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THE HADRONIC WAVE FUNCTION IN QUANTUM CHROMODYNAMICS*

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ABSTRACT

The underlying link between hadronic phenomena in quantum chromodynamics at large and small distance is the hadronic wavefunction. We present[1] the theoretical and empirical constraints on the hadronic wavefunction and hadronic structure functions; the predictions of perturbative QCD for the large transverse momentum tail of the Fock state infinite momentum wavefunction $\psi(k_{\perp i},x_i,s_i)$; the valence Fock state meson wavefunctions from the meson decay; the evolution equations of the distribution amplitudes; and a simplified model for the basic wavefunctions. In particular we obtain a new type of low energy theorem for the pion wavefunction from the $\pi^\circ \to \gamma\gamma$. This result together with the constraint on the valence wavefunction from the $\pi^\circ \to \mu\nu$ decay, leads to the probability of finding the valence $|q\bar{q}\rangle$ state. All these constraints allow us to construct a possible model which describes hadronic wavefunctions, probability amplitudes, and distributions. We compare our results with data for form factors and the deep inelastic processes. This work represents a first attempt to construct a model of hadronic structure which is consistent with data and QCD at large and small distances.

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I. GENERAL CONSIDERATIONS

The underlying link between hadronic phenomena in quantum chromodynamics at large and small distance is the hadronic wavefunction. In this paper we will discuss the theoretical and empirical constraints on the structure of the Fock state wavefunctions of mesons and nucleons. We define the states at equal $\tau = t + z$ on the light-cone using the light-cone gauge $A^+ = A^0 + A^3 = 0$. The amplitude to find n (on-mass-shell) quarks and gluons in a hadron with 4-momentum P directed along the Z-direction and spin projection S_z is defined ($k^\pm = k^0 \pm k^3$) (see Fig. 1)

$$\psi_{S_z}^{(n)}(x_i, \vec{k}_{\perp i}, s_i) \quad , \quad x_i \equiv \frac{k_i^+}{P^+} \quad ,$$

where by momentum conservation $\sum_{i=1}^{n} x_i = 1$ and $\sum_{i=1}^{n} \vec{k}_{\perp i} = 0$. The s_i specify the spin-projection of the constituents. The state is off the light-cone energy shell,

$$P^- - \sum_{i=1}^{n} k_i^- = \frac{M^2 - \sum_{i=1}^{n} \frac{\vec{k}_{\perp i}^{\,2} + m_i^2}{x_i}}{P^+} < 0 \quad .$$

The valence Fock states (which in fact dominate large momentum transfer exclusive reactions) are the $|q\bar{q}\rangle$ (n=2) and $|qqq\rangle$ (n=3) components of the meson and baryon. For each fermion or anti-fermion constituent $\psi_{S_z}^{(n)}(k_{\perp i}, x_i, s_i)$ multiplies the spin factor $\frac{u(\vec{k}_i)}{\sqrt{k_i^+}}$ or $\frac{v(\vec{k}_i)}{\sqrt{k_i^+}}$. The wavefunction normalization condition is

$$\sum_{(n)(s_i)} \int |\psi_{S_z}^{(n)}(k_{\perp i}, x_i, s_i)|^2 [d^2 k_\perp][dx] = 1 \quad , \tag{1}$$

where

$$[d^2k_\perp] \equiv 16\pi^3\ \delta^{(2)}\left(\sum_i k_{\perp i}\right) \prod_{i=1}^{n} \frac{d^2k_{\perp i}}{16\pi^3} \quad ,$$

and

$$[dx] = \delta\left(1 - \sum_i x_i\right) \prod_{i=1}^{n} dx_i \quad .$$

By studying the wavefunctions themselves, one could in principle understand not only the origin of the standard structure functions, but also the nature of multi-particle longitudinal and transverse momentum distributions, helicity dependences, as well as the effects of coherence. For example, the standard quark and gluon structure functions (probability distributions) which control large momentum transfer inclusive reactions at the scale Q^2 are

$$G_{a/H}(x_a,Q^2) \equiv d_a^{-1}(Q^2) \sum_{n,s_i,s_z} \int^{k_{\perp a}^2 < Q^2} |\psi_{S_z}^{(n)}(k_{\perp i},x_i,s_i)|^2 [d^2k_\perp][dx] \cdot \delta(x - x_a) \quad , \tag{2}$$

where $d_a^{-1}(Q^2)$ is due to the wavefunction renormalization of the constituent a. Note that only terms which fall-off as $|\psi|^2 \sim (k_{\perp a}^2)^{-1}$ (modulo logs) contribute to the Q^2 dependence of the integral. These contributions are analyzable by the renormalization group and correspond in perturbative QCD to quark or gluon pair production or fragmentation processes associated with the struck constituent a. In general, unless x is close to 1, all Fock states in the hadron contribute to $G_{a/H}$. Multi-particle probability distributions are simple generalizations of Eq. (2).

