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RECENT EXPERIMENTAL RESULTS ON THE BEAM-BEAM EFFECTS IN STORAGE RINGS AND AN ATTEMPT OF THEIR INTERPRETATION *)

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ABSTRACT

The latest available experimental results on the luminosity, the space charge parameters, and the beam blowup as functions of particle energy and beam current are reviewed. The comparison with the phenomenological diffusion theory are done and useful scaling law are derived.

1. INTRODUCTION

Although there are a number of papers[1-4] on the beam-beam phenomena, the importance of the problem which implies the most severe limitation on the beam currents of the storage ring as well as recent availability of new experimental results[5-8] and theoretical approach[9] make it quite feasible to add to the list.

In this work I suggest some scaling laws for the luminosity, space charge parameters, and beam size as functions of particle energy, maximum beam current, and the number of bunches. These scaling laws are derived from the latest experimental data available now. For more details the reader is referred to a somewhat wider (and more comprehensive) version[10] of this work.

2. MAIN RELATIONSHIPS AND ASSUMPTIONS

Before discussing recent experimental results observed on different electron storage rings it is useful to look first at the conditions in which they are obtained and the assumptions under which they are interpreted.

Among the machine parameters entering into expressions for the luminosity $\mathscr{L}$ and the space charge parameters ξ_y and ξ_x the particle energy E, the number of bunches B, and the revolution frequency f are known with great accuracy. The luminosity $\mathscr{L}$ and the beam current i can be measured directly.

On the other hand, several other parameters such as the values of horizontal and vertical β-functions at the interaction point β_x, β_y are very difficult to measure. Although one can expect that β_x, β_y should be modified by the beam-beam force, these functions are changed only in the second order of the preturbation theory and therefore usually are assumed to be equal to their theoretical value at the zero current. The same holds for the horizontal beam emittance ϵ_x and consequently for the horizontal beam size $\sigma_x = \sqrt{\epsilon_x \beta_x}$.

Experimental data on the beam-beam effect are obtained on different machines virtually in quite different conditions.

a) The investigation of the beam-beam limitations. Measurements of this kind are done during special machine physics runs. The main goal of these measurements is to achieve the maximum possible luminosity for given parameters by increasing the currents to the point where the lifetime of the beam starts to decrease sharply. To maximize the luminosity of the ring both currents are usually maintained pretty much the same.

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• One tries to do the same with the vertical size of the beam. At least at SPEAR this condition was met by means of adjustment of the phase between the rf cavities positioned symmetrically around the interaction point.[11)]

• Experimental data obtained in this situation should be more sensitive to the particle distribution at large amplitudes (to the tails of distributions) rather than to the distribution in the core of the beam.

b) The investigation of the storage ring performance. Measurements of this kind are usually done during high energy physics runs in a parasitic mode. Maximum luminosity is achieved in this case under a restrained condition of the beam lifetime being unaffected or almost unaffected by beam-beam phenomena. These measurements should be more sensitive to the distribution in the core of the beam.

In all of the storage rings the longitudinal size of the bunch σ_ℓ is much less than β_y. If this condition were not fulfilled, different particles along the bunch would experience different focusing and the results could be distorted by this effect. As we shall see later, it is assumed usually that the distribution of the particles is Gaussian, at least in the core. This assumption one needs to be able to calculate the space charge parameters from the measured luminosity and current.

An experimental fact observed on all the machines is that the horizontal size of the bunch is not influenced by the beam-beam interaction[2,7)] with the accuracy $\lesssim 10\%$.

