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A COSMOLOGICAL LOWER BOUND ON THE HIGGS BOSON MASS*

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ABSTRACT

Cosmological considerations imply that the Weinberg-Salam Higgs boson mass $m_H \gtrsim 9$ GeV. If this bound were violated, the symmetry breaking phase transition would occur only after extreme supercooling, resulting in too high a ratio of entropy to baryon number.

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The Weinberg-Salam theory of weak and electromagnetic interactions contains one parameter which is not yet experimentally determined--the mass of the Higgs boson. Some time ago it was noted that when this mass is sufficiently small the scalar self-coupling is weak enough for the radiative corrections to the effective potential to have a significant effect.[1] In the one-loop approximation the effective potential takes the form[2]

$$V = \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4!}\phi^4 + B\phi^4\left[\ln\frac{\phi^2}{M^2} - \frac{25}{6}\right] \qquad (1)$$

where

$$B = \frac{3}{64}\left(\frac{e^2}{4\pi}\right)^2\left(\frac{2+\sec^4\theta_W}{\sin^4\theta_W}\right) - \frac{1}{16\pi^2}\Sigma f_i^4 \quad , \qquad (2)$$

and M is an arbitrary mass related to λ by

$$\left.\frac{\partial^4 V}{\partial\phi^4}\right|_{\phi=M} = \lambda \quad . \qquad (3)$$

The first term in B represents the contribution of gauge boson loops, while the second arises from loops involving fermions with Yukawa couplings f_i. Since the Yukawa coupling is proportional to the fermion mass, the latter term is non-negligible only if there are quarks or leptons much more massive than those so far discovered. The contribution from scalar loops is negligible for the case in which we are interested and has been omitted. Assuming that there are no heavy fermions and taking $\sin^2\theta_W = 0.23$, one finds $B = 1.74\times 10^{-4}$.

For sufficiently small λ, V has a minimum at a non-zero value of ϕ, which we denote by σ. (Experimently, $\sigma = (\sqrt{2}\,G)^{-1/2} = 246$ GeV.) By choosing $M=\sigma$, we may rewrite Eq. (1) in the form

$$V = B\left\{\frac{1}{2}\alpha\sigma^2\phi^2 - \frac{1}{4}(\alpha+2)\phi^4 + \phi^4\ln(\phi^2/\sigma^2)\right\} \quad . \tag{4}$$

The Higgs mass is then given by $m_H^2 = (4-\alpha)(2B\sigma^2)$. If $\alpha > 0$, the effective potential has an additional minimum at $\phi = 0$. In order that the symmetry breaking state $\phi = \sigma$ be the absolute minimum, we must require that $\alpha < 2$; this gives the lower bound on m_H obtained by S. Weinberg.[1] In this paper we will consider the case $(0 < \alpha < 2)$ in which there is a metastable $SU_2 \times U_1$ symmetric vacuum, and show that cosmological considerations can be used to obtain a more stringent lower bound.

We assume that the universe was at one time very hot $(T \gg \sigma)$. At such temperatures V must be modified[3,4] by the addition of a term

$$V_T = \frac{3}{2\pi^2}T^4\left\{2I(g\phi/2T) + I\left(\sqrt{g^2+g'^2}\,\phi/2T\right)\right\} \quad , \tag{5}$$

where

$$I(y) = \int_0^\infty dx\, x^2 \ln\left[1 - e^{-\sqrt{x^2+y^2}}\right] \quad . \tag{6}$$

(The contribution from scalar loops is again negligible and has been omitted. We have also neglected all fermion loop contributions.) From the approximation

$$V_T = -\frac{\pi^2}{10}T^4 + \frac{1}{32}(g'^2 + 3g^2)T^2\phi^2 + \mathcal{O}(T\phi^3) \tag{7}$$

valid for $T \gg \phi$, it is clear that at $T \gg \sigma$ the minimum at $\phi = \sigma$ disappears. Thus, a hot universe would begin in an $SU_2 \times U_1$ symmetric phase. As it cooled, there would be a transition to the spontaneously broken phase, with the critical temperature T_c being that at which the values of the effective potential at the two minima are equal. One finds $T_c = k(\alpha)(2-\alpha)^{1/4}\sigma$, where $k(\alpha)$ varies monotonically from .081 to .087 as α varies from 2 to 0. The phase transition is first-order and would proceed by the formation and growth of bubbles of the new phase. During this process the expansion of the universe would cause the temperature to continue to fall, eventually rising again with the release of the latent heat of the phase transition.

