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A POSSIBLE $SU(4)_c \times SU(3)_f \times U(1)$ MODEL*

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ABSTRACT

An anomaly free model of strong and electro-weak interactions involving leptons and quarks in the $SU(4)_c \times SU(3)_f \times U(1)$ gauge theory is constructed. After spontaneous symmetry breaking, it reduces itself into the quantum chromodynamics for strong interactions and a broken $SU(3) \times U(1)$ model for electro-weak interactions. As a limiting case it gives the same results as those of the Weinberg-Salam model in the low energy region. The Weinberg angle is bounded by $\sin^2\theta_W < 1/4$ and becomes slightly less than 30° in the limiting case. Below the mass scale of $SU(4)_c$ breaking there exists an inequality between Weinberg angle and the strong coupling constant, which is consistent with experiments. A correction to the neutral current of Weinberg-Salam model is suggested.

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A new conserved quantum number is introduced in this model and there should exist several new fermions with masses lighter than 160 GeV. The Kobayashi-Maskawa expression of Cabibbo mixing for quarks may be obtained in the model generalized to include several generations of fermions.

I INTRODUCTION

Recent neutrino induced neutral current experiments are in agreement with the expectations based on the Weinberg-Salam model.[1] The Weinberg angle θ_W is found to be $\sin^2\theta_W = 0.230 \pm 0.009$ by the experimenters.[2] Beyond the Weinberg-Salam model one may ask:

(1) Is there any symmetry higher than $SU(2)_L \times U(1)$ for the electro-weak interaction?

(2) Whether $\sin^2\theta_W$ being slightly less than one fourth has special physical meaning?

(3) How to unify the Weinberg-Salam model with the strong interaction?

If the answer to the first question is "no," the next problem to be solved is the grand unification of the Weinberg-Salam model with the strong interaction. In this way one may construct a model of grand unification, such as the SU(5) model suggested by Georgi and Glashow.[3]

If one thinks that the answer to the first question is "yes," this leads to another question: What kind of group is the higher symmetry? There are several considerations which may become the motivations to choose it:

(1) It must ensure left-right symmetry before spontaneous symmetry breaking.

(2) It must include both leptons and quarks in a single model.

(3) It must give some restriction to the Weinberg angle.

There are many possibilities. One of them is the SU(3) × U(1) group for the electro-weak interactions. In a previous paper[4] a model with the SU(3) × U(1) gauge group was proposed. This model is left-right symmetric before spontaneous symmetry breaking and anomaly free. It gives the same results as those of the Weinberg-Salam model in the low energy region for a limiting case. The Weinberg angle is bounded by $\sin^2\theta_W \leq 1/4$ in this model and $\sin^2\theta_W$ becomes slightly less than 1/4 in the limiting case.

In this paper we discuss a way of unifying the strong and electro-weak interactions by embedding this SU(3) × U(1) model into a larger one, $SU(4)_c \times SU(3) \times U(1)$, where the main results of Ref. 4, including the interesting property of $\sin^2\theta_W \leq 1/4$, are preserved and several further consequences are obtained.

Before discussing this model we will briefly analyze the construction of the SU(3) × U(1) group, which will be helpful in understanding the motivation of an extension to SU(4) × SU(3) × U(1).

When one wants to embed the $SU(2)_L \times U(1)$ model into an SU(3) × U(1) model, a naive requirement is that ν_L and e_L will correspond to the first two components of a left-handed triplet and e_R will correspond to the third component of the right-handed triplet of the SU(3) group. There are two possibilities.

Case A: ν_L and e_L belong to the representation $\underline{3}_L$ and e_R belongs to the representation $\underline{3}_R$ of the SU(3) group. This case is investigated by B. W. Lee, S. Weinberg, R. E. Shrock, G. Segre, J. Weleys and many others[5] in details.

Case B: ν_L and e_L belong to the representation $\underline{3}_L$ while e_R belongs to the representation $\underline{3}^*_R$, the conjugate representation of $\underline{3}$. This case is investigated in Ref. 4.

These two cases lead to different consequences summarized in Table I.

In the expressions for the charge operator, $\hat{I}_3$ and $\hat{I}_8$ are the third and the eighth generators of the SU(3) group respectively while Y is the generator of the U(1) group. Of course the Y assignment of the fermion multiplets are different in these two cases. In case B, there exists an additional conserved quantum number called the weak strangeness, S_W, coming from an unbroken U(1) symmetry after spontaneous symmetry breaking. There are several heavy particles to be discovered in this case too. However, most of them have non-vanishing values of S_W while the known particles in the Weinberg-Salam model have $S_W = 0$. We may call these particles with $S_W \neq 0$ the weak strange particles. Some of them have "exotic" values of charge, for example, $Q = 2$ for a heavy vector boson and $Q = 5/3$ for a heavy quark.

The most interesting character of the case B is that the upper bound on the Weinberg angle is close to the measured value. Of course the existence of the conservation of weak strangeness gives many new physical predictions for the high energy electro-weak interactions and is interesting too. However, because the left-handed and right-handed fermions belong to different kinds of the representations of the SU(3) group, this kind of model cannot easily be embedded into a simple SU(6) model of grand unification. So we have to study other ways of connecting this kind of model with the strong interaction. One attractive idea is that the color group may be an SU(4) group and the lepton number may be treated as the fourth color as suggested by Pati and Salam.[6] Adopting this idea, we will extend this kind of SU(3) × U(1) model to an $SU(4)_c \times SU(3) \times U(1)$ model.

II FUNDAMENTAL STRUCTURE OF THE MODEL

The local gauge groups considered in this model are the SU(4) color group, the SU(3) flavor group and the U(1) group. Their generators, the corresponding gauge fields and the coupling constants are denoted by

$$\hat{I}'_j,\ C^j_\mu,\quad j = 1, \ldots, 15, \qquad g'' \text{ for } SU(4)$$

$$\hat{I}_i\ \ A^i_\mu,\quad i = 1, \ldots, 8, \qquad g \text{ for } SU(3)$$

$$\hat{F},\ \ B_\mu, \qquad g' \text{ for } U(1)$$

respectively. Besides the local symmetry there is another global U(1) symmetry whose generator will be denoted by $\hat{S}$. This global U(1) will combine with an Abelian subgroup in SU(4) × SU(3) × U(1) to give a new conserved quantum number S_W after spontaneous symmetry breaking. We will use four numbers in a bracket (m, n, F, S) to denote the representations for these four groups respectively. For example, ($\underline{4}$, $\underline{3}$, -1, 1) means the representation $\underline{4}$ for the SU(4), the representation $\underline{3}$ for the SU(3), F = -1 for the local U(1) and S = 1 for the global U(1) groups.

For simplicity we discuss the model involving only one generation of fermions. It can easily be extended to the case involving several generations. The fermions form four left-handed multiplets and four right-handed multiplets.

$$
\begin{array}{llll}
\psi_L: & (\underline{4}, \underline{3}, 1, 1) & \psi_R: & (\underline{4}, \underline{3}^*, 1, -1) \\
S_L: & (\underline{4}, \underline{1}, 1, -3) & S_R: & (\underline{4}, \underline{1}, 1, 3) \\
\psi'_L: & (\underline{4}, \underline{3}^*, -1, -1) & \psi'_R: & (\underline{4}, \underline{3}, -1, 1) \\
S'_L: & (\underline{4}, \underline{1}, -1, 3) & S'_R: & (\underline{4}, \underline{1}, -1, -3)
\end{array}
\tag{2.1}
$$

After spontaneous symmetry breaking one SU(3) symmetry, one local U(1) symmetry and one global U(1) symmetry remain unbroken. The unbroken SU(3) group is a subgroup on the first three dimensions of the SU(4) group and becomes the color gauge group for quarks. The generators of the local U(1) and the global U(1) groups are the charge

$$\hat{Q} = \hat{I}_3 - \sqrt{3}\,\hat{I}_8 + \sqrt{\frac{2}{3}}\,\hat{I}'_{15} + \frac{1}{2}\,\hat{F} \tag{2.2}$$

and the weak strangeness

$$\hat{S}_W = \frac{2}{\sqrt{3}}\,\hat{I}_8 - \frac{1}{2}\,\hat{F} + \frac{1}{6}\,\hat{S} \tag{2.3}$$

respectively.