It is instructive first to see how one can derive the relevant parameters from the measured ones.

a) First of all, assuming σ_x to be equal to $\sqrt{\epsilon_x \beta_x}$, one can find beam aspect ratio σ_y/σ_x from the measured luminosity:

$$\sigma_y/\sigma_x = i^2 \big/ 4\pi e^2 f B \sigma_x^2 \mathscr{L} \quad . \tag{1}$$

b) The horizontal space charge parameter then is:

$$\xi_x = e i \beta_x \big/ 2\pi f B E \sigma_x^2 (1 + \sigma_y/\sigma_x) \quad . \tag{2}$$

c) After eliminating σ_y from expressions for the luminosity and the vertical space charge parameter one gets:

$$\xi_y = 2e^3 \mathscr{L} \beta_y \big/ E i (1 + \sigma_y/\sigma_x) \quad . \tag{3}$$

Let us review the recent experimental results obtained on different storage rings.

3. RECENT EXPERIMENTAL RESULTS

3.1 SPEAR: dependence on energy (H. Wiedemann)[5)]

Recently a set of new measurements of the maximum luminosity and the beam current versus machine energy was undertaken by H. Wiedemann. The range of energy variation was from 0.6 to 3.7 GeV and is much wider than in all previous experiments. The data were taken during the special runs of the SPEAR dedicated to machine physics. Much work was done to adjust all the machine parameters to achieve maximum luminosity. Special attention was paid to balance the vertical sizes of electron and positron bunches to avoid the loss of the luminosity due to the flip-flop effect.

Table 1

Dependence of SPEAR parameters on the particle energy E.[a] The fit is done[b] by a function $f = kE^q$.

f	k	q	
$\mathscr{L}_{max}$	0.033	6.6	*c)*
i_{max}	1.2	3.6	*d)*
σ_y/σ_x	0.5	-1.0	
ξ_x	0.022	0.87	
ξ_y	0.011	2.3	

a) in GeV
b) H. Wiedemann, these Proceedings
c) in 10^{30} cm^{-2} sec^{-1}
d) in ma

Table 2

Dependence of SPEAR parameters on the beam current i.[a] The fit is done by a function $f = ki^q$.

f	E GeV	k	q	Comment
$\mathscr{L}_{max}$ (10^{30} cm^{-2} sec^{-1})	1.5	0.030	1.95	*b)*
	2.5	0.046	1.55	*b)*
	3.7	0.054	1.45	*b)*
	1.95	0.052	1.41	β_y=10 cm *c)*
	1.95		1.45	β_y=20 cm *c)*
σ_y			0.59	
σ_x			0	
ξ_y	2.4		0.33	

a) in ma
b) high energy physics runs
c) machine physics runs

The fit by a power law to recent data seems to give quite different slopes, especially for the vertical space charge parameter, than ones for the previous measurements.[4] The difference may be attributed to the fact that the energy range in the work[2] was much narrower (from approximately 1.2 to 2.5 GeV). Table 1 summarizes the results of fitting to these measured and calculated data.

3.2 SPEAR: dependence on the beam current

Table 2 summarizes the data picked up from SPEAR logbooks by M. Cornacchia.[8] The data were mostly taken during regular physics runs of the machine. The fits to the data taken at high energy physics runs are recalculated. Instead of fitting data by the least square method, the maximum luminosity was fitted.

3.3 ADONE (S. Tazzari)[6]

Table 3 summarizes the dependencies of the maximum luminosity and the beam current versus energy which were taken from the report by S. Tazzari.[6] The space charge parameters of this machine were kept approximately equal to each other. The fit for the space charge parameters is derived from the calculated values plotted in the work.[6] The number of bunches in ADONE can be and was changed. The data taken with 1 and 3 bunches do not contradict the assumption

$$\xi_y \sim 1/\sqrt{B}$$

Table 3

Dependence of ADONE parameters on the particle energy E (in GeV). The fit is done [a] by a function $f = kE^q$.

f	k	q	Comment
$\mathscr{L}_{max}$	0.64	7	in 10^{30} cm^{-2} sec^{-1}
$\xi_x \simeq \xi_y$	0.068	1.57	
i_{max} (in ma)	105	4.34	3 bunches, strong beam - weak beam
	42.4	4.12	1 bunch, strong beam - weak beam

a) S. Tazzari, ibid., see Reference 2, pp. 128-135.

