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VACUUM INSTABILITY AND REFINED LIMITS ON THE HIGGS MESON MASS*

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ABSTRACT

Exact numerical computations of the decay of a metastable vacuum at zero temperature are presented and are used to calculate a lower bound for the Higgs mass. The dependence of the bound on the Weinberg mixing angle and on chiral symmetry breaking is analyzed.

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In the last few years, various attempts have been made to place a lower bound on the mass of the Higgs boson in the Weinberg-Salam model based upon the fact that the present Universe sponteneously breaks $SU(2) \times U(1)$. In the one loop approximation, the effective potential $V(\varphi)$ for the theory is given by [1]:

$$V(\varphi) = (2A-B)\sigma^2\varphi^2 - A\varphi^4 + B\varphi^4 \ln \frac{\varphi^2}{\sigma^2} \tag{1}$$

where φ is the classical part of the Higgs field; $\varphi = 0,\sigma$ correspond to minima of $V(\varphi)$; and A and B are constants with

$$B = \frac{3e^4}{1024\pi^2} \frac{1}{\sin^4\theta} \left(2 + \frac{1}{\cos^4\theta}\right) \quad . \tag{2}$$

θ is the Weinberg mixing angle and $\sigma \simeq 248$ GeV. The Higgs meson mass is given by

$$M_H^2 = \left.\frac{d^2V}{d\varphi^2}\right|_{\varphi=\sigma} = 4\sigma^2(3B - 2A) \quad , \tag{3}$$

and the energy gap between the $\varphi = 0$ and $|\varphi| = \sigma$ vacua is given by

$$\varepsilon = V(\sigma) - V(0) = (A - B)\sigma^4 \quad . \tag{4}$$

The desired $\varphi = \sigma$ state is energetically favorable if $\varepsilon < 0$; this requires A to be less than B, or

$$M_H > \sqrt{4B}\,\sigma \equiv M_{cr} \quad , \tag{5}$$

the lower bound on M_H stated by Weinberg [1]. However, as was pointed out by Frampton [2] and re-emphasized by Linde [3], it is still possible that $A > B$ and $M_H < M_{cr}$ if for some reason the vacuum began in the $\varphi = \sigma$ state and if the decay rate from the $\varphi = \sigma$ false vacuum to the $\varphi = 0$ true vacuum was so small that the lifetime of the metastable $\varphi = \sigma$ state was greater than the age of the Universe. (These computations only take into account the zero temperature effective potential.)

The decay of a false vacuum occurs through a tunneling process in which bubbles of a certain critical size are spontaneously formed and expand in linear dimension at a speed approaching the speed of light. Following Coleman's treatment of the barrier penetration problem [4], the tunneling is computed as a classical motion in the imaginary time direction. The tunneling is described by the O(4) symmetric solution to the equation

$$\frac{\partial^2 \varphi}{\partial t^2} + \nabla^2 \varphi = \frac{dV}{d\varphi} \tag{6}$$

with the boundary condition that $\varphi = \sigma$ at $t^2 + x^2 \to \infty$. If $\underline{A}$ is the action associated with the O(4) symmetric solution, the number of tunnelings that already occured should be given approximately by:

$$N = R \exp(-A) \tag{7}$$

where R is proportional to the volume of space-time in the backward light cone, estimated to be $\sim 10^{150-170} = e^{345-390}$. If the present Universe is symmetry breaking but the $\varphi = \sigma$ vacuum is only a metastable vacuum, it must be that $N \ll 1$ or $A \gg 390$.