Since quarks are degenerate for three colors and the color index is unimportant in many discussions, we may omit it and express the fermion multiplets as

$$
\psi_L = \begin{bmatrix} u_L \\ d_L \\ h_L \\ \hline\hline \nu_L \\ e_L \\ E_L \end{bmatrix}, \quad
\psi_R = \begin{bmatrix} g_R \\ h_R \\ d_R \\ \hline\hline N_R \\ E_R \\ e_R \end{bmatrix}, \quad
\psi'_L = \begin{bmatrix} d'_L \\ u'_L \\ w_L \\ \hline\hline e'_L \\ \nu'_L \\ \xi_L \end{bmatrix}, \quad
\psi'_R = \begin{bmatrix} x_R \\ w_R \\ u'_R \\ \hline\hline \zeta_R \\ \xi_R \\ \nu'_R \end{bmatrix}
\tag{2.4}
$$

$$
S_L = \begin{bmatrix} g_L \\ \hline\hline N_L \end{bmatrix}, \quad
S_R = \begin{bmatrix} u_R \\ \hline\hline \nu_R \end{bmatrix}, \quad
S'_L = \begin{bmatrix} x_L \\ \hline\hline \zeta_L \end{bmatrix}, \quad
S'_R = \begin{bmatrix} d'_R \\ \hline\hline e'_R \end{bmatrix}
\tag{2.5}
$$

where the symbols above the double lines denote three colors of quarks while the symbols under the double lines denote leptons.

The quantum number assignments of various states are listed in Table II.

This model is anomaly free. The proof can be done in the similar way as in Ref. 4.

III SPONTANEOUS SYMMETRY BREAKING

Six multiplets of the Higgs fields are introduced to realize the spontaneous symmetry breaking. Their transformation properties are

$$\begin{aligned} &\Phi_A: \ (\underline{1}, \underline{3}, 0, -2), \quad \Phi_D: \ (\underline{1}, \underline{3}, -2, -2) \\ &\Phi_B: \ (\underline{1}, \underline{6^*}, 0, -2), \quad \Phi_E: \ (\underline{4}, \underline{1}, 1, 3) \\ &\Phi_C: \ (\underline{15}, \underline{3}, 0, -2), \quad \Phi_F: \ (\underline{1}, \underline{3}, 2, 4) \end{aligned} \tag{3.1}$$

respectively.

The self interaction potential of the Higgs fields is chosen to be

$$\begin{aligned} V = &\sum_{\ell=A}^{F} [-a_\ell \ \mathrm{tr}\ \Phi_\ell^+ \Phi_\ell + b_\ell (\mathrm{tr}\ \Phi_\ell^+ \Phi_\ell)^2] \\ &+ c(\mathrm{tr}\ \Phi_A^+ \Phi_D)(\mathrm{tr}\ \Phi_D^+ \Phi_A) \\ &+ d[\phi_{Di}^* \phi_{Dj} \phi_{B\{ik\}}^* \phi_{A\ell} + \phi_{Di} \phi_{Dj}^* \phi_{B\{ik\}} \phi_{A\ell}^*]\varepsilon_{ik\ell} \\ &+ e[\phi_{E\mu}^* \phi_{E\nu} \phi_{C\mu\nu i}^* \phi_{Ai} + \phi_{E\mu} \phi_{E\nu}^* \phi_{C\mu\nu i} \phi_{Ai}^*] \\ &+ f[\phi_{Ai} \phi_{Dj} \phi_{Fk} + \phi_{Ai}^* \phi_{Dj}^* \phi_{Fk}^*]\varepsilon_{ijk} \end{aligned} \tag{3.2}$$

where a's, b's, c, d, e and f > 0, ϕ_{Ai}, $\phi_{B\{ik\}}$, $\phi_{C\mu\nu i}$, ϕ_{Di}, ϕ_{Ei} and ϕ_{Fi} are the components of Φ_A, Φ_B, Φ_C, Φ_D, Φ_E and Φ_F respectively. As shown in Appendix A, it leads to a stable minimum for V. The vacuum expectation values of the Higgs fields may be taken to be

$$\langle\phi_A\rangle_0 = v_A\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\times\begin{pmatrix}1\\0\\0\end{pmatrix}, \qquad \langle\phi_D\rangle_0 = v_D\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\times\begin{pmatrix}0\\0\\1\end{pmatrix},$$

$$\langle\phi_B\rangle_0 = \frac{v_B}{\sqrt{2}}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\times\begin{pmatrix}0&0&0\\0&0&1\\0&1&0\end{pmatrix}, \qquad \langle\phi_E\rangle_0 = v_E\begin{pmatrix}0\\0\\0\\1\end{pmatrix}\times\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}, \tag{3.3}$$

$$\langle\phi_C\rangle_0 = \frac{v_C}{\sqrt{12}}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-3\end{pmatrix}\times\begin{pmatrix}1\\0\\0\end{pmatrix}, \qquad \langle\phi_F\rangle_0 = v_F\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\times\begin{pmatrix}0\\1\\0\end{pmatrix},$$

respectively, where all v's are positive and determined by the coefficients in V. Owing to the stability of the minimum, no pseudo-goldstone appears after spontaneous symmetry breaking and all the remaining Higgs particles are massive. They may be rather heavy by a suitable choise of the coefficients in the self interaction potential. One may easily verify that the color SU(3) symmetry, the electromagnetic U(1) symmetry and the weak strange U(1) symmetry remain unbroken after spontaneous symmetry breaking.

There are 24 gauge bosons in this model. After spontaneous symmetry breaking all gauge bosons other than eight gluons and the photon get masses. The mass terms of the vector bosons have the form

$$\begin{aligned}&\frac{1}{2}g^2(v_A^2+v_B^2+v_C^2+v_F^2)W^+W^- + \frac{1}{2}g^2(v_A^2+v_B^2+v_C^2+v_D^2)V^+V^-\\&+\frac{1}{2}g^2(4v_B^2+v_D^2+v_F^2)U^{++}U^{--} + \frac{1}{2}g''^2(\frac{8}{3}v_C^2+v_E^2)\sum_{i=1}^{3}C_i^{2/3}C_i^{-2/3}\\&+\frac{1}{4}g^2(v_A^2+v_B^2+v_C^2)|A^3+\frac{1}{\sqrt{3}}A^8|^2 + \frac{1}{4}g^2v_D^2|\frac{2}{\sqrt{3}}A^8+\frac{2g'}{g}B|^2\\&+\frac{1}{4}g^2v_E^2|\frac{g'}{g}B-\sqrt{\frac{3}{2}}\frac{g''}{g}C^{15}|^2 + \frac{1}{4}g^2v_F^2|-A^3+\frac{1}{\sqrt{3}}A^8+\frac{2g'}{g}B|^2\end{aligned} \tag{3.4}$$

where

$$W^{\pm} = \frac{1}{\sqrt{2}} (A^1 \mp iA^2), \qquad V^{\pm} = \frac{1}{\sqrt{2}} (A^4 \pm iA^5),$$

$$U^{\pm\pm} = \frac{1}{\sqrt{2}} (A^6 \pm iA^7), \qquad C_1^{\pm 2/3} = \frac{1}{\sqrt{2}} (C^9 \mp iC^{10}), \tag{3.5}$$

$$C_2^{\pm 2/3} = \frac{1}{\sqrt{2}} (C^{11} \mp iC^{12}), \qquad C_3^{\pm 2/3} = \frac{1}{\sqrt{2}} (C^{13} \mp iC^{14}).$$

The masses of these particles are

$$\begin{aligned} m_W^2 &= \frac{1}{2} g^2 (v_A^2 + v_B^2 + v_C^2 + v_F^2) \quad , \\ m_V^2 &= \frac{1}{2} g^2 (v_A^2 + v_B^2 + v_C^2 + v_D^2) \quad , \\ m_U^2 &= \frac{1}{2} g^2 (4v_B^2 + v_D^2 + v_F^2) \ , \\ m_C^2 &= \frac{1}{2} g''^2 (\frac{8}{3} v_C^2 + v_E^2) \ , \end{aligned} \tag{3.6}$$

respectively. From (3.6) we get the inequalities

$$m_V^2 < m_U^2 + m_W^2 \quad , \quad m_U^2 < m_V^2 + 3m_W^2 \tag{3.7}$$

Three neutral gauge bosons will get masses from the last four terms in (3.4), we denote them by Z_1, Z_2 and Z_3 respectively. Eight gluons and the photon remain massless. The electromagnetic field is

$$A = \frac{1}{\sqrt{1 + \frac{1}{\lambda^2} + \frac{1}{\mu^2}}} \left(\frac{1}{2} A^3 - \frac{\sqrt{3}}{2} A^8 + \frac{1}{\lambda} C^{15} + \frac{1}{\mu} B \right) \tag{3.8}$$

where

$$\lambda = \sqrt{6}\, \frac{g''}{g} \quad , \quad \mu = 2\, \frac{g'}{g} \quad . \tag{3.9}$$

The quantum numbers of these bosons are listed in Table III. The bosons with a star are new particles introduced in this model. The six $C_i^{\pm 2/3}$

are so-called lepto-quarks. They have fractional charges and couple quarks to leptons. The V and U bosons have non-vanishing weak strangeness. We will call this kind of particles the weak strange particles.

