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TRIPLE GLUON COUPLING, ADLER-BELL-JACKIW ANOMALY, AND POLARIZED DEEP INELASTIC SCATTERING*

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ABSTRACT

We discuss an unusual effect of triple gluon coupling and the Adler-Bell-Jackiw anomaly on the flavor singlet part of the polarized deep inelastic scattering structure function $\nu G_1(Q^2,x)$. Namely, the x-integral $I_S(Q^2)$ of this function is Q^2-independent both in parton model and leading logarithm calculations, but the first order non-leading logarithm calculation produces a term growing like $(-\ell n\, \ell n\, Q^2)$, dominating over the parton model contributions at large Q^2. The detection of this unusual term will amount to an experimental confirmation of the existence of triple gluon coupling and the Adler-Bell-Jackiw anomaly. Technically, this term comes from a new axial vector gluon operator which we introduce in the Wilson expansion. Other results of this paper include a discussion of mass-sensitive and mass-insensitive structure functions and the derivation of the expression for, and the relations between, some of these structure functions.

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1. INTRODUCTION

Much is known[1] about deep inelastic inclusive lepton nucleon scattering processes when the beam and the target are unpolarized. When both of them are polarized, experimental information[2] is as yet very scanty and theoretical consideration becomes much more complicated. Instead of only three (two) structure functions for neutrino (electron) scattering as is the case in the unpolarized case, we have now nine (four) structure functions. Is there any interesting physics that can be fished out from this complicated mess?

We shall show that there is.[3] Specifically, the x-integral $I_S(Q^2)$ of the flavor-singlet part of the structure function $\nu G_1(Q^2,x)$ behaves in an abnormal and unexpected manner, because of the existence of triple gluon coupling and the Adler-Bell-Jackiw (ABJ) anomaly.[4] If we can detect this peculiar Q^2-variation, then we will have an experimental way of confirming not only the existence of triple gluon coupling, so central in showing the non-Abelian nature of the theory, but also in confirming experimentally the existence of such a highly theoretical object as the ABJ anomaly. For that reason we think that it is worthwhile to try to measure $I_S(Q^2)$ experimentally.

The peculiar effect mentioned above is the following. Both in the parton model and in the leading log approximation (LLA) calculation in quantum chromodynamics (QCD), $I_S(Q^2)$ is independent of Q^2; viz., it scales. In the first non-leading log order (NLLA), one would normally expect an order $\bar{\alpha}_S(Q^2)$ (running coupling constant) correction which vanishes at large momentum transfer Q^2. Instead, one finds a correction term of order $(-\ln\ln Q^2)$, dominating over the parton or LLA contributions for large Q^2. Moreover, the sign of this term is opposite to the sign of the parton or LLA terms. This makes it hopeful for this special effect to be detectable and recognizable.

The detailed explanation of the origin of this effect is contained in Section 4. Briefly, it is as follows. $I_S(Q^2)$ actually measures the total helicity carried by the quarks and anti-quarks. This helicity turns out to be Q^2-independent in LLA because of chirality conservation and charge conjugation invariance. In the meantime, the total gluon helicity grows like $\ln Q^2$ as a consequence of the triple gluon coupling, though chirality conservation prevents the pair-created quarks to inherit any of this growing helicity, thus making it undetectable by the photon in LLA calculations. But on account of the ABJ anomaly, chirality conservation is broken in NLLA, and this growing helicity of the gluons can be passed, in a diluted form, to the quarks. Thus we see in NLLA a $\ln \ln Q^2$ growth of quark helicity. Moreover, it is a special signature of the anomaly that the helicity is flipped when it is passed from the gluon to the quark. This accounts for the minus sign associated with the $\ln \ln Q^2$ term.

The final formula for $I_S(Q^2)$ containing this $-\ln \ln Q^2$ term appears in Eq. (71). It is obtained by the Wilson expansion and renormalization group techniques. To carry out such a calculation, we find it necessary to introduce a gluon axial vector operator $a_\rho(x)$ hitherto ignored. This operator is the same one used in discussions of topological solutions of Yang-Mills fields.[5] It mixes with the normal quark axial vector operator on account of the ABJ anomaly and triple gluon coupling. The property of this operator and the calculation of $I_S(Q^2)$ will be discussed in Section 4.

In Section 3, we obtain results for all the structure functions in terms of the Wilson coefficients and parton distribution coefficients. From this we are able to derive some relations for these other structure functions.

The calculations in Section 4 will be carried out by putting the quark masses equal to zero. It turns out to be a non-trivial question whether such may be done even in the leading twist, or even parton model, calculations. This problem together with the definition of the structure functions and their parton model expressions are given in Section 2.

Finally, the mathematical evaluation of an ordered exponential is given in Appendix A and graphical discussions of the ABJ anomaly for QCD is contained in Appendix B.