Frampton attempted to estimate $\underline{A}$ by assuming: (1) that the bubble thickness is small compared to the bubble radius; and (2) that the energy density inside the bubble is $-\varepsilon$. He thus obtained an approximate expression for A given by

$$A = -\frac{1}{2}\pi^2 R^4 \varepsilon + 2\pi^2 R^3 S_1 \tag{8}$$

where R is the radius of the bubble and $-\varepsilon$ and S_1 are the volume and surface energy densities, respectively. The stationary value of the action is therefore given by

$$A = \frac{27}{2} \pi^2 S_1^4 / \varepsilon^3 \quad . \tag{9}$$

From the approximation Frampton obtained a lower bound on the Higgs mass $M_H > .7M_{cr}$. Linde noted that assumption (1) is only satisfied if the energy difference ε between the two vacua is much less than the energy barrier between them. Linde suggested that an exact solution would yield a much lower bound on the mass of the Higgs meson. (For $\theta = 35^o$, Linde claimed an exact calculation should yield $M_H > 260$ MeV in good agreement with the results reported in this paper. The details of how Linde arrived at the bound have not been published.)

In this paper the results of an exact numerical computation of the decay rate as a function of M_H are reported. The solution to eq. (6) was computed by assuming an O(4) symmetric solution so that the equation becomes:

$$\frac{d^2\varphi}{dr^2} + \frac{3}{r}\frac{d\varphi}{dr} = \frac{dV}{d\varphi} \quad . \tag{10}$$

The variable r can be considered to be a time variable and the desired "bounce" solution corresponds to the classical motion of a ball rolling down the inverted potential, $-V(\varphi)$, under the influence of a frictional force beginning from an initial position $\varphi(0)$ and ending at $\varphi(\infty) = \sigma$. By guessing a series of trial initial positions, $\varphi(0)$, and numerically integrating eq. (10) forward in time, the degree of undershoot or overshoot at $\varphi(\infty) = \sigma$ could be determined and an improved trial initial position found. The procedure was determined to within one part in 10^8

within 40 choices of trial initial positions. The action associated with the bounce solution was then computed, converging to within 1 part in 10^6 after 40 trials.

The action was computed as a function of M_H first using the value of the Weinberg mixing angle, $\theta = 35^o$, as did Linde and Frampton. The value of M_H for which $N \simeq 1$ was found to be $M_H > 240$-260 MeV, in good agreement with the result predicted by Linde [3]. A study of the bounce solution reveals that there are two contributions to the discrepancy between this result and that found for the thin-wall approximation. Firstly, near the lower bound, $M_H = 240$ MeV, the exact initial position for the bounce solution was found to be $\varphi(0) = .98\sigma$, far from the value of $\varphi(0) = 0$ that Frampton assumed. If $\varepsilon' = V(\sigma) - V(.98\sigma)$ replaced ε in eq. (9) -- still assuming a thin wall -- the action estimated by Frampton would be changed by

$$A_{new} = A_{old} \times (\varepsilon/\varepsilon')^3 \quad . \tag{11}$$

For $M_H \sim 260$ MeV, the correction is a factor of 10^{14}, far from insignificant ! The second correction to Frampton's is due to the fact, as Linde suggested, that the wall of the bubble is not really thin. However, it is not appropriate to compare the barrier height to ε, it should be compared to ε'. For the value $M_H = 260$ MeV, the barrier height is found to be .003 which is small compared to $\varepsilon' = 3.52$ even though $\varepsilon' << \varepsilon$, so the thin-wall approximation is not correct. It is more difficult to estimate the effect of the second correction, but it is important to be aware of both corrections to the bounce solution.

The results obtained for the bounce solution and the bound on the Higgs meson mass are sensitive to the value of the Weinberg angle. In fig. 1, the action $\underline{A}$ is shown as a function of M_H for various values of $\sin^2\theta$, including the value $\theta = 35^o$ used in the computations of Frampton and Linde. The dotted horizontal lines bound the region of values where tunneling is expected to become significant. From the curves one observes that the bound on M_H rises and the slope of the curve decreases as θ decreases. For the presently accepted value of $\sin^2\theta = .23$, fig. 1 indicates that M_H must be greater than 450-470 MeV.

