2 \sim (k_{\perp a}^2)^{-1}$ (modulo logs) contribute to the Q^2 dependence of the integral; in general, unless x is close to 1, all Fock states in the hadron contribute to $G_{a/H}$. These contributions are analyzable by the renormalization group and correspond in perturbative QCD to quark or gluon pair production or fragmentation processes associated with the struck constituent a. Multiparticle probability distributions are simple generalizations of Eq. (2).

Recently, it has been shown that exclusive processes such as form factors and large angle elastic scattering can be systematically analyzed in perturbative QCD.[2] For example, the (helicity conserving) hadronic form factors to leading order in m^2/Q^2 and to all orders in $\alpha_s(Q^2)$ take the form

$$F\left(Q^2\right) = \int [dx]\,[dy]\; \phi^\dagger\left(x, \tilde{Q}_x, s'\right)\; T_H(x,y,Q)\; \phi\left(y, \tilde{Q}_y, s\right) \ , \tag{3}$$

where $\tilde{Q}_x = \left(\min_i x_i\right) Q$, T_H is the hard scattering amplitude for the virtual photon to scatter the valence quarks from p to p + q; it can be expanded in powers of $\alpha_s(Q^2)$. The quantity $\phi(x, Q)$ is the "distribution amplitude" for finding the valence quark with light-cone fraction x_i in the hadron at relative separation $b_\perp^2 \sim O(1/Q^2)$.

$$\phi\left(x_i, Q, s_i\right) \equiv \prod_{i=1}^{n} \left[d_i^{-1}\left(Q^2\right)\right]^{\frac{1}{2}} \int^{k_{\perp i}^2 < Q^2} \left[d^2 k_\perp\right] \psi^{(n)}\left(x_i, k_{\perp i}, s_i\right) \quad , \tag{4}$$

The large Q^2 dependence of ϕ(i.e., the large $k_\perp$ tail of ψ) is in fact completely determined by the operator product expansion near the light-cone,[3] and in QCD can be calculated from the perturbative expansion in the irreducible kernel for the quark constituents.[1] To order $\alpha_s(Q^2)$ one only requires single gluon exchange, and we find

$$\phi\left(x_i, Q^2\right) = \phi\left(x_i, Q_0^2\right) + \frac{C_F}{\beta} \int_{Q_0^2}^{Q^2} \frac{d\ell_\perp^2}{\ell_\perp^2} \;[dy]\; \alpha_s\left(\frac{\ell_\perp^2}{y_i\left(1 - y_1\right)}\right) \times \left[V\left(x_i, y_i\right) - \delta(x - y)\right] \phi\left(y_i, \ell_\perp^2\right) \quad , \tag{5}$$

where

$$V(x,y) = 2\left\{x_1 y_2 \theta\left(y_1 - x_1\right)\left(\delta_{h_1\bar{h}_2} + \frac{\Delta}{y_1 - x_1}\right) + \left(1 \leftrightarrow 2\right)\right\} = V(y,x) \quad . \tag{6}$$

This result[4] is derived in the region where $\ell_\perp^2/[y_1(1-y_1)]$ is large compared to the off-shell energy $\langle \mathcal{E} \rangle$ in the wavefunction.
(b) Exact boundary conditions for the valence Fock state meson wavefunctions from the meson decay amplitudes. The leptonic decays of mesons give an important constraint on the valence $|q\bar{q}\rangle$ wavefunction at the origin,[2]

$$\lim_{Q\to\infty} \phi_M\left(x_i, Q^2\right) = a_0 x_1 x_2 = \begin{cases} \dfrac{3}{\sqrt{n_C}}\, f_\pi x_1 x_2 & \text{for } \pi \\ \dfrac{3\sqrt{2}}{\sqrt{n_C}}\, f_\rho x_1 x_2 & \text{for } \rho_L \end{cases} , \tag{7}$$

where $f_\pi \simeq 93$ MeV is the pion decay constant for $\pi^+ \to \mu\nu$ and $f_\rho \simeq 107$ MeV is the leptonic decay constant from $\rho^0 \to e^+e^-$. The analogous result holds for all zero helicity mesons. Because the $Q \to \infty$ distribution amplitude has zero anomalous dimension, this constraint is independent of gluon radiative correction and can be applied directly to the non-perturbative wavefunction

$$a_0 = 6 \int_0^1 [dx]\left[d^2k_\perp\right] \psi_M^{\text{non-pert}}\left(x_i, k_\perp\right) \quad . \tag{8}$$

