SLAC-PUB-2545
June 1980
(T/E)

NEUTRINO MASS IN THE SO(10) MODEL WITHOUT RIGHT-HANDED HEAVY NEUTRAL LEPTON*

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ABSTRACT

We discuss a special construction of SO(10) model which has the following features: (1) No right-handed heavy neutral lepton; (2) all Higgs multiplets are in 16×16 or $16 \times \overline{16}$; (3) two kinds of combination of the left- and right-handed neutrinos in one family have a very small mass difference.

Submitted to Physical Review Letters

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* Work supported in part by the Department of Energy, contract DE-AC03-76SF00515 and by the National Science Foundation, grant PHY77-22864.

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In the SO(10) grand unification model of the weak, electromagnetic and strong interactions[1,2], right-handed antineutrino $(\nu_R)^c$ (with left-handed chirality) and left-handed neutrino are in the same representation 16. Neutrinos, thus, are likely to have a Dirac mass comparable with the masses of charged leptons or quarks. To keep the left-handed neutrino having a small mass in the model-as experiments hint, the simplest way is to give the right-handed neutrino a terribly huge mass.[1,2] A minimal SU(10) model[2] only uses Higgs 10, 45 and 16 multiplets to make such a mechanism work and at the same time keeps the reasonable mass relation between charged lepton and down quark in the simplest SU(5) model[3] unchanged. Though to give the right-handed neutrino a huge mass is the simplest way, nevertheless it is hard to say that this is the unique way to keep a small mass for the left-handed neutrino in the model. Other possibilites exist and are worth studying. Also, because of the necessity of introducing a Higgs 16-plet in the minimal SO(10) model[2], the idea of dynamic symmetry breakdown[4] can hardly work because, as we know, if Higgs are composite particles made of two fermions they can only be in 16×16 or $16\times\overline{16}$. Recently, Yasue[5] invested an artificial rule for specifying the pattern of vacuum expectation values to avoid Dirac neutrino mass at tree level. This rule may not be the best, it gives a bad mass relation, $m_e = 3m_d$, but it works in suppressing the big Dirac neutrino mass[6] in the SO(10) model. Maybe there are some other undiscovered rules which avoid big Dirac neutrino mass and also give better mass relations, but we do not know. Anyway, a heavy neutral lepton (also a Higgs 16-plet) is not a necessary part of the

SO(10) model. Unfortunately, in the original version of Yasue's paper[5], there is a 16-plet which is unnecessary.

In principle, the SO(10) model is a model which can produce any kind of neutrino mass pattern one wants because it may have three possible neutrino mass terms with three parameters a, b, and c as follows:

$$a\,\bar{\nu}_R \nu_L + b\,\overline{(\nu^c)}_R \nu_L + c\,\bar{\nu}_R (\nu^c)_L + \text{h.c.} \tag{1}$$

If $a \neq 0$, we have a Dirac mass; if b and/or $c \neq 0$, we have a Majorana mass.[2] We can rewrite Eq. (1) as

$$\bar{\psi}_R M \psi_L + \text{h.c.} \tag{2}$$

where

$$\psi = \begin{pmatrix} \nu \\ \nu^c \end{pmatrix}, \quad M = \text{[illegible]} \,. \tag{3}$$

Notice the two diagonal elements of mass matrix are the same because of the CPT theorem. To diagonalize Eq. (4) is a trivial job[7]

$$U_R M U_L^+ = M_d \equiv \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} e^{i\phi} \tag{4}$$

where U_R and U_L are unitary matrices, $\phi = \arg(\det M)$. We define

$$\begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}_L = U_L \begin{pmatrix} \nu \\ \nu^c \end{pmatrix}_L \tag{5}$$

$$\begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}_R = U_R \begin{pmatrix} \nu \\ \nu^c \end{pmatrix}_R \tag{6}$$

If $\phi = 0$, the mass term Eq. (4) can be rewritten as

$$m_1\bar{\chi}_1\chi_1 + m_2\bar{\chi}_2\chi_2 \tag{7}$$

which has an obvious unitary property where

$$\chi_1 = \chi_{1L} + \chi_{1R} , \quad \chi_2 = \chi_{2L} + \chi_{2R} . \tag{8}$$

If $\phi \neq 0$, then we have to do a global chiral transformation to get rid of it.[8] Hereafter we will suppose $\phi = 0$ and a, b, c are real numbers for simplicity. Then there are three interesting extreme cases.