Recently, it has been shown that exclusive processes such as form factors and large angle elastic scattering can be systematically analyzed in perturbative QCD.[2,3] For example, the (helicity conserving) hadronic form factors to leading order in m^2/Q^2 and to all orders in $\alpha_s(Q^2)$ take the form

$$F(Q^2) = \int [dx]\,[dy]\,\phi^\dagger(x,s',\tilde{Q}_x)\,T_H(x,y,Q)\,\phi(y,s,\tilde{Q}_y) \quad ,$$

with

$$\tilde{Q}_x = Q\cdot\min_i(x_i) \quad , \tag{3}$$

where T_H is the hard scattering amplitude for the virtual photon to scatter the valence quarks from p to p+q;* it can be expanded in powers of $\alpha_s(Q^2)$,

$$T_H \sim \left(\frac{\alpha_s(Q^2)}{Q^2}\right)^{n-1} f_n(x,y)\left[1 + O\left(\alpha_s(Q^2)\right)\right] \quad ,$$

where n is the number of valence constituents. The quantity $\phi(x,Q)$ is the "distribution amplitude" for finding the valence quark with light-cone fraction x_i in the hadron at relative separation $b_\perp^2 \sim O(1/Q^2)$. In fact,

$$\phi(x_i,s_i,Q) \equiv \prod_{i=1}^{n}\left[d_i^{-1}(Q^2)\right]^{\frac{1}{2}} \int^{k_{\perp i}^2 < Q^2} [d^2k_\perp]\,\psi^{(n)}(k_{i\perp},x_i,s_i) \tag{4}$$

Although the complete specification of hadronic wavefunctions clearly will require a solution of the non-perturbative bound state problem in QCD, there is a large number of properties of the wavefunctions which

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*Note that because T_H conserves hadronic helicity $s_i' = s_i$. Further, to leading order in $1/Q^2$ only the terms with $\sum_i s_i = s_z$ contribute in ϕ.

can be derived from the theory and experimental phenomena. In this paper we will discuss the following constraints:

(a) The predictions of perturbative QCD for the large transverse momentum tail of the Fock state infinite momentum wavefunction $\psi(k_{\perp i},x_i)$. These results, which also follow from the operator product expansion near the light-cone,[3] lead to evolution equations for the process-independent distribution amplitudes $\phi(x_i,Q)$ which control large transverse momentum exclusive reactions such as form factors, and for the distribution functions $G_{q/H}(x_i,Q)$ and $G_{g/H}(x_i,Q)$ which control large transverse momentum inclusive reactions.[4]

(b) Exact boundary conditions for the valence Fock state meson wavefunctions from the meson decay amplitudes. In particular we show how the $\pi^\circ \to \gamma\gamma$ decay amplitude for massless quarks specifies the pion wavefunction at <u>zero</u> $k_\perp$. This is a new type of low energy theorem for the pion wavefunction which is consistent with chiral symmetry and the triangle anomaly for the axial vector current.[5] This large-distance result, together with the constraint on the valence wavefunction at short distance from the $\pi \to \mu\nu$ leptonic decay amplitude, leads to a number of new results for the parametrization of the pion wavefunction. In particular, we show that the probability of finding the valence $|q\bar{q}\rangle$ state in the total pion wavefunction is ~ 0.2 to 0.25, for a broad range of confining potentials.

(c) As noted above the wavefunction for the Fock states of the hadrons on the light-cone (or at infinite momentum frame) $\psi^{(n)}_{S_z}(k_{\perp i},x_i,s_i)$ completely specify the quark and gluon particle content of the hadrons. The coherent aspects of the wavefunction are required for constructing the distribution amplitudes which are not only necessary for exclusive processes, but also for the multi-particle, high twist subprocesses which enter inclusive reactions and control transverse momentum smearing effects. We show that the evolution equations which specify the large Q^2 behavior of the distribution amplitudes and of incoherent distribution functions G are correctly applied for $Q^2 \gtrsim \langle\mathcal{E}\rangle$, where $\langle\mathcal{E}\rangle$ is the mean value of the off-shell (light-cone/infinite momentum frame) energy in the Fock state wavefunction

$$\mathcal{E} \equiv \sum_i \mathcal{E}_i \equiv \sum_{i=1}^{n} \left(\frac{\vec{k}_\perp^2 + m^2}{x} \right)_i \quad .$$

To first approximation, $\langle\mathcal{E}\rangle$ is the "starting point" Q_0^2 for evolution due to perturbative effects in QCD. A more detailed discussion is given in Section II.

