SLAC-PUB-2515, Erratum/Addendum
May 1980
(T/E)

MINIMAL RELATIVISTIC THREE PARTICLE EQUATIONS*

James V. Lindesay and H. Pierre Noyes
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

(Submitted to the International Conference on the Few Body Problem, Eugene, Oregon, August 17-23, 1980.)

ON THE RELATIVISTIC EFIMOV EFFECT

James V. Lindesay
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ERRATUM:

Please note the following changes:

The term in the integrand for $\hat{W}^{(J=0)}(k;M)$ which reads

$$\frac{\sqrt{s'}}{\left(\frac{1}{a}\right) - \sqrt{m^4 - \frac{s^4}{4}}} \quad \text{should read} \quad \frac{\sqrt{s'}}{\left(\frac{1}{a}\right) - \sqrt{m^2 - \frac{s'}{4}}}$$

ADDENDUM:

If one defines $\alpha \equiv |1/ma|$, the following relation can be shown to hold for the number of bound states N:

$$N \leq \frac{1}{\alpha^2}\int dk \int dk' \; R(k,k';M)$$

where

$$R(k,k';M) = 4\pi\sqrt{\frac{\sqrt{ss'}}{\varepsilon\varepsilon'}} \log\left(\frac{\sqrt{m^2+(k+k')^2}+\varepsilon+\varepsilon'-M}{\sqrt{m^2+(k-k')^2}+\varepsilon+\varepsilon'-M}\right)$$

and the variables k and k' range from 0 to $(M^2-m^2)/2M$. Thus N is finite for all $M < 3m$ and $\alpha \neq 0$.

---

* Work supported by the Department of Energy, contract DE-AC03-76SF00515.

SLAC-PUB-2515
May 1980
(T/E)

MINIMAL RELATIVISTIC THREE PARTICLE EQUATIONS*

James V. Lindesay and H. Pierre Noyes
Stanford Linear Accelerator Center, Stanford, CA. 94305

(Submitted to the International Conference on the Few Body Problem, Eugene, Oregon, August 17 - 23, 1980.)

ON THE RELATIVISTIC EFIMOV EFFECT*

James V. LINDESAY, Stanford Linear Accelerator Center, Stanford, CA 94305

We show that a relativistic, 3-particle equation with minimal 2-body input has unique solutions for bound states, and exhibits the Efimov effect (that there is an infinite accumulation of 3-particle bound states as the scattering length between the pairs increases without bound). The theory is Lorentz invariant and unitary. The physical input (for the three equal mass case) is in one parameter μ, the mass of the 2-body bound (or virtual) states. The Efimov effect for the nonrelativistic theory occurs for $(|a|/R) \to \infty$, where a is the scattering length and R an effective range parameter, whereas in this theory the behavior is for $\mu \to 2m$, where m is the mass of any one of the three particles.

When attempting to obtain bound state solutions to zero range on-shell Faddeev equations[1] using nonrelativistic kinematics one encounters divergent integral equations due to the infinite limit of integration. However, relativistic kinematics in a theory which preserves the clustering properties gives finite, well-defined results which reduce to the nonrelativistic equations, only with finite integral cutoffs. This momentum cutoff results from the kinematic condition that in the pair center of mass system the spectator can have any momentum $0 \le |k_\alpha| \le \infty$, which transforms into a finite range of spectator momentum in the CMS of the total system[2]. Since the Faddeev equations involve a finite (fixed number) particle theory, we do not encounter infinite self energies, and have a unitary theory if 2-body unitarity is preserved.

The results given below involve the assumptions that there are three equal mass distinct particles which point interact only via s-waves, and which form 2-body bound states of mass μ via a separable unitary t matrix. The homogeneous equation for the J=0 bound state energies M is

$$\hat{W}^{(J=0)}(k;M) = -4\pi \int_0^{\frac{M^2-m^2}{2M}} \frac{dk'}{\varepsilon'} \frac{k'}{k} \frac{\sqrt{s'}}{(\frac{1}{a})-\sqrt{m^4-\frac{s^4}{4}}} \log\left(\frac{\sqrt{m^2+(k+k')^2}+\varepsilon+\varepsilon'-M}{\sqrt{m^2+(k-k')^2}+\varepsilon+\varepsilon'-M}\right) \hat{W}^{(J=0)}(k';M)$$

$$\varepsilon = \sqrt{k^2+m^2},\ \varepsilon' = \sqrt{k'^2+m^2},\ s' = M^2+m^2-2M\varepsilon'$$