The formation of bubbles of true vacuum is a tunneling process. Callan and Coleman[5] have shown that its rate at zero temperature can be obtained by solving the Euclidean equation

$$0 = \left(\frac{\partial^2}{\partial t^2} + \nabla^2\right)\phi - \frac{\partial V}{\partial \phi} \tag{8}$$

with the boundary condition that ϕ approach 0 as $x^2 + t^2$ goes to infinity. The probability per unit time per unit volume of bubble nucleation is given by $f_0 = C\exp(-A_0)$, where A_0 is the four-dimensional Euclidean action corresponding to the tunneling solution of Eq. (8) with the least action, which we assume to be O(4) symmetric. The determination of C requires calculation of radiative corrections; it is of the form $\gamma\sigma^4$, with γ a dimensionless number expected to be of order unity.[6] Solving Eq. (8) by computer gives the values of A_0 shown in Fig. 1.

The extension of this calculation to finite temperature requires some modifications. First, V must be replaced by the finite temperature effective potential. Second, the Euclidean problem becomes one with periodicity $1/T$ in imaginary time, so we can only require $O(3)$ rather than $O(4)$ symmetry. For large T we expect the dominant solution to be independent of t with an exponent of the form $A(T) = E(T)/T$, where E is the energy calculated using the finite temperature effective potential.[7] The behavior of $E(T)/T$ as a function of T and α is shown in Fig. 2. For sufficiently small T solutions with approximate $O(4)$ symmetry and $A \approx A_0$ will dominate.

We see that nearly all choices of α in the range we are considering lead to extremely small bubble nucleation rates and thus to a rather long lifetime for the metastable symmetric state. It has been argued[8] that this lifetime must be short compared to 10^{10} years (the generally accepted age of the universe), but this reasoning seems rather imprecise. The figure of $\sim 10^{10}$ years is obtained by assuming adiabatic expansion throughout the history of the universe and so cannot be used to place a limit on how long the universe could have remained in a metastable state.[9] However, the continued expansion of the universe would lead to a supercooling which could have observable consequences. Current theories show that the present baryon number to entropy ratio of $\sim 10^{-8}$ can be explained as a relic of CP- and baryon number-violating processes at energies of the order of 10^{14} GeV.[10] If at some time after the baryon number excess was produced the universe supercooled to a temperature T_{sc} and then rose to T_{rec} (of order T_c) after the release of the latent heat, the baryon number to entropy ratio would have been reduced by a factor of $(T_{rec}/T_{sc})^3$. Thus the observed baryon number excess puts a bound on how much supercooling is acceptable; certainly T_{sc} cannot be more than two or three orders of magnitude below T_c. We shall see that such extreme supercooling is quite possible in a rapidly expanding universe.

The extent of supercooling depends on the rate of expansion of the universe. With a Robertson-Walker metric (in comoving coordinates) $d\tau^2 = dt^2 - R^2(t)d\vec{x}^2$, the expansion is governed by the equation[11]

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3M_P^2}\rho \tag{9}$$

where $M_P = 1.2 \times 10^{19}$ GeV is the Planck mass. The energy density may be written as $\rho = \rho_0 + (\pi^2/30)\mathcal{N}T^4$. Here $\mathcal{N}$ is the number of effectively massless degrees of freedom, with fermion degrees of freedom each counting 7/8; for three families of leptons and quarks, $\mathcal{N} = 106.75$. The vacuum energy density has the same effect as a cosmological constant; to agree with observation we must take $\rho_0 = 0$ for the spontaneously broken minimum and therefore $\rho_0 = \frac{1}{4}(2-\alpha)B\sigma^4$ for the symmetric state. Note that for small temperatures ($T < .033\,(2-\alpha)^{1/4}\sigma$) the energy density is dominated by the vacuum energy, leading to an R which grows exponentially with time.

