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# CP VIOLATION IN THE SIX-QUARK MODEL*

Mark B. Wise†
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

## ABSTRACT

Some of the recent work on CP violation in the six-quark model is reviewed.

## INTRODUCTION

CP violation has been observed in the $K^o - \bar{K}^o$ system. Nonzero values for the quantities $\eta_{+-}$ and $\eta_{oo}$, defined by

$$\eta_{+-} \equiv \frac{\langle\pi^+\pi^-|H_{eff}^{|\Delta S|=1}|K_L\rangle}{\langle\pi^+\pi^-|H_{eff}^{|\Delta S|=1}|K_S\rangle} \quad \text{and} \quad \eta_{oo} \equiv \frac{\langle\pi^o\pi^o|H_{eff}^{|\Delta S|=1}|K_L\rangle}{\langle\pi^o\pi^o|H_{eff}^{|\Delta S|=1}|K_S\rangle} \quad , \tag{1}$$

are an indication of CP violation. Experimentally[1] $|\eta_{+-}| = (2.274\pm.022)\times10^{-3}$ and $|\eta_{oo}| = (2.32\pm.09)\times10^{-3}$. In the standard model of strong, weak and electromagnetic interactions based on the gauge group SU(3) ⊗ SU(2) ⊗ U(1) the quarks get their masses through Yukawa couplings to the Higgs fields. The resulting mass matrices for the quarks can be made diagonal, with real positive elements, by performing unitary transformations on the left-handed and right-handed quark fields. In the minimal model, which contains only one Higgs doublet, CP violation can then appear in the Lagrangian in only two ways.

In the strong interaction part of the Lagrangian density there is a term $\theta\epsilon^{\mu\nu\lambda\sigma}G^a_{\mu\nu}G^a_{\lambda\sigma}$, where $G^a_{\mu\nu}$ is the gluon field strength tensor, $a\epsilon\{1, \ldots 8\}$, and $\mu,\nu\epsilon\{0,1,2,3\}$. Such a term violates both P and CP invariance. An electric dipole moment for the neutron also violates both P and CP. The stringent experimental upper limit, $|D_n| \lesssim 10^{-24}$ cm, on the electric dipole moment of the neutron[2] gives rise to an upper bound on $\theta$ which shows that strong interaction violation of CP is much too small to explain the CP violation observed in the kaon system.[3,4]

In the six-quark model CP violation can also occur in the weak interaction portion of the Lagrangian through the coupling of the quarks to the W-bosons.[5] This part of the Lagrangian density has the form

$$\mathscr{L}_I = \frac{g}{2\sqrt{2}} J_\mu^{(+)} W^{(-)\mu} + \text{h.c.} \quad , \tag{2}$$

where $W_\mu^{(-)}$ is the charged W-boson field, g is the gauge coupling of the weak SU(2) group and $J_\mu^{(+)}$ is the charged weak current. In the

* Supported by the Department of Energy, contract DE-AC03-76SF00515.
† Present address: Department of Physics, Harvard University, Cambridge MA 02138.

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six-quark model

$$J_\mu^{(+)} = (\bar{u}\,\bar{c}\,\bar{t})\,\gamma_\mu(1-\gamma_5)\,\mathcal{U}\begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad . \tag{3}$$

$\mathcal{U}$ is a $3\times 3$ unitary matrix which arises from the diagonalization of the quark mass matrices. u, c and t denote the quark fields with charge $+2/3$ and d, s and b the quark fields with charge $-1/3$. In general a $3\times 3$ unitary matrix is specified by nine independent real parameters. However, five of the parameters used to specify $\mathcal{U}$ can be absorbed into the phases of the quark fields. Consequently $\mathcal{U}$ can be written in terms of only four real quantities. Three of these are Euler-type angles denoted by $\theta_1$, $\theta_2$ and $\theta_3$ and the fourth is a phase denoted by $\delta$. With the standard choice of quark field phases[5]

$$\mathcal{U} = \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{pmatrix} \quad , \tag{4}$$

where $s_i \equiv \sin\theta_i$ and $c_i \equiv \cos\theta_i$ for $i\varepsilon\{1,2,3\}$. The signs of the quark fields are chosen so that $\theta_1$, $\theta_2$ and $\theta_3$ all lie in the first quadrant. Then the quadrant of the phase $\delta$ has physical significance and cannot be specified by convention. Experimental information from $\beta$-decay give $s_1^2 \approx 0.05$. Combining this with experimental information on semileptonic hyperon decays provides the limit $s_3 \lesssim 0.5$ on violations of universality.[6]

By readjusting the phases of the quark fields the phase $\delta$ can be moved from one part of the matrix $\mathcal{U}$ to another. However, it is impossible to render the matrix $\mathcal{U}$ real by readjusting the phases of the quark fields. Consequently there is CP violation when $\delta$ differs from zero. In the next two sections the phenomenology of weak interaction CP violation in the $K^o - \bar{K}^o$ and $B_d^o - \bar{B}_d^o$ systems is discussed.

