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NON-STANDARD ASSIGNMENTS OF THE τ LEPTON WITHIN SU(2)$\otimes$U(1)*

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ABSTRACT

Within the context of SU(2)$\otimes$U(1), alternative multiplet assignments for the τ lepton and possible accompanying neutral leptons are investigated. Currently available experimental data is sufficient to rule out many alternatives to the standard assignment of the left-handed τ to a weak isospin doublet (with massless ν_τ partner) and the right-handed τ to a singlet.

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I. Introduction

In recent years evidence from electron-positron annihilation experiments have established the existence of a charged heavy lepton, called the τ, beyond a reasonable doubt.[1] Since the τ's discovery, a great deal of experimental evidence on its properties, decay branching ratios, etc. has accumulated. These data are consistent with the τ being a sequantial lepton - in particular, with the τ fitting into the standard $SU(2)\otimes U(1)$ model[2] with a weak $SU(2)$ assignment to a right-handed singlet and a left-handed doublet comprised of the τ and a massless neutrino, ν_τ.

Although this standard multiplet assignment might be preferred on aesthetic grounds to possible alternatives, one cannot *a priori* rule out alternative multiplet assignments: for example, assignments which involve a heavy neutral partner of the τ, assignments which place the τ in a right-handed doublet, etc. In this paper we systematically consider a number of the more obvious alternative multiplet structures within $SU(2)\otimes U(1)$ and show that many of these alternatives are inconsistent with currently available experimental data.

Nearly all of these alternatives would be trivially ruled out if mass eigenstates were necessarily identical to weak eigenstates. However, this need not be so: there can be Cabibbo-like mixing in the leptonic sector analogous to the well-known Cabibbo mixing among quarks. Experimental data place constraints on these leptonic Cabibbo angles: for example, an upper limit on the τ lifetime (equivalent to a lower limit on the τ decay width) implies that the τ must have some minimum coupling to a light neutral lepton. Because Cabibbo mixing is constrained to be unitary, mixing involving the τ generally affects mixing of the μ and e multiplets also. Therefore, μ and e physics measurements also place relevant constraints on the leptonic mixing. For example, in multiplet assignments which do not have the GIM mechanism[3], the experimental limits on μ-e neutral currents restrict the allowed mixing. In order to fully rule out a proposed multiplet structure, it is necessary to show that it is ruled out for any values of the mixing angles. One does this generally by showing that the various constraints on the mixing due to τ physics and due to μ-e physics are inconsistent.

The experimentally established facts about the τ which we primarily use for this purpose are:

(1) The τ lifetime is less than 1.4×10^{-12} seconds.[4] This fact, combined with the τ mass[5] of $1782 \pm {}^{3}_{4}$ MeV and branching ratios for $\tau \to \nu e \bar{\nu}$ and/or $\tau \to \nu\pi$, implies a lower bound on the strength of the τ to ν coupling.

(2) The Michel ρ parameter for the e^- energy spectrum in $\tau \to \nu e \bar{\nu}$ equals[6] 0.72 ± 0.15. This value was deduced taking into account radiative corrections, so as to make it directly comparable to the (non-radiatively corrected) theoretical value which, e.g., is 0.75 in the standard model with a purely V-A current connecting the τ and the ν.

(3) Muon neutrinos, ν_μ, couple to the τ with a strength (coupling squared) which is at most[7] 2.5% of the ν_μ to μ^- coupling.

(4) The upper limit[8] on the sum of branching ratios $B(\tau^- \to e^-e^+e^-) + B(\tau^- \to e^-\mu^+\mu^-) + B(\tau^- \to \mu^-e^+e^-) + B(\tau^- \to \mu^-\mu^+\mu^-)$ is 0.017.

(5) The upper limit[6] on the mass of the ν in $\tau \to \nu e \bar{\nu}$ is 250 MeV.

As for μ and e physics our arguments rely particularly on:

(1) The strength of the ν_μ-e^- coupling is at most[7] 0.3% of that for ν_μ to μ^-.

(2) μ-e universality is known to hold to a few percent. Based on the measured $\pi \to \mu\bar{\nu}$ and $\pi \to e\bar{\nu}$ rates, the e to ν exceeds the μ to ν coupling strength[9] by 3.2 ± 1.9%.

(3) Based on the lack of μ to e conversion on nuclei[10], μ-e neutral currents are at most 1.2×10^{-8} of full strength neutral currents.[11]

(4) The Michel ρ parameter in μ decay[12] is 0.7518 ± 0.0026. Therefore, both e^- and μ^- couple with better than 99% left-handed chirality.