3.4 PETRA (G. Voss)[7]

The data from the measured specific luminosity $\mathcal{L}/i^2$ during high energy physics experiments were fitted with the help of the blowup function σ_y assumed to behave according to the following:

$$\sigma_y^2 = \sigma_0^2 + \left(\frac{ai}{\sigma_y}\right)^2 \qquad (4)$$

Here σ_0 is the value of σ_y at zero current i and a is a parameter. From the data taken at different energies, a is found to be:

$$a = \text{const}/E^4 \qquad (5)$$

The values of aspect ratio of the beam emittances are estimated to be of the order of several percent at all energies.

4. SCALING IN e^+e^- STORAGE RINGS

It is useful first to go through main assumptions which will be used in the following considerations.

First of all we shall consider one dimensional model of the beam-beam interaction. Although the phenomenon is essentially multidimensional, the justification of this model at least in the first approximation comes from the experimental observations that the vertical size of the bunch is most strongly affected by the interaction while the horizontal size of the bunch seems to be affected very little if any. One may argue about the loss of some particular multidimensional features like the Arnold diffusion, sideband resonances, and the like. All of these effects seem to be small compared to the main rough effect.

Secondly, we assume that at least some number of particles behave stochastically. The reason for such a behavior can be nonlinearities in the machine lattice, nonlinearity of the electromagnetic beam-beam force, combined action of many close-lying resonances, presence of a stochastic layer in the phase space of particle motion, etc. Note that I do not include in this list the change of particle amplitude due to radiation quantum fluctuations making thus the consideration equally applicable to proton storage rings.

We shall use in forthcoming considerations an assumption that both beams are identical. This assumption is not mandatory for the derivations but is justified by experimental conditions and makes all formulae more straightforward. Also everywhere where it is appropriate I will use Gaussian distribution, linear force, etc. Although more exact calculations can be fulfilled sometimes they do not seem to be necessary due to oversimplifying assumptions made above already.

4.1 Beam blowup according to diffusion theory

According to the main assumption a certain part of the particle motion due to the beam-beam interaction can be described as stochastic and hence can be considered as an additional source of diffusion (in addition to all other sources which do not depend on the beam-beam force).

The beam blowup for the core of the bunch is found as follows:[9]

$$\sigma_y^2 = \sigma_0^2 + \frac{2\pi^2 e^2 \tau \beta_y^2 h^2 \sigma_0^2 i^2}{f B E^2 \sigma_y^2 \sigma_x^2 (1 + \sigma_y/\sigma_x)^2} \qquad (6)$$

where τ is the vertical damping time and h is a fitting parameter. First of all we see here exactly the same formula (4) that was postulated in the work.[7] Comparing (6) with (4), we find

$$a = \frac{\pi e \beta_y h \sigma_0}{E\sigma_x(1+\sigma_y/\sigma_x)} \sqrt{\frac{2\tau}{fB}} \qquad (7)$$

An expression (6) can also be found in the paper[12] which gives to parameter h the physical meaning of the probability of finding the particle in a stochastic layer.

Expression similar to (6) was also derived by J. Rees[13] from the assumption

$$\sigma_y^2 = \sigma_0^2 + fB\tau\beta_y^2\theta_{rms}^2 \quad ,$$

where θ_{rms} is the effective r.m.s scattering angle of a particle in the vertical plane.

4.2 Scaling laws

Expressions (6,7) contain only one unknown parameter h. Let us consider it as a phenomenological parameter which should be determined from experimental data. One way to do this is to use PETRA results[7] to satisfy E^{-4} decrease for the value a.

Since we are interested now in maximum values of the luminosity and the current, we derive from (4) that asymptotically at large current i (for the case $\sigma_y \ll \sigma_x$) $\sigma_y^4 \simeq a^2 i^2$ or

$$\sigma_y \sim \sqrt{i}/E^2 \qquad (8)$$

The maximum possible value of σ_y limited by particle losses and beam lifetime should be some constant which can be written as $\sqrt{A_y\beta_y}$ where A_y is an effective vertical acceptance of the storage ring. From formula for the beam lifetime[14] for Gaussian distribution we would find that σ_y is constant with the logarithmic accuracy. Let us see now what consequences follow from these assumptions.