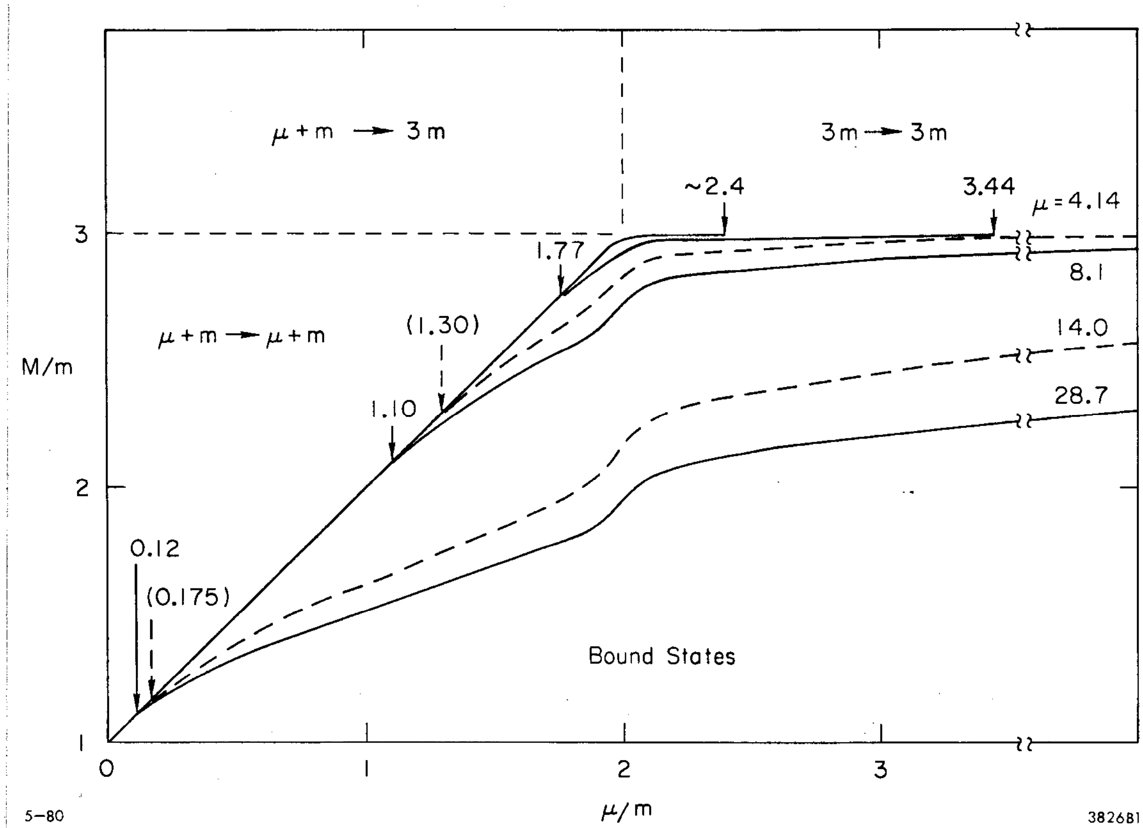
where the scattering length can be obtained from $1/a^2 = |m^2-(\mu^2/4)|$ with $a < 0$ for $\mu > 2m$. The solutions were obtained by Gaussian quadratures using Jacobi polynomials as weight functions. The lowest energy eigensolutions to the equation given above are represented by the solid curves on the graph below. The dashed curves represent the lowest energy solutions to the equations when only two of the pairs interact. This corresponds to replacing -4π by -2π in the equation. The boundaries of the graph are defined by the kinematic limits for the existence of bound states. For $M \ge 3m$ there is 3-particle elastic scattering if $\mu > 2m$ and breakup if $\mu < 2m$. For $M \ge m + \mu$ there is 2-particle (bound state $\mu + m$) elastic scattering and rearrangement. One feature of the curves is the behavior for $\mu \sim 2m$. As μ decreases through 2m, the bound state energy begins to drop sharply (more tightly bound system) until the relativistic kinematic cutoff levels the curve.