In the following section, we discuss eight models which possess a non-standard multiplet structure and in which all neutral leptons are either massless or more massive than the τ. We show that only one of these alternative models, a slight variation on the standard model, is consistent with experiment. Then, in the last section we discuss briefly other kinds of models which can be ruled out with present or soon to be available data and state our conclusions.

II. Alternative Models

By allowing complete freedom in the choice of weak-electromagnetic gauge group, representations of that group, and yet undiscovered charged and neutral leptons, one can produce an infinity of different leptonic models. To avoid this unmanageable situation, one restricts one's attention to a limited number of structures selected on the basis of aesthetic criteria such as simplicity.

In this section we consider models within the standard $SU(2)\otimes U(1)$ gauge group (with gauge bosons W, Z, γ) and with all leptons in SU(2) doublets or singlets. For the e, μ, ν_e, and ν_μ we assume the standard multiplet structure

$$\begin{pmatrix}\nu_e \\ e\end{pmatrix}_L ; \quad \begin{pmatrix}\nu_\mu \\ \mu\end{pmatrix}_L ; \quad (e)_R ; \quad (\mu)_R$$

of left-handed weak doublets and right-handed singlets. The left-handed assignments have long been established experimentally. Placing the e^- in a right-handed singlet rather than a doublet is also required, particularly by the polarized electron-nucleon asymmetry measurements.[13] The assignment of μ^-_R to a singlet is not uniquely required by experiment, although with the right-handed μ^- in a right-handed doublet experiments would place sharp limits on some of the resulting mixing angles. The right-handed singlet assignment is chosen on the grounds of simplicity.

All leptons are treated as spin ½ point Dirac particles. The e, μ, and τ (and their antiparticles) are assumed to be the only charged leptons. We take the τ^- to be a lepton and the τ^+ to be an antilepton rather than the other way around.

The different models we consider then differ in their neutral lepton content. We do not add neutral leptons in SU(2) singlets beyond necessity. More precisely, we only include singlet neutral leptons when necessary to allow for a mass for a neutral present in a doublet.

There are then nine cases with the τ and new neutral leptons, the mass of which is either zero or greater than the τ mass.

We now proceed to discuss all these models and to briefly give the arguments which show the status of the various models vis-a-vis experiment. In the following we use primes (e', τ', etc.) to indicate weak eigenstates; unprimed symbols (e, τ, etc.) to denote mass eigenstates. The symbol ν refers to massless neutral leptons, while N refers to leptons with $m_N \geqslant m_\tau$.

1. The Standard Model

The multiplet structure is:

$$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L ; \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L ; \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}_L ; (e)_R ; (\mu)_R ; (\tau)_R .$$

The neutral leptons are taken as massless and need not have right-handed components. Since the neutrals have the same mass, without loss of generality, one can set all mixing angles to zero: i.e., the weak eigenstates can be defined to be equal to the mass eigenstates. The standard model is consistent with all well-established experimental facts of τ and μ-e physics.

2. Superfluous Heavy Neutral Model

The multiplet structure is:

$$\begin{pmatrix}\nu'_e\\ e\end{pmatrix}_L ; \begin{pmatrix}\nu'_\mu\\ \mu\end{pmatrix}_L ; \begin{pmatrix}\nu'_\tau\\ \tau\end{pmatrix}_L ; (N'_\tau)_L ; (e'')_R ; (\mu'')_R ; \begin{pmatrix}N_\tau\\ \tau''\end{pmatrix}_R .$$

With all mixing angles negligible or zero, this model differs from the standard model only in the presence of the right-handed (N_τ, τ) doublet.

Neutral currents involving the τ will be purely vector as a consequence, and with no mixing this is the handle by which this model eventually might be ruled out.

In general there will be mixing. If one sets inter-generational Higgs couplings (e.g., $\bar{e}\tau\phi$) to zero, only ν_τ and N_τ will mix: $e'' = e$, $\mu'' = \mu$, $\tau'' = \tau$, $\nu_e' = \nu_e$. $\nu_\mu' = \nu_\mu$. If we further restrict ourselves to singlet and doublet Higgs bosons, the mixing among left-chirality neutrals is

$$\nu'_{\tau L} = \left(\frac{m_\tau}{m_{N_\tau}}\right) N_{\tau L} + \left(1 - \frac{m_\tau^2}{m_{N_\tau}^2}\right)^{\frac{1}{2}} \nu_{\tau L} \ . \tag{1}$$

For m_{N_τ} close to m_τ, the τ-ν_τ coupling strength is reduced and the τ lifetime gets longer, with the experimental limit becoming relevant. But with $m_{N_\tau} \to \infty$ (or triplet Higgs), the mixing becomes negligible. This situation cannot be ruled out by current experiment.