An interesting limiting case is

$$g^2 \ll g'^2, \quad g''^2 \quad \text{i.e. } \lambda^2, \quad \mu^2 \gg 1$$
$$v_F^2 \ll v_A^2 + v_B^2 + v_C^2 \ll v_D^2, \quad v_E^2 \ . \tag{3.10}$$

The physical meaning of the limiting case is that the coupling constant of the SU(3) group is much smaller than those of the SU(4) color group and the U(1) group; the mass scales of $SU(4)_c$ and $SU(3)_f$ breaking are much higher than that for the secondary $SU(2)_f$ breaking.

In this limiting case the masses of the three massive neutral bosons are approximately

$$m_{Z_1}^2 = \frac{2}{3} g^2 \left(v_A^2 + v_B^2 + v_C^2\right) \left[1 - \frac{1}{3}\left(\frac{1}{\lambda^2} + \frac{1}{\mu^2}\right)\right]$$

$$m_{Z_2}^2 = \frac{1}{4} g^2 \left[\mu^2 v_D^2 + \frac{\lambda^2 + \mu^2}{4} v_E^2 - \sqrt{\left(\mu^2 v_D^2 + \frac{\lambda^2 + \mu^2}{4} v_E^2\right)^2 - \lambda^2 \mu^2 v_D^2 v_E^2}\right] \tag{3.11}$$

$$m_{Z_3}^2 = \frac{1}{4} g^2 \left[\mu^2 v_D^2 + \frac{\lambda^2 + \mu^2}{4} v_E^2 + \sqrt{\left(\mu^2 v_D^2 + \frac{\lambda^2 + \mu^2}{4} v_E^2\right)^2 - \lambda^2 \mu^2 v_D^2 v_E^2}\right]$$

Z_1 can be expressed as

$$Z_1 \approx \frac{1}{2}\left(\sqrt{3}\, A^3 + A^8\right) - \frac{1}{\sqrt{3}}\left(\frac{1}{\mu} B + \frac{1}{\lambda} C^{15}\right) + \frac{1}{2\sqrt{3}}\left(\frac{1}{\lambda^2} + \frac{1}{\mu^2}\right)\left(A^3 - \sqrt{3}\, A^8\right) \tag{3.12}$$

Its dominant component is just $Z = \frac{1}{2}(\sqrt{3}\, A^3 + A^8)$, so it may be treated as the particle corresponding to the Z boson in the Weinberg-Salam model. Comparing with the mass formula in the Weinberg-Salam model

$$M_Z^2 = M_W^2/\cos^2\theta_W \tag{3.13}$$

we obtain to the first order of approximation for the $1/\lambda^2$ and $1/\mu^2$ expansion that

$$\sin^2\theta_W = \frac{1}{4}\left[1 - \left(\frac{1}{\lambda^2} + \frac{1}{\mu^2}\right)\right] \tag{3.14}$$

The limiting condition (3.10) ensures that $\sin^2\theta_W$ is slightly less than $\frac{1}{4}$ and is consistent with experiment.

Both Z_2 and Z_3 are much heavier than Z_1. Their dominant components are B and C^{15}. For two special cases they get simple expressions.

Case A: When $\mu^2 v_D^2 >> \frac{1}{4}(\lambda^2 + \mu^2)v_E^2$, we get

$$m_{Z_2}^2 \approx \frac{1}{8}\, g^2\lambda^2 v_E^2 \quad , \quad m_{Z_3}^2 \approx \frac{1}{2}\, g^2\mu^2 v_D^2 \quad , \tag{3.15}$$

$$Z_2 \approx C^{15} \quad , \qquad Z_3 \approx B \quad . \tag{3.16}$$

Case B: When $\mu^2 v_D^2 << \frac{1}{4}(\lambda^2 + \mu^2)v_E^2$, we get

$$m_{Z_2}^2 \approx \frac{1}{2}\, g^2 \frac{\lambda^2\mu^2}{\lambda^2 + \mu^2}\, v_D^2 \quad , \qquad m_{Z_3}^2 \approx \frac{1}{8}\, g^2(\lambda^2 + \mu^2)v_E^2 \quad , \tag{3.17}$$

$$Z_2 \approx \frac{1}{\sqrt{\lambda^2 + \mu^2}}(\lambda B + \mu C^{15}) \quad , \quad Z_3 \approx \frac{1}{\sqrt{\lambda^2 + \mu^2}}(-\mu B + \lambda C^{15}) \quad , \tag{3.18}$$

The masses of gauge bosons are shown in Fig. 1. It manifests the existence of three mass scales. They relate themselves to the breaking of $SU(4)_c$, $SU(3)_f$ and $SU(2)_f$ respectively.

The interaction Lagrangian among the fermions and the Higgs fields has the form

$$\begin{aligned}
\mathscr{L}_{FH} = \sum_{i=A,B,C} & \left[f_i \left(\bar{\psi}_R \Phi_i \psi_L + \bar{\psi}_L \Phi_i^+ \psi_R \right) \right. \\
& \left. + f_i' \left(\bar{\psi}_L' \Phi_i \psi_R' + \bar{\psi}_R' \Phi_i^+ \psi_L' \right) \right] \\
& + \sum_{i=A,C} \left[h_{i_1} \left(\bar{S}_R \Phi_i^+ \psi_L + \bar{\psi}_L \Phi_i \bar{S}_R \right) \right. \\
& \left. + h_{i_1}' \left(\bar{S}_L' \Phi_i^+ \psi_R' + \bar{\psi}_R' \Phi_i S_L' \right) \right] \\
& + \sum_{i=A,C} \left[h_{i_2} \left(\bar{S}_L \Phi_i \psi_R + \bar{\psi}_R \Phi_i^* S_L \right) \right. \\
& \left. + h_{i_2}' \left(\bar{S}_R' \tilde{\Phi}_i \psi_R' + \bar{\psi}_L' \Phi_i^* S_R' \right) \right] \\
& + h_F \left(\bar{S}_R' \Phi_F^+ \psi_L + \bar{\psi}_L \Phi_F S_R' \right) \\
& + h_F' \left(\bar{S}_R \tilde{\Phi}_F \psi_L' + \bar{\psi}_L' \Phi_F^* S_R \right)
\end{aligned} \tag{3.19}$$

where Φ_A and Φ_C in the first terms are expressed in the matrix form

$$\begin{aligned}
\langle \Phi_A \rangle_0 &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} , \\
\langle \Phi_C \rangle_0 &= \frac{1}{\sqrt{24}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix} \times \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} .
\end{aligned} \tag{3.20}$$

