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FINITE SPATIAL VOLUME APPROACH TO FINITE TEMPERATURE FIELD THEORY*

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ABSTRACT

A relativistic quantum field theory at finite temperature $T = \beta^{-1}$ is equivalent to the same field theory at zero temperature but with one spatial dimension of finite length β. This equivalence is discussed for scalars, for fermions and for gauge theories. The relationship is checked for free field theory. The translation of correlation functions between the two formulations is described with special emphasis on the non-local order parameters of gauge theories. Possible applications are mentioned.

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It is well known that quantum field theory in n spatial dimensions is related to the statistical mechanics of a classical n + 1 dimensional system [1]. This "equivalence" has proven useful in studying properties of both systems [1]. The analysis of the statistical mechanics of a quantum system has traditionally been more difficult than either a zero temperature quantum system or a finite temperature classical system [2]. This paper discusses the "equivalence" of a relativistic quantum field theory in n spatial dimensions at finite temperature $T = \beta^{-1}$ to the same field theory, at zero temperature but with one of the n spatial dimensions (say the z direction) finite and of length β, $(0 \le z \le \beta)$. (We shall call this a "finite volume system").

Scalar Theories

As an example consider a scalar theory in n + 1 dimensions with fields $\phi(\vec{x})$, and with canonically conjugate fields $\pi(\vec{x})$ and with a Hamiltonian, H given by

$$H = \int d^n x \left(\frac{\pi^2}{2} + \frac{(\nabla\phi)^2}{2} + V(\phi) \right) \tag{1}$$

Where $V(\phi)$ is a polynomial in ϕ. Let us denote by $\{|n\rangle, \varepsilon_n\}$ the set of eigenstates and eigenvalues of H. Suppose the system is heated to a temperature $T = \beta^{-1}$. Then all states of the system are excited, each with probability of $e^{-\beta\varepsilon_n}$. The partition function is given by:

$$Z(\beta) = e^{-\beta F(\beta)} = \sum_n e^{-\beta\varepsilon_n} = \sum_n \langle n| e^{-\beta H} |n\rangle \tag{2}$$

where $F(\beta)$ is the free energy of the system. The basis can be charged from eigenstates $|n\rangle$ of H to eigenstates $|\phi\rangle$ of field operators $\phi(\vec{x})$. Equation (2) then becomes:

$$Z(\beta) = \sum_{\phi} \langle \phi | e^{-\beta H} | \phi \rangle \quad . \tag{3}$$

Equation (3) is now written in a path integral form [3]:

$$Z(\beta) = e^{-\beta F(\beta)} = N \int_{\substack{\phi(\vec{x},0) \\ =\phi(\vec{x},\beta)}} \prod_{\vec{x},t} d\phi(\vec{x},t) \, \exp\left[-\int_0^\beta d\tau \int d^n x \, \mathscr{L}_E\right] \tag{4}$$

where N is a normalization factor[1] and where $\mathscr{L}_E$ is the Euclidean Lagrangian

$$\mathscr{L}_E = \frac{(\partial_0\phi)^2}{2} + \frac{(\nabla\phi)^2}{2} + V(\phi) \tag{5}$$

Thus the partition function is evaluated via a Euclidean path integral over finite time $0 \leq \tau \leq \beta$ with periodic boundary conditions.

It is clear that as $T \to 0$, equation (4) becomes the Euclidean generating functional for the zero temperature theory:

$$Z = \hat{N} \int d\phi \, \exp\left[-\int_{-\infty}^{\infty} d\tau \int d^n x \, \mathscr{L}_E\right] \tag{6}$$

Notice that $\vec{x}$ and τ are dummy variables of integration in equation (4). Let us define

$$\vec{x}_\perp = (x_1, \ldots x_{n-1}) \quad \text{and} \quad z = x_n \tag{7}$$

then interchange τ and z and interchange $\phi(\vec{x}_\perp, z, \tau)$ with $\phi(\vec{x}_\perp, \tau, z)$ in equation (4). We then find:

$$Z(\beta) = N \int_{\substack{\phi(z=0) \\ =\phi(z=\beta)}} \prod_{0\leq z\leq\beta} d\phi(\vec{x},\tau) \, \exp\left[-\int_{-\infty}^{\infty} d^{n-1}x \, d\tau \int_0^\beta dz \, \mathscr{L}_E\right] \tag{8}$$

Equation (8) is (apart from a normalization factor[1]) precisely the Euclidean path integral for a field theory at zero temperature ($-\infty < \tau < \infty$) but in "finite spatial volume" ($-\infty < x_\perp < \infty$, $0 \leq z \leq \beta$) with periodic boundary conditions.

