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# SCALING OF DAMPING RINGS FOR COLLIDING LINAC BEAM SYSTEMS*

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## ABSTRACT

The scaling of damping rings for colliding linear accelerator beams is discussed. It was found that low repetition rates should be used to achieve the highest luminosity. Only for very high values of the repetition rate above $10^4$ $sec^{-1}$ we find the luminosity to increase again with pulse repetition rate. In this regime, however, damping rings are not useful anymore and the operating cost for the facility is very high without gain in luminosity as compared to low repetition rates. Therefore for reasons of economics as well as performance low pulse repetition rates should be chosen and research and development should be denoted to solve tolerance problems encountered at these low pulse rates.

## INTRODUCTION

In recent years much effort has been spent to evaluate the feasibility of colliding linac beams as a means to reach high center of mass energies for research in high energy physics[1-4]. The concept of electron positron storage rings which has been very successful in the past is heading for a severe "collision" as the energy is increased with our concern on energy consumption as well as the cost to supply that energy. This comes from the fourth power dependence of the synchrotron radiation power with the particles energy. In contrast the energy required for a colliding linac beam system increases only linear with the beam energy since the synchrotron radiation is eliminated. For both kinds of colliding beam facilities the luminosity is given by

$$\mathscr{L} = \frac{N^2 \nu_{rep}}{4\pi\sigma^2} \tag{1}$$

where N is the number of particles per bunch ($N^+ = N^- = N$), $\nu_{rep}$ is the number of collisions per unit time and $\sigma$ is the beam radius. Here we assume for simplicity a round beam. We also assume in both cases a gaussian or at least bell shaped density distribution up to about $2\sigma$.

In general we find that the number of particles per bunch is about 2 to 3 orders of magnitude larger in a storage ring than in present day linear accelerators. This is specially true for positrons. The repetition rate in a storage ring is very high (larger than $10^4$ $sec^{-1}$) and it will be difficult and costly to raise the pulse rates of linear accelerators to the order of $10^4$ pps or more.

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In order to still get a useful luminosity a very small beam size $\sigma$ is required. The beam size is given by $\sigma = (\beta^*\varepsilon)^{\frac{1}{2}}$ with $\beta^*$ the betatron function at the interaction point and $\varepsilon$ the beam emittance. The betatron function $\beta^*$ can be reduced by not more than about one order of magnitude below values in storage rings because of limitations in the focussing system. This leaves us with the beam emittance which has to be made much smaller than in storage rings. Fortunately this is possible in principle and we will discuss in this paper the limitations associated with the production of small emittance beams.

Electrons and more so positrons are produced with a beam emittance much larger than required for the use in colliding linac beams. Therefore, it was suggested to "cool" the beams in specially designed storage rings. The cooling is achieved by the synchrotron radiation and the way the lost energy is replenished by the rf accelerating system. Particles radiate synchrotron radiation photons along their trajectories and therefore loose transverse momenta as they emit a photon. In the accelerating cavity, however, the lost momentum is replaced only along the longitudinal axis. Therefore the net effect on the particle due to photon emission and rf acceleration is a reduction in transverse momentum or a damping of the transverse beam size. There is however also an excitation effect due to the quantized emission of photons. This causes an increase in the energy spread of the beam which in turn together with the focussing properties of the lattice causes quantum fluctuations of the transverse momenta of the particles and therefore an increase in beam size. Both damping and quantum excitation lead to an equilibrium beam size. By proper choice of the storage ring parameters the beam size or beam emittance can be made very small. This choice of parameters is possible only in a dedicated storage ring. In a storage ring for high energy physics the demand for high luminosity requires large beam sizes in order not to exceed the destructive forces when the beams collide.