After spontaneous symmetry breaking the mass terms of the fermions are derived from (3.3), (3.19) and (3.20). They have the form

$$\frac{1}{\sqrt{2}}(-f_A v_A + f_B v_B - \frac{1}{\sqrt{12}} f_C v_C)(\bar{d}_L d_R + \bar{d}_R d_L)$$
$$+ \frac{1}{\sqrt{2}}(-f'_A v_A + f'_B v_B - \frac{1}{\sqrt{12}} f'_C v_C)(\bar{w}_L w_R + \bar{w}_R \bar{w}_L)$$
$$+ \frac{1}{\sqrt{2}}(-f_A v_A + f_B v_B + \frac{3}{\sqrt{12}} f_C v_C)(\bar{e}_L e_R + \bar{e}_R e_L)$$
$$+ \frac{1}{\sqrt{2}}(-f'_A v_A + f'_B v_B + \frac{3}{\sqrt{12}} f'_C v_C)(\bar{\xi}_L \xi_R + \bar{\xi}_R \xi_L)$$
$$+ \frac{1}{\sqrt{2}}(f_A v_A + f_B v_B + \frac{1}{\sqrt{12}} f_C v_C)(\bar{h}_L h_R + \bar{h}_R h_L)$$
$$+ \frac{1}{\sqrt{2}}(f'_A v_A + f'_B v_B + \frac{1}{\sqrt{12}} f'_C v_C)(\bar{u}'_L u'_R + \bar{u}'_R u'_L)$$
$$+ \frac{1}{\sqrt{2}}(f_A v_A + f_B v_B - \frac{3}{\sqrt{12}} f_C v_C)(\bar{E}_L E_R + \bar{E}_R E_L) \qquad (3.21)$$
$$+ \frac{1}{\sqrt{2}}(f'_A v_A + f'_B v_B - \frac{3}{\sqrt{12}} f'_C v_C)(\bar{\nu}'_L \nu'_R + \bar{\nu}'_R \nu'_L)$$
$$+ (h_{A1} v_A + \frac{1}{\sqrt{12}} h_{C1} v_C)(\bar{u}_L u_R + \bar{u}_R u_L) + (h'_{A1} v_A + \frac{1}{\sqrt{12}} h'_{C1} v_C)(\bar{x}_L x_R + \bar{x}_R x_L)$$
$$+ (h_{A1} v_A - \frac{3}{\sqrt{12}} h_{C1} v_C)(\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L) + (h'_{A1} v_A - \frac{3}{\sqrt{12}} h'_{C1} v_C)(\bar{\zeta}_L \zeta_R + \bar{\zeta}_R \zeta_L)$$
$$+ (h_{A2} v_A + \frac{1}{\sqrt{12}} h_{C2} v_C)(\bar{g}_L g_R + \bar{g}_R g_L) + (h'_{A2} v_A + \frac{1}{\sqrt{12}} h'_{C2} v_C)(\bar{d}'_L d'_R + \bar{d}'_R d'_L)$$
$$+ (h_{A2} v_A - \frac{3}{\sqrt{12}} h_{C2} v_C)(\bar{N}_L N_R + \bar{N}_R N_L) + (h'_{A2} v_A - \frac{3}{\sqrt{12}} h'_{C2} v_C)(\bar{e}'_L e'_R + \bar{e}'_R e'_L)$$
$$+ h_F v_F(\bar{d}_L d'_R + \bar{d}'_R d_L + \bar{e}_L e'_R + \bar{e}'_R e_L) + h'_F v_F(\bar{u}'_L u_R + \bar{u}_R u'_L + \bar{\nu}'_L \nu_R + \bar{\nu}_R \nu'_L)$$

The terms involving v_F lead to the mixing between ψ, S and ψ', S'.

For the masses of fermions the following remarks may be made:

1. In each generation of fermions, there are eight weak strange fermions and eight ordinary fermions. One may assume that the choice

of coefficients ensures the weak strange fermions to be much heavier than the ordinary ones, thus in the low energy region only the fermions with $S_W = 0$ can be observed. Eight ordinary fermions are doublet degenerate. In general, one generation of fermions in this model includes two generations of fermions in ordinary classification. The existence of τ lepton and b quark implies that there are at least two generations existing in this model. This means that one may expect the existence of the fourth generation of fermions in ordinary classification.

2. We will discuss the generalized Cabibbo mixing of fermions. If we have n degenerate states d_i, i=1, ..., n. The mass term of these states can be expressed as

$$\bar{d}_{iL}\, m_{ij}\, d_{jR} + \bar{d}_{jR}\, m^{*}_{ji}\, d_{iL} \tag{3.22}$$

From mass matrix $M = (m_{ij})$, one may construct two Hermitian matrices MM^{+} and $M^{+}M$ and diagonalize them by means of certain unitary transformations U and V

$$UMM^{+}U^{+} = VM^{+}MV^{+} = (m_1^2 \ldots m_n^2) \tag{3.23}$$

This implies that the transformation

$$d_L \longrightarrow U^{+}d_L \qquad d_R \longrightarrow V^{+}d_R \tag{3.24}$$

will diagonalize the mass matrix M. Since the charged weak current is left-handed, only U relates itself to the generalized Cabibbo mixing. If one use $U_{-1/3}$ and $U_{2/3}$ to denote the U matrices for -1/3 charged and 2/3 charged quarks respectively, then the unitary matrix $U = U_{-1/3}\, U^{+}_{2/3}$ describe the Cabibbo mixing among quarks. There are n^2 parameters appearing in the U matrix. 2n-1 of them can be eliminated by the choice of relative phases and n(n-1)/2 of them can be related to the rotation angles in an n-dimensional space. So, there are $n(n-3)/2 + 1$ phases appearing in the general expression of mixing, which may lead to the CP violation in weak interaction for $n \geq 3$ and is just the description given by Kobayashi-Maskawa.[7]

In this model n is an even number. For $n = 2$ if one denotes the mass matrix as

$$M = \begin{pmatrix} \alpha & \gamma \\ \delta & \beta \end{pmatrix} \quad , \tag{3.25}$$

then the eigenvalues of mass matrix are

$$m = \frac{1}{2}\left[\sqrt{(\alpha + \beta)^2 + (\gamma - \delta)^2} \pm \sqrt{(\alpha - \beta)^2 + (\gamma + \delta)^2}\right] \quad . \tag{3.26}$$

and the mixing angles have the forms

$$\tan^2\theta_L = \frac{2(\alpha\delta + \beta\gamma)}{\alpha^2 - \beta^2 + \gamma^2 - \delta^2} \quad , \quad \tan^2\theta_R = \frac{2(\alpha\gamma + \beta\delta)}{\alpha^2 - \beta^2 - \gamma^2 + \delta^2} \quad , \tag{3.27}$$

If one relates four ordinary quarks in this case as u, d, c and s, the Cabbibo angle θ_C can be expressed as

$$\theta_C = \theta_{L-1/3} - \theta_{L2/3} \tag{3.28}$$

In this model, $\delta = 0$ holds and α, β, γ's can be obtained from (3.21). Since there exist at least two generations in this model, so the mixing should happen for $n = 4$ case in this model.

3. There exist right-handed neutrinos in this model. Since they do not couple with charged leptons via W boson, there is no contradiction to the experiment. Since the massive neutrinos are permited in this model, in general, there may be the Cabbibo type mixing among neutrinos, which will manifest itself as the phenomenon of neutrino oscillation. This model predicts that the oscillation among different flavors of

neutrinos may happen while the oscillation between ν and $\bar{\nu}$ is forbidden. This prediction is consistent with the recent experiments.[8]

IV INTERACTIONS BETWEEN THE FERMIONS AND THE GAUGE BOSONS

The gauge interaction Lagrangian for fermions can be written as

$$\begin{aligned}
\mathscr{L} = {} & \bar{\psi}_L \gamma^\mu(\partial_\mu + ig\, \hat{I}_i\, A^i_\mu + ig''\, \hat{I}'_j\, C^j_\mu + ig'\tfrac{1}{2}\, B_\mu)\psi_L \\
& + \bar{\psi}_R \gamma^\mu(\partial_\mu - ig\, \hat{I}^*_i\, A^i_\mu + ig''\, \hat{I}'_j\, C^j_\mu + ig'\tfrac{1}{2}\, B_\mu)\psi_R \\
& + \bar{S}_L \gamma^\mu(\partial_\mu + ig''\, \hat{I}'_j\, C^j_\mu + ig'\tfrac{1}{2}\, B_\mu)S_L \\
& + \bar{S}_R \gamma^\mu(\partial_\mu + ig''\, \hat{I}'_j\, C^j_\mu + ig'\tfrac{1}{2}\, B_\mu)S_R \\
& + \bar{\psi}'_L \gamma^\mu(\partial_\mu - ig\, \hat{I}^*_i\, A^i_\mu + ig''\, \hat{I}'_j\, C^j_\mu - ig'\tfrac{1}{2}\, B_\mu)\psi'_L \\
& + \bar{\psi}'_R \gamma^\mu(\partial_\mu + ig\, \hat{I}_i\, A^i_\mu + ig''\, \hat{I}'_j\, C^j_\mu - ig'\tfrac{1}{2}\, B_\mu)\psi'_R \\
& + \bar{S}'_L \gamma^\mu(\partial_\mu + ig''\, \hat{I}'_j\, C^j_\mu - ig'\tfrac{1}{2}\, B_\mu)S'_L \\
& + \bar{S}'_R \gamma^\mu(\partial_\mu + ig''\, \hat{I}'_j\, C^j_\mu - ig'\tfrac{1}{2}\, B_\mu)S'_R
\end{aligned} \tag{4.1}$$