On the other hand we can also obtain an exact low-energy constraint on $\psi(x_i, k_\perp = 0)$ for the pion in the chiral limit $m_q \to 0$. The $\gamma^*\pi \to \gamma$ vertex defines the $\pi^0 - \gamma$ transition form factor $F_{\pi\gamma}(Q^2)$ (Fig. 2a)

$$\Gamma_\mu = -ie^2 F_{\pi\gamma}(Q^2)\, \epsilon_{\mu\nu\rho\sigma} P_\pi^\nu \epsilon^\rho q^\sigma \quad . \tag{9}$$

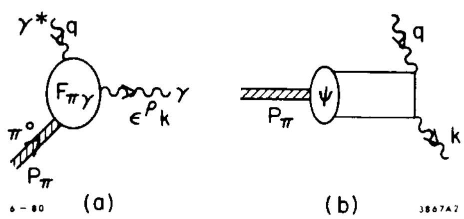

Fig. 2. (a) The π-γ transition from factor $F_{\pi\gamma}(Q^2)$; (b) the lowest order diagram which contributes to $F_{\pi\gamma}(Q^2)$.

If $m_q \to 0$, then the valence $|q\bar{q}\rangle$ contribution to $F_{\pi\gamma}(Q^2)$ is (Fig. 2b)

$$F_{\pi\gamma}(Q^2) = 2\sqrt{n_C}\left(e_u^2 - e_d^2\right)\left\{\int_0^1 dx_1 \int_0^{x_1^2 Q^2} \frac{dk_\perp^2}{16\pi^2} \frac{\psi\left(x_i, k_\perp\right)}{Q^2 x_1} + \left(x_1 \leftrightarrow x_2\right)\right\} . \tag{10}$$

In fact, gauge-invariance requires that the valence $|q\bar{q}\rangle$ state should give exactly ½ of the total decay amplitudes at $q^2 \to 0$.[5] Thus from the $\pi \to \gamma\gamma$ decay rate and Eq. (10), we find

$$\int_0^1 [dx]\, \psi\left(x_1, k_\perp = 0\right) = \int_0^1 dx_1\, \psi\left(x_1, k_\perp = 0\right) = \frac{\sqrt{n_C}}{f_\pi} \quad . \tag{11}$$

This is a new type of low-energy theorem for the pion wavefunction which is consistent with chiral symmetry and the triangle anomaly for the axial vector current. This large-distance result, together with the constraint on the valence wavefunction at short distance from the $\pi\to\mu\nu$ leptonic decay amplitude, leads to a number of new results for the parametrization of the pion wavefunction, which we discuss below.

(c) We can show that the evolution equations which specify the large Q^2 behavior of the distribution amplitudes and incoherent distribution functions G are correctly applied for $Q^2 \gtrsim \langle\mathcal{E}\rangle$, where $\langle\mathcal{E}\rangle$ is the mean value of the off-shell energy in the Fock state wavefunction, $\mathcal{E}\equiv\sum_i \mathcal{E}_i \equiv \sum_{i=1}^{n}\left((\vec{k}_\perp^2+m^2)/x\right)_i$ i.e., $\langle\mathcal{E}\rangle$ is a measure of the "starting point" for evolution due to perturbative effects in QCD*

In order to organize the predictions for hadronic matrix elements and all of the distribution functions and amplitudes, we shall make the following prescription:

(i) We assume the Fock state wavefunction $\psi^{(2)}$ for the 2-quark state in the non-perturbative domain depends only on the off-shell energy variable $\mathcal{E}$. [This ansatz, which is true for non-relativistic theories, can be justified, if we use the Bethe-Salpeter equation with an instantaneous energy independent kernel.[6]] For the n-particle case, we shall assume the Fock state wavefunction $\psi^{(n)}$ is a symmetric function of the $\mathcal{E}_i$, i.e., $\psi^{(n)}=\psi^{(n)}(\mathcal{E}_i)$. Although we have no strong argument for this form, we shall use it as an illustration of the effect of the non-perturbative wavefunction. Thus we find

$$G_{a/H}^{\text{non-pert}}\left(x_a\right) \xrightarrow[x_a\to 1]{} \left(1-x_a\right)^{2n_s-1} \quad g\left(\mathcal{E}^i_{min}\right) \quad , \tag{12}$$

where $n_s=\min(n_H-n_a)$ is the minimum number of spectator constituents in the hadron H after removing the particle (or subcomposite) a, and $\mathcal{E}^i_{min}=m_\perp^2/x_i$ is the minimum value of $\mathcal{E}_i$.[7] Notice that if we can neglect the quark masses [i.e., for $(1-x_a) \gg m^2/\langle k_\perp^2\rangle$] we obtain the spectator rule[8]

$$G_{a/H}^{\text{non-pert}}\left(x_a\right)= C_{a/H}\left(1-x_a\right)^{2n_s-1} \quad . \tag{13}$$

