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TRANSVERSE WAKE FIELD EFFECTS ON INTENSE BUNCHES WITH APPLICATION TO THE SLAC LINEAR COLLIDER*)

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ABSTRACT

The theory of transverse beam break-up is summarized and briefly discussed in this paper. It is then applied to the SLAC linear accelerator to give the various design tolerances for beam injection and alignment of accelerator components for the linear collider operation.

INTRODUCTION

The problem of beam break-up of an intense bunch of particles in a linear accelerator has been studied before in the literatures.[1,2] When a point charge travels off axis down a linac structure it interacts with the walls of the structure and leaves behind a transverse wake field which will deflect particles traveling behind the point charge. If an intense bunch of particles travels through a sturcture, the transverse wake field generated by the head of the bunch will deflect the tail particles further away from the axis of the sturcture, thus leading to the beam break-up phenomenon.

Existing analysis of this problem assumes no or weak focusing forces provided by the quadrupole magnets. This assumption sometimes does not apply to the case of interest. For example the recently proposed SLAC linear collider[3] requires a strong focusing force to retain the beam emittance in order to optimize the luminosity. In this work, we analyzed the beam break-up problem with betatron focusing taken into account.

The physical picture of beam break-up in the presence of a strong betatron focusing is different from that otherwise. In the absence of wake fields, the transverse motion of a particle is a simple harmonic motion with a certain betatron frequency ω_β. This simple harmonic oscillator of natural frequency ω_β is driven by the wake field force. Since the wake field force, being proportional to the transverse offset from the structure axis of the preceding particles, has a time dependence of frequency ω_β, a condition for resonant driving of the harmonic oscillator is satisfied and the beam break-up results.

For the single pass collider, the particle bunch must be controlled in such a way that the transverse deviation of the bunch trajectory from the axis of the linac structure does not exceed a certain tolerance value. Exceeding this tolerance would lead to a growth in the projected area in transverse phase space occupied by all particles in the bunch and thus a

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reduction in the achievable luminosity. Two tolerances have been considered: the tolerance on the error at injection and the misalignment tolerance of the linac structure relative to the beam trajectory. The results of this study are summarized and discussed in this paper.[4]

INJECTION TOLERANCES

In what follows, we ignore the transverse dimensions of the bunch. We calculate the displacement of a particle in the bunch, $x(z,s)$, as a function of z, the longitudinal position relative to the center of the bunch, and s, the distance from the beginning of the linac. The approximation of zero transverse dimension for the bunch is valid if the transverse dimension of the bunch is much smaller than the diameter of the linac irises. Under this approximation $x(z,s)$ is taken to be the displacement of the center of a slice through the bunch at the position z.

The transverse force at z depends on the displacement of all preceding charges with $z' > z$ and is given by

$$F_x(z,s) = e^2 \int_z^\infty dz' \; \rho(z') \, W(z'-z) \, x(z',s) \tag{1}$$

where ρ is the line density of particles in the bunch ($\int \rho \, dz$ is normalized to the total number of particles in the bunch), $e \cdot W \cdot x$ is the transverse field produced by a point charge displaced from the axis by x at a distance $z'-z$ behind the point charge. All properties of the linac structure are contained in the wake function W. We have assumed that the displacement of a particle changes sufficiently slowly with s so that the average W of the structure can be used.[5] We also assume that the bunch length is much shorter than the betatron wavelength so that the retardation in the transverse field from the head to the tail of the bunch can be ignored.

The equation of motion for $x(z,s)$ can be written as

$$\frac{d}{ds}\left\{\gamma(s) \frac{d}{ds} x(z,s)\right\} + \left(\frac{2\pi}{\lambda(s)}\right)^2 \gamma(s) \, x(s,z)$$

$$= r_0 \int_z^\infty dz' \; \rho(z') \, W(z'-z) \, x(z',s) \quad . \tag{2}$$

where $\gamma(s)$ is the energy of beam at position s in units of mc^2, m being the rest mass of the particle, $\lambda(s)$ is the instantaneous wavelength of betatron focusing at position s, and $r_0 = e^2/mc^2$ is the classical radius of the particle. We have assumed that the betatron focusing is provided by a smooth function rather than coming from a series of discrete quadrupoles. We assume that the energy of the beam increases linearly with s as a result of acceleration in such a way that $\gamma(s) = \gamma_0(1+Gs)$ with $\gamma_0 mc^2$ the beam energy at injection and G the acceleration gradient. We let the strength of the focusing force in the linac scale with beam energy so that the instantaneous betatron wavelength remains constant $\lambda(s) = \lambda_0$.