## DAMPING RING PARAMETERS

The normalized beam emittance in a storage ring is given by

$$\psi = \psi_D \, (1-e^{-2n}) + \psi_o \, e^{-2n} \tag{2}$$

Here we define $\psi = \varepsilon E$ where $\varepsilon \cdot \pi$ is the area of the beam in the phase plane as given by particles with amplitude and angles of value $\sigma$ and $\sigma'$ or less. $\psi_D$ is the equilibrium beam emittance in the storage ring and $\psi_o$ the beam emittance of the injected beam. $n = t/\tau$ where $t$ is the total time the particles are in the damping ring and $\tau$ the transverse damping time. The equilibrium emittance of the damping ring is given by[5]:

$$\psi_D = \frac{1}{2} \frac{C_q}{(mc^2)^2} E_D^3 \frac{\langle \mathcal{H}/\rho_D^3 \rangle}{\langle 1/\rho_D^2 \rangle} \tag{3}$$

where $E_D$ is the energy and $\rho_D$ the bending radius of the damping ring. The factor 1/2 appears because we assume full coupling of the horizontal and vertical betatron motion. The quantity $\mathcal{H}$ is determined by the lattice of the ring. For a great variety of strong focussing lattices it was found[6] that $\mathcal{H}$ reaches a minimum for a betatron phase advance per cell of 120 to 160°. Since we are interested in the minimum beam emittance we take from Ref. 6

$$<\mathcal{H}> = H \cdot L_m^3/\rho_D^2 \tag{4}$$

with $H_{min} = 1.25$. $L_m$ is the length of the bending magnets. With $F_m$ the magnet filling factor we have $<1/\rho_D^2> = F_m/\rho_D^2$ and with B the magnetic field in the bending magnet we get:

$$E_D/\rho_D = .29975\ B \tag{5}$$

Here and for the rest of this report we measure the energy in GeV, lengths in meter and magnetic fields in tesla. Equation (3) then becomes:

$$\psi_D = \frac{C_q\ H\ 0.29975^3}{2(mc^2)^2} \frac{(B\ L_m)^3}{F_m} = C_\psi \frac{(B\ L_m)^3}{F_m} \tag{3a}$$

with $C_\psi = 1.26 \cdot 10^{-11}$ Tesla$^{-3}$m$^{-2}$.

The total damping time n is determined by

$$n = \frac{N_B\ N_S}{\nu_{rep}\tau} \tag{6}$$

where $N_B$ is the number of bunches being damped in a damping ring at any one time and $N_S$ the number of damping rings involved. From storage ring theory we have $\tau(\sec) = 2111 \cdot F_m E_D^3/\rho_D^2$ and together with Eq. (5) we get:

$$n = 56.86\ F_m\ B^3 \rho_D\ N_B\ N_S \tag{6a}$$

We have tacitly assumed so far that we can always achieve a betatron phase advance per cell of 120 to 160° independent of the energy of the damping ring. There are however limits on the strength of the quadrupoles. We assume a simple FODO lattice with a quadrupole length of just half the length of the bending magnets. We have then with $2\mu$ the betatron phase per cell[7]:

$$\frac{1}{\sin\mu} \frac{4\ F_m}{k\ L_m^2} = \frac{4\ F_m}{L_m^2} \frac{E_D}{.29975\ g} = \frac{4\ F_m\ B\rho_D}{L_m^2\ g} \tag{7}$$

where k is the quadrupole strength and g the quadrupole gradient in Tesla/m. If we insert Eq. (7) into Eq. (6a) we get:

$$n = \frac{14.21}{\sin\mu} \frac{g\, N_B\, N_S}{\nu_{rep}} \cdot (B\ L_m)^2 = \frac{1}{2}\,\tilde{n}(B\ L_m)^2 \tag{8}$$

From Eq. (8) we see that the quadrupole gradient should be as large as possible to maximize n, e.g., the damping effect.

Using $BL_m = X$ we get from Eqs. (8), (3a) and (2):

$$\psi = \frac{C_\psi}{F_m} X^3(1-e^{-\tilde{n}X^2}) + \psi_o\, e^{-\tilde{n}X^2} \tag{9}$$

It is obvious that there is a minimum beam emittance $\psi$ between X = 0 and very large values of X.

Having chosen values for g, $\mu$, $\psi_o$ and $F_m$ we can find an optimum value for $X = BL_m$ as a function of the repetition rate $\nu_{rep}/N_B N_S$. Consequently we find $\rho_D$ from Eq. (7). We still have to make a choice on B since only $BL_m$ is determined. From Eq. (7) we find that for economic reasons a large value for the magnetic field should be chosen to reduce the size of the damping ring ($\rho_D \sim B^{-3}$) and because of Eq. (5) to reduce also the energy ($E_D \sim B^{-2}$).