1) $b \neq 0$, $a = c = 0$. We get $m_1 \neq 0$, $m_2 = 0$

$$\chi_1 = \nu_L + (\nu^c)_R . \tag{9}$$

This means only left-handed neutrino get mass because $(\nu^c)_R = (\nu_L)^c$. χ_1 is a linear combination of the left-handed neutrino and its anti-neutrino , so people call χ_1 a left-handed Majorana neutrino because $\chi_1^c = \chi_1$. The SU(5) model[3] serves as such an example if there is Higgs 10 or 15-plet. In the SU(5) model the right-handed neutrino does not exist at all.

2) c is extremely large but a and b are very small. Then we get

$$m_1 \approx 0, \quad m_2 \approx c \tag{10}$$

$$\chi_1 \approx \nu_L + (\nu^c)_R , \quad \chi_2 \approx \nu_R + (\nu^c)_L . \tag{11}$$

This means we have a very heavy right-handed neutral lepton χ_2 and a very light left-handed neutrino χ_1.

3) a is small but b and c are much smaller. Then χ_1 and χ_2 are both not pure left- or right-handed and $|m_1-m_2| \ll m_1$ or m_2. As an example, when $b=c$

$$\chi_1 = \nu + \nu^c , \quad \chi_2 = \nu - \nu^c . \tag{12}$$

In case 3) we may have observable neutrino-antineutrino oscillation[9], a process which violates lepton number (and fermion number) conservation directly.

Let us discuss how we can make case 3) work in the SO(10) model. We will present a model which produces case 3). Incidently, this model does not need a Higgs 16-plet.

To express everything more clearly, we discuss some mathematical points first.[1] One of the maximal decompositions of SO(10) is down to $SU(2)_L \times SU(2)_R \times SO(6)$. SO(6) then can breakdown to color SU(3) and hypercharge U(1). Because color is an unbroken symmetry, all mass terms must be in color singlets. However they may have different $SU(2)_L \times SU(2)_R$ properties. In fact, a, b and c terms are in (2,2), (3,1) and (1,3) respectively with quantum number $(T_3^L, T_3^R) = (\frac{1}{2}, -\frac{1}{2})$, (1,0) and (0,1) respectively. All known fermions can be put in 16-plets. As we know, $16\times16 = 10+120+126$, $16\times\overline{16} = 1+45+210$. These representations transform under the subgroup $SU(2)_L \times SU(2)_R \times SO(6)$ as follows:[5]

$$16 = (2,1,4) + (1,2,4^*) \tag{13}$$

$$
\begin{aligned}
10 &= (2,2,1) + (1,1,6) \\
120 &= (2,2,1) + (3,1,6) + (1,3,6) + (2,2,15) + (1,1,10) \\
&\quad + (1,1,10^*) \\
126 &= (1,1,6) + (2,2,15) + (3,1,10) + (1,3,10^*) \\
45 &= (2,2,6) + (1,1,15) + (1,3,1) + (3,1,1) \\
210 &= (1,1,1) + (1,1,15) + (1,3,15) + (3,1,15) + (2,2,6) \\
&\quad + (2,2,10) + (2,2,10^*) \ .
\end{aligned}
\tag{14}
$$

There are color singlets in SO(6) representation 1 and 15 (with hypercharge $y = 0$), and 10 ($y = -2$).

Yasue[5] identifies that under $SU(2)_L \times SU(2)_R \times SU(3)_c$, Weinberg's Higgs doublet[10] transforms as (2,2,1) which appears in representations 10, 120 (twice) and 126 and gives gauge boson W_L and fermion masses at tree level. Yasue's point is supposing that the vacuum expectation values (VEV) of these three Higgs multiplets are developed in such a way that the total VEV of their linear combination is proportional to the electric charge operator in the 16 representation

$$
\begin{aligned}
&< \Phi(10{+}126) > \sim Q_{16} \\
&< \Phi(120) > \sim Q_{16} \quad .
\end{aligned}
\tag{15}
$$

Here VEV of 120 only contributes to the antisymmetric part of the mass matrix, but the other two contribute to the symmetric part. Thus he gets zero mass at tree level because the neutrino is a neutral particle. Before figuring out other mechanisms, we follow Yasue's rule, Eq. (15), here and suppose the Dirac mass of neutrino is zero at tree level by whatever reasons.