Free Scalars

The above analysis could be applied to free scalars at temperature T. We would conclude that the free energy density of an ideal base gas at temperature $T = \beta^{-1}$ is equal to the ground state energy density of a free scalar theory (at zero temperature) but in "finite volume" (i.e., with one dimension of length β). Let us see how this works.

The free energy per unit volume of an ideal base gas of mass m at inverse temperature β is given by [4]:

$$\frac{\hat{F}(\beta)}{V} = \frac{1}{\beta}\int d^n k \; \ln(1 - e^{-\beta\omega_k}) \tag{9}$$

Here V represents the volume of space. This is not identical to the free energy appearing in equation (4). $\hat{F}(\beta)$ differs from $F(\beta)$ by an overall additive constant due to normal ordering. The point is that

$$\begin{aligned} e^{-\beta F} &= \sum_n \langle n| \exp\left[-\beta\int d^n x\left(\frac{\pi^2}{2} + \frac{(\nabla\phi)^2}{2} + \frac{m^2\phi^2}{2}\right)\right]|n\rangle \\ &= \sum_n \langle n| \exp\left[-\beta\int d^n k\left(a^\dagger(k)a(k) + \frac{1}{2}\right)\omega_k\right]|n\rangle \\ &= \exp\left[-\beta\int d^n k \; \frac{1}{2}\,\omega_k\right]\sum_n \langle n|\exp\left[-\beta\int d^n k \; a^\dagger(k)a(k)\omega_k\right]|n\rangle \\ &= \exp\left[-\beta\int d^n k \; \frac{1}{2}\,\omega_k\right]e^{-\beta\hat{F}} \end{aligned} \tag{10}$$

where $|n\rangle$ are Fock states for H and $a^\dagger(k)$, $a(k)$ are the usual creation and annihilation operators for these states.

It is $F(\beta)/V$ that is equal to the energy density, ε_0 of the "finite volume" system;

$$\varepsilon_0(\beta) = \frac{2\pi}{\beta}\int d^{n-1}k \sum_{n=-\infty}^{\infty} \frac{1}{2}\,\omega(k_\perp, n) \tag{11}$$

where

$$\omega(k_\perp, n) = \left(k_\perp^2 + \left(\frac{2\pi n}{\beta}\right)^2 + m^2\right)^{1/2} \tag{12}$$

is the frequency corresponding to the momentum

$$\vec{k}_n = \left(k_1, \ldots k_{n-1}, \frac{2\pi n}{\beta}\right) \equiv \left(\vec{k}_\perp, \frac{2\pi n}{\beta}\right) \tag{13}$$

Setting $F(\beta)/V$ equal to ε_0 and using equation (9) and (10) for F we obtain[1]

$$\frac{1}{\beta}\int d^n k \, \ell n\left(1 - e^{-\beta\omega_k}\right) = \frac{\pi}{\beta}\int d^{n-1}k \sum_n \omega(k_\perp, n) - \frac{1}{2}\int d^n k \, \omega_k \tag{14}$$

The left-hand side of equation (14) is finite. The right-hand side is the difference of two infinite quantities; $\varepsilon_0(\beta) - \varepsilon_0(\infty)$. To check the validity of (14) we can regularize it by taking $\partial^2/\partial(m^2)^2$ of both sides. (This works for n = 1 and 2. For n > 2 higher derivatives are required.) As a result:

$$-\frac{1}{4}\int \frac{d^n k}{\omega_k^3 \left(e^{\beta\omega_k} - 1\right)} \left\{1 + \frac{\beta\omega_k}{1 - e^{-\beta\omega_k}}\right\} \tag{15}$$

$$= -\frac{\pi}{4\beta}\int d^{n-1}k \sum_n \left(k_\perp^2 + \left(\frac{2\pi n}{\beta}\right)^2 + m^2\right)^{-3/2} + \frac{1}{8}\int d^n k \, (k^2 + m^2)^{-3/2}$$