## NUMERICAL RESULTS

Numerical computations have been performed along the lines discussed in the previous section.

The following free parameters have been chosen:

| | |
|---|---|
| $B = 2$ Tesla | $g = 100$ Tesla/m |
| $F_m = 0.5$ | $2\mu = 130^o$ |
| $\beta^* = 0.01$ m | $N = 5\cdot10^{10}$ |
| $\psi_o = 5\cdot10^{-6}$ m GeV | |

The parameters of the damping ring depend very sensitive on the choice of the emittance of the injected beam. In this report it is assumed that particles are not recycled after the collision. Therefore $\psi_o$ is given by the emittance of the positrons as determined by the positron target and the initial focussing system. Studies performed at SLAC on the limitations in target design and parameters on the initial focussing system show that the required number of positrons can be obtained only if the positron emittance is as large as about:

$$\psi_o = 5\cdot10^{-6}\ \text{m GeV} \tag{10}$$

Various recycling methods have been proposed to regain the energy of the colliding beams after collision and/or the positrons which are not easy to produce in large numbers and small emittance[2,8].

For a realistic colliding linac beam facility, however, recycling the beams does not seem to be feasible. In order to get a useful luminosity a substantial amount of beam strahlung has to be tolerated which leads to an energy spread of the beam in the order of a percent. This energy spread is increased according to the deceleration in the recycling process. Apart from the inability of a storage ring to accept that large an energy spread it is very unlikely that an intense beam reaches the low energy end of the linear accelerator. To avoid beam break up a strong focussing system is required along the linear accelerator. For increasing energy spread in the beam this focussing system gets less effective and will cause loss of the beam. We therefore assume in this paper that the positrons have to be generated from a target for each colliding pulse.

For the above mentioned free parameters we now get for each value of the repetition rate $\nu_{rep}/N_B N_S$ a minimum beam emittance. Using this emittance we can calculate an optimum normalized luminosity as a function of $\nu_{rep}/N_B N_S$. The result is shown in Fig. 1.

At very large repetition rates we find a steep increase in luminosity. This is the regime where a damping ring is not useful because the minimum equilibrium beam emittance obtainable for these pulse repetition rates is larger than $\psi_0$. This however, is also the regime where it is very expensive to run a colliding linac beam facility because the operating power increases linear with the pulse repetition rate. At lower repetition rates we find the surprising result that the luminosity decreases as $\nu_{rep}$ is increased. This result reflects the fact that for increasing repetition rate there is less and less time for the beam to damp and therefore the ratio of $\nu_{rep}/\psi$ and with it the luminosity decreases.

From Fig. 1 we therefore find that high luminosity is reached where the operating costs are low. There is however another limit. As the repetition rate is reduced the beam emittance and therefore the beam spot size is reduced too. A practical lower limit will be reached due to either stability tolerances, or beam beam disruption, or excessive beam strahlung or emittance growth in the final focussing system or some other reason.

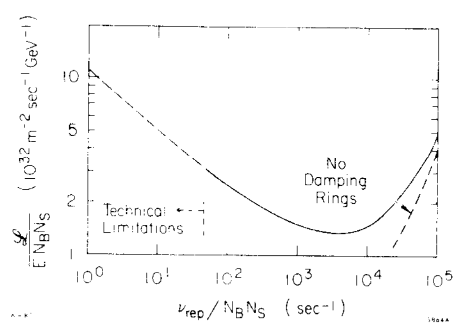


Fig. 1 Optimum luminosity for linear colliding beam facilities.