In order to organize the predictions for hadronic matrix elements and all of the distribution functions and amplitudes, it is very convenient to assume a simplified model for the basic wavefunctions. We shall make the following prescription:

(i) We assume the Fock state wavefunction $\psi^{(2)}$ for the 2-quark state in the non-perturbative domain depends only on the off-shell energy variable $\mathcal{E}$. [This ansatz, which is true for non-relativistic theories, can be justified, if we utilize the Bethe-Salpeter equation with an instantaneous energy independent kernel.[6]] For the n-particle state, weshall assume the Fock state wavefunction $\psi^{(n)}$ is a symmetric function of the $\mathcal{E}_i$, i.e., $\psi^{(n)} = \psi^{(n)}(\mathcal{E}_i)$. Although we have no strong argument for this form, we shall use it as an illustration of the effect of the non-perturbative wavefunction.

(ii) An (approximate) connection between the equal-time wavefunction in the center of mass frame and the infinite momentum frame wavefunction can be established by equating the energy propagator $M^2 - \mathcal{E} = M^2 - \left(\sum_{i=1}^{n} k_i^\mu\right)^2$ in the two frames:

$$M^2 - \mathcal{E} = \begin{cases} M^2 - \left(\sum_{i=1}^{n} q^0_{(i)}\right)^2 & \sum_{i=1}^{n} \vec{q}_i = 0 \quad [\mathrm{CM}] \\ M^2 - \sum_{i=1}^{n} \left(\dfrac{\vec{k}_\perp^2 + m^2}{x}\right)_i & \sum_i \vec{k}_{\perp i} = 0 \quad \sum_i x_i = 0 \quad [\mathrm{IMF}] \end{cases} , \tag{5}$$

Thus the rest frame wavefunction $\psi_{CM}(\vec{q}_{(i)})$ which controls binding and hadronic spectroscopy implies a form for the IMF wavefunction $\psi_{IMF}(x_i, k_{\perp i})$ if we kinematically identify

$$x_i \equiv \frac{k_i^+}{P^+} \leftrightarrow \frac{(q^0 + q^3)_i}{\sum_j^n q_j^0} , \tag{6}$$

and

$$\vec{k}_{\perp i} \leftrightarrow \vec{q}_{\perp i} .$$

For a two particle state, there is thus a possible connection;

$$\psi_{IMF}\left(\frac{k_\perp^2 + m^2}{1 - x^2} - m^2\right) \leftrightarrow \psi_{CM}(\vec{q}^{\,2}) , \tag{7}$$

$$(x = x_1 - x_2) .$$

An equivalent result was also obtained recently by Karmonov[7] using a different method.

Before we discuss a concrete model, we will give results which are insensitive to the specific form of ψ. For example, if $\psi \equiv \psi(\mathscr{E}_i)$ we find

$$G^{\text{non-pert}}_{a/H}(x_a) \xrightarrow[x_a \to 1]{} (1 - x_a)^{2n_s - 1} F(\mathscr{E}^i_{min}) \quad , \tag{8}$$

where $n_s = \min(n_H - n_a)$ is the minimum number of spectator constituents in the hadron H after removing the particle (or subcomposite) a, and $\mathscr{E}^i_{min} = m_i^2/x_i$ is the minimum value of $\mathscr{E}_i$. This result which follows from the definition Eq. (2) by changing variables from $d^2k_{\perp i}$ to $d^2k_{\perp i}/x_i$ is independent of the form of $\psi(\mathscr{E}_i)$ as long as it is square-integrable under $[d^2k_\perp]$. Examples of this result for $G_{q/M}$ and $G_{q/B}$ have recently been given by de Rujula and Martin.[8] Notice that if we can neglect the quark masses (i.e., for $(1 - x_a) >> \frac{m^2}{\langle k_\perp^2 \rangle}$) we obtain the spectator rule proposed in Ref. 9,

$$G^{\text{non-pert}}_{a/H}(x_a) = C_{a/H}(1 - x_a)^{2n_s - 1} \quad ,$$
$$\left(x_a \sim 1 \quad , \quad (1 - x_a) >> \frac{m^2}{\langle k_\perp^2 \rangle} \right) \quad . \tag{9}$$

In fact if we neglect $\frac{m^2}{\langle k_\perp^2 \rangle}$ the non-perturbative contribution can dominate the perturbative prediction in the $x \sim 1$ domain! For example, the perturbative power-law behavior is

$$\Delta G^{\text{pert}}_{q/M} \underset{x \to 1}{\sim} \alpha_s^2 (1 - x)^2 \quad , \tag{10}$$

$$\Delta G^{\text{pert}}_{q/B} \underset{x \to 1}{\sim} \alpha_s^4 \begin{cases} (1 - x)^3 & \text{parallel q and B helicity} \\ (1 - x)^5 & \text{anti-parallel q and B helicity} \end{cases} \tag{11}$$