We shall discuss the interactions between the $S_W = 0$ fermions and the gauge vector bosons first. For this purpose, the terms involving $S_W \neq 0$ fermions are neglected and the following substitutions are made in (4.1).

$$\psi_L \rightarrow \begin{bmatrix} u_L \\ d_L \\ -- \\ == \\ \nu_L \\ e_L \\ -- \end{bmatrix}, \quad \psi_R \rightarrow \begin{bmatrix} -- \\ -- \\ d_R \\ == \\ -- \\ -- \\ e_R \end{bmatrix}, \quad \psi'_L \rightarrow \begin{bmatrix} d'_L \\ u'_L \\ -- \\ == \\ e'_L \\ \nu'_L \\ -- \end{bmatrix}, \quad \psi'_R \rightarrow \begin{bmatrix} -- \\ -- \\ u'_R \\ == \\ -- \\ -- \\ \nu'_R \end{bmatrix},$$

$$S_L \rightarrow \begin{bmatrix} -- \\ == \\ -- \end{bmatrix}, \quad S_R \rightarrow \begin{bmatrix} u_R \\ == \\ \nu_R \end{bmatrix}, \quad S'_L \rightarrow \begin{bmatrix} -- \\ == \\ -- \end{bmatrix}, \quad S'_R \rightarrow \begin{bmatrix} d'_R \\ == \\ e'_R \end{bmatrix}, \tag{4.2}$$

Only the W boson appears in the charged weak interaction, this is reasonable on account of the conservation of the weak strangeness. The charged weak interactions has the form

$$\begin{aligned} & i\frac{g}{\sqrt{2}}\left[\bar{u}_L\gamma^\mu W^+_\mu d_L + \bar{d}_L\gamma^\mu W^-_\mu u_L - \bar{u}'_L\gamma^\mu W^+_\mu d'_L - \bar{d}'_L\gamma^\mu W^-_\mu u'_L\right] \\ & + i\frac{g}{\sqrt{2}}\left[\bar{\nu}_L\gamma^\mu W^+_\mu e_L + \bar{e}_L\gamma^\mu W^-_\mu \nu_L - \bar{\nu}'_L\gamma^\mu W^+_\mu e'_L - \bar{e}'_L\gamma^\mu W^-_\mu \nu'_L\right] \end{aligned} \tag{4.3}$$

where the terms involving quarks imply the sum over three colors.

The interactions involving C^j_μ, j=1, ..., 8 are just the color SU(3) interactions of QCD. The interactions involving C^j_μ, j=9, ..., 14 give the transition between quarks and leptons. Since the masses of $C_i^{\pm 2/3}$ are very heavy, they can appear only at rather high energies.

The neutral interactions consist of four terms involving photon, Z_1, Z_2 and Z_3 respectively. The interaction involving the photon gives the electric charge to be

$$e = \frac{1}{2}\, g \frac{1}{\sqrt{1 + \frac{1}{\lambda^2} + \frac{1}{\mu^2}}} \tag{4.4}$$

Comparing with another definition of the Weinberg angle in the Weinberg-Salam model

$$e = g \sin\theta_W \tag{4.5}$$

we obtain

$$\sin^2\theta_W = \frac{1}{4}\,\frac{1}{1 + \frac{1}{\lambda^2} + \frac{1}{\mu^2}} < \frac{1}{4} \tag{4.6}$$

in contrast to (3.16). We can treat (3.15) and (4.5) as two definitions of the Weinberg angle. In the limiting case, when Z_1 is much lighter than Z_2, they lead to the same expression of $\sin\theta_W$ as shown in (3.16) and (4.6).

From the expression (4.6) of θ_W, a lower bound on the strong coupling constant may be obtained:

$$\alpha_s > \frac{2}{3}\,\frac{\alpha}{(1 - 4\sin^2\theta_W)} \tag{4.7}$$

However, α_s is a running constant. It decreases as Q^2 increases, the inequality (4.7) should hold for any energy below the mass scale of the SU(4) breaking. Using the experimental value $\sin^2\theta_W = 0.230 \pm 0.009$, it becomes

$$\alpha_s > 0.06 \tag{4.8}$$

and is consistent with the experimental estimation of α_s.

We may use the inequality (4.8) to estimate the mass scale of SU(4) breaking. If we use the estimation of $\alpha_s \approx 0.23$ for $Q \sim 30$ GeV in the formula

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2N_f)\ln\frac{Q^2}{\Lambda^2}} \tag{4.9}$$

the upper limit of this mass scale may be estimated as $\sim 10^5$ GeV and $\sim 10^6$ GeV for $N_f = 5$ and 6 respectively. Of course, the estimation value will increase as the number of flavors increases.

Now we discuss the neutral currents in this model. The interaction involving Z_1 manifests itself very much like the usual neutral current. In the limiting case of (3.10), to lowest order it can be expressed as

$$\mathscr{L}_{eff} = 4 \frac{G_F}{\sqrt{2}} J_{1\mu} J_1^\mu \tag{4.10}$$

where

$$J_{1\mu} \approx J_\mu^3 - \sin^2\theta_W J_\mu^{e.m.} \tag{4.11}$$

It is just the well-known formula in the Weinberg-Salam model. To the next order of approximation J_1 becomes

$$J_{1\mu} = J_\mu^3 - \sin^2\theta_W J_\mu^{e.m.}$$
$$- \frac{2}{3} \frac{v_A^2 + v_B^2 + v_C^2}{v_D^2} \frac{1}{\lambda^2 + \mu^2} \frac{\mu}{\lambda} \left(\frac{\lambda}{\mu} \sqrt{\frac{2}{3}} J'^{15}_\mu - \frac{\mu}{\lambda} \sqrt{\frac{2}{3}} J^F_\mu \right) \tag{4.11a}$$

Since $(v_A^2 + v_B^2 + v_C^2)/v_D^2 \approx m_W^2/m_V^2 << 1$ and $\lambda^2 >> 1$, the correction term in (4.11a) is smaller in magnitude than the main terms.

The effective Lagrangian involving Z_2 and Z_3 can be expressed as

$$\mathscr{L}_{eff} = 4 \frac{GF}{\sqrt{2}} (r_2 J_{2\mu} J_2^\mu + r_3 J_{3\mu} J_3^\mu) \tag{4.12}$$

with $r_3 << r_2 << 1$.

When $\mu^2 v_D^2 \gg \frac{1}{4}(\lambda^2 + \mu^2)v_E^2$, and $J_{2\mu}$, r_2, $J_{3\mu}$ and r_3 take the forms

$$J_{2\mu} = \sqrt{\frac{2}{3}}\, J'^{15}_{\mu} \quad , \quad r_2 = \frac{1}{4}\lambda^2 \frac{m_W^2}{m_{Z_2}^2} \; ,$$
$$J_{3\mu} = J^F_{\mu} \quad , \quad r_3 = \frac{\mu^2}{16}\frac{m_W^2}{m_{Z_3}^2} = \frac{m_W^2}{16 m_V^2} \; . \tag{4.13}$$

Since $r_3 \ll r_2$, the Z_2 interaction is more important than the Z_3 interaction. Owing to that the effective Z_1 charge and Z_2 charge of neutrino are 1/2 and -1/2 respectively, the contribution of Z_2 interaction to the neutrino induced neutral current experiment can be described effectively as a correction for the $J_{1\mu}$ current

$$J_{1\mu} \longrightarrow J_{1\mu} - r_2\, J_{2\mu} = J_{1\mu} - r_2 \sqrt{\frac{2}{3}}\, J'^{15}_{\mu} \tag{4.14}$$

One may note that the correction term in (4.11a) is much smaller than that from Z_2 current and only the Z_2 correction should be considered. Since the effective Z_2 charges of quarks and leptons are 1/6 and -1/2 respectively, this correction is easy to be observed in more accurate experiments.