Then we arrange the VEV of 45 to break SO(10) down to $SU(2)_L \times SU(2)_R \times SU(3) \times U(1)$, i.e. we give the (1,1,15) of 45 a huge VEV at 10^{14} GeV.[11] We give (1,3,15) of 210 a VEV larger than 10^5 GeV (maybe as large as 10^{14} GeV) to break $SU(2)_L \times SU(2)_R$ down to $SU(2)_L \times U(1)_R$.[12] Because this $U(1)_R$ symmetry survives up to energies larger than 300 GeV, the right-handed neutrino cannot get a big Majorana mass. Now we still have a harmful residual symmetry $T_3^L + T_3^R$ after developing all the VEV described. To break this symmetry without breaking the rule , Eq. (15), we have to give (2,2,10) and/or (2,2,10*) of 210 a VEV as large as that of Weinberg's Higgs doublets. At this step, we already give all the Weinberg Higgs doublets in the table, Eq. (14), approximately the same VEV. Now we finish our model construction. We admit that our assignment of VEV is not natural in the sense that we cannot give any deep argument for such an assignment. Of course, this is the case in any Higgs mechanism of spontaneous breakdown of the symmetry.

It sounds fussy to use three Higgs multiplets 120, 126, and 210 here to play the role of 16-plet Higgs in the simplest SO(10) model.[2] However this is not really a defect because in our feeling the five Higgs multiplets are more or less equal in the sense that they all come from 16×16 and $16 \times \overline{16}$. Having 10-plet and 45-plet already, to add Higgs 120, 126, and 210 can hardly be called a step that seriously complicates the model. But to add Higgs 16-plet is to add essentially a new factor. So we do not feel uncomfortable because we use five Higgs multiplets.

Mixing angles among gauge bosons with the same electric charge and Weinberg's angle can be calculated generally and clear results can be obtained in simplified cases.[5] We shall not go into details here.

Having no mass at the tree level, the neutrino gets a Dirac mass at the one loop level[5] (Fig. 1)

$$a \sim \frac{\alpha}{\pi} \varepsilon m_\ell \sim 10^{-5}\ m_\ell \tag{16}$$

where ε is the mixing between W_L^+ and W_R^+

$$\varepsilon \sim \frac{M_L}{M_R} \sin\theta_W \sim 10^{-2} - 10^{-3}\ . \tag{17}$$

Also the neutrino gets a Majorana mass at the two loop level. We present such a Feynman diagram for left-handed neutrino in Fig. 2. We notice that the Majorana coupling $\overline{(\nu^c)}_R \nu_L$ and $\overline{(\nu^c)}_L \nu_R$ transform as (3,1,10) and (1,3,10*) of 126 respectively. A 126 can be found in the product 10×45^2 as well as in the product $126\times 210\times 45^3$. In respect to the transformation property under the subgroup $SU(2)_L\times SU(2)_R\times SO(6)$, (3,1,10) and (1,3,10*) are also included in the products $(2,2,15)\times(2,2,10)\times(1,1,15)^3$ and $(2,2,15)\times(2,2,10^*)\times(1,1,15)^3$ respectively. The effective Lagrangian[13] of this diagram is

$$\frac{1}{M^6}\, \Phi^2 \phi_{45}^3 \phi_{126} \phi_{210} \overline{(\nu^c)}_R \nu_L + \text{h.c.} \tag{18}$$

where M is the unification mass scale about 10^{14} GeV. As we know, $G\langle\Phi\rangle \sim m_u$, here G is the Yukawa coupling constant between the Φ and u quark. The coupling constant between 10-plet and fermions is about m_u/M_L. The (2,2,15) of 126 and (2,2,10) of 210 gets VEV about

$V = 300$ GeV. Thus we have

$$b \sim c \sim \frac{V^2}{M^3} \frac{m_u^3}{M_L} \cdot \lambda_1 \cdot \lambda_2 \cdot \left(\frac{\alpha}{\pi}\right)^2 \tag{19}$$

where $\alpha = e^2/4\pi$ is the grand unification coupling constant. λ_1 and λ_2 are the two coupling constants in Higgs potential as shown in the diagram.