[In addition QCD evolution increases the exponent of Eqs. (9), (10) and (11) by $\sim 4C_F\,\xi(Q^2)$ where $\left(C_F = \frac{4}{3},\ \beta = 11 - \frac{2}{3}n_f,\ \text{for } n_c = 3\right)$

$$\xi(Q^2) = \frac{1}{4\pi}\int_{Q_0^2}^{Q^2} \frac{d^2k_\perp}{k_\perp^2}\,\alpha_s(k_\perp^2)$$

$$\cong \frac{1}{\beta}\log\frac{\log Q^2/\Lambda^2}{\log Q_0^2/\Lambda^2}\ .]$$

Since flavor and spin are correlated in the baryon wavefunction, perturbative QCD predicts $\Delta G_{u/P} \neq 2\Delta G_{d/P}$. In fact if we assume the baryon wavefunction satisfies SU(6) symmetry (which is a rigorous result for $\phi_B(x_i,Q)$, $(Q \to \infty)$, we have $\Delta G_{u/P} = 5\Delta G_{d/P}$ for $x \to 1$.[10] The question of whether the non-perturbative or perturbative contribution dominates the structure functions at $x \to 1$ can thus be studied using spin and flavor correlations. In the case of meson structure function, perturbative QCD also predicts a contribution to the longitudinal structure function $F_L^\pi(x,Q^2) \underset{x \to 1}{\sim} (1-x)^0 F_\pi(Q^2)$.[10,11] Evidence for the presence of the perturbative term has recently been given from measurement of the angular distribution at large $x_{\bar{q}}$ in the reaction $\pi N \to \mu^+\mu^- X$.[12]

II. GENERAL CONSTRAINTS ON MESON WAVEFUNCTIONS AND THEIR APPLICATION TO A SIMPLE MODEL

In this section we will discuss the constraints on meson wavefunctions imposed by their decay constants. The leptonic decays of the mesons give an important constraint on the valence $|q\bar{q}\rangle$ wavefunction at the origin. As shown in Ref. 2

$$\lim_{Q\to\infty} \phi_M(x_i,Q) = a_0 x_1 x_2 = \begin{cases} \dfrac{3}{\sqrt{n_c}}\, f_\pi\, x_1 x_2 & \text{for } \pi \\ \dfrac{3\sqrt{2}}{\sqrt{n_c}}\, f_\rho\, x_1 x_2 & \text{for } \rho_L \end{cases} , \tag{12}$$

where $f_\pi \cong 93$ MeV is the pion decay constant for $\pi^+ \to \mu^+\nu$ and $f_\rho \cong 107$ MeV is the leptonic decay constant from $\rho^0 \to e^+e^-$. The analogous result holds for all zero helicity mesons. Because the $Q^2 \to \infty$ distribution amplitude has zero anomalous dimension, this constraint is independent of gluon radiative correction and can be applied directly to the non-perturbative wavefunction:

$$a_0 = 6\int [dx][d^2k_\perp]\ \psi_M^{\text{non-pert}}(k_\perp,x) \tag{13}$$

On the other hand we can also obtain an exact low energy constraint on $\psi(k_\perp = 0,x)$ for the pion in the chiral limit $m_q \to 0$. The $\gamma^*\pi^\circ \to \gamma$ vertex defines the $\pi^\circ - \gamma$ transition form factor $F_{\pi\gamma}(Q^2)$ ($q^2 = -Q^2$, see Fig. 2a)

$$\Gamma_\mu = -ie^2\, F_{\pi\gamma}(Q^2)\, \varepsilon_{\mu\nu\rho\sigma}\, P_\pi^\nu\, \epsilon^\rho\, q^\sigma \quad , \tag{14}$$

where

$$F_{\pi\gamma}(0) = \frac{1}{4\pi^2}\, n_c \left(e_u^2 - e_d^2\right) \frac{1}{f_\pi} \quad . \tag{15}$$

This result, derived by the Schwinger, Adler, Bell and Jackiw,[5] gives for $n_c = 3$ the $\pi^\circ \to \gamma\gamma$ decay rate, $\Gamma_{\pi^\circ\to\gamma\gamma} = \frac{\pi\alpha^2}{2}\, m_\pi^3\, F_{\pi\gamma}^2(0) = 7.63$ eV compared to $\Gamma_{\text{expt}} = (7.95 \pm 0.55)$ eV.