If f(t) is the rate of bubble nucleation per unit time per unit (physical) volume, the fraction of space remaining in the symmetric phase at time t is[12]

$$p(t) = \exp\left\{-\int_{t_0}^{t} dt_1 f(t_1) R^3(t_1) V(t_1,t)\right\} \tag{10}$$

where

$$V(t_1,t) = \frac{4\pi}{3}\left[\int_{t_1}^{t} dt_2 \frac{1}{R(t_2)}\right]^3 \tag{11}$$

is the coordinate volume at time t of a bubble formed at time t_1. (The bubbles are formed with a negligible initial radius and expand with a speed which rapidly approaches that of light.) It is convenient to convert from time to temperature. If we assume adiabatic expansion (RT = constant), then Eq. (9) leads to

$$\dot{T} = -\chi T g(T)$$
$$\chi = \left(\frac{8\pi}{3}\frac{\rho_0}{M_P^2}\right)^{1/2} \tag{12}$$

where $g(T) \approx 1$ for small T. This gives

$$p(T) = \exp\left\{-b\int_T^{T_c} dT_1 \frac{e^{-A(T_1)}}{g(T_1)T_1^4}\left[\int_T^{T_1}\frac{dT_2}{g(T_2)}\right]^3\right\} \tag{13}$$

with $b = (4\pi/3)(\gamma\sigma^4/\chi^4)$. Numerically, $b \approx 10^{74}/(2-\alpha)^2$. As T falls from T_c, A(T) decreases to a minimum at a temperature T* and then rises, leveling off at A_0. The potentially dominant contributions to the T_1 integral in Eq. (13) come from the regions $T_1 \approx T^*$ and $T_1 \approx 0$. Approximating the integral by the sum of these gives (for $T < T^*$)

$$p(T) \approx \exp\left\{-b\left[\frac{\sqrt{2\pi}\ he^{-A(T^*)}}{T^*\sqrt{A''(T^*)}}\left(\frac{T^*-T}{T^*}\right)^3 + e^{-A_0}\ \ln\left(\frac{T_c}{T}\right)\right]\right\} \tag{14}$$

where h is a correction factor of order unity. Thus, if $A(T^*) < \ln b$, $p(T)$ decreases rapidly to 0 and supercooling ceases by $T \approx T^*$. On the other hand, if $A(T^*) > \ln b$, the bubbles formed at high temperature are not sufficient to complete the phase transition and supercooling ceases only when the effect of the low temperature bubbles becomes large; since this effect grows only logarithmically, supercooling will continue to exceedingly small temperatures.

Therefore, to avoid excessive supercooling we must require (1) that $A(T) < \ln b$ for some T and (2) that the temperature at which this happens be not too far below the critical temperature (certainly no less than $10^{-3}\ T_c$). The first condition alone excludes essentially the entire range of parameters in which we are interested; even for α as small as 0.01, $A(T^*)$ is too large by a factor of seven. The second requirement eliminates even the theoretically attractive case $\alpha = 0$;[13] the universe would supercool to $T \approx 3 \times 10^{-8}\ T_c$ before $A(T) \approx \ln b$.

We have implicitly assumed that the effective potential (1) with its finite temperature corrections continues to give a good description of the relevant physics as the temperature decreases several orders of magnitude below the critical temperature. Witten[14] has suggested that this may not be the case if in this temperature range the universe undergoes a phase transition from unbroken to broken chiral symmetry. He argues that the effect of the broken chiral symmetry would be to facilitate the Weinberg-Salam phase transition, and that $\alpha = 0$ may not in fact be ruled out. Except for very small α (no more than 0.01), our arguments and conclusions concerning positive α would not be affected.

Although the data shown in Figs. 1 and 2 were obtained using $\sin^2\theta_W = 0.23$, variation of θ_W well beyond the present experimental uncertainty has little effect. For $0.13 < \sin^2\theta_W < 0.91$, the qualitative conclusions are unchanged. Also, it is easy to show that the effect of heavy fermion loops, which we have omitted, would be to further inhibit bubble formation.