Equation (2) can be solved by an interaction procedure. The solution $x(z,s)$ is expanded in a series of powers in terms of the wake field

$$x(z,s) = \sum_{n=0}^{\infty} x^{(n)}(z,s) \quad . \tag{3}$$

For the case of an injection error, the zeroth order solution $x^{(o)}$ is obtained by setting the wake field W to 0 in Eq. (2) and demanding the initial conditions $x(z,s) = x_o$ and $dx(z,s)/ds = 0$ at $s = 0$. The n-th order term $x^{(n)}$ is then obtained from the $(n-1)^{th}$ order term $x^{(n-1)}$ by the recurrence relation

$$x^{(n)}(z,s) = \frac{r_o}{\gamma_o k_o} \int_0^s ds' \frac{\sin k_o(s-s')}{(1+Gs)^{\frac{1}{2}}(1+Gs')^{\frac{1}{2}}} \int_z^{\infty} dz' \, \rho(z') W(z'-z) x^{(n-1)}(z',s') \tag{4}$$

where we have defined $k_o = 2\pi/\lambda_o$. We have made the adiabatic approximation that the betatron wave number k_o is much larger than the energy gradient G. The zeroth order term is given by

$$x^{(o)}(z,s) = \frac{x_o}{\sqrt{1+Gs}} \cos k_o s \tag{5}$$

where the factor $\sqrt{1+Gs}$ is the usual adiabatic damping term.

If the beam energy at the end of acceleration, $\gamma_f mc^2$, is much higher than the beam energy at injection, the solution for $x^{(n)}$ of all order n is found to be

$$x^{(n)}(z,s) = \frac{x_o}{\sqrt{1+Gs}} \left(\frac{r_o}{{}_o k_o}\right)^n I_n(s)\, R_n(z) \quad , \tag{6}$$

where

$$R_n(z) = \int_z^{\infty} dz_1 \, \rho(z_1) W(z_1 - z) \int_{z_1}^{\infty} dz_2 \, \rho(z_2) W(z_2 - z_1) \ \cdots \int_{z_{n-1}}^{\infty} dz_n \, \rho(z_n) W(z_n - z_n - 1) , \tag{7}$$

and

$$I_n(s) = \frac{1}{n!} e^{ik_o s} \left[\frac{1}{2iG} \ell n(1+Gs)\right]^n \tag{8}$$

To apply to the linac collider, we will approximate ρ by a rectangular distribution and W by a linear function, i.e.,

$$\rho = \begin{cases} N/\ell & \text{for} \quad |z| < \frac{\ell}{2} \\ 0 & \text{for} \quad |z| \geq \frac{\ell}{2} \end{cases} \tag{9}$$

$$W = W_o \frac{z}{\ell} \quad . \tag{10}$$

From Eq. (7), we find

$$R_n(z) = \frac{1}{(2n)!} \left[NW_o \left(\frac{1}{2} - \frac{z}{\ell}\right)^2 \right]^n \quad . \tag{11}$$

Knowing $R_n(z)$ and $I_n(s)$, we can substitute Eq. (6) into Eq. (3) to obtain

$$x(z,L) = \frac{x_o}{\sqrt{1+Gs}} e^{ik_oL} \sum_{n=0}^{\infty} \frac{1}{n!(2n)!} \left(\frac{\eta}{2!}\right)^n \quad , \tag{12}$$

where we have evaluated x at the end of linac $s = L$ and have defined a dimensionless strength parameter

$$\eta = \frac{Lr_oNW_o}{k_o(\gamma_f - \gamma_o)} \ln \frac{\gamma_f}{\gamma_o} \cdot \left(\frac{1}{2} - \frac{z}{\ell}\right)^2 \quad . \tag{13}$$

There is no closed form for Eq. (12), but one can find an asymptotic expression in the strong wake field limit $|\eta| >> 1$:

$$x(z,L) \simeq \frac{x_o}{\sqrt{6\pi(1+Gs)}} |\eta|^{-1/6} \exp\left(\frac{3\sqrt{3}}{4} |\eta|^{1/3}\right) \cos\left[k_oL - \frac{\eta}{|\eta|}\left(\frac{3}{4} |\eta|^{1/3} - \frac{\pi}{12}\right)\right] \quad . \tag{14}$$