If $m_q \to 0$, then the valence $|q\bar{q}\rangle$ contribution to $F_{\pi\gamma}(Q^2)$ is (Fig. 2b, see Ref. 2)

$$F_{\pi\gamma}(Q^2) = 2\sqrt{n_c}\left(e_u^2 - e_d^2\right) \cdot \int_0^1 [dx] \int \frac{d^2k_\perp}{16\pi^3}\, \psi(x_i,k_{\perp i})$$

$$\cdot \left\{ \frac{\left(\vec{q}_\perp x_2 + \vec{k}_\perp\right) \times \vec{\epsilon}_\perp}{\left(\vec{q}_\perp \times \vec{\epsilon}_\perp\right)\left(\vec{q}_\perp x_2 + \vec{k}_\perp\right)^2} + (x_1 \longleftrightarrow x_2) \right\} \tag{16}$$

$$= 2\sqrt{n_c}\left(e_u^2 - e_d^2\right)\left\{\int_0^1 dx_1 \int_0^{x_1^2Q^2} \frac{dk_\perp^2}{16\pi^2}\, \frac{\psi(k_\perp,x_i)}{Q^2x_1} + x_1 \longleftrightarrow x_2\right\} . \tag{17}$$

In fact, as shown in Ref. 13, gauge-invariance requires that the valence $|q\bar{q}\rangle$ state should give exactly ½ of the total decay amplitude for $q^2 \to 0$. Thus from Eqs. (15) and (17), we find

$$\int_0^1 dx_1\, x_1\, \psi(0,x_1) = \frac{\sqrt{n_c}}{2f_\pi} \quad . \tag{18}$$

Therefore the pion wavefunction is constrained both at large and small distances.

In order to implement these constraints it is convenient to construct a simple model of the hadronic wavefunction. By using the connection (7) for the two particle state from the harmonic oscillator model[6] we can get the wavefunction in the infinite momentum frame

$$\psi^{(2)}(k_\perp,x_i,s_i) = A\, \exp\left[-R^2\left(\frac{\vec{k}_\perp^2 + m_q^2}{x_1} + \frac{\vec{k}_\perp^2 + m_q^2}{x_2}\right)\right] . \tag{19}$$

Perhaps the simplest generalization for the n-particle Fock state wavefunctions in the non-perturbative domain is the Gaussian form:

$$\psi^{(n)}_{S_z}(k_{\perp i}, x_i, s_i) = A_n \exp\left[-R_n^2 \mathscr{E}\right]$$

$$= A_n \exp\left[-R_n^2 \sum_{i=1}^{n} \left(\frac{\vec{k}_\perp^2 + m^2}{x}\right)_i\right] \qquad (20)$$

The parametrization is taken to be independent of spin. The full wavefunction is the $\psi^{(n)}_{S_z}(k_{\perp i}, x_i, s_i)$ multiplied by the free spinor $u(k_i, s_i)/\sqrt{k_i^+}$ or $v(k_i, s_i)/\sqrt{k_i^+}$. The Gaussian model corresponds to a harmonic oscillator-confining potential $V \propto \vec{r}^{\,2}$ in the CM frame. This ansatz for the wavefunction has the additional analytic simplicity of (a) factorizing in the kinematics of each constituent and (b) satisfying a "cluster" property when the constituents are grouped into any rearrangement of subcomposites A,B,..., e.g., for two subcomposites

$$\psi^{(n)} = A_n \, e^{-R_n^2 \mathscr{E}_A} \, e^{-R_n^2 \mathscr{E}_B} , \qquad (21)$$

where

$$\mathscr{E}_A = \sum_{i \in A} \left(\frac{\vec{k}_\perp^2 + m^2}{x}\right)_i = \frac{\vec{k}_{\perp A}^2 + M_A^2}{x_A} ,$$

$$\sum_{i \in A} \vec{k}_{\perp i} = \vec{k}_{\perp A} ,$$

$$\sum_{i \in A} x_i = x_A ,$$

and

$$\mathscr{E} = \mathscr{E}_A + \mathscr{E}_B .$$

The actual form of the non-perturbative wavefunction in QCD is undoubtedly more complicated than the form Eq. (20). The only clear constraint is that the non-perturbative wavefunction falls-off faster at large $k_\perp$ than the perturbative contributions $\Delta\psi \sim O\left(\frac{1}{k_\perp}, \frac{1}{k_\perp^2}\right)$ in order that the operator product expansion at short distances and near the light-cone dominates large momentum transfer reaction. A model based on a linear confining potential $V \propto r$ in the CM frame would give a non-perturbative wavefunction $\psi \sim O\left(\frac{1}{k_\perp^6}\right)$ at large $k_\perp$.

If we adopt the Gaussian form for the meson wavefunction Eq. (19) then constrains (13) and (18) imply ($m_q^2 R^2 << 1$, $n_c = 3$)

$$R = \frac{1}{4\pi f_\pi} \simeq 0.17 \text{ fm} \quad ,$$

and

$$A = \frac{\sqrt{3}}{f_\pi} \cdot e^{2m_q^2 R^2} \quad . \tag{22}$$

The probability of finding the valence $q\bar{q}$ state in the pion is thus

$$P(q\bar{q}) = \int [dx][d^2k_\perp]|\psi(k_\perp, x_i)|^2 = \frac{1}{4} \quad . \tag{23}$$

Alternatively, if we use a power law form

$$\psi(k_\perp, x_i) = \frac{A_\alpha}{\left(\frac{k_\perp^2 + m_q^2}{x(1-x)} + \mu^2\right)^\alpha} \tag{24}$$

we find ($m_q^2 << \mu^2$)

$$P(q\bar{q}) = \frac{1}{2}\,\frac{\alpha - 1}{2\alpha - 1} \quad , \tag{25}$$

which again leads to ¼ for large α. For the linear potential case, where $\alpha = 3$, we have $P(q\bar{q}) = 1/5$.