## THE $K^o$ - $\bar{K}^o$ SYSTEM

Within the phase convention where the $K \to \pi\pi$ $(I=0)$ amplitude, $A_o$, is chosen real, the quantities $\eta_{+-}$ and $\eta_{oo}$ are approximately given by $\eta_{+-} = \varepsilon + \varepsilon'$ and $\eta_{oo} = \varepsilon - 2\varepsilon'$. The quantity $\varepsilon$ determines the eigenstates $K_S$ and $K_L$ in terms of the $K^o$ and $\bar{K}^o$ states. Explicitly

$$K_S = \frac{1}{\sqrt{2(1+|\varepsilon|2)}}\left[(1+\varepsilon)K^o + (1-\varepsilon)\bar{K}^o\right] \tag{5a}$$

$$K_L = \frac{1}{\sqrt{2(1+|\varepsilon|2)}}\left[(1+\varepsilon)K^o - (1-\varepsilon)\bar{K}^o\right] . \tag{5b}$$

To first order in CP violating quantities

$$\varepsilon = \frac{i\left(\mathrm{Im}\,\Gamma_{12}/2 + i\,\mathrm{Im}\,M_{12}\right)}{\tfrac{1}{2}\left(\Gamma_S-\Gamma_L\right) + i\left(m_S-m_L\right)} \quad , \tag{6}$$

where $M_{12}$ and $\Gamma_{12}$ are the $K^o - \bar{K}^o$ mass and width transition matrix elements. In the phase convention $A_o$ real

$$\varepsilon \approx \frac{e^{i\pi/4}}{2\sqrt{2}} \left(\frac{\mathrm{Im}\,M_{12}}{\mathrm{Re}\,M_{12}}\right) \quad , \tag{7}$$

where the experimental values of the $K_S$ and $K_L$ masses and widths have been used. In addition $m_S - m_L = 2\,\mathrm{Re}\,M_{12}$ was used. The quantity $\varepsilon'$ measures the deviation of $\eta_{+-}/\eta_{oo}$ from unity and is defined by

$$\varepsilon' = \frac{i}{\sqrt{2}}\, e^{i(\delta_2-\delta_o)}\, \frac{\mathrm{Im}\,A_2}{A_o} \quad , \tag{8}$$

where $A_2$ is the $K \to \pi\pi$ $(I=2)$ amplitude and $\delta_2$ and $\delta_o$ are the $\pi\pi$ $(I=2)$ and $\pi\pi$ $(I=0)$ phase shifts.

The CP violation parameters $\varepsilon$ and $\varepsilon'$ can be computed from the matrix elements of the effective Hamiltonians for $\Delta S = 2$ $K^o - \bar{K}^o$ mixing and $\Delta S = 1$ weak nonlepontic decays. These effective Hamiltonians are derived by a four step process in which the W-boson t-quark, b-quark and c-quark are treated as heavy and their fields removed from explicitly appearing in the theory. The choice of quark field phases made in Eq. (4) puts the CP violating phase only in the couplings of the heavy quarks to the W-bosons. Therefore CP violation in the effective Hamiltonian for $\Delta S = 1$ weak nonleptonic decays only occurs from so called penguin-type diagrams which contain a heavy quark loop. These diagrams are purely $I = \tfrac{1}{2}$ and may be responsible for the $\Delta I = \tfrac{1}{2}$ rule in nonleptonic kaon and hyperon decays.[7] The penguin-type diagrams give an imaginary CP violating part to the $K \to \pi\pi$ $(I=0)$ amplitude $A_o$. Therefore, the choice of quark fields made in Eq. (4) does not correspond to the phase convention where $A_o$ is real. Transforming the strange quark field, $s \to e^{i\xi}s$, to make $A_o$ real causes the amplitude $A_2$ to pick up on imaginary part. Consequently

$$\varepsilon' \approx \frac{1}{20\sqrt{2}}\, e^{i\pi/4}(-\xi) \quad , \tag{9}$$

where the experimental relation $\mathrm{Re}\,A_2/A_o \approx 1/20$ has been used. The phase which has been approximated by $\pi/4$ follows from the $\pi\pi$ phase shifts and has the experimental value $(37\pm6)^o$.