If equation (14) is valid then equation (15) must also be correct. For n = 1, 2 both sides of equation (15) converge. I have checked numerically that for a very large range of values for βm equation (15) does, in fact hold. It is interesting to note that the left-hand side of equation (15) converges more rapidly for large β whereas the right-hand side converges better for small β.

Fermions

Formally we have only shown the above equivalence for scalar theories. For photons and for massive spin 1 particles we expect the formalism to work since both the free energy of the infinite temperature system and the energy density of the "finite volume" system are multiplied by the same overall factor--the number of degrees of freedom. However for Fermions we must be careful. The energy density of the finite volume system is multiplied by a factor of 4 (in 3 + 1 dimensions) but the free energy of the finite temperature Fermi gas is entirely different (see equation (17) below). Thus we must rethink our equivalence for Fermi systems.

Consider a Fermion field theory with fields $\psi(\vec{x})$, conjugate fields $\psi^{\dagger}(\vec{x})$ and a Hamiltonian $H(\psi, \psi^{\dagger})$. The partition function for this system at temperature $T = \beta^{-1}$ is given by [5]

$$Z(\beta) = e^{-\beta F(\beta)} = \int_{\psi(\vec{x},0) = -\psi(\vec{x},\beta)} \prod_{\vec{x},\tau} d\psi(\vec{x},\tau)\ d\psi^{\dagger}(\vec{x},\tau)\ \exp\left[-\int_0^{\beta} d\tau \int d^n x\ \mathscr{L}_E(\psi,\psi^{\dagger})\right] \tag{16}$$

where $\mathscr{L}_E$ is the Euclidean action for the system. The key point is that ψ must have antiperiodic rather than periodic boundary conditions. The O(n) invariance of the $\mathscr{L}_E$ assures that we can interchange τ and z as we did for the scalar case. As a result a Fermion field theory at finite temperature $T = \beta^{-1}$ is equivalent to the same theory at zero temperature with one direction of finite length β but with antiperiodic boundary conditions.

Free Fermions

We can now see what happens for free fermions. The free energy per unit volume of an ideal Fermi gas is given by:

$$\frac{\hat{F}(\beta)}{V} = \frac{\eta}{\beta} \int d^n k\ \ln(1 + e^{-\beta\omega_k}) \tag{17}$$

where η is the number of fermi degrees of freedom. Note the + sign in equation (17). The "finite volume" system with antiperiodic boundary conditions allows momenta:

$$\vec{k} = \left(\vec{k}_\perp , \frac{2\pi(n + \frac{1}{2})}{\beta}\right) \qquad n = 0, \pm 1, \ldots \tag{18}$$

i.e., half integer momenta in the z direction. This leads to an energy density

$$\varepsilon_0(\beta) = - \frac{2\pi}{\beta} \int d^{n-1}k \sum_{n=-\infty}^{\infty} \frac{n}{2}\, \tilde{\omega}(k_\perp , n) \tag{19}$$

where

$$\tilde{\omega}(k_\perp , n) = \left(k_\perp^2 + \left(\frac{2\pi(n + \frac{1}{2})}{\beta}\right)^2 + m^2\right)^{1/2} \tag{20}$$

Note the minus sign in equation (19) and the sum over half integers. In analogy with equation (10) we can relate $F(\beta)$ to $\hat{F}(\beta)$ and then equate $F(\beta)/V$ to ε_0. The result is:

$$\frac{1}{\beta} \int d^n k \, \ln(1 + e^{-\beta\omega_k}) = - \frac{\pi}{\beta} \int d^{n-1}k \sum_n \tilde{\omega}(k_\perp , n) + \frac{1}{2} \int d^n k \, \omega_k \tag{21}$$

We now take $\partial^2/\partial(m^2)^2$ of both sides of equation (21) and, in analogy with the scalar case the resulting equality has been numerically checked.