Let us consider the implications of these results for exclusive large momentum transfer processes. As seen in Eq. (3) we require the behavior of the distribution amplitude $\phi(x_i,s_i,Q^2)$ defined in Eq. (4), which is the probability for finding valence quarks at relative transverse separation $b_\perp \sim O(1/Q)$. The large Q^2 dependence of ϕ (i.e., the large $k_\perp$ tail of ψ) is in fact completely determined by the operator product expansion near the light-cone, and in QCD can be calculated from the perturbative expansion in the irreducible kernel for the quark constituents. To order $\alpha_s(Q^2)$ one only requires single gluon exchange, and we find, using the evolution equation of Ref. 2

$$\phi(x_i,Q^2) = \phi(x_i,Q_0^2) + \frac{C_F}{\beta}\int_{Q_0^2}^{Q^2} \frac{d\ell_\perp^2}{\ell_\perp^2} \int [dy]\, \alpha_s\left(\frac{\ell_\perp^2}{y_1(1-y_1)}\right) \cdot \tag{26}$$

$$\cdot\ [V(x_i,y_i) - \delta(x-y)]\,\phi(y_i,\ell_\perp^2) \quad ,$$

where[14]

$$\begin{aligned} &V(x,y) \\ &= 2\left\{x_1y_2\theta(y_1 - x_1)\left(\delta_{h_1\bar{h}_2} + \frac{\Delta}{y_1 - x_1}\right) + (1 \longleftrightarrow 2)\right\} \\ &= V(y,x) \quad . \end{aligned} \tag{27}$$

This result is derived in the region where $\frac{\ell_\perp^2}{y_1(1-y_1)}$ is large compared to the off-shell energy $\langle\mathscr{E}\rangle$ in the wavefunction. Thus the natural starting point for the evolution of the distribution amplitude is $\frac{Q_0^2}{x_1(1-x_1)} \sim \langle \mathscr{E} \rangle$, i.e., to the first approximation we can identify

$$\phi(x_i,Q^2) = \phi^{\text{non-pert}}(x_i) + \frac{C_F}{\beta}\int_{x_1(1-x_1)\langle \mathscr{S} \rangle}^{Q^2} \frac{d\ell_\perp^2}{\ell_\perp^2}\int [dy]\,\alpha_s\left(\frac{\ell_\perp^2}{y_1(1-y_1)}\right)\cdot \tag{28}$$

$$\cdot\ [V(x,y) - \delta(x-y)]\,\phi(y_i,\ell_\perp^2) \quad ,$$

where

$$\phi^{\text{non-pert}}(x_i) = \int [d^2k_\perp]\psi^{\text{non-pert}}(\vec{k}_\perp^2,x_i) \quad . \tag{29}$$

Assuming the wavefunction given by Eqs. (19) and (22), the shape and the normalization of $\phi^{\text{non-pert}}(x_i)$ depends only upon the quark mass. This dependence is illustrated in Fig. 3. The pion form factor[15] in QCD is then as shown in Fig. 4, for values of the quark mass and of the QCD scale parameter Λ_{eff}^2. The application of perturbative QCD for $\frac{\ell_\perp^2}{y_1(1-y_1)} \gtrsim \langle \mathscr{E} \rangle$ is reasonable here, since $\langle \mathscr{E} \rangle \sim 0.7\ \text{GeV}^2$ for this wavefunction is much larger than QCD Λ_{eff}^2.

A similar analysis is being applied to the baryon wavefunction and its structure distribution function. These results will be given elsewhere.

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14. Higher order kernels entering the evolution equation include all two-particle irreducible amplitudes for $q\bar{q} \to q\bar{q}$. However, these corrections to $V(x_i,y_i)$ are all suppressed by powers of $\alpha_s(Q^2)$, because they are irreducible.

15. The data are from the analysis of electroproduction $e^-p \to e^- + \pi^+ + n$; C. Bebek et al., Phys. Rev. D13, 25 (1976).

FIGURE CAPTIONS

1. The amplitude to find n (on-mass-shell) quarks and gluons in a hadron.
2. (a) The π-γ transition form factor $F_{\pi\gamma}(Q^2)$.