3. Economy Model

The multiplet structure is:

$$\begin{pmatrix}\nu_e \\ e'\end{pmatrix}_L \ ; \ \begin{pmatrix}\nu_\mu \\ \mu'\end{pmatrix}_L \ ; \ (\tau')_L \ ; \ (e)_R \ ; \ (\mu)_R \ ; \ (\tau)_R \ .$$

The name, "Economy Model" is due to Cabibbo[14], and refers to the lack of new neutral leptons. If all mixing angles equal zero, the model is trivially ruled out as then the τ does not decay.

In general, however, the τ can mix into the μ and e doublets on the left:

$$
\begin{aligned}
e_L' &\cong \left(1 - \frac{\varepsilon_e^2}{2}\right) e_L + \varepsilon_e \tau_L \\
\mu_L' &\cong \left(1 - \frac{\varepsilon_\mu^2}{2}\right) \mu_L + \varepsilon_\mu \tau_L - \varepsilon_e \varepsilon_\mu e_L ,
\end{aligned}
\tag{2}
$$

and thereby is allowed to decay by coupling to ν_e and $\bar{\nu}_\mu$. Here and in later models, we have expanded the exact expressions to lowest significant order in ε_e, ε_μ; we also ignore possible complex phases which do not affect the phenomenology we are considering. We have found that more careful analysis which avoids these approximations yields the same results.

With mixing there are τ-μ and τ-e neutral currents, and their consequences in terms of $\Gamma(\tau \to \nu e \bar{\nu}) \neq \Gamma(\tau \to \nu \mu \bar{\nu})$ and $\tau \to$ three charged leptons were used by Altarelli et al.[11] and Horn and Ross[15], respectively, to rule out the model. Here we simply note that the width for the purely charged current process $\tau \to \nu\pi$ would be[16]

$$
\begin{aligned}
\Gamma(\tau \to \nu\pi) &= (\varepsilon_e^2 + \varepsilon_\mu^2) \frac{(f_\pi \cos\theta_c G_F)^2}{16\pi} m_\tau^3 \left(1 - \frac{m_\pi^2}{m_\tau^2}\right)^2 \\
&= (\varepsilon_e^2 + \varepsilon_\mu^2)/(2.64\times 10^{-12}\text{sec})
\end{aligned}
\tag{3}
$$

and the measured[17] $B(\tau \to \nu\pi) = 11.7 \pm 2.2\%$ and lifetime limit[4] yield

$$\varepsilon_e^2 + \varepsilon_\mu^2 > 0.18 . \tag{4}$$

But the limit on μ-e conversion[10] requires that:

$$\varepsilon_e^2 \varepsilon_\mu^2 < 1.2\times 10^{-8} \tag{5}$$

while μ-e universality[9,17] implies:

$$\varepsilon_\mu^2 - \varepsilon_e^2 = 0.032 \pm 0.019 \ . \tag{6}$$

Therefore

$$\varepsilon_e^2 + \varepsilon_\mu^2 = [(\varepsilon_\mu^2-\varepsilon_e^2)^2 + 4\varepsilon_e^2\varepsilon_\mu^2]^{\frac{1}{2}} < 0.070 \ , \tag{7}$$

contradicting Eq. (4). The model is inconsistent with experiment for any values of the mixing angles.

4. Left-Handed Heavy Neutral Model

The multiplet structure is:

$$\begin{pmatrix} \nu_e \\ e' \end{pmatrix}_L ; \begin{pmatrix} \nu_\mu \\ \mu' \end{pmatrix}_L ; \begin{pmatrix} N_\tau \\ \tau' \end{pmatrix}_L ; (e)_R ; (\mu)_R ; (\tau)_R ; (N_\tau)_R \ .$$

As in the Economy Model, the τ can decay only if there is mixing. We define ε_e, ε_μ as in Eq. (2). Equations (4) and (6) still hold but we no longer have a constraint on $\varepsilon_e^2\varepsilon_\mu^2$ as this model possesses a GIM mechanism[3] which prevents lepton-family-changing neutral currents. Still, the experimental limit[7] on ν_μ production of τ requires:

$$\varepsilon_\mu^2 \leqslant 0.025 \ , \tag{8}$$

which when combined with Eq. (6) implies[18]

$$\varepsilon_e^2 + \varepsilon_\mu^2 = -(\varepsilon_\mu^2-\varepsilon_e^2) + 2\varepsilon_\mu^2 < 0.056 \tag{9}$$

again contradicting Eq. (4) and ruling out the model.[19]

5. Right-Handed τ Doublet Model

The multiplet assignment is:

$$\begin{pmatrix}\nu_e\\ e'\end{pmatrix}_L ;\ \begin{pmatrix}\nu_\mu\\ \mu'\end{pmatrix}_L ;\ (\tau')_L ;\ (e'')_R ;\ (\mu'')_R ;\ \begin{pmatrix}\nu_\tau\\ \tau''\end{pmatrix}_R .$$

In the absence of mixing this model is consistent with all experimental facts except that the right-handed charged current coupling of τ to ν_τ makes the Michel parameter $\rho = 0$ in contrast to the experimental[6] value 0.72 ± 0.15. The question is if the mixing on the left can be made large enough within the other experimental constraints to get an acceptable value of ρ.