* Work supported by the Department of Energy, contract DE-AC03-76SF00515.

We conclude that the results presented are well defined and reasonable, with minimal input. The kinematic regions for elastic scattering and breakup will be examined. It should be noted that more physical input will be needed to include crossing, as is discussed by Noyes[3] in a contribution to this conference.

*Work supported by the Department of Energy, contract number DE-AC03-76SF00515.

1. H. P. Noyes, SLAC-PUB-2358, January 1980 (Rev.).
2. D. D. Brayshaw, Phys. Rev. D18, 2638 (1978).
3. H. P. Noyes, contribution to this conference.



ON UNITARITY, CROSSING, AND UNIQUENESS in a Lorentz-invariant 3-particle context*

H. Pierre NOYES, Stanford Linear Accelerator Center, Stanford, CA 94305

Renormalizable quantum field theories in the weak coupling limit are manifestly covariant, crossing invariant, and preserve unitarity order by order. The resulting perturbation series is not uniformly convergent; general non-perturbative solutions have not been constructed. S-matrix theories, although both Lorentz and crossing invariant, introduce a non-linear dynamics which implies an infinite number of degrees of freedom; unique solutions have not been proved to exist. Brayshaw's[1] finite range and separable models, which are exactly unitary and preserve the cluster property, imply a non-unique phenomenology; no attempt is made to include crossing. Here we attempt an alternative approach which retains these advantages with the ultimate aim, not yet achieved, of incorporating the same parameter content as field theory and reducing to the renormalized perturbation series in the weak coupling limit.

The program started with the observation[2,3] that the zero range or "on shell" limit of the Faddeev equations implies only free particle scattering wave functions, and hence can be made Lorentz invariant simply by using relativistic kinematics. The Faddeev summation convention in the multiple scattering series excludes "self-energy diagrams," and guarantees unitarity as was shown by Freedman, Lovelace and Namyslowski[4]. The zero range limit of the Karlsson-Zeiger equations[5] defines unique solutions in terms of two particle phase shifts, binding energies, and reduced widths[6]. The equations can be derived directly from a zero range boundary condition on the three particle asymptotic wave function[7] and the same technique yields four particle equations[8] which Vanzani finds equivalent to one of the standard forms[9]. However, the unitarity of the zero range KZ equations has been proved only if the two particle amplitudes $t(z-\tilde{p}^2)$ when continued to negative energies have no singularities other than bound state poles. Under the same restriction, the zero range KZ and Faddeev equations are equivalent, and the FLN unitarity proof also holds. The scattering length approximation $q \text{ ctn } \delta = -1/a$ meets the restriction, but Lindesay[10] has shown that the non-relativistic equations then diverge, an example of the well known Thomas[11] singularity. Introduction of a Castillejo-Dalitz-Dyson resonance in the physical region makes the equations convergent, but non-unique. Further, Bugg[12] has shown that the analytic continuation of the empirical nucleon-nucleon amplitudes exhibit the expected negative energy singularities arising from meson exchanges. Hence, even as a phenomenology, the non-relativistic zero range theory is not applicable to nuclear physics.

Fortunately for our program, requiring the cluster property restricts the spectator momentum k in the three particle zero momentum system to the range[1] $0 \le k \le (M^2-m^2)/2M$ where M is the invariant four-momentum and m the mass of the spectator. Hence the relativistic theory converges even in the scattering length case. Physically, the theory is finite because we use particle functions as a basis. These functions can be obtained from the field functions by a non-local operator which smears them out over a region of size h/mc. Since the particle functions, rather than the field functions, go over to the Schroedinger particle functions in the non-relativistic limit, this is consistent with our approach. At a deeper level, we note that the same restriction arises from requiring that the invariant two-particle subsystem four-momentum squared $s = 4(q^2+m^2)$ be non-negative, an obvious requirement from the point of view of general relativity.