Equation (19) gives an extremely small number. Even for τ neutirno, set $m_t > 10^2$ GeV, we get $b \sim c \sim 10^{-27}$ eV which is experimentally uninteresting. However our estimate is a very rough one. A serious calculation may find some dramatic factors to increase this number to the experimentally sensitive region (larger than 10^{-12} eV) because the diagram is so complicated. For instance, if two of the masses in M^3 in Eq. (19) are substituted by M_R^2 or some light Higgs mass, such a dramatic change will occur. If ν_τ and ν_e have a big mixing, b and c will be experimentally sensitive.

In conclusion, it is possible to construct an SO(10) model without 16-plet Higgs and heavy leptons but giving neutrino mass light enough.

Acknowledgment

The author would like to thank Dr. N. Nakazawa and Dr. B. Morel for enjoyable discussions, and Professors S. Weinberg and H. Georgi for very good suggestions. Work supported in part by the Department of Energy, contract DE-AC03-76SF00515 and by the National Science Foundation, grant PHY77-22864.

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Figure Captions

Fig. 1 The one loop diagram that gives neutrino a Dirac mass through the mixing between right- and left-handed gauge bosons.

Fig. 2 The two loop diagram that gives neutrino a Majorana mass. The waved lines are gauge bosons. The dotted lines are Higgs with their dimensions of SO(10) representations along the lines. Notation "x" means VEV. The numbers in parentheses show the transformation properties of the components under the subgroup $SU(2)_L \times SU(2)_R \times SO(6)$ which develop VEV. Φ is the Higgs linear combination in Eq. (15).

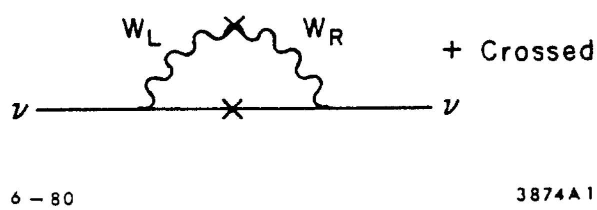
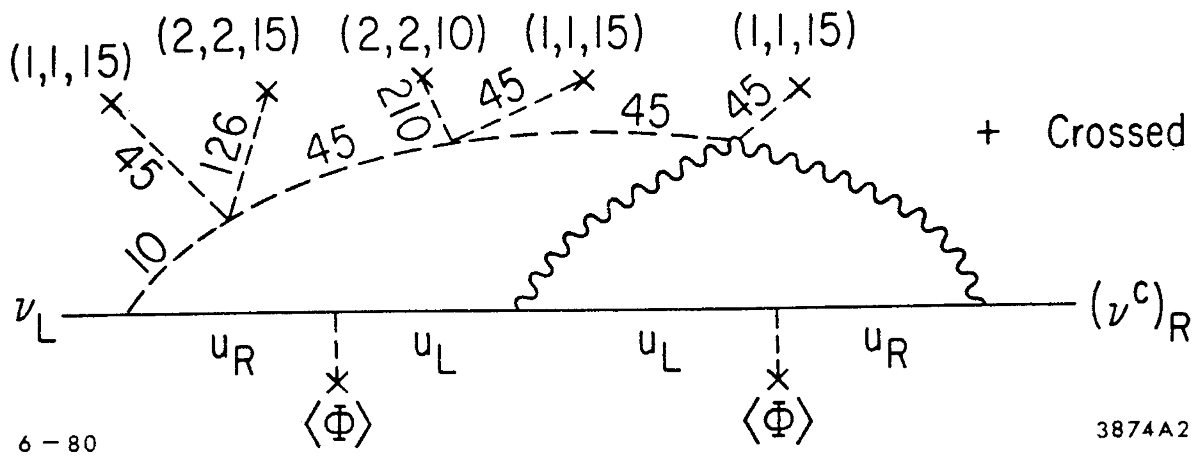


Fig. 1



(1,1,15)
(2,2,15)
(2,2,10)
(1,1,15)
(1,1,15)
45
126
45
210
45
45
45
10
+ Crossed
ν_L
u_R
u_L
u_L
u_R
$(\nu^c)_R$
$\langle\Phi\rangle$
$\langle\Phi\rangle$
6 – 80
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Fig. 2