We parametrize e', μ', and τ' as in Eq. (2) with parameters $\varepsilon_{\mu L}$ and ε_{eL}, and similarly on the right for e'', μ'', and τ'' with $\varepsilon_{\mu R}$ and ε_{eR}. The limit from μ-e conversion[10] now implies

$$(\varepsilon_{eL}\varepsilon_{\mu L})^2 + (\varepsilon_{eR}\varepsilon_{\mu R})^2 \leq 1.2\times10^{-8} . \tag{10}$$

Furthermore, the ρ parameter measurements from μ decay indicate that

$$\varepsilon_{eR}^2 + \varepsilon_{\mu R}^2 < 0.01 . \tag{11}$$

The restriction from μ-e universality is instead of Eq. (6)

$$\varepsilon_{\mu L}^2 - \varepsilon_{eL}^2 = 0.032 \pm 0.019 \pm 0.01 , \tag{12}$$

with the extra $\pm$ 0.01 due to possible μ-ν_τ or e-ν_τ right-handed couplings. Combining[18] Eqs. (10) and (12)

$$\varepsilon_{\mu L}^2 + \varepsilon_{eL}^2 = [(\varepsilon_{\mu L}^2-\varepsilon_{eL}^2)^2 + 4\varepsilon_{\mu L}^2\varepsilon_{eL}^2]^{\frac{1}{2}} \leq 0.08 . \tag{13}$$

The ρ parameter in $\tau \rightarrow \nu e \bar{\nu}$, including neutral current contributions, is

$$\rho = \frac{3}{4} \frac{\frac{3}{4}\varepsilon_{eL}^2 + \varepsilon_{\mu L}^2 + \frac{3}{4}\varepsilon_{eR}^2}{\frac{3}{4}\varepsilon_{eL}^2 + \varepsilon_{\mu L}^2 + \frac{3}{4}\varepsilon_{eR}^2 + (1-\varepsilon_{eR}^2-\varepsilon_{\mu R}^2)} < 0.066 \ . \tag{14}$$

Equation (14) is still inconsistent with experiment[6], and the model is ruled out in general.

6. Ambidextrous τ Model

The multiplet structure is:

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \ ; \ \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \ ; \ \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \ ; \ (e'')_R \ ; \ (\mu'')_R \ ; \ \begin{pmatrix} \nu_\tau \\ \tau'' \end{pmatrix}_R \ .$$

Without mixing, the τ-ν_τ current is pure vector, resulting in a ρ parameter of 0.375. Even with mixing, the τ'' must be more than 99% τ because of the constraints on right-handed currents in muon decay. Neutral current contributions on the right are negligible compared to charged current contributions and therefore in $\tau \rightarrow \nu e \bar{\nu}$,

$$\rho = \frac{3}{4}\left(\frac{1}{1 + (1-\varepsilon_{eR}^2-\varepsilon_{\mu R}^2)}\right) \leqslant 0.38 \ , \tag{15}$$

which is more than two standard deviations from the measured[5] $\rho = 0.72 \pm 0.15$ and the model is ruled out in general.

Note that if we limit ourselves to singlet and doublet Higgs we have a mass relationship,

$$m_{\nu_\tau}^2 = \varepsilon_{eR}^2 m_e^2 + \varepsilon_{\mu R}^2 m_\mu^2 + (1-\varepsilon_{eR}^2-\varepsilon_{\mu R}^2) m_\tau^2 \ . \tag{16}$$

This follows from the Higgs coupling to $\bar{\ell}_L \tau_R$ being equal to that for $\bar{\nu}_{\ell L} \nu_{\tau R}$ for ℓ = e, μ, or τ. If $m_{\nu_\tau} \ll m_\mu$, then the chirality constraint in $\mu \to \nu e \bar{\nu}$ demands small $(\varepsilon_{eR})^2$ and $(\varepsilon_{\mu R})^2$. But then Eq. (16) makes $m^2_{\nu_\tau} \approx m^2_\tau$, contradicting our assumption of small m_{ν_τ}. Therefore, with singlet and doublet Higgs the requirement that $m_{\nu_\tau} \ll m_\mu$ cannot be met.