Gauge Theories

We discussed earlier that the equivalence of ε_0 to $\hat{F}/V$ seems to work for photons and for free massive spin 1 particles. It is straightforward to generalize the formal equivalence to the massive spin 1 case. For photons and for interacting gauge theories in general, the proof is more tricky due to gauge fixing problems.

At zero temperature it is well known that the generating functional for a gauge theory can be written as

$$Z = \int dA_\mu^a \exp\left[- \int d^{n+1}x \, \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a\right] \tag{22}$$

Here $A^a_\mu(\vec{x},\tau)$ are the gauge potentials with field strengths $F^a_{\mu\nu}$ and $1/4\ F^a_{\mu\nu}\ F^a_{\mu\nu}$ denotes the Euclidean Lagrangian:

$$\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} = \frac{1}{2} (E^2 + B^2) \tag{23}$$

To calculate the finite temperature partition function [3, 6] we can work in $A_o = 0$ gauge. In this gauge [7] $A^a_i(x)$ and $E^a_i(x)$ are canonically conjugate variables; and we work only in the gauge invariant sector of the Hilbert space,[2] i.e., we deal only with states which satisfy Gauss' Law:

$$D_i E^a_i |\psi\rangle = 0 \tag{24}$$

Any state $|\chi\rangle$ can be projected onto this gauge invariant subspace via:

$$|\chi\rangle_{\text{invariant}} = \sum_{\lambda^a(\vec{x})} \exp\left[i\int d^3x\ D\,E^a\lambda^a\right]|\chi\rangle \equiv P|\chi\rangle \tag{25}$$

Where the sum is over all gauge functions $\lambda^a(\vec{x})$ which vanish at spatial infinity. The Hamiltonian is given by

$$H = \int d^3x\ \frac{E^2 + B^2}{2} \quad \text{with} \quad B^2 = \frac{1}{2} F_{ij}\ F_{ij} \tag{26}$$

The partition function is calculated by summing only over the gauge invariant states:

$$\begin{aligned} Z(\beta) &= \sum_{\substack{n \\ \text{gauge} \\ \text{invariant}}} \langle n| e^{-\beta H} |n\rangle \\ &= \sum_{\substack{m \\ \text{all} \\ \text{states}}} \langle m| e^{-\beta H} \sum_{\lambda^a(\vec{x})} \exp\left[i\int d^3x\ D\,E^a\lambda^a\right]|m\rangle \end{aligned} \tag{27}$$

(Only one projection P is needed since $P^2 = P$ and P commutes with H.) Equation (27) can be written as:

$$Z(\beta) = \int dA_i^a(\vec{x})\ d\lambda_a(\vec{x})\ \langle A_i^a| e^{-\beta\hat{H}} |A_i^a\rangle \tag{28a}$$

Where

$$\hat{H} = \int d^3x \left[\frac{E^2 + B^2}{2} - iDE^a\left(\frac{\lambda^a}{\beta}\right)\right] \tag{28b}$$

Note that λ^a is strictly a function of $\vec{x}$. It is time independent.

$Z(\beta)$ can now be written in path integral form. Letting

$$A_o^a(\vec{x}) \equiv \frac{\lambda^a(\vec{x})}{\beta} \tag{29}$$

We find, apart from an overall normalization[1]

$$Z(\beta) = \int_{\substack{A_i^a(\vec{x},0)\\ =A_i^a(\vec{x},\beta)}} \prod_{\vec{x},\tau} dA_i^a(\vec{x},\tau) \prod_{\vec{x}} dA_o(\vec{x}) \exp\left[-\int_0^\beta d\tau \int d^3x\ \mathscr{L}_E\right] \tag{30}$$

where $\mathscr{L}_E$ is the Euclidean action

$$\mathscr{L}_E = \frac{1}{2}\left\{(\partial_o A_i - \partial_i A_o + A_o \times A_i)^2 + B^2\right\} \tag{31}$$