When $\mu^2 v_D^2 \ll \frac{1}{4}(\lambda^2 + \mu^2)v_E^2$ we get

$$J_{2\mu} = \frac{1}{2}J^F_{\mu} + \sqrt{\frac{2}{3}}\, J'^{15}_{\mu} \quad , \quad r_2 = \frac{1}{4}\frac{\lambda^2\mu^2}{\lambda^2+\mu^2}\frac{m_W^2}{m_{Z_2}^2} = \frac{m_W^2}{4 m_V^2} \; ,$$
$$J_{3\mu} = \sqrt{\frac{2}{3}}\frac{\lambda}{\mu}\, J'^{15}_{\mu} - \frac{1}{2}\frac{\mu}{\lambda}\, J^F_{\mu} \quad , \quad r_3 = \frac{1}{4}\frac{\lambda^2\mu^2}{\lambda^2+\mu^2}\frac{m_W^2}{m_{Z_3}^2} \; . \tag{4.15}$$

The correction term in (4.11a) is much smaller than that from Z_2 current too in this case. Since the effective Z_2 charge is of the form

$$Q'_{Z_2} = \frac{1}{2}F + \sqrt{\frac{2}{3}}\, I'^{15} \tag{4.16}$$

which gives different values to two kinds of fermions, we may use them to distinguish these two kinds of fermions ψ, S and ψ', S'. The effective Z_2 charges for different fermions are given in Table IV. Owing to the mixing between these two kinds of fermions, this effect might not be explicit. However, one may expect the existence of some difference between them.

Two interesting remarks may be made:

1. Under the tree approximation there is no restriction about the fermion masses. If one calculate the radiative correction, the stability of vacuum will give a bound to fermion masses.[9,10] Using the formula given in Ref. 9 one finds that the masses of E, ξ, h and w are bounded by

$$m < 2 \times \left[\frac{3}{4}\left(1 + \frac{1}{4\cos^4\theta_W}\right)\right]^{1/4} m_W \approx 160 \text{ GeV} \tag{4.17}$$

The masses of other weak strange fermions are bounded by

$$m_N,\ m_\zeta < \left[\frac{3}{2}\left(1 + \frac{1}{4\cos^4\theta_W}\right)\right]^{1/4} m_W + \left(\frac{1}{123}\right)^{1/4} 2|\sin\theta_W|\sqrt{\frac{\alpha_s}{\alpha}}\, m_W \quad , \tag{4.18}$$

$$m_g,\ m_x < \left[\frac{3}{2}\left(1 + \frac{1}{4\cos^4\theta_W}\right)\right]^{1/4} m_W + \left(\frac{27}{41}\right)^{1/4} 2|\sin\theta_W|\sqrt{\frac{\alpha_s}{\alpha}}\, m_W \quad .$$

These estimations depend on the value of α_s. If one takes $\alpha_s \approx 0.20$ in (4.18) and (4.19), the upper limits become m_N, m_ζ < 210 GeV and m_g, m_x < 440 GeV respectively. However, (4.17) means that there exist at least four weak strange fermions in one generation lighter than 160 GeV. This prediction can be verified experimentally.

2. Because the quantum number F for ψ and S is different to that for ψ' and S', the radiative decay between two degenerate states is allowed

in this model. For example if e and e' mix with each other and form two leptons e_1 and e_2, $m_{e_1} < m_{e_2}$, one may calculate the partial width of the decay mode $e_2 \to e_1 + \gamma$. In the tree approximation it is forbidden. But in the one loop approximation, it is allowed. Using the method given in,[11] one may obtain the branching ratio to be

$$R \equiv \frac{\Gamma(e_2 \to e_1\gamma)}{\Gamma(e_2 \to e_1\,\bar{\nu}_1\,\nu_2)} = \frac{3\alpha}{\pi^2}\left(\frac{m_W}{m_V}\right)^4 \eta \tag{4.20}$$

where η is a function of the mixing angles. For the case B discussed last section, η can be expressed as

$$\eta = (\frac{1}{3}\, s_+ + c_+\, s_+ + c_-\, s_-)^2 + (s_- + \frac{5}{3}\, c_-\, s_+ + \frac{1}{3}\, c_+\, s_-)^2 \quad , \tag{4.21}$$

where

$$\begin{aligned} s_+ &= \frac{1}{2}(\sin 2\theta_L + \sin 2\theta_R) \quad , \quad s_- = \frac{1}{2}(\sin 2\theta_L - \sin 2\theta_R) \quad , \\ c_+ &= \frac{1}{2}(\cos 2\theta_L + \cos 2\theta_R) \quad , \quad c_- = \frac{1}{2}(\cos 2\theta_L - \cos 2\theta_R) \quad , \end{aligned} \tag{4.22}$$

θ_L and θ_R are the left-handed and the right-handed mixing angles respectively. For several special values of mixing angles, η may vanish. But in general η is of the order of unity. If one takes $\eta \sim 1$ and identifies e_2 and e_1 as muon and electron respectively, the lower limit of m_V can be obtained as

$$m_V > 28\; m_W \tag{4.23}$$

from the experimental upper limit $R < 3.6 \times 10^{-9}$. This estimation is consistent with the requirement of the limiting case discussed above

$$\frac{m_V^2}{m_W^2} \approx \frac{v_D^2}{v_A^2 + v_B^2 + v_C^2} >> 1$$

If one identifies e_2 as τ instead of μ, the lower limit of m_V will be much lighter than that obtained from μ.

V ADDITIONAL CONSERVATION LAWS

In this model there exist several additional conservation laws. We discuss them separately.

1. The Conservation of the Quark Number

Many models of the grand unification predict that the proton decays. But in this model the proton may be stable. We will give a simple proof of this. One may introduce a new global U(1) symmetry in this model. Whose generator is denoted by $\hat{R}$ and the R-assignments of various multiplets are

$$R = 1 \quad \text{for} \quad \psi,\ S,\ \psi',\ S',\ \Phi_E$$

$$R = 0 \quad \text{for} \quad A^i_\mu,\ B_\mu,\ C^j_\mu,\ \Phi_A,\ \Phi_B,\ \Phi_C,\ \Phi_D,\ \Phi_F \quad .$$

One may easily verify that the invariance of this global symmetry holds before spontaneous symmetry breaking. After spontaneous symmetry breaking it is broken too. But it will combine with $\hat{I}'_{15}$ and form a new global U(1) symmetry, whose generator is the quark number

$$\hat{N} = \sqrt{\frac{3}{2}}\ \hat{I}'_{15} + \frac{3}{4}\hat{R}$$

$\hat{N}$ is conserved after spontaneous symmetry breaking. Since the particles with non-vanishing N are:

$N = 1$: quarks, $C_i^{2/3}$, several particles in Φ_C and Φ_E ,

$N = -1$, anti-quarks, $C_i^{-2/3}$, several particles in Φ_C and Φ_E^+ ,

and all these bosons with non-vanishing N are heavy, the proton may be stable.

One may note that the global U(1) symmetry R is not an original one. The possibility of introducing such a symmetry depends on the existence of the U(1) symmetries S and F, which rules out the existence of terms involving $\varepsilon_{\mu\nu\lambda\sigma}$ structure in the Higgs potential. So the stability of proton depends on the U(1) symmetries S and F. If, for example, S is partially conserved, proton may be long lived.

2. The Conservation of Weak Strangeness

In this model there exist four weak strange vector bosons and eight weak strange fermions in each generation. They are listed below:

	$s_W = -1$	$s_W = 1$
vector boson	V^+, U^{++}	V^-, U^{--}
lepton	N^o, E^+	ξ^{-2}, ζ^{-1}
quark	$g^{2/3}$, $h^{5/3}$	$w^{-4/3}$, $x^{-1/3}$

All of them are massive and probably heavier than the known particles. The conservation of weak strangeness requires that:

(1) Weak strange particles can be produced only in pair.