In Fig. 1, we have plotted $x(z,L)$ across the bunch for values of $k_oL = 0$, $\pi/2$, π and $3\pi/2$ (Modulus 2). The wake field strength is such that the value of η at the very tail of the bunch is equal to 150. The vertical scale is in units of $(\sqrt{\gamma_o/\gamma_f})x_o$. It is clear from Fig. 1 that the distortion of the bunch can be very large. For the parameter used in Fig. 1, 15% at the head of the bunch is dominated by $x^{(0)}$; 40% in the middle of the bunch is dominated by $x^{(1)}$ and the tail 45% is dominated by $x^{(2)}$. The third order term $x^{(3)}$ is not negligible only at the very tail of the bunch.

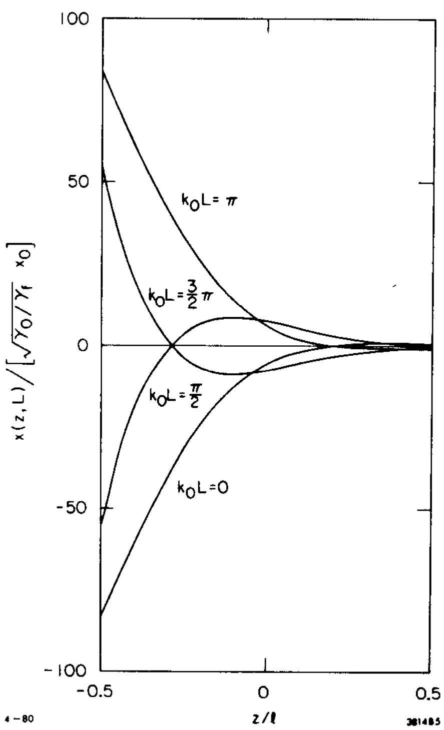

Fig. 1. The transverse bunch shape at the end of acceleration for four different values of total betatron phase: 0, $\pi/2$, π, and $3\pi/2$. The wake-field strength parameter $\eta = 150$ at the tail of the bunch.

MISALIGNMENT TOLERANCE

In the previous analysis, we have assumed that the accelerator sturcture is perfectly aligned and the wake field is produced as a consequence of beam injection with a displacement error. In this section, we will study the effect caused by misalignment of the accelerator pipe. We assume the beam is injected into the linac with perfect precision.

The equation of motion can be written as

$$\frac{d}{ds}\left\{\gamma(s)\ \frac{d}{ds}\ x(z,s)\right\} + \left(\frac{2\pi}{\lambda(s)}\right)^2 \gamma(s)\ x(z,s)$$
$$= r_o \int_z^\infty dz'\ \rho(z')W(z'-z)\left[x(z',s) - d(s)\right] \tag{15}$$

where d(s) is the transverse position error of the linac structure at position s. Compared with Eq. (2), Eq. (15) contains an additional force term on the right hand side that comes from the pipe misalignment.

The zeroth order solution to Eq. (15) is $x^{(o)} = 0$ since the beam is assumed to be injected without error. The trajectory of the head of the bunch strictly follows $x^{(o)}$ and is, therefore, a perfect straight line. The first order perturbation term comes solely from the linac structure misalignment d_i:

$$x^{(1)}(z,s) = -\sum_{\substack{i\\(s>s_i)}} \frac{r_o d_i \ell_i}{\gamma_o k_o} \cdot \frac{1}{(1+Gs)^{\frac{1}{2}}(1+Gs_i)^{\frac{1}{2}}} \cdot \sin\left[k_o(s-s_i)\right] \cdot R_1(z)\ , \tag{16}$$

where the quantity $R_n(z)$ has been defined in Eq. (7).