   (b) The lowest order diagram which contributes to $F_{\pi\gamma}(Q^2)$.
3. The distribution amplitude $\phi^{\text{non-pert}}(x_i)$ for a Gaussian wave function with the different values of the quark mass m_q.
4. QCD prediction for the meson form factor.

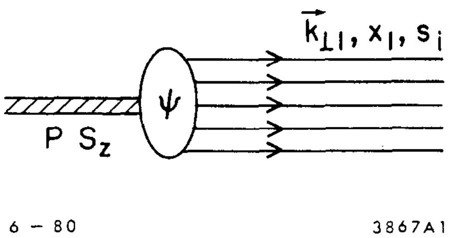


Fig. 1

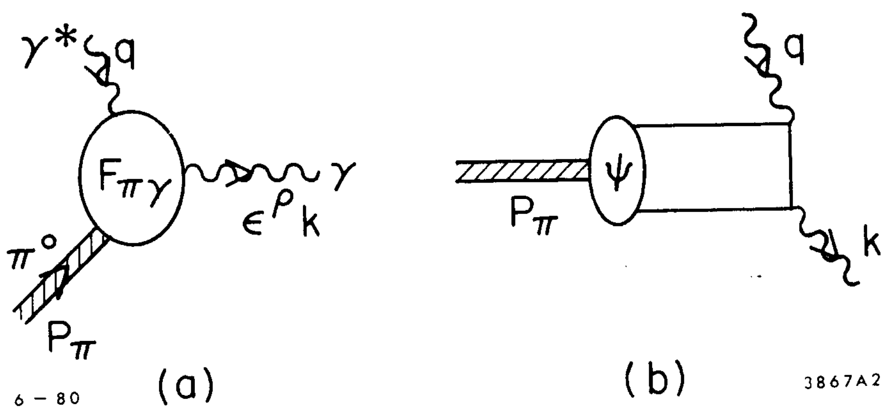