As an example we get for $E = 350$ GeV and $\nu_{rep}/N_B N_S = 200\ sec^{-1}$ the following parameters:

| N | $5 \cdot 10^{10}$ | | $10^{11}$ | |
|---|---|---|---|---|
| $\sigma_z$ (mm) | 1 | 2 | 1 | 2 |
| $\mathscr{L}_o/N_B N_S \left(\frac{10^{36}}{m^{-2} sec^{-1}}\right)$ | .07 | .07 | .28 | .28 |
| $\delta$ | .084 | .024 | .096 | .048 |
| D | .38 | .76 | .76 | 1.52 |

Here $\delta$ is the energy spread due to beam strahlung, D the disruption parameter and $\sigma_z$ the bunch length.[4)]

The actual luminosity may be enhanced due to the beam beam pinch effect. For the disruption parameters listed an increase in the luminosity by a factor 2 to 6 can be expected. Beyond that the luminosity can be further increased only by increasing the number of bunches $N_B$ and/or the number of damping rings $N_S$ involved. In either case the actual pulse rate scales like $N_B \cdot N_S$ and so does the electric power required to run the facility.

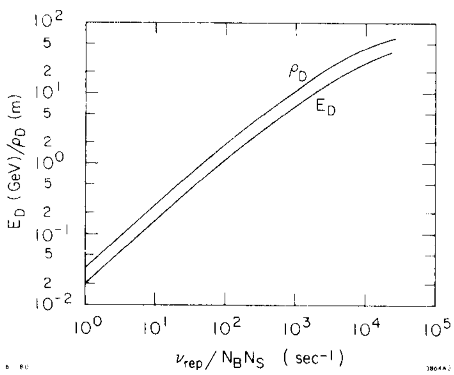


Fig. 2 Energy and radius of damping rings.

In Fig. 2 we show the optimum bending radius $\rho_D$ and the energy $E_D$ of the damping ring. We observe that the damping rings get rather sizable as the repetition rate goes up. The beam emittance is shown in Fig. 3 and the length of the bending magnet in Fig. 4. The length of the bending magnets and the quadrupoles ($L_q = L_m/2$) are rather short but should not create any technical difficulty.

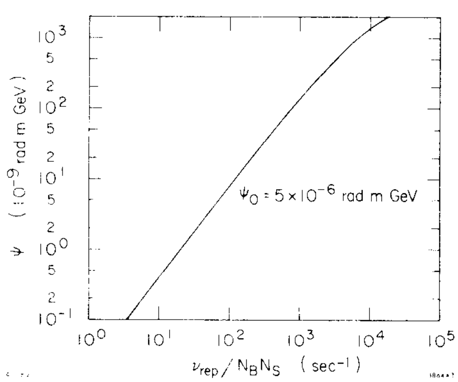


Fig. 3 Minimum beam emittance in damping rings.

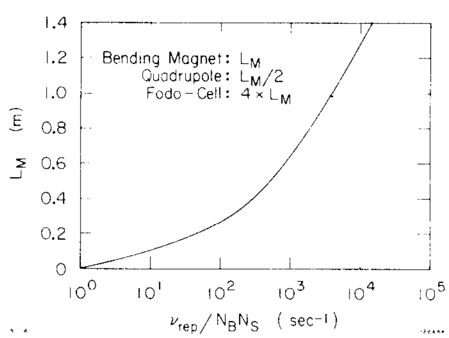


Fig. 4 Magnet length for damping rings.

## CONCLUSION

Optimum parameters for damping rings in colliding linac beam facilities have been calculated. It was found that small pulse repetition rates should be chosen for maximum luminosity. This is because for high repetition rates there is not time enough in the damping ring to get a small beam emittance. Clearly at very low repetition rates there must be other limitations like the spot size at the collision point, the small size of the damping ring which makes injection and ejection impossible, emittance growth in linac and final focus system and the beam strahlung to name only a few. Still the parameters for repetition rates in the order of $\nu_{rep}/N_B N_S \approx 10^2$ to $10^3$ sec$^{-1}$ seem to be feasible. Since, however, the electrical power required to operate a colliding linac beam facility increases proportional to the pulse rate every effort should be made to design systems with low repetition rate. From Fig. 1 we find that for any value of $\nu_{rep}$ we can get high luminosity if we choose large values for the number of bunches $N_B$ and/or for the number of damping rings. Both choices however have their problems depending on the size of the damping rings and have to be decided on for a specific facility.

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