The path integral is over all $A_i^a(\vec{x},\tau)$ which are periodic in time and over $A_o^a(\vec{x})$ which is constant in time. Due to gauge invariance the integral can be extended to all periodic functions $A_o^a(\vec{x},\tau)$, keeping $A_i^a(\vec{x},\tau)$ periodic.[3] Thus, apart from an overall factor:

$$Z(\beta) = \int_{\substack{A_\mu^a(\vec{x},0)\\ =A_\mu^a(\vec{x},\beta)}} \prod_{\vec{x},\tau} dA_\mu^a(\vec{x},\tau) \exp\left[-\int_0^\beta d\tau \int d^3x\ \mathscr{L}_E\right] \tag{32}$$

$Z(\beta)$ is now in a manifestly $0(4)$ invariant form. Thus at this stage we can interchange τ and z and find that $Z(\beta)$ is equivalent to the generating functional for the zero temperature theory in finite spatial volume in one direction, and with periodic boundary conditions.

Correlation Functions and Order Parameters in Gauge Theories

Let us consider the theory of free photons at a temperature $T = \beta^{-1}$. Suppose we put two static external sources into the system and ask for their interaction energy $V_\beta(r)$ as a function of their separation r. The answer is well known:

$$V_\beta(r) \sim \frac{1}{r} \tag{33}$$

We have shown above that this theory is equivalent to the zero temperature theory with one spatial direction (say z) of length β. Let us suppose $r \gg \beta$ and ask for the separation energy of two static sources in this system. The situation is shown in Fig. 1(a). If we use periodicity to make the system infinite in the z direction, we have two "lines" of charge consisting of charges separated by a distance β, and the two lines separated by a distance r. The situation is shown in Fig. 1(b).

For $r \gg \beta$ we have two line charges separated by a distance r and

$$V(r) \sim \ell n\, r \tag{34}$$

in contrast with equation (35).

From this simple example it is clear that to calculate the energy of separation in the two formulations of the theory is not the same thing. To straighten out this situation let us ask how one calculates the finite temperature separation energy $V_\beta(r)$ in the usual (finite imaginary time) formulation of the theory. (The following remarks apply to all gauge theories--abelian and non-abelian.)

At zero temperature one considers a Wilson loop of large time T and of spatial length r and one evaluates:

$$W = \langle 0| \mathrm{TrP} \exp \left[i \oint A \cdot d\ell \right] |0\rangle \tag{35}$$

Where TrP denotes the trace of the path ordered exponential and $\oint$ is a loop integral around the Wilson Loop. The energy of separation is evaluated from the formula.

$$W \underset{T\to\infty}{\sim} e^{-TV(r)} \tag{36}$$

Due to Euclidean invariance, this Wilson loop may be taken totally as a spatial Wilson loop, with the same result.

At finite temperature equation (35) must be modified. The energy of a single source at position $\vec{x}$ is obtained by evaluating the Loop variable

$$L = \frac{\int dA_\mu \; e^{-S_\beta} \; \mathrm{TrP} \; \exp\left[i\int_0^\beta d\tau \; A_o(\vec{x},\tau)\right]}{\int dA_\mu \; e^{-S_\beta}} \tag{37}$$

where

$$S_\beta = \int_0^\beta d\tau \int d^3x \; \mathscr{L}_E$$

and $A_o \equiv A_o^a \tau_a$ where τ^a are the generating matrices for the group. L is invariant under periodic gauge transformations. L is a useful order parameter for the gauge theory [8]. In fact

$$L = e^{-BE} \tag{38}$$

where E is the energy of a single source. L = 0 corresponds to the confining situation where E is infinite.[4] The energy of separation of two sources at $\vec{x}_1$ and $\vec{x}_2$ is obtained by evaluating

$$W = \frac{\int dA_\mu \; e^{-S_\beta} \; \mathrm{TrP} \; \exp\left[i\int_0^\beta A_o(\vec{x}_1,\tau) \; d\tau\right] \mathrm{TrP} \; \exp\left[i\int_0^\beta A_o(\vec{x}_2,\tau) \; d\tau\right]}{\int dA_\mu \; e^{-S_\beta}} \tag{39}$$

W is the correlation function for 2 loops at spatial positions $\vec{x}_1$ and $\vec{x}_2$ with $r = |\vec{x}_1 - \vec{x}_2|$. If L is non-zero then

$$\frac{W}{L^2} = e^{-\beta V(r)} \tag{40}$$

To evaluate V(r) using the "finite spatial volume" approach we must therefore evaluate the same quantity as (39) but with τ and z interchanged. In other words we must evaluate the correlation function of two spatial periodic loops in the z direction, from 0 to β.