7. Heavy Ambidextrous Model

The multiplet structure is:

$$\begin{pmatrix} \nu_e \\ e' \end{pmatrix}_L ; \begin{pmatrix} \nu_\mu \\ \mu' \end{pmatrix}_L ; \begin{pmatrix} N_\tau \\ \tau' \end{pmatrix}_L ; (e'')_R ; (\mu'')_R ; \begin{pmatrix} N_\tau \\ \tau'' \end{pmatrix}_R .$$

The argument used to rule out the Heavy Neutral Model (No. 4 above) also rules out this model.

8. Backward Heavy Neutral Model

The multiplet structure is:

$$\begin{pmatrix} \nu'_e \\ e' \end{pmatrix}_L ; \begin{pmatrix} \nu'_\mu \\ \mu' \end{pmatrix}_L ; (\tau')_L ; (N'_\tau)_L ; (e'')_R ; (\mu'')_R ; \begin{pmatrix} N_\tau \\ \tau'' \end{pmatrix}_R .$$

We define the mixing of the charged leptons as before and parametrize the neutrals on the left by

$$\begin{aligned} \nu'_{eL} &\approx (1-\delta_e^2/2)\nu_{eL} + \delta_e N_{\tau L} - \delta_e \delta_\mu \nu_{\mu L} \\ \nu'_{\mu L} &\approx (1-\delta_\mu^2/2)\nu_{\mu L} + \delta_\mu N_{\tau L} . \end{aligned} \tag{17}$$

The mixing among left-handed neutrals prevents us from ruling out this model as we did the Economy Model. For, μ-e universality can be enforced even with very unequal mixing of the e and μ on the left by introducing compensating mixing ($\delta_e^2 - \delta_\mu^2 = \varepsilon_{\mu L}^2 - \varepsilon_{eL}^2$) of the neutral leptons, and then we lack a restriction on $\varepsilon_{eL}^2 - \varepsilon_{\mu L}^2$. In general the massless components of ν'_{eL} and $\nu'_{\mu L}$ are not even orthogonal, but the experimental limit of 0.3% on ν_μ production of electrons[7] requires

$$(\delta_e \delta_\mu)^2 < 0.003 \ , \tag{18}$$

so that the nonorthogonality is very small.

We still have that the $\tau \to \nu\pi$ charged current decay only occurs through charged lepton mixing on the left and as in Eq. (4)

$$\varepsilon_{eL}^2 + \varepsilon_{\mu L}^2 > 0.18 \ .$$

The limit[7] on ν_μ production of the τ implies $\varepsilon_{\mu L}^2 < 0.025$, so $\varepsilon_{eL}^2 > 0.155$. Then the very small limit on μ-e conversion, $\varepsilon_{eL}^2 \varepsilon_{\mu L}^2 < 1.2\times 10^{-8}$, forces

$$\varepsilon_{\mu L}^2 < 10^{-7} \ , \tag{19}$$

so that the τ mixes on the left almost entirely with the electron.

Now the process $\tau \to \nu e \bar{\nu}$ in this model can be either $\tau \to e\nu_\mu \bar{\nu}_\mu$ (through neutral currents alone because the τ and μ don't mix to the 10^{-7} level on the left) or $\tau \to \nu_e e \bar{\nu}_e$ (through charged or neutral currents). Relative to the standard model the rate for the first process, $\tau \to e\nu_\mu \bar{\nu}_\mu$, is

$$\left(\frac{\varepsilon_{eL}^2}{4} + \frac{\varepsilon_{eR}^2}{4}\right)\left(1-\delta_\mu^2\right) \ ,$$

while for the second process, $\tau \to \nu_e e \bar{\nu}_e$, the rate is

$$\left(\frac{\varepsilon_{eL}^2}{4} + \frac{\varepsilon_{eR}^2}{4}\right)\left(1-\delta_e^2\right) .$$

The rate for the neutral current process $\tau \to e\mu\bar{\mu}$ for the left-handed muons alone is, in the same units,

$$\left(\frac{\varepsilon_{eL}^2}{4} + \frac{\varepsilon_{eR}^2}{4}\right)\left(1-2\sin^2\theta_W\right)^2 .$$

Therefore

$$\frac{\Gamma(\tau \to e\mu\bar{\mu})}{\Gamma(\tau \to e\nu\bar{\nu})} \geqslant \frac{(1-2\sin^2\theta_W)^2}{2-\delta_e^2-\delta_\mu^2} \geqslant 0.25 \qquad (20)$$

using[20] $\sin^2\theta_W < 0.25$. Experimentally this last ratio[8] is $< (3.3\times10^{-4})/(16.5 \pm 1.5\times10^{-2}) = 0.003$, and the model is ruled out in general.