(2) They are weakly decaying and the decay chains end in a final state with the lightest weak strange particle.

(3) The lightest weak strange particle is stable.

(4) The weak strange bosons do not couple directly with ordinary fermion pair.

Since the weak strange vector bosons are much heavier than W boson while some of the weak strange fermions are lighter than 160 GeV, the lightest weak strange particle should be a fermion. It may be produced in high energy e^+e^- collision and manifests itself as either a stable lepton or a stable hadron. The weak strange fermions interact with each other via the right-handed current coupling with the W boson. The transition between weak strange quark and lepton will be suppressed by the propagator of V or U. This means that, for example, if the lightest weak strange particle is a lepton, the width of the lightest weak strange quark ought to be smaller than that estimated in ordinary way.

It is interesting that the masses of all weak strange fermions with exotic charges (ξ^{--}, $h^{5/3}$ and $w^{-4/3}$) are bounded by $m < 160$ GeV, the existence of these particles can be observed significantly in the e^+e^- experiment. However, owing to the weak strangeness conservation, the existence of weak strange particles will not give significant influence to the weak interaction processes below the threshold of the weak strange particles.

In addition, there is another possibility that the existence of the global U(1) symmetry is not original and is derived from special structure of the Lagrangian. In this case there might be several terms violating the weak strangeness conservation, which will then become partial.

VI SUMMARY AND REMARKS

The main results of this model can be summarized as:

(1) The electro-weak interaction can be connected to the strong interaction via the SU(4) × SU(3) × U(1) model. In this model there exist the same number of left-handed and right-handed multiplets of the fermions before spontaneous symmetry breaking.

(2) This model is anomaly free.

(3) As a limiting case, it gives the same results as those of the Weinberg-Salam model and is in agreement with experiment. The Weinberg angle θ_W is bound by the relation $\sin^2\theta_W \leq 1/4$ and the neutral

current for ordinary fermions reduces to that of the Weinberg-Salam model in the limiting case.

(4) There are some deviation from the Weinberg-Salam model concerning the predictions in neutral currents. They are colorless and flavorless but possibly different between the two kinds of fermions and can be verified by more accurate experiments.

(5) A new conserved quantum number S_W, called the weak strangeness, is introduced in the present model. The model predicts many particles with non-vanishing weak strangeness. All of them are likely to be heavier than known fermions. They can be produced only in pairs and the lightest weak strange particle is stable. Some of the weak strange particles have unusual charges and can easily be identified experimentally.

(6) There exists a relation between Weinberg angle and the strong coupling constant. A more accurate value of the Weinberg angle can be used to get the bound of the strong coupling constant and to estimate the upper limit for the mass scale of symmetry breaking.

(7) Quark number may be conserved and proton may be stable in this model.

(8) There are two types of fermions introduced in this model. They will mix with each other after spontaneous symmetry breaking. It manifests itself as the Cabibbo-type mixing of fermions.

(9) The neutrinos may be massive and there may exist the oscillation between different kinds of neutrinos. The oscillation between neutrino and anti-neutrino is excluded in this model.

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APPENDIX A

The Form of the Higgs Potential

There are six Higgs multiplets introduced in this model. Their transformation properties are

$$\begin{aligned} \Phi_A &: (\underline{1}, \underline{3}, 0, -2) \quad , \\ \Phi_B &: (\underline{1}, \underline{6}^*, 0, -2) \quad , \\ \Phi_C &: (\underline{15}, \underline{3}, 0, -2) \quad , \\ \Phi_D &: (\underline{1}, \underline{3}, -2, -2) \quad , \\ \Phi_E &: (\underline{4}, \underline{1}, 1, 3) \quad , \\ \Phi_F &: (\underline{1}, \underline{3}, 2, 4) \quad . \end{aligned} \tag{A.1}$$

We will discuss the form of the Higgs potential which can realize the spontaneous symmetry breaking and the stability of the breaking.

The potential will have the standard potentials for every Higgs multiplet.

$$\begin{aligned} V &= V_o + V_I \\ V_o &= \sum_{\ell=A}^{F} \left[-a_\ell \, \mathrm{tr}\, \Phi_\ell^+ \Phi_\ell + b_\ell \left(\mathrm{tr}\, \Phi_\ell^+ \Phi_\ell \right)^2 \right] \end{aligned} \tag{A.2}$$

where a'_ℓ, $b_\ell > 0$. In Eq. (A.2) the minimum of V_o takes place at $\mathrm{tr}\ \Phi_\ell^+ \Phi_\ell = a_\ell/2b_\ell$.

For simplicity, we will introduce the following notation. If ξ is a n-vector in an n-dimensional complex space with the components ξ_i, i=1, ..., n. $R(\xi)$ is used to denote

$$R\ (\xi) = \sqrt{\xi_i^* \ \xi_i} \quad . \tag{A.3}$$

For two vectors ξ and ζ in the same linear space, the scalar product of them can be expressed as

$$(\xi,\zeta) = \xi_i^+ \ \zeta_i = R(\xi)\ R(\zeta)\ e^{i\phi} \cos\theta \quad . \tag{A.4}$$

We will use the indices i, j, k, ... for the SU(3) group and the indices μ, ν, λ, ... for the SU(4) group.

In order to remove the degeneracy of spontaneous symmetry breaking the additional term V_I of the Higgs potential must be introduced. We introduce a correlation term of Φ_A and Φ_D as

$$c(\mathrm{tr}\ \Phi_A^+ \ \Phi_D)(\mathrm{tr}\ \Phi_D^+ \ \Phi_A) \tag{A.5}$$

According to (A.4) it becomes

$$c\ R^2(\Phi_A)\ R^2(\Phi_D)\ \cos^2\theta = c\ v_A^2\ v_D^2\ \cos^2\theta$$

If $c > 0$ the minimum takes place at $\cos^2\theta = 0$. This means that the non-vanishing vacuum expectation values of Φ_A and Φ_D should take place at different components. We may use a transformation to ensure that the non-vanishing vacuum expectation values take place at the i = 1 and i = 3 components of Φ_A and Φ_D respectively. We make both v_A and v_D be positive by suitable choice of the phases.

We further introduce an additional term as

$$d[\phi^*_{Di}\ \phi_{Dj}\ \phi^*_{B\{ik\}}\ \phi_{A\ell}\ \varepsilon_{jk\ell} + \phi_{Di}\ \phi^*_{Dj}\ \phi_{B\{ik\}}\ \phi^*_{A\ell}\ \varepsilon_{jk\ell}] \qquad \text{(A.6)}$$

where $d > 0$. Since the minimum takes place at $\langle\phi_{Di}\rangle_0 = v_D\ \delta_{i3}$ and $\langle\phi_{Ai}\rangle_0 = v_A\ \delta_{i1}$, it becomes effectively

$$= -2d\ v_D^2\ v_A\ \mathrm{Re}\phi_{B\{32\}}$$

It becomes a minimum as $\mathrm{Re}\phi_{B\{32\}} > 0$. However, the self potential V_o makes the restriction that the components of Φ_B agree:

$$\mathrm{tr}\ \Phi_B^+\ \Phi_B = v_B^2$$

This means that the minimum takes place at

$$\phi_{B\{32\}} = v_B > 0$$

which is just adopted in the model.