It is clear from the simple example of free abelian electrodynamics discussed above that this is not the same as evaluating the potential energy V(r) of two static sources in finite spatial volume. This would correspond to evaluating a temporal Wilson loop for $T >> r$ rather than a Wilson loop in z. Figure 2 shows the two situations. It is clear from our simple example that at finite temperature the spatial and temporal Wilson loops are not equivalent order parameters.

The lesson to be learned from this example is quite general. Although the "finite time" and the "finite spatial volume" approaches are equivalent, one must carefully translate all physical quantities from one formulation to the other. This is done by expressing any physical quantity in the path integral language, interchanging z and τ and performing the required transformation on the fields (e.g., $A_o \rightarrow A_3$ in the above example).

Summary

I have discussed the equivalence of a finite temperature field theory to a zero temperature, "finite volume" theory. The ultimate utility of such an approach is in one's ability to use it to calculate quantities which cannot be calculated in the standard approach. I know of no such cases at present. However, over the years much folklore and intuition has developed for understanding zero temperature Hamiltonian systems. Symmetries are particularly useful to study in a Hamiltonian formulation. I thus feel that in the least, this approach should be useful for using our understanding of Hamiltonian systems to further our understanding of finite temperature systems.[5]

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FOOTNOTES

1. The normalization factor, N is temperature dependent. In fact the usual derivation of the path integral formulation shows that for a system with time T and spatial volume V, N is given by: $N \sim \eta^{T} \Delta^{VT}$ where η and Δ are infinite. In equation (4) $N \sim \eta^{\beta} \Delta^{L^3\beta}$ with $V = L^3$ whereas the partition function for the "finite volume system" has a normalization factor $\hat{N} \sim \eta^{L} \Delta^{L^3\beta}$. Thus the finite temperature and finite volume partition functions are equivalent only up to a total normalization factor. It is easy to show that for spatial dimensions $n > 1$ this fact is irrelevant whereas for $n = 1$, $F(\beta)/V$ and ε_0 of equations (9) and (11) are only equal up to an additive constant: $\hat{F}/V = \varepsilon_0 + \mathrm{Const}/\beta$ (for $n = 1$). Derivatives of $\hat{F}/V$ and ε_0 with respect to m^2 are unchanged and thus the analysis of equation (15) holds for $n = 1$ as well.

2. We shall ignore the issue of θ states which are discussed in detail in Ref. [6].

3. This is proved by noting that for any periodic A_o and for all periodic A_i one can find a time-dependent $\hat{A}_o$ such that the gauge transformation $(\hat{A}_o, A_i) \rightarrow (A_o, \hat{A}_i)$ keeps A_i periodic. The resulting Jacobion is one; and only an overall a factor multiplies the integral.

4. This is certainly the case on a lattice. In any case, there is a symmetry (Z(n) for SU(n)) in the action which implies that $L = 0$. The case $L \neq 0$ corresponds to spontaneous breaking of this symmetry. This situation is being studied in more detail. Further results on L are given in Refs. [6] and [8].

5. A lattice quantum Statistical Mechanics problem near a critical point can be approximated by a finite temperature field theory. This approach may then also be useful for studying these theories.

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FIGURE CAPTIONS

Fig. 1(a) Two sources separated by a distance $r >> \beta$ in finite spatial volume. (b) The situation in Fig. 1(a) is extended, by periodicity to $-\infty < z < \infty$.

Fig. 2 Two types of Wilson loops in finite volume: $\langle W_1 W_2 \rangle$ and $\langle W \rangle$.