The results were computed assuming the form of the effective potential given by eq. (1). However, one might also consider whether the breaking of chiral symmetry might effect the results found above. As was pointed out by Witten [5] and analyzed numerically by this author [6], as the temperature of the Universe decreases to about a few hundred MeV, previously massless quarks with an $SU(6) \times SU(6)$ chiral symmetry gain a mass and chiral symmetry is broken, presumably through a second order phase transition. When chiral symmetry is broken, $\bar{q}q$ should get a vacuum expectation value and $SU(6) \times SU(6)$ breaks down to diagonal $SU(6)$, the chiral breaking also breaks the weak interaction $SU(2) \times U(1)$ symmetry down to $U(1)$. The $\bar{q}q = \bar{q}_R q_L$ is an $SU(2) \times U(1)$ doublet that couples to the Higgs meson and a vacuum expectation value in $\bar{q}q$ leads to a linear term in the effective Lagrangian. The term is given by

$$L = -\frac{\varphi}{\sigma} \sum_i m_i \bar{q}_i q_i \qquad (12)$$

where the sum ranges over quark flavors. From the work of Gell-Mann Oakes and Renner [7], one expects

$$L = -\frac{\varphi}{\sigma} \frac{f_\pi^2 m_\pi^2}{\sqrt{2}\,(m_u + m_d)} \sum m_i \approx -\varphi(100 \text{ MeV})^3 . \qquad (13)$$

Before the additional term, there is a continuous set of asymmetric vacua of equivalent energy; the energy degeneracy of the asymmetric vacua is split by the new term and it may be assumed that the Universe lies in the minimum energy asymmetric vacuum. The effect of the new term is complicated since it alters the shape of the barrier and depresses the minimum energy of the asymmetric vacuum; the asymmetric minimum is not at $\varphi = \sigma$ and the computation of the Higgs mass or effective quartic coupling must be adjusted for this fact. However, because the coefficient in eq. (13) is small compared to the others in the effective potential, one expects the magnitude of its effect to be small. In fig. 2, the effect of the Higgs meson coupling to the quarks has been computed numerically for $\sin^2\theta = .23$. From the results it appears that the effect of chiral symmetry breaking on the action of the bounce solution is less than 1% along most of the curve and is less than .1% over the region in which the bound on the Higgs mass must be computed. Thus, the bound on the Higgs mass is changed by only .1%.

The bounds obtained above presume that by some mechanism the Universe has been cooled and has ended in the metastable $\varphi = \sigma$ state rather than the stable $\varphi = 0$ vacuum that one would expect. At this time, no specific mechanism has been proven, although there do seem to be some possible scenarios [8]. If a mechanism can be found, the following conclusions can be drawn:

(1) For there to be $SU(2) \times U(1)$ symmetry breaking, M_H must be greater than 450 MeV for $\sin^2\theta = .23$ or 600 MeV if $\sin^2\theta$ is found to be as low as .20.

(2) Chiral symmetry breaking affects the lower bound on the value of M_H by less than .1%.