In a contribution to this Conference, Lindesay[13] presents the three particle bound state spectrum for the minimal theory of three equal mass particles in which either two or three bind to make a "meson" of mass μ. If, in the first case, we call the distinguished particle which binds to either of the others an "antiparticle," this amounts to a Fermi-Yang[14] model for the meson. One iteration gives the term $\bar{\delta}_{ab}\, t_a G_0 t_b$ which at the double bound state pole in $t_a t_b$ gives the "crossed" diagram for meson-particle scattering, since G_0 is simply the particle-antiparticle-particle propagator. To get the direct diagram, we must insert a unitary three particle scattering amplitude as a new channel, and require it to have a pole at M=m, i.e. require the meson and particle to bind making a three particle bound state with the mass m. An obvious choice is $t = ((-M^2)^{\frac{1}{2}}-m)^{-1}$. It is easy to show that with this added channel the FLN unitarity proof still holds. Two iterations then yield $t_a G_0 t_0 G_0 t_b$, which at the double pole is the direct diagram. Whether this is <u>precisely</u> equivalent to the lowest order of some field theory is still under investigation.

In the special case that $\mu=m$, if we also insist on a three body bound state at M=m, the upper limit on the spectator momentum $(M^2-m^2)/2M$ vanishes, and there is no integral equation! Examining the double pole, we find that the kinematics requires s=0; in this case the direct and the crossed diagrams coincide. We can then claim that if particle and antiparticle bind to make mass m, and this meson again binds with a particle to make mass m, we have "bootstrapped" the particle as a bound state of particle antiparticle and particle[15]. If this argument is accepted, we have found in our theory an analogy to the lowest level of the combinatorial hierarchy[16].

It should be noted that, so far, our one parameter model does not have the same content as field theory, since the latter allows both g^2 and μ to be picked arbitrarily. To insure two-particle unitarity we have had to choose $e^{i\delta}\sin\delta/q = (-1/a - iq)^{-1}$. If we introduce a second parameter by multiplying this by g^2, then unitarity is lost unless we interpret the formula as an elastic amplitude $(\eta e^{2i\delta}-1)/2iq$ with an open inelastic channel. It is easy to show that for $\eta \le 1$ we must require $g^2 \le 1$. But such an amplitude makes sense for us only is a four particle theory, e.g. with $m \pm m \to m + m + (m\bar{m} = \mu)$, or a five particle theory, e.g. with $m + \mu \to m + (m\bar{m} + m\bar{m} = 2\mu)$. So we must go to such theories before we can reach our initial objective. To approach QED we must first show that a $\mu \to 0$ limit makes sense, and introduce spin. Nevertheless, we believe that the results presented at this Conference are encouraging enough to continue with this difficult task.

*Work supported by the Department of Energy, contract number DE-AC03-76SF00515.

1. D. D. Brayshaw, Phys. Rev. D18, 2638 (1978).
2. H. P. Noyes, in Few Particle Problems, I. Slaus et al., eds. North Holland, Amsterdam, 1972, p. 122.
3. H. P. Noyes, Foundations of Physics 5, 37 (1975) [Erratum 6, 125 (1976)].
4. D. Z. Freedman, C. Lovelace and J. Namyslowski, Nuovo Cimento 43A, 258 (1966).
5. B. R. Karlsson and E. M. Zeiger, Phys. Rev. D11, 939 (1975).
6. H. P. Noyes and E. M. Zeiger, in Few Body Nuclear Physics, G. Pisent, V. Vanzani & L. Fonda, eds., Intl. Atomic Energy Auth., Vienna, 1978, p. 153.
7. H. P. Noyes, SLAC-PUB-2358, January 1980 (Rev.).
8. H. P. Noyes, unpublished, 1979.
9. V. Vanzani, private communication to HPN, 1979.
10. J. V. Lindesay, private communication to HPN, 1979.
11. L. H. Thomas, Phys. Rev. 47, 903 (1935).
12. D. V. Bugg, Nucl. Phys. B5, 29 (1968).
13. J. V. Lindesay, contribution to this Conference.
14. E. Fermi and C. N. Yang, Phys. Rev. 76, 1739 (1949).
15. H. P. Noyes, SLAC-PUB-2277, March 1979.
16. T. Bastin, H. P. Noyes, J. Amson and C. W. Kilmister, International Journal of Theoretical Physics (in press).