9. Heavy Left-Light Right Model

The multiplet structure is:

$$\begin{pmatrix}\nu_e\\ e'\end{pmatrix}_L ; \begin{pmatrix}\nu_\mu\\ \mu'\end{pmatrix}_L ; \begin{pmatrix}N_\tau\\ \tau'\end{pmatrix}_L ; (e'')_R ; (\mu'')_R ; \begin{pmatrix}\nu''_\tau\\ \tau''\end{pmatrix}_R ; (N''_\tau)_R .$$

Without mixing, the model involves a purely right-handed τ-ν_τ coupling and therefore a predicted ρ parameter in $\tau \to \nu e\bar{\nu}$ which disagrees with experiment. There can be little (< 1%) coupling of the e and/or μ on the right to ν_τ through mixing because of the ρ parameter in μ decay. On the left, μ-e universality and the limits on ν_μ production of τ then imply[21]

$$\varepsilon_{eL}^2 + \varepsilon_{\mu L}^2 = 2\varepsilon_{\mu L}^2 - (\varepsilon_{\mu L}^2 - \varepsilon_{eL}^2) \leqslant 0.05 - (0.032 \pm 0.019 \pm 0.01)$$
$$\leqslant 0.066 \ . \tag{21}$$

But the lifetime limit and branching ratio for $\tau \to \nu\pi$ require that the total coupling strength of the τ to a neutrino be > 18% of full strength. The coupling strength on the right must then be > 11.4% of full strength.

The fact that the charged-current coupling of the τ to a neutrino is predominantly right-handed cannot yet be directly translated into a limit on the ρ parameter in $\tau \to \nu e\bar{\nu}$; for both neutral and charged currents are involved in this process. Fortunately, the contribution to $\tau \to \nu e\bar{\nu}$ from neutral currents (which all involve a right-handed τ) and from the charged current contribution to $\tau_R \to e_R\nu_\tau\bar{\nu}_\tau$ can be related to $\tau \to e\mu\mu$:[22]

$$\Gamma(\tau_R \to e_R\nu_e\bar{\nu}_e) + \Gamma(\tau_R \to e_R\nu_\mu\bar{\nu}_\mu) + \Gamma(\tau_R \to e_R\nu_\tau\bar{\nu}_\tau)$$
$$\leqslant \frac{3}{(1-2\sin^2\theta_W)^2}\,\Gamma(\tau_R \to e_R\mu_L\bar{\mu}_L) \leqslant 12\Gamma(\tau \to e\mu\bar{\mu}) \leqslant 4\times10^{-3}\,\Gamma(\tau \to \text{all}) \ . \tag{22}$$

These contributions to $\tau \to \nu e\bar{\nu}$ then can be neglected. We are left with the charged current induced processes $\tau_R \to \nu_R e_L\bar{\nu}_L$ and $\tau_L \to \nu_L e_L\bar{\nu}_L$, for which the bounds on charged current couplings derived above imply that

$$\rho \leqslant \frac{3}{4}\,\frac{0.066}{0.066 + 0.114} = 0.275 \ , \tag{23}$$

in contradiction with the measured $\rho = 0.72 \pm 0.15$.

III. Discussion of More General Cases

The nine models above are the only models which meet the criteria stated at the beginning of Section II. We now relax some of these criteria, producing more general classes of models. We will not present detailed arguments for each of these models, but will simply discuss results and the key facts that lead to them.

The multiplet structure for the electron and muon sectors remains as before. For the sake of brevity from here on we only list the τ sector multiplet structure.

A. Intermediate Mass Neutrals

If we relax the constraint that the neutral leptons associated with the τ be either zero-mass neutrinos or have a mass greater than that of the τ, we have seven additional models with the following multiplet structures: (L° denotes an intermediate mass neutral lepton, $0 < m_{L^\circ} < m_\tau$):

1'. $\begin{pmatrix} L^\circ \\ \tau' \end{pmatrix}_L$; $(\tau')_R$; $(L^\circ)_R$

2'. $(\tau')_L$; $(L^{\circ\prime})_L$; $\begin{pmatrix} L^\circ \\ \tau' \end{pmatrix}_R$

3'. $\begin{pmatrix} L^\circ \\ \tau' \end{pmatrix}_L$; $\begin{pmatrix} L^\circ \\ \tau'' \end{pmatrix}_R$

4'. $\begin{pmatrix} L^\circ \\ \tau' \end{pmatrix}_L$; $\begin{pmatrix} \nu''_\tau \\ \tau'' \end{pmatrix}_R$; $(L^{\circ\prime\prime})_R$