The other CP violation parameter $\varepsilon$ arises from the matrix elements of the effective Hamiltonian for $K^o - \bar{K}^o$ mixing.[8,9] The six-quark model parameters $\theta_2$, $\theta_3$ and $\delta$ may be fit to the experimental values of the $K^o - \bar{K}^o$ mass difference and the CP violation parameter $\varepsilon$;

however, no prediction can be made for these quantities. The measured phase of $\varepsilon$ forces $\delta$ to lie in the upper half plane for small $s_3$.[10] For $s_3$ near the universality bound of .5 there is also a small region of allowed angles for which $\delta$ lies in the lower half plane.[11,12] The region with $\delta$ in the lower half plane exists only for $\cos\delta<0$ while the region with $\delta$ in the upper half plane exists for both $\cos\delta<0$ and $\cos\delta>0$.[11,12,13,14]

Both $\varepsilon$ and $\varepsilon'$ are proportional to the combination of angles $c_2s_2s_3\sin\delta$ and their ratio $\varepsilon'/\varepsilon$ is fairly insensitive to the values of the six-quark model parameters. There have been two approaches to estimating $\varepsilon'/\varepsilon$. In order to understand these computations it is necessary to know a few facts about the effective Hamiltonian for $\Delta S = 1$ weak nonleptonic decays. In the leading logarithmic approximation the effective Hamiltonian is a sum of Wilson coefficients multiplied by local four-quark operators:[7,15,16] $\mathcal{H}_{eff}^{|\Delta S|=1} = \sum_{i=1}^{6} C_iQ_i +$ h.c.. The operator $Q_6$ has a chiral structure $(V-A)\otimes(V+A)$ and is induced by penguin-type diagrams with a heavy quark loop.[17] The Wilson coefficient $C_6$ is small in magnitude compared with those of the $(V-A)\otimes(V-A)$ operators. However, the $(V-A)\otimes(V+A)$ chiral structure of $Q_6$ leads to enhanced matrix elements and this operator may make important contributions to nonleptonic decay amplitudes. If this is the case then an understanding of the $\Delta I=\frac{1}{2}$ rule is possible since $Q_6$ is purely $I=\frac{1}{2}$. The ratio $\mathrm{Im}\,C_6/\mathrm{Re}\,C_6$ is of order $c_2s_2s_3\sin\delta$ while the ratio of imaginary to real parts of the Wilson coefficients of the familiar $(V-A)\otimes(V-A)$ operators are of order $10^{-2}\ c_2s_2s_3\sin\delta$. Thus it is the matrix elements of $Q_6$ which contribute the largest CP violating imaginary part to the amplitude $A_0$. Let f be the fraction of the $K\to\pi\pi$ $(I=0)$ amplitude that arises from the matrix elements of $Q_6$. Then the isospin zero amplitude has a phase $\xi$ where

$$\xi = \mathrm{Im}\,C_6\,\frac{\langle\pi\pi(I=0)|Q_6|K^0\rangle}{A_0\ e^{i\delta_0}} \tag{10a}$$

$$= f\,\frac{\mathrm{Im}\,C_6}{\mathrm{Re}\,C_6}\quad . \tag{10b}$$

Recall that $\varepsilon'$ is proportional to $\xi$ (cf., Eq. (9)). One approach to estimating $\varepsilon'$ assumes that penguin-type diagrams are responsible for the $\Delta I=\frac{1}{2}$ rule and therefore chooses a large value for f. $\varepsilon'$ is then calculated from Eqs. (9) and (10b) using a leading logarithmic calculation of $C_6$. Of course, the value of f is renormalization point dependent. It is not known what value of the renormalization point mass, $\mu$, corresponds to the value of f used. Therefore several different values of $\mu$ are used in the computation of $C_6$ to get an idea of the uncertainties involved. This approach typically finds $\varepsilon'/\varepsilon$ to be of order a fraction of a percent although the uncertainties are large.[15] The second approach to estimating $\varepsilon'$ recognizes that a leading logarithmic calculation of $\mathrm{Re}\,C_6$ is very uncertain since it depends on integrations over virtual momenta primarily in the range $\mu^2 \lesssim p^2 \lesssim m_c^2$. In this approach Eq. (10a) is used to estimate $\xi$. The amplitude $A_0$ is taken from experiment and a quark-model-type or vacuum insertion estimate of the matrix element $\langle 2\pi(I=0)|Q_6|K^0\rangle$ is used. This approach also involves an implicit choice of $\mu$, namely that for which the matrix element estimate is correct. Predictions for $\varepsilon'$ are, however, now not as sensitive to the value of the renormalization mass, $\mu$, used to compute $C_6$ since $\mathrm{Im}\,C_6$ is less sensitive to variations in $\mu$ than $\mathrm{Re}\,C_6$ . In general the values of $\varepsilon'/\varepsilon$ obtained in this way are somewhat smaller than those obtained by the first approach.[16,18,19]