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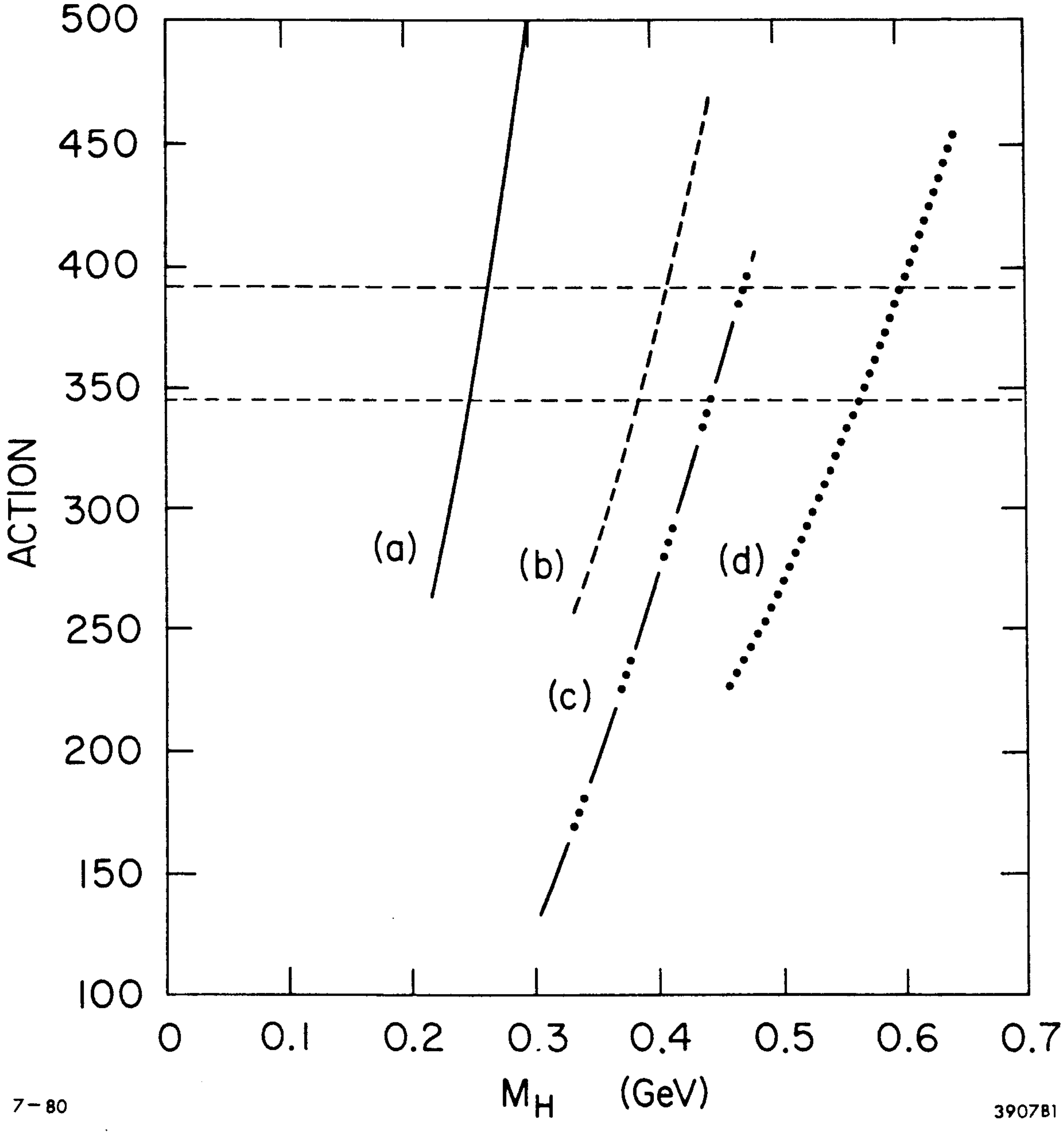
FIGURE CAPTIONS

Fig. 1. Bounce action is plotted versus the Higgs meson mass for various values of the Weinberg mixing angle, θ. From left to right the curves correspond to: (a) $\theta = 35^{o}$; (b) $\sin^2\theta = .25$; (c) $\sin^2\theta = .23$; and (d) $\sin^2\theta = .20$. The horizontal dotted lines indicate the band of values for the action where tunneling from the metastable $\varphi = \sigma$ vacuum becomes significant.

Fig. 2. Bounce action is plotted versus the Higgs meson mass with chiral symmetry breaking effects taken into account. The middle heavy line corresponds to the curve found if there is no chiral symmetry breaking and the curve to the right corresponds to the same computation with chiral symmetry breaking taken into account as discussed in the text of the paper.