For example, $n_s=1$, for the meson case; $n_s=2$ for the baryon case.† We can see that the non-perturbative contribution can dominate the perturbative predication in the $x\sim 1$ domain.[1]

(ii) An (approximate) connection between the equal-time wavefunction in the rest frame and in the infinite momentum frame wavefunction can be established by equating the energy propagator $M^2-\mathcal{E}=M^2-\left(\sum_{i=1}^{n} k_i^\mu\right)^2$ in the two frames.

* The actual limit of the $k_\perp^2$ integration is $k_\perp^2/[z(1-z)] \gtrsim \langle\mathcal{E}\rangle$, where z is the fraction of the radiated gluon. The correct argument of α_s is $\alpha_s\left(k_\perp^2/(y-z)\right)$. [y is the fraction of the quark.]

† In addition, QCD evolution increases the exponent of Eq. (13).[1]

If we kinematically identify

$$\frac{\left(q^0+q^3\right)_i}{\sum_j^n q_j^0} \leftrightarrow x_i \equiv \frac{k_i^+}{P^+}\,, \quad \vec{q}_{\perp i} \leftrightarrow k_{\perp i}\,, \tag{14}$$

Then the rest frame wavefunction $\psi_{CM}(\vec{q}_i)$ which controls binding and hadronic spectroscopy implies a form for the IMF wavefunction $\psi_{IMF}(x_i,k_{\perp i})$. For two particle state there is a possible connection[9]

$$\psi_{CM}(\vec{q}^{\,2}) \leftrightarrow \psi_{IMF}\left(\frac{k_\perp^2+m^2}{4\,x_1x_2} - m^2\right) \quad . \tag{15}$$

In order to implement these constraints it is convenient to construct a simple model of the hadronic wavefunction. By using the connection (15) from the harmonic oscillator model[6] we can get the wavefunction at the infinite momentum frame

$$\psi^{(2)}\left(x_i,k_\perp,s_i\right) = A\exp\left[-R^2\left(\frac{k_\perp^2+m^2}{x_1} + \frac{k^2+m^2}{x_2}\right)\right] \quad . \tag{16}$$

It is certainly a function of $\mathscr{E}$. From Eqs (7), (11) and (16) we can obtain ($m_q^2R^2 << 1$)

$$R = \frac{1}{4\pi f_\pi} \simeq 0.17\text{ fm}, \quad A \simeq \frac{\sqrt{3}}{f_\pi} \quad . \tag{17}$$

The probability of finding the valence $|q\bar{q}\rangle$ state in the pion is thus

$$P(q\bar{q}) = \int\left[dx\right]\left[d^2k_\perp\right]\left|\psi\left(x_i,k_\perp\right)\right|^2 = 1/4 \quad . \tag{18}$$

Alternatively, if we use a power law form

$$\psi\left(x_i,k_\perp\right) = \frac{A_\alpha}{\left(\frac{k_\perp^2+m_q^2}{x(1-x)}+\mu^2\right)^\alpha}\,, \tag{19}$$

we find ($m_q^2 << \mu^2$)

$$P(q\bar{q}) = \frac{1}{2}\;\frac{\alpha-1}{2\alpha-1}\,, \tag{20}$$

which again leads to 1/4 for large α. For the linear potential case, where $\alpha = 3$, we have $P(q\bar{q}) = 1/5$. The distribution amplitude for the Gaussian form depends only upon the quark mass. In Fig. 3 we give the prediction of the perturbative QCD[10] for the pion form factor. Note that $\langle\mathscr{E}\rangle \sim 0.7$ GeV^2 is reasonable compared to the QCD Λ^2. For the multiparticle case, perhaps the simplest generation for the Fock state wavefunctions in the non-perturbative domain is the Gaussian form