Now we introduce the additional term involving Φ_A, Φ_C and Φ_E as

$$e[\phi^*_{E\mu}\ \phi_{E\nu}\ \phi^*_{C\mu\nu i}\ \phi_{Ai} + \phi_{E\mu}\ \phi^*_{E\nu}\ \phi_{C\mu\nu i}\ \phi^*_{Ai}] \qquad \text{(A.7)}$$

with the coefficient $e > 0$. Using a transformation in the SU(4) group to ensure that $\langle\phi_{E\mu}\rangle_0 = v_E\ \delta_{\mu 4}$ with $v_E > 0$ it becomes

$$2e\ v_E^3\ v_A\ \mathrm{Re}\phi_{C441}$$

The components of Φ_C can also be denoted by the index $\alpha=1, \ldots, 15$ instead of μ and ν. We change the notation as $\phi_{\mu\nu i} \to \phi_{(\alpha)i}$, then we have $\phi_{C441} \to -\frac{\sqrt{3}}{2}\ \phi_{C(15)1}$ and this additional term becomes

$$-\sqrt{3}e\ v_E^2\ v_A\ \mathrm{Re}\phi_{C(15)1}$$

Using the same argument discussed above we obtain that the minimum takes place at

$$\phi_{C(15)1} = v_C > 0 \quad .$$

The additional term involving Φ_A, Φ_D and Φ_F is chosen to be

$$f[\phi_{Ai}\,\phi_{Dj}\,\phi_{Fk} + \phi^*_{Ai}\,\phi^*_{Dj}\,\phi^*_{Fk}]\varepsilon_{ijk} \tag{A.8}$$

where $f > 0$. Since the minimum takes place at $\langle\phi_{Ai}\rangle_0 = v_A\,\delta_{i1}$ and $\langle\phi_{Di}\rangle = v_D\,\delta_{i3}$, it becomes effectively

$$-2f\,v_A\,v_D\,\mathrm{Re}\phi_{F2}$$

and becomes a minimum at $\mathrm{Re}\phi_{F2} = v_F > 0$.

In summary, if the self interaction potential of the Higgs fields has the form

$$\begin{aligned} V = \sum_{\ell=A}^{E} & [-a_\ell\ \mathrm{tr}\ \Phi^+_\ell\,\Phi_\ell + b_\ell(\mathrm{tr}\ \Phi^+_\ell\,\Phi_\ell)^2] \\ & + c(\mathrm{tr}\ \Phi^+_A\,\Phi_D)\ (\mathrm{tr}\ \Phi^+_D\,\Phi_A) \\ & + d[\phi^*_{Di}\,\phi_{Dj}\,\phi^*_{B\{ik\}}\,\phi_{A\ell} + \phi_{Di}\,\phi^*_{Dj}\,\phi_{B\{ik\}}\,\phi^*_{A\ell}]\varepsilon_{jk\ell} \\ & + e[\phi^*_{E\mu}\,\phi_{E\nu}\,\phi^*_{C\mu\nu i}\,\phi_{Ai} + \phi_{E\mu}\,\phi^*_{E\nu}\,\phi_{C\mu\nu i}\,\phi^*_{Ai}] \\ & + f[\phi_{Ai}\,\phi_{Dj}\,\phi_{Fk} + \phi^*_{Ai}\,\phi^*_{Dj}\,\phi^*_{Fk}] \end{aligned} \tag{A.9}$$

with a's, b's, c, d, e and $f > 0$, then it leads to a minimum capable of generating the spontaneous symmetry breaking adopted in this model.

Now we discuss the stability of the spontaneous symmetry breaking. There are 8, 6 and 4 dimensional degeneracies appearing in the choice of the breaking components for Φ_E, Φ_D and Φ_A respectively. These $7 + 5 + 3 = 15$ superfluous components can be removed by the choice of gauge and make 15 gauge bosons massive. So no pseudo-goldstone will appear after the spontaneous symmetry breaking and all remaining components of Higgs fields will get masses. In other words, the spontaneous symmetry breaking is stable.

One may note that under the symmetries discussed above the terms like $\mathrm{tr}\ \Phi_\ell^+ \Phi_\ell\ \mathrm{tr}\ \Phi_m^+ \Phi_m$ $(\ell \neq m)$, $\phi^*_{Fi}\ \phi_{Fj}\ \phi^*_{B\{ik\}}\ \phi_{A\ell}\ \varepsilon_{jk\ell}$, $\phi^*_{Ai}\ \phi_{Aj}\ \phi^*_{B\{ik\}}\ \phi_{A\ell}\ \varepsilon_{jk\ell}$, ... etc. can also be introduced in the Higgs potential. However, the existence of such terms will not alter the results obtained above and the only remark should be mentioned is that the values of the coefficents of these terms are bounded by the requirement of the stability of the breaking.

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Table I

	Case A	Case B
Charge operator	$\hat{Q} = \hat{I}_3 + \frac{1}{\sqrt{3}} \hat{I}_8 + \hat{Y}$	$\hat{Q} = \hat{I}_3 - \sqrt{3}\, \hat{I}_8 + \hat{Y}$
$\sin^2\theta_W$	$\frac{3}{4} \frac{1}{1 + \frac{3 g^2}{g'^2}}$	$\frac{1}{4} \frac{1}{1 + \frac{g^2}{g'^2}}$
Boundary	$< \frac{3}{4}$	$< \frac{1}{4}$
New conserved quantum number	No	Weak strangeness
Additional heavy particles	Yes	Yes
With exotic charges	No	Some of them
To be embedded into an SU(6) model	Easily	Can not

Table II

	ψ, S								ψ', S'							
	ν	e	N	E	u	d	g	h	ν'	e'	ξ	ζ	u'	d'	w	x
Q	0	-1	0	1	2/3	-1/3	2/3	5/3	0	-1	-2	-1	2/3	-1/3	-4/3	-1/3
S_W	0	0	-1	-1	0	0	-1	-1	0	0	1	1	0	0	1	1

Table III

	Gluons	$C_i^{2/3}$	$C_i^{-2/3}$	γ	Z_1	Z_2	Z_3
Q	0	2/3	-2/3	0	0	0	0
S_W	0	0	0	0	0	0	0
	QCD	*	*	W-S	W-S	*	*

	W^+	W^-	V^+	V^-	U^{++}	U^{--}
Q	1	-1	1	-1	2	-2
S_W	0	0	-1	1	-1	1
	W-S	W-S	*	*	*	*

Table IV

ψ, S		ψ', S'	
lepton	quark	lepton	quark
0	2/3	-1	-1/3

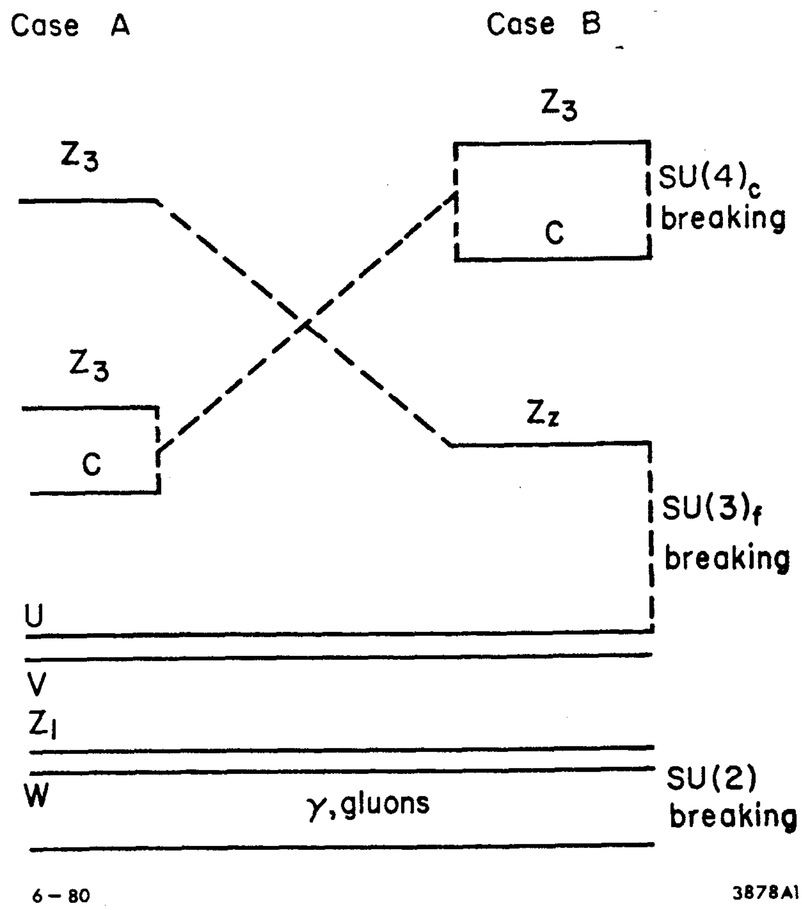
Case A
Case B
Z3
Z3
SU(4)c
breaking
C
Z3
C
Zz
SU(3)f
breaking
U
V
Z1
W
γ, gluons
SU(2)
breaking
6 – 80
3878A1

Fig. 1