5'. $\begin{pmatrix} \nu'_\tau \\ \tau \end{pmatrix}_L$; $(L^{\circ\prime})_L$; $\begin{pmatrix} L^\circ \\ \tau'' \end{pmatrix}_R$

$$6'. \quad \begin{pmatrix} L^{\circ\prime} \\ \tau \end{pmatrix}_L ; \; (N'_\tau)_L ; \; \begin{pmatrix} N''_\tau \\ \tau'' \end{pmatrix}_R ; \; (L^{\circ\prime\prime})_R$$

$$7'. \quad \begin{pmatrix} N'_\tau \\ \tau \end{pmatrix}_L ; \; (L^{\circ\prime})_L ; \; \begin{pmatrix} L^{\circ\prime\prime} \\ \tau'' \end{pmatrix}_R ; \; (N''_\tau)_R .$$

Strictly speaking we cannot use the chirality constraints or mass limits derived from $\tau \to \nu e \bar{\nu}$ as they exist in published form to rule out these models. The chirality constraint was derived on the basis of the assumption that the τ couples only to a massless neutral lepton in $\tau \to \nu e \bar{\nu}$ - an assumption violated in these models. Similarly, the mass limit, $m_{L^\circ} \leqslant 250$ MeV, was derived under the assumption that the τ couples to only one neutral lepton lighter than the τ; in these models, on the contrary, the τ can, via mixing, couple both to L° and to ν_e, ν_μ, or ν_τ.

However, we believe on the basis of qualitative heuristic arguments that an appropriate reanalysis of the raw unpublished data would produce constraints sufficient to rule out most of these models for most values of m_{L°. For example, we expect the chirality constraint to be stronger for higher neutral lepton masses. The published value of the ρ parameter corresponds to a "hard" electron energy spectrum. Both adding in a right-handed (V+A) chirality component to the τ-L° current and raising the mass of L° tend to "soften" this spectrum. Thus, the higher m_{L° is, the less (V+A) component one can include and still produce the "hard" spectrum observed experimentally.

If m_{L° is sufficiently close to m_τ, the phase space for any decay involving L° will be negligible and the model will be functionally equivalent to a model in which L° is replaced by a heavy N_τ. For example, in model 1', as in the Left-Handed Heavy Neutral Model, the coupling to ν_e and ν_μ is constrained to be < 0.056. If $m_{L^\circ} > 1.0$ GeV, the phase space for $\tau \to L^\circ e^- \bar{\nu}_e$ is less than 10% of the phase space for the zero-mass case. The total rate for $\tau \to \nu e^- \bar{\nu}_e$ plus $\tau \to L^\circ e^- \bar{\nu}_e$ would then be inconsistent with the experimental limit on the lifetime.[4] A similar argument for model 3' shows that $m_{L^\circ} > 1100$ MeV in that model.

In the limit that m_{L° is very close to m_τ, 1', 2', 3', and 4' are ruled out. Models 6' and 7' are allowed in this limit if one is willing to accept an apparent $G_{F\ \mathrm{lepton}}$ substantially less than the standard-model $G_{F\ \mathrm{quark}}$ (one could avoid this discrepancy by introducing an appropriate non-standard multiplet structure in the quark sector also). In this limit, model 5' is functionally equivalent to the Superfluous Heavy Neutral Model and is therefore allowed.

If m_{L° is very close to zero, the models are functionally equivalent to models obtained by replacing L° by a massless neutrino. Cases 1' and 6' will then be allowed; all others are inconsistent with experiment.

One should also note that, unless mixing is appropriately restricted, L° can decay (into, e.g., $e^+e^-\nu_e$) if $m_{L^\circ} > 2m_e$ in these models. This provides further constraints on these models. Other authors have used astrophysical considerations to constrain the number of neutrinos and their masses.[23]

B. Models with Extra Neutrals

To all the above models, one can add extra neutral leptons, which must be in singlets if one does not add extra charged leptons. If one adds massless neutrinos to the Economy Model, Right-Handed Doublet Model, or Ambidextrous τ Model, they are still ruled out. In these models the mixing with the extra neutrinos may be defined away by a redefinition of the neutrino mass eigenstates. The Backward Heavy Neutral Model with extra neutrinos can also be ruled out. The other five models with extra neutrinos reduce to the Standard Model or to the Superfluous Heavy Neutral Model for appropriate values of the mixing angles and are therefore allowed.