The present experimental limit is $|\varepsilon'/\varepsilon| \lesssim 1/50$, however, upcoming experiments should be able to determine $\varepsilon'/\varepsilon$ to the fraction of a percent level.[20] The measurement of a nonzero value for $\varepsilon'/\varepsilon$ in these experiments would be qualitative evidence that penguin-type diagrams are responsible for the $\Delta I = \frac{1}{2}$ rule. In addition information on the six-quark model parameters would be obtained from a measurement of $\varepsilon'/\varepsilon$. The theoretical estimates of $\varepsilon'/\varepsilon$ predict that it is almost real and has the same sign as $\sin\delta$. If $\varepsilon'/\varepsilon$ is measured to be negative then we would have very tight constraints on the angles $\theta_2$, $\theta_3$ and $\delta$ because the allowed region for these angles is very small when $\delta$ lies in the lower half plane.[11,12]

CP violation in low energy systems is characterized by the combination of angles $s_2 s_3 \sin\delta$. The experimental value of $\varepsilon$ implies that $s_2 s_3 \sin\delta$ is of order $10^{-3}$. Estimates of $D_n$, the electric dipole moment of the neutron,[21] indicate that $|D_n| \approx 10^{-30}$ cm when $s_2 s_3 \sin\delta \approx 10^{-3}$. The electric dipole moment is very small because first order weak diagrams do not contribute to it. Therefore, unlike strong interaction CP violation, CP violation in the weak couplings of the quarks to the W-bosons can be responsible for the observed values of $\eta_{+-}$ and $\eta_{oo}$ without giving too large an electric dipole moment to the neutron.

## THE $B_d^o - \bar{B}_d^o$ SYSTEM

At the present time observation of CP violation has been confined to the neutral kaon system. It may be possible to also observe CP violation in B meson decays. The analysis of the $B_d^o - \bar{B}_d^o$ system is similar to that of the neutral kaon system. The eigenstates are

$$B_1 = \frac{1}{\sqrt{2(1+|\varepsilon|^2)}} \left[(1+\varepsilon)B_d^o + (1-\varepsilon)\bar{B}_d^o\right] \tag{11a}$$

$$B_2 = \frac{1}{\sqrt{2(1+|\varepsilon|^2)}} \left[(1+\varepsilon)B_d^o - (1-\varepsilon)\bar{B}_d^o\right] \quad . \tag{11b}$$

Since CP $B_d^o = \bar{B}_d^o$ and CP $\bar{B}_d^o = B_d^o$ the eigenstates $B_1$ and $B_2$ would also be CP eigenstates if $\varepsilon = 0$. To lowest nontrivial order in CP violating quantities

$$\varepsilon = \frac{i\left(\mathrm{Im}\,\Gamma_{12}/2 + i\,\mathrm{Im}\,M_{12}\right)}{\frac{1}{2}\left(\Gamma_{B_1} - \Gamma_{B_2}\right) + i\left(m_{B_1} - m_{B_2}\right)} \quad , \tag{12}$$

where $\Gamma_{12}$ and $M_{12}$ are the $B_d^o - \bar{B}_d^o$ width and mass transition matrix elements. Recall that in low energy systems CP violating quantitites are characterized by the combination of angles $s_2 s_3 \sin\delta$. For heavier systems like the B mesons this is no longer true. To leading order in the large W-boson and t-quark masses the box diagram for $B_d^o - \bar{B}_d^o$ mixing gives rise to a mass transition matrix element which is proportional to the combination of angles $s_1^2 s_2^2 (c_1 s_2 s_3 - c_2 c_3 e^{i\delta})^2$.

Then, for small $s_3$, $\mathrm{Im}M_{12}/\mathrm{Re}M_{12} \approx \tan 2\delta$, which can be large if $\delta$ is not small.[22] Unfortunately a calculation of the absorbtive part of the box diagram reveals that the leading contribution to the width transition matrix element is proportional to the same combination of angles so that $\varepsilon$ is almost pure imaginary in the region where $B_d^o - \bar{B}_d^o$ mixing is large.[23] A purely imaginary $\varepsilon$ can be transformed away by readjusting the quark field phases so that CP violating physical quantities, like the charge asymmetry in the number of same sign dilepton events from semileptonic $B_d^o - \bar{B}_d^o$ decays

$$\frac{\ell^{++} - \ell^{--}}{\ell^{++} + \ell^{--}} = \frac{-4\mathrm{Re}\ \varepsilon\left(1 + |\varepsilon|^2\right)}{\left(1 + |\varepsilon|^2\right)^2 + 4\left(\mathrm{Re}\ \varepsilon\right)^2} , \qquad (13)$$

vanish when $\varepsilon$ has no real part. It may be better to look for CP violation in processes where CP violation coming from the decay amplitudes also plays a role.[24]

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