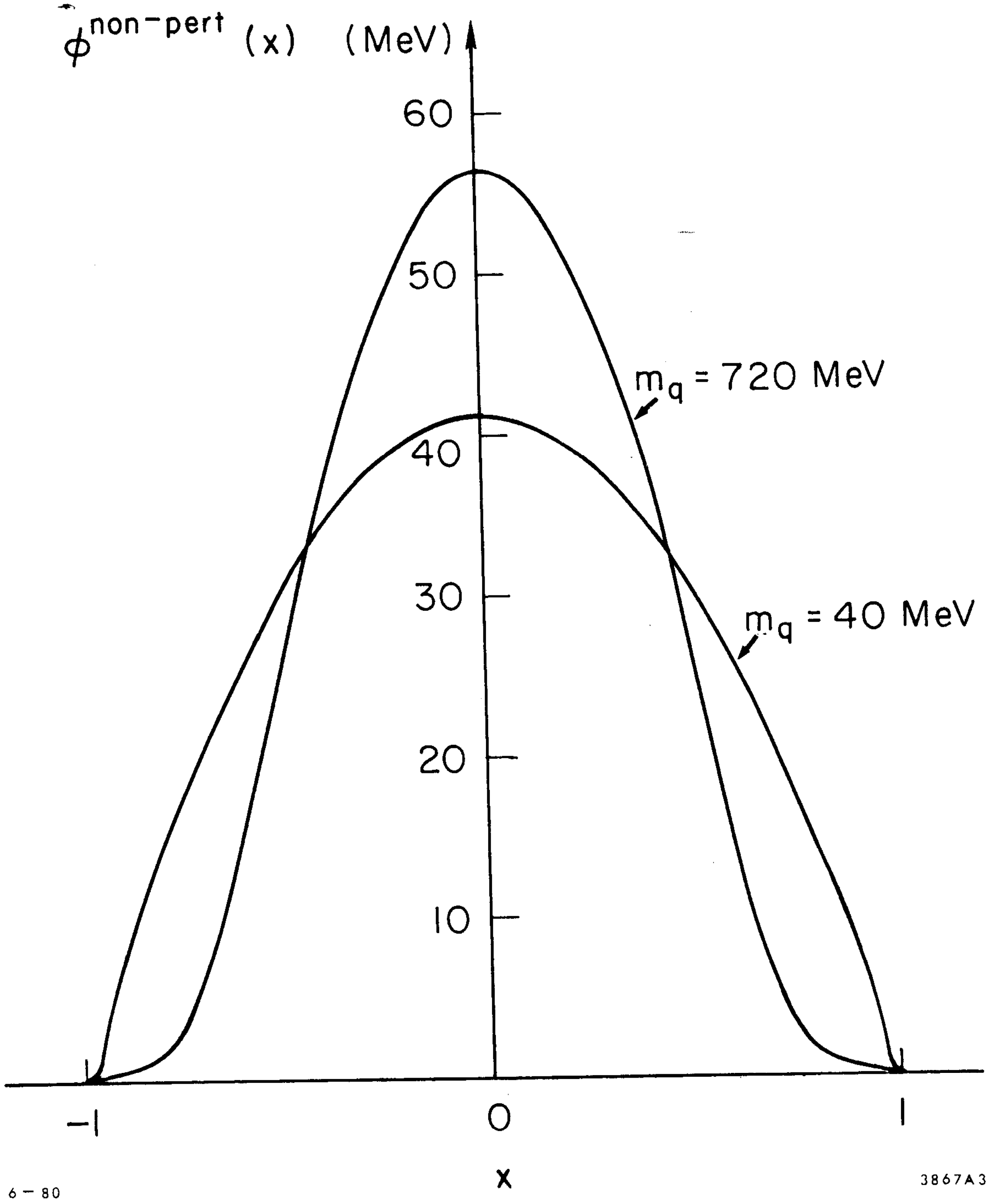
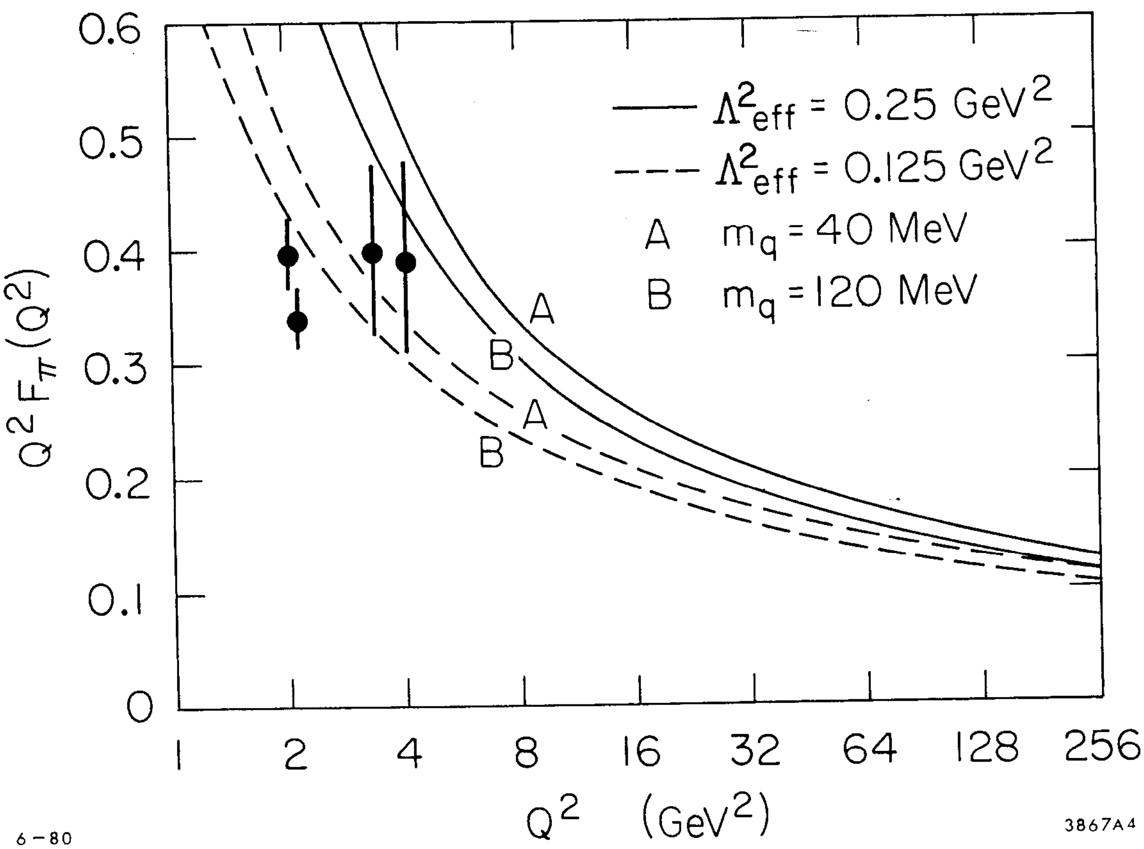


Fig. 2



$\phi^{non-pert}(x)$ (MeV)
60
50
40
30
20
10
m_q = 720 MeV
m_q = 40 MeV
-1
0
1
x
6 - 80
3867A3

Fig. 3



0.6
0.5
0.4
0.3
0.2
0.1
0
$Q^2 F_\pi(Q^2)$
Λ^2_{eff} = 0.25 GeV^2
Λ^2_{eff} = 0.125 GeV^2
A m_q = 40 MeV
B m_q = 120 MeV
A
B
A
B
1
2
4
8
16
32
64
128
256
Q^2 (GeV^2)
6-80
3867A4

Fig. 4