We thus have a lower bound for the Higgs mass

$$m_H^2 \gtrsim 8B\sigma^2 \tag{15}$$

with B given by Eq. (2). For $\sin^2\theta_W = 0.23$ and no heavy fermions, this gives $m_H \gtrsim 9$ GeV.

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FIGURE CAPTIONS

Fig. 1. A_0 as a function of α.

Fig. 2. (a) $E(T)/T$ as a function of temperature for several values of α.

(b) $E(T^*)/T^*$ as a function of α. T^* is the temperature at which $E(T)/T$ reaches its minimum value.

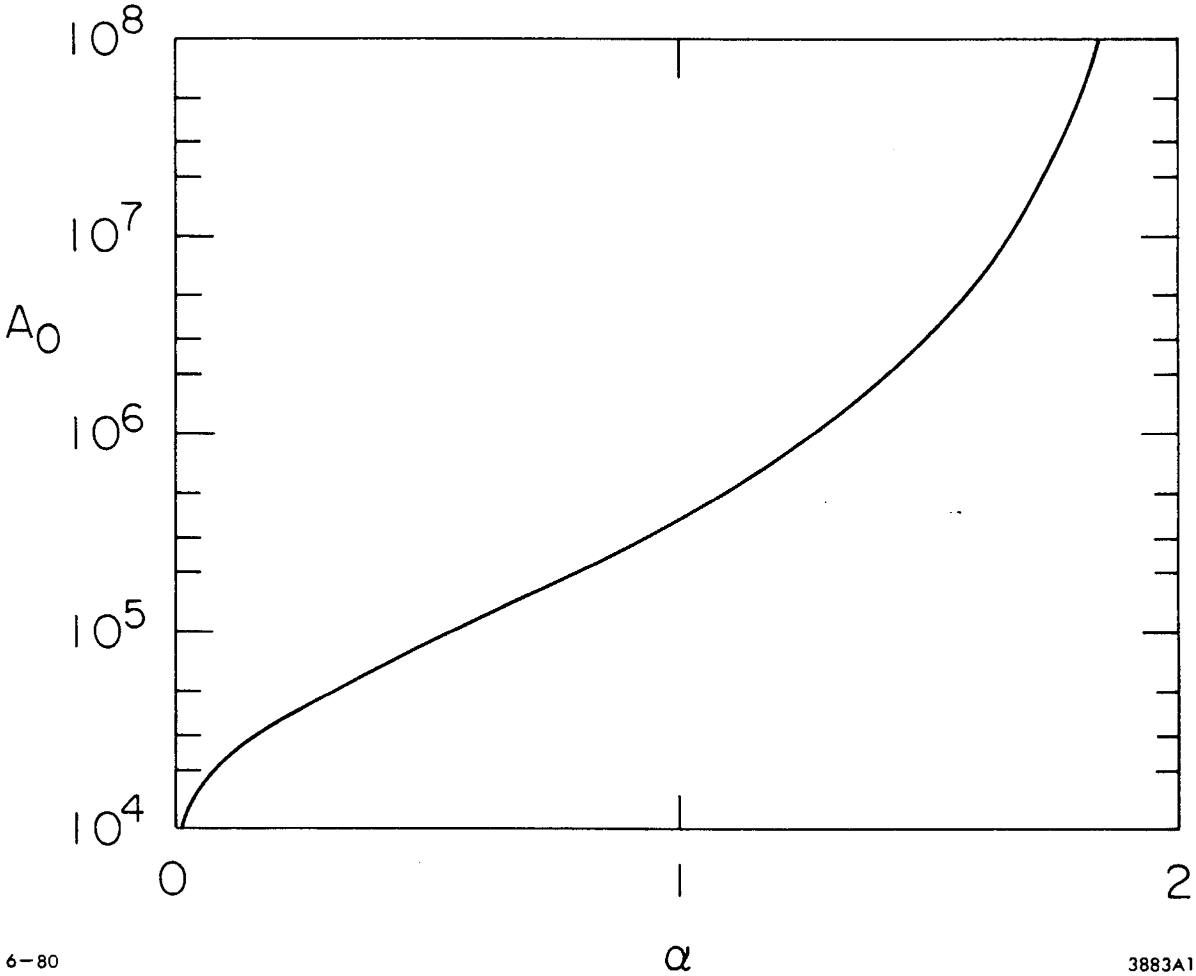

Fig. 1

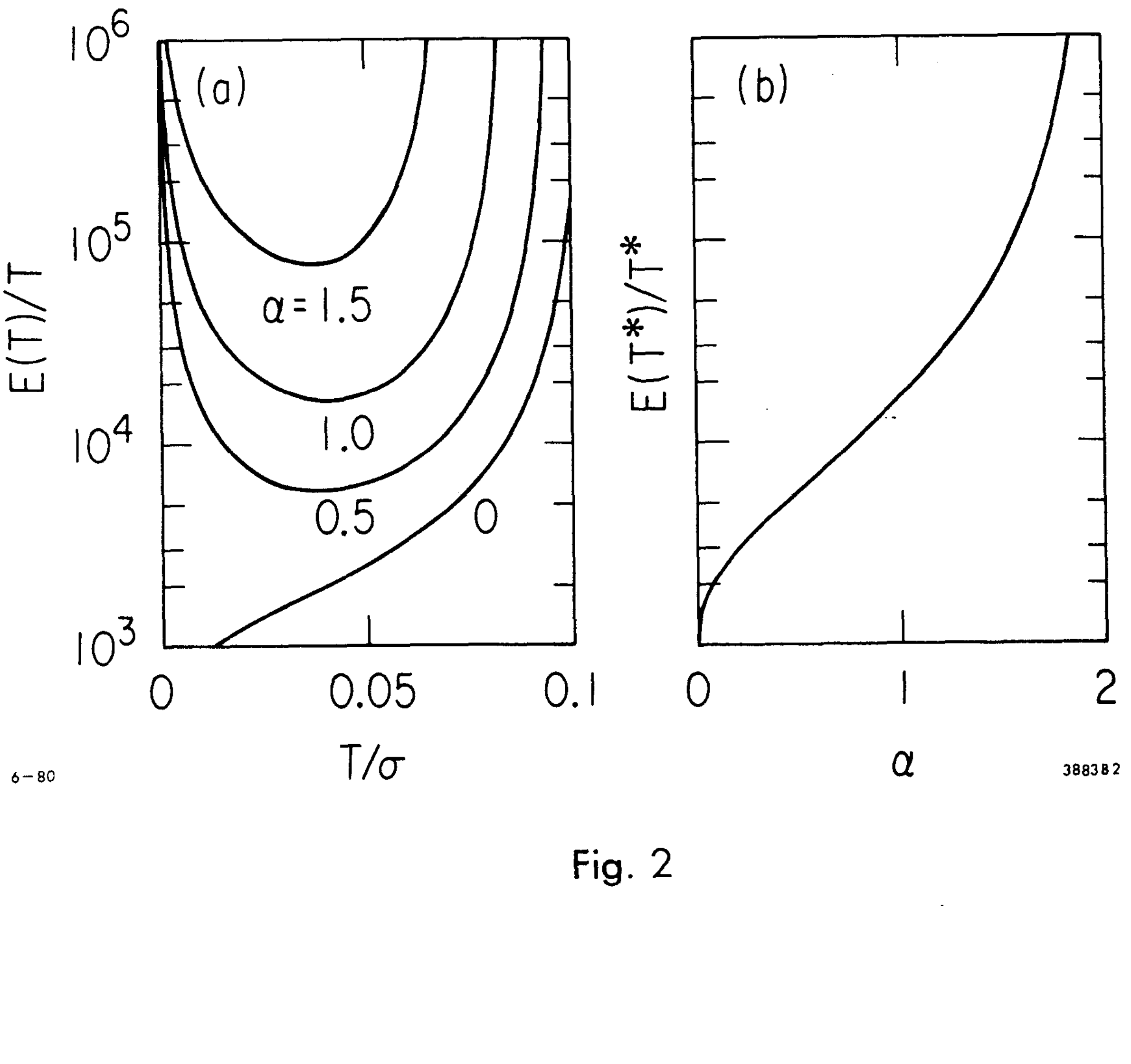

Fig. 2