In order to apply this to the linac collider, it is necessary to carry out the perturbation calculation up to the second order in the wake field. The second order term can be obtained through the use of Eq. (4):

$$x^{(2)}(z,s) = \sum_{\substack{i\\(s>s_i)}} \frac{r_o^2 d_i \ell_i}{2\gamma_o^2 k_o^2 G} \cdot \frac{\ln\left[(1+Gs)/(1+Gs_i)\right]}{(1+Gs)^{\frac{1}{2}}(1+Gs_i)^{\frac{1}{2}}} \cdot \cos\left[k_o(s-s_i)\right] \cdot R_2(z)\ . \tag{17}$$

The emittance growth due to misalignment can be substantially reduced by empirically controlling the injection offset x_o and angle x_o' at the beginning of the linac. By choosing proper values of x_o and x_o', it is possible to cancel either the first order misalignment contribution, Eq. (16), or the second order misalignment contribution Eq. (17), by a corresponding contribution from x_o and x_o'. For example, if the second order alignment term dominates, one might choose:

$$\begin{pmatrix} x_o \\ x_o' \end{pmatrix} = \frac{4G}{\ln^2(1+GL)} \sum_i d_i \ell_i \frac{\ln\left[(1+GL)/(1+Gs_i)\right]}{(1+Gs_i)^{\frac{1}{2}}} \begin{pmatrix} \cos k_o s_i \\ k_o \sin k_o s_i \end{pmatrix} \tag{18}$$

so that the second order contribution from the injection offset and angle cancels the second order contribution from the misalignments. With x_o and x'_o from Eq. (18), the first order term is given by the sum of misalignment and injection contributions. If we assume the misalignment errors d_i are uncorrelated from one linac section to the next, we can make an rms estimate to obtain:

$$\langle x^{(1)2} \rangle = \frac{1}{6N_c} \langle d^2 \rangle \left(\frac{r_o L}{\gamma_f k_o} \right)^2 \ln \frac{\gamma_f}{\gamma_o} \cdot R_1^2(z) \tag{19}$$

where $\langle d^2 \rangle^{1/2}$ is the rms misalignment of the accelerator sections relative to the trajectory of a weak beam, N_c is the total number of accelerator sections in the linac.

APPLICATION TO THE LINEAR COLLIDER

The design parameters for the collider mode of operation are:[3)]

$$\begin{aligned} N &= 5 \times 10^{10} & \sigma_x &= 70\ \mu m \\ \gamma_o &= 2.4 \times 10^3\ (1.2\ \text{GeV}) & \ell &= 3.5\ \text{mm} \\ \gamma_f &= 10^5\ (50\ \text{GeV}) & W_o &= 5.9 \times 10^5\ m^{-3} \\ L &= 3 \times 10^3\ m & N_c &= 240 \\ \lambda_o &= 100 \end{aligned} \tag{20}$$

We first study the tolerance on injection conditions. The value of η, according to Eq. (13), is 37 at $z = 0$, and 94 at $z = -\sigma_z = -(\ell/2\sqrt{3})$ and 150 at $z = -\ell/2$. The bunch shape for this case has been shown in Fig. 1. With an injection displacement of x_o, the values of the magnitude of $x(z,L)$ are $1.5x_o$ for $z = 0$ and $6.1x_o$ for $z = -\sigma_z$. If we demand that $x(z,L)$ at $z = -\sigma_z$ be less than the transverse beam size σ_x at the end of acceleration, we obtain the tolerance on the injection displacement $|x_o| < 11\ \mu m$. This tolerance in injection error x_o is a criterion on the injection jittering since a static injection error can always be compensated by a set of trajectory kickers before injection. The corresponding tolerance on the jittering of the injection angle is $\pm 0.7\ \mu rad$.

The effect of accelerator misalignment on the emittance growth can be minimized by injecting the beam with empirically determined offset x_o and angle x'_o. Since the second order contribution dominates, for the collider linac the optimum choice of x_o and x'_o is given by Eq. (18). The expected rms value of the required injection offset is $\langle x_o^2 \rangle^{1/2} = 0.35 \langle d^2 \rangle^{1/2}$. After optimizing by controlling the injection conditions, the resultant beam size growth, $\langle x^{(1)2} \rangle^{1/2}$ is found to be $0.25 \langle d^2 \rangle^{1/2}$ at the bunch center and $0.62 \langle d^2 \rangle^{1/2}$ at σ_z behind the bunch center. For the beam size growth at $z = -\sigma_z$ to be less than the transverse beam size σ_x at the end of the linac, we demand a misalignment tolerance of $\langle d^2 \rangle^{1/2} = 0.11$ mm.

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