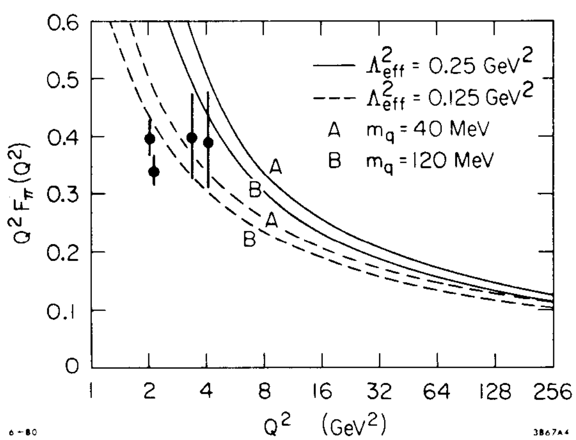

Fig. 3. QCD prediction for the meson form factor for the distribution function $\phi(x_i)$.[11]

$$\psi_{S_z}^{(n)}\left(x_i,k_{\perp i},s_i\right) = A_n\exp\left(-R_n^2\mathscr{E}\right) = A_n\exp\left[-R_n^2\sum_{i=1}^{n}\left(\frac{\vec{k}_\perp^2+m^2}{x}\right)_i\right] \quad . \tag{21}$$

The parametrization is taken to be independent of spin. This ansatz for the wavefunction has the additional analytic simplicity of (a) factorizing in the kinematics of each constituent and (b) satisfying a "cluster" property.

A similar analysis has been applied to the baryon wavefunction; we find

$$\phi\left(x_i, Q_0^2\right) \propto x_1 x_2 x_3 \exp\left[-R^2\left(\frac{m_1^2}{x_1}+\frac{m_2^2}{x_2}+\frac{m_3^2}{x_3}\right)\right] \quad . \qquad (22)$$

In addition, we can consider the sea quark effects and the high twist effects. These results will be given elsewhere.

We conclude that theoretical and empirical constraints on wavefunctions and structure functions have been presented. In particular we obtain a new type of low-energy theorem and the probability of finding the valence $|q\bar{q}\rangle$ state in the total pion wavefunction is ~0.2 to 0.25, for a broad range of confining potentials. This work represents a first attempt to construct a model of hadronic wavefunction and hadronic structure which is consistent with data and QCD at large and small distances.

I would like to thank S. J. Brodsky, G. P. Lepage and M. Barnett for their help in preparing this talk. I also enjoyed the hospitality of the Aspen Center for Physics where I prepared the talk.

REFERENCES

1. S. J. Brodsky, T. Huang and G. P. Lepage, SLAC-PUB-2540.
2. G. P. Lepage and S. J. Brodsky, Phys. Rev. D (to be published); Phys. Lett. 87B, 359 (1979); Phys. Rev. Lett, 43 545 (1979); Erratum ibid., 43, 1625 (1979); SLAC-PUB-2447.
3. S. J. Brodsky, Y. Frishman, G. P. Lepage and C. Sachradja, Phys. Lett. 91B, 239 (1980); A. V. Efremov and A. V. Radyushkin, Dubna preprints JINR-E2-11535,-11983 and -12384; A. Duncan and A. H. Mueller, Phys. Lett. 90B, 159 (1980); Phys. Rev. D21, 1636 (1980).
4. Higher order kernels entering the evolution equation include all two-particle irreducible amplitudes for $q\bar{q}\to q\bar{q}$. However, these corrections to $V(x_i,y_i)$ are all suppressed by powers of $\alpha_s(Q^2)$, because they are irreducible.
5. J. Schwinger, Phys. Rev. 82, 664 (1951); S. L. Adler, Phys. Rev. 177, 2426 (1969); J. S. Bell and R. Jackiw, Nuovo Cimento 60A, 47 (1969); R. Jackiw, Lectures on a Current Algebra and its Applications, Princeton University Press, 1972, pp. 97-230.
6. For example, see Elementary Particle Theory Group, Beijing University, Acta Physics Sinica 25, 415 (1976).
7. Examples of this result for $G_{q/M}$ and $G_{q/B}$ have recently been given by A. De Rujula and F. Martin, MIT preprint CTP 851 (1980).
8. S. J. Brodsky and R. Blankenbecler, Phys. Rev. D10, 2973 (1974).
9. An equivalent result was also obtained by V. A. Karmanov, ITEP-8, Moscow (1980).
10. The $Q^2\to\infty$ asymptotic result was first obtained by G. R. Farrar and D. R. Jackson, Phys. Rev. Lett. 43, 246 (1979).
11. The data are from C. Bebek et al., Phys. Rev. D13, 25 (1976).