If instead one adds more heavy neutral leptons ($m \geqslant m_\tau$), the Economy Model, Backward Heavy Neutral Model, and Right-Handed τ Doublet Model are still ruled out. The Left-Handed Heavy Neutral Model, Heavy Ambidextrous Model, and Heavy Left-Light Right Model with extra heavy neutrals can be made consistent with experimental lepton data by appropriate mixing with the heavy neutrals to preserve μ-e universality; however, the apparent $G_{F\ \mathrm{lepton}}$ will differ substantially from $G_{F\ \mathrm{quark}}$. The other three models with extra heavy neutrals are allowed for appropriate values of the mixing angles.

If one adds both neutrinos and neutrals with mass $\geqslant m_\tau$, the Economy Model, Right-Handed τ Model, and Backward Heavy Neutral Model will be ruled out; the other six models will be allowed.

If one adds neutrals with intermediate mass, one has the difficulties discussed above.

C. Anomalous τ Lepton Number

If there is no mixing among lepton generations, it is a matter of convention whether the τ^+ or τ^- be considered lepton or antilepton. With inter-generational mixing, however, the distinction is real: is it the neutral partner of the τ^- or the τ^+ which mixes with ν_e, ν_μ?

For example, in the Left-Handed Heavy Neutral Model, the multiplet structure would be

$$\begin{pmatrix} \tau^+ \\ N_\tau \end{pmatrix}_R \; ; \; (\tau^+)_L \; ; \; (N_\tau)_L \; .$$

τ^+ could not mix with e^- or μ^-, and mixing among the neutral leptons would violate GIM (since I_3 differs for N_τ and ν_e or ν_μ). (Note that we require τ^+ in a right-hand doublet so that τ^- will be in a left-hand doublet.)

If one similarly reverses the τ lepton number in our canonical nine models, the conclusions will not change - all but the Standard Model and the Superfluous Heavy Neutral Model will be ruled out.

D. Other Possibilities

Placing the μ in a right-handed doublet with a heavy neutral partner alters some of the above results: for appropriate mixing angles, the Ambidextrous τ Model would be allowed. The Left-Handed Heavy Neutral Model, Heavy Ambidextrous Model, and Heavy Left-Right Model would also be allowed if one will accept $G_{F\ \mathrm{lepton}} \neq G_{F\ \mathrm{quark}}$. (The chirality structure of the μ-μ neutral current, and hence the right-handed assignment of the μ will be probed by currently planned experiments.)

One need not restrict oneself to $SU(2)\otimes U(1)$ doublets and singlets; e.g., one could assign the τ to:

$$\begin{pmatrix} L^+ \\ \nu_\tau \\ \tau^- \end{pmatrix}_L \; ; \quad (L^+)_R \; ; \quad (\tau^-)_R \; .$$

Or one could abandon the conventional $SU(2)\otimes U(1)$ gauge-theoretic framework altogether.

Although these may be real possibilities, we will not consider them here. We have also not discussed the interesting phenomena encompassed in "neutrino oscillations."

Conclusion

We have discussed the Standard Model and eight plausible variations involving the τ $SU(2)\otimes U(1)$ multiplet structure and have shown that all but two of these models are inconsistent with experiment. Until τ-τ neutral currents are measured, it will apparently not be possible to discriminate between these two possibilities.

We have briefly discussed wider classes of models beyond the nine canonical models. For the most part, these models appear to be ruled out except when they are essentially equivalent to our two allowed canonical models.

While existing information from experiment does not uniquely require the Standard Model, it does rule out the bulk of the simple alternatives and justifies a strong prejudice in favor of the Standard Model.

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16. One could also use $\tau \to \rho\nu$ or $\tau \to \mu\bar{\nu}\nu$ or $\tau \to e\bar{\nu}\nu$ to convert the lifetime limit to a limit on the ν-τ coupling at this point rather than $\tau \to \pi\nu$. However, these alternatives would be less clean: e.g., $\tau \to e\bar{\nu}\nu$ can involve both charged and neutral currents.

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18. Here and elsewhere, we conservatively take two standard deviations from the central value in deriving the inequality.

19. This result has been previously reported by F. J. Gilman at the 1978 Kyoto Summer Institute reporting on work by D. H. Miller and F. J. Gilman (SLAC-PUB-2226, 1978) and by G. Altarelli at the 1978 Tokyo Conference (G. Altarelli in Proceedings of the 19th International Conference on High Energy Physics, edited by S. Homma, M. Kavaguchi, and H. Miyazawa (Physical Society of Pana, 1979), p. 411).

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21. As in Eq. (12), the extra $\pm$ 0.01 is due to possible (but necessarily small because of the constraint from the μ-decay ρ

parameter) e-ν or μ-ν right-handed couplings.

22. $\tau_R \rightarrow e_R \nu_\tau \bar{\nu}_\tau$ involves both charged and neutral currents which interfere destructively so as to produce the same total rate as would be produced by the neutral current amplitude alone.

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