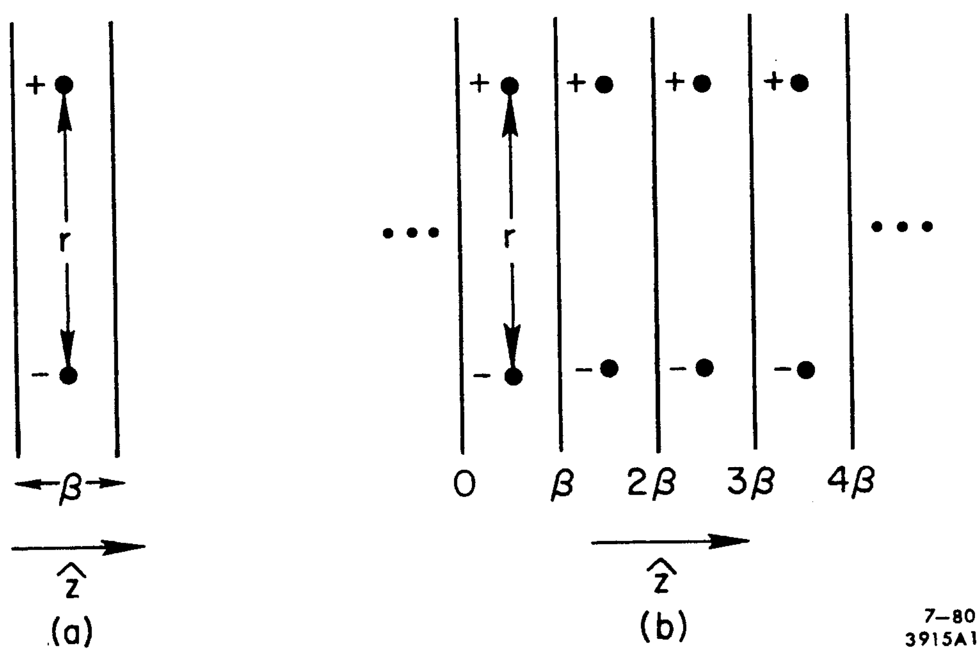

Fig. 1

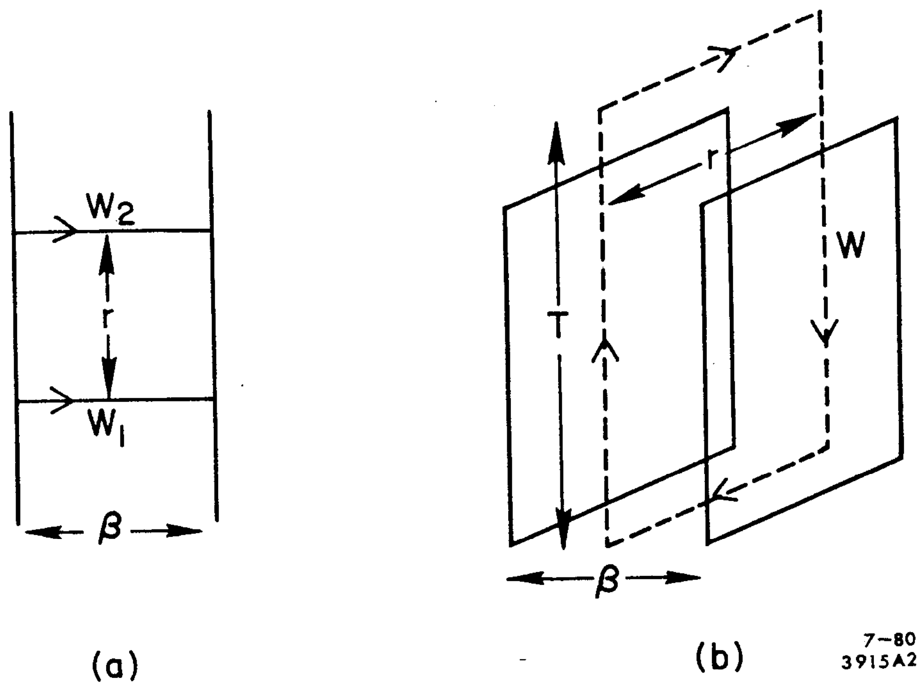

Fig. 2