Consider first the situation where the limitation arises from the beam lifetime. Assuming σ_y = const in expression (8) we immediately get

$$i_{max} \sim E^4 \qquad (9)$$

With the help of this expression we also get the following scaling laws (note that for the electron storage ring $\sigma_x \sim E$):

$$\mathscr{L}_{max} \sim E^7 \qquad (10)$$

$$\xi_{ymax} \sim E^2 \qquad (11)$$

$$\xi_{xmax} \sim E \qquad (12)$$

$$\sigma_y/\sigma_x \sim 1/E \qquad (13)$$

Let us now turn to experiments in which beam lifetime limit has not been reached yet. At a given energy one gets from the same expressions

$$\sigma_x\sigma_y \sim i^{1/2} \qquad (14)$$

$$\xi_{ymax} \sim i^{1/2} \qquad (15)$$

$$\mathscr{L}_{max} \sim i^{3/2} \qquad (16)$$

For the strong beam - weak beam case we have observations made on ADONE.[6] Expression (6) in this case should be rewritten for the blowup of the weak beam by an unperturbed strong beam:

$$\sigma_y^2 = \sigma_0^2 + \frac{2\pi^2 e^2 \tau \beta_y^2 h^2 i^2}{fBE^2\sigma_x^2(1+\sigma_y/\sigma_x)^2} \qquad (17)$$

Assuming the same dependence of a on E we have in this case

$$\sigma_y^2 \simeq \frac{i^2}{BE^{10}} \simeq \text{const} \tag{18}$$

The last equality corresponds to conditions of the ADONE experiment.[6] Hence

$$i_{max} \sim E^5\sqrt{B} \tag{19}$$

$$\mathscr{L}_{max} \sim E^9 B \tag{20}$$

$$\xi_{ymax} \sim E^3/\sqrt{B} \tag{21}$$

$$\xi_{xmax} \sim E^2/\sqrt{B} \tag{22}$$

The scaling (19) seems to be in quite good agreement with the experimental data[6] on the strong beam - weak beam interaction at ADONE both on E and on B. On the other hand, ξ_y and ξ_x were maintained equal. That makes the comparison of the energy dependence meaningless. The dependence on B does not contradict the experiment.

5. SUMMARY OF THE EXPERIMENT AND THEORY COMPARISON

Tables 4-6 present the summary of the theoretical and experimental values for different parameters relevant for the beam-beam interaction. Keeping in mind the number of assumptions and the approximations made the agreement seems to be astonishingly good.

Table 4

The power q in the power law $f(E) \sim E^q$

Parameter f	Experiment			Theory	Comment	
	SPEAR	ADONE	PETRA			
$\mathscr{L}_{max}$	6.6	7		7		(10)
i_{max}	3.6	4.5		4	strong-strong	(9)
i_{max}		4.12 , 4.34		5	weak-strong	(19)
ξ_{ymax}	2.3	1.5		2		(11)
ξ_{xmax}	0.9			1		(12)
σ_y/σ_x	-1			-1		(13)
a			-4			(5)

Table 5

The power q in the power law $f(i) \sim i^q$

Parameter f	Experiment		Theory	*a)*
	SPEAR	PETRA		
$\mathscr{L}_{max}$	1.4		1.5	(16)
$\mathscr{L}_{spmax}$		-0.5	-0.5	
$\sigma_x\sigma_y$	0.6		0.5	(14)
ξ_{ymax}	0.4		0.5	(15)

a) equation number

Table 6

The power q in the power law $f(B) \sim B^q$.

Parameter	Experiment	Theory	*a)*
	ADONE		
i_{max}	0.8	0.5	(19) *b)*
ξ_{ymax}	-0.8	-0.5	(21) *b)*

a) equation number
b) strong beam-weak beam

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