500
450
400
350
300
250
200
150
100
ACTION
(a)
(b)
(c)
(d)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
M_H (GeV)
7-80
3907B1

Fig. 1

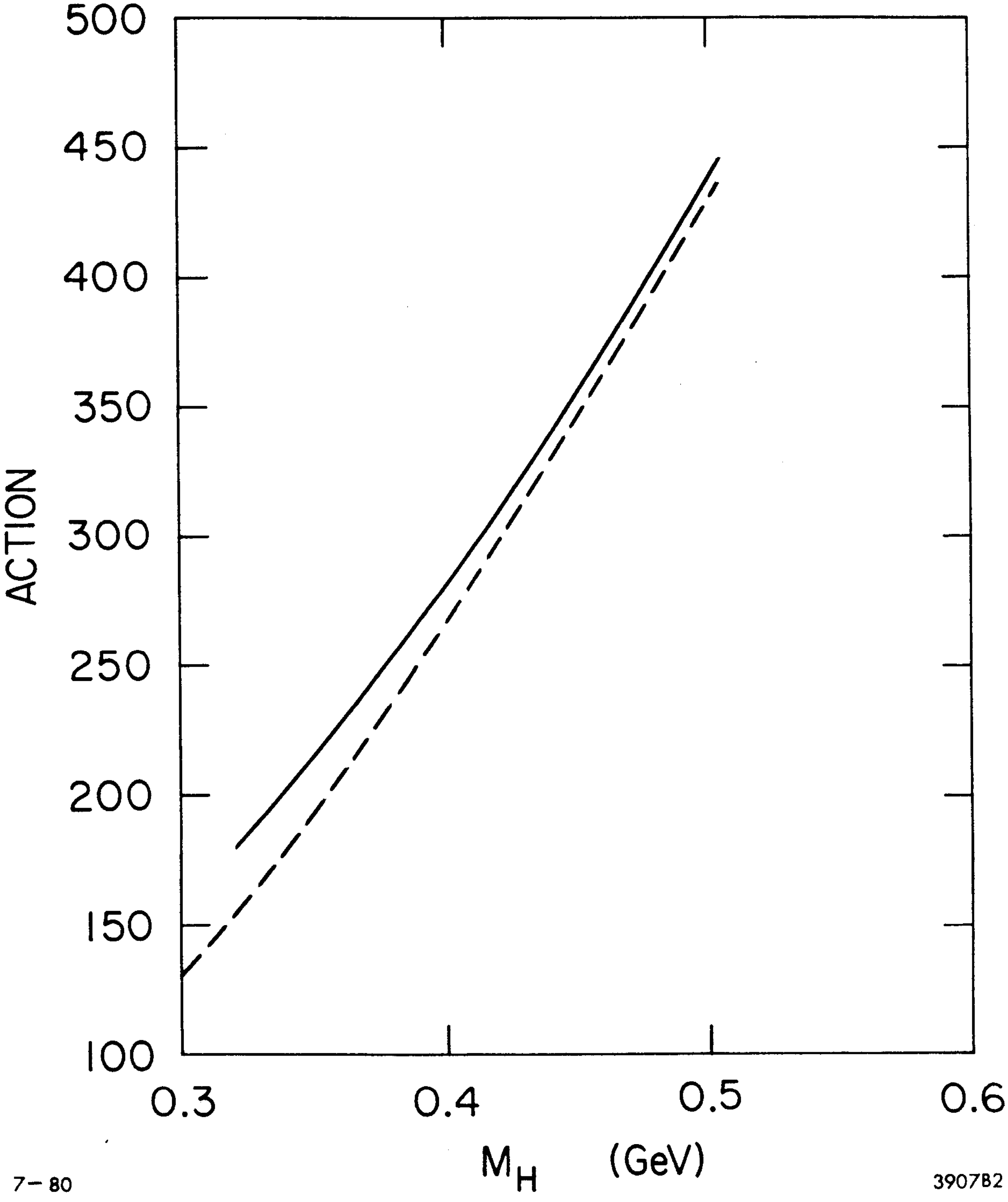

500
450
400
350
300
250
200
150
100
ACTION
0.3
0.4
0.5
0.6
M_H (GeV)
7-80
3907B2

Fig. 2