2. STRUCTURE FUNCTIONS AND PARTON MODEL RESULTS

The inclusive cross section for a neutrino to scatter from a polarized nucleon of momentum P^μ, mass M, and polarization direction S^μ, is determined by the current-current correlation function

$$\begin{aligned} W_{\mu\nu} &= \frac{1}{4M\pi}\int d^4x\, e^{iq\cdot x}\, \langle P,S \left| \left[J^+_\mu(x), J_\nu(o)\right] \right| P,S\rangle \\ &= -g_{\mu\nu}W_1 + \frac{1}{M^2}P_\mu P_\nu W_2 - \frac{i}{2M^2}\varepsilon_{\mu\nu\rho\sigma}\, q^\rho P^\sigma W_3 \\ &+ \frac{i}{M}\varepsilon_{\mu\nu\rho\sigma}\, q^\rho S^\sigma G_1 + \frac{i}{M^3}\varepsilon_{\mu\nu\rho\sigma}\, q^\rho\left(q\cdot PS^\sigma - q\cdot SP^\sigma\right)G_2 \\ &+ \frac{1}{M} g_{\mu\nu} q\cdot SG_3 - \frac{1}{M}\left(P_\mu S_\nu + P_\nu S_\mu\right)G_4 \\ &+ \frac{1}{M^3}\left[P_\mu P_\nu q\cdot S - \frac{1}{2}\left(P_\mu S_\nu + P_\nu S_\mu\right)P\cdot q\right] G_5 + \frac{i}{M}\varepsilon_{\mu\nu\rho\sigma}\, P^\rho S^\sigma G_6 \end{aligned} \tag{1}$$

Here J_μ is the charged weak current operator, and q^μ is the momentum transferred from the initial neutrino. We use metric $g_{00} = +1$, $\varepsilon_{0123} = +1$, and normalize S so that $S^2 = -1$, $S \cdot P = 0$. Terms proportional to q_μ or q_ν have been dropped from the above expression and will continue to be dropped below.

The tensor in Eq. (1) is related to the discontinuity of the time ordered product matrix element

$$\widetilde{W}_{\mu\nu} = \frac{i}{4M\pi} \int d^4x \; e^{iq\cdot x} \langle P,S|T\left(J^+_\mu(x) J_\nu(o)\right)|P,S\rangle$$

$$= -g_{\mu\nu}\widetilde{W}_1 + \frac{1}{M^2} P_\mu P_\nu \widetilde{W}_2 + \ldots = \frac{1}{4M\pi} T_{\mu\nu} \qquad (2)$$

$$W_{\mu\nu} = -i \; \mathrm{Disc}\left(\widetilde{W}_{\mu\nu}\right) \qquad (3)$$

In Eq. (2), we have written out explicitly only two out of the nine structure functions. The rest is as in (1), except that we will put a tilde on top of the structure functions appearing in Eq. (2).

For antineutrino scattering, Eqs. (1) and (2) are still valid if we interchange J^+ and J. Consequently,

$$\widetilde{W}^\nu_{\mu\nu}(q,P) = \widetilde{W}^{\bar{\nu}}_{\nu\mu}(-q,P)$$

$$W^\nu_{\mu\nu}(q,P) = -W^{\bar{\nu}}_{\nu\mu}(-q,P) \quad ,$$

and

$$\widetilde{W}^\nu_i(Q^2,x) = +\widetilde{W}^{\bar{\nu}}_i(Q^2,-x) \qquad (i=1,2,3)$$

$$\widetilde{G}^\nu_i(Q^2,x) = +\widetilde{G}^{\bar{\nu}}_i(Q^2,-x) \qquad (i=1,4) \qquad (4)$$

$$\widetilde{G}^\nu_i(Q^2,x) = -\widetilde{G}^{\bar{\nu}}_i(Q^2,-x) \qquad (i=2,3,5,6) \quad ,$$

where $Q^2 = -q^2$, $x = Q^2/2q\cdot P$. The relations for the untilde structure functions are as in (4) except with all the signs reversed.

For electroproduction, one simply replaces the weak current by the electromagnetic current. Because of parity conservation and current conservation, the correlation functions $W_{\mu\nu}$ (and $\tilde{W}_{\mu\nu}$) contain in this case only four structure functions: W_1, W_2, G_1 and G_2. All the other terms should be dropped from Eqs. (1) and (2). The crossing symmetry properties are the same as in (4) if we replace both the neutrino and the antineutrino interactions by electromagnetic interactions.

It is straightforward to calculate the neutrino scattering structure functions in the naive parton model. If s is the polarization vector and $p = \xi P$ is the momentum of the incoming quark, and if q is the momentum of the incoming virtual boson, then Fig. 1 yields

$$T_{\mu\nu} = \sum_i \sum_{s=\pm S} \int_0^1 \frac{d\xi}{\xi} q_s^i(\xi) \left(T_a^i + T_b^i\right)_{\mu\nu} \tag{5}$$

with

$$\frac{1}{2}(T_a)_{\mu\nu} = \frac{1}{4}\mathrm{Tr}\left[(1+\gamma_5)\gamma_\nu(m+\gamma p)(1+\gamma_5\gamma s)(1+\gamma_5)\gamma_\mu\left(m+\gamma(q+p)\right)\right]\Big/\left(m^2-(q+p)^2-i\epsilon\right)$$

$$= 2\left\{im\,[\mu,\nu,q+p,s] - m\langle\mu,s,\nu,q+p\rangle + \langle\mu,p,\nu,q+p\rangle - i[\mu,\nu,q,p]\right\}\Big/\left(m^2-(q+p)^2-i\epsilon\right)$$

and

$$T_b(q,p)_{\mu\nu} = T_a(-q,-p)_{\nu\mu} \quad , \tag{6}$$

where

$$[\mu,\nu,\alpha,\beta] \equiv \varepsilon_{\mu\nu\alpha\beta}$$

$$[\mu,\nu,\alpha,A] \equiv [\mu,\nu,\alpha,\beta]A^\beta$$

$$\langle\mu,\nu,\alpha,\beta\rangle = g_{\mu\nu}g_{\alpha\beta} + g_{\mu\beta}g_{\nu\alpha} - g_{\mu\alpha}g_{\nu\beta}$$

Here i sums over all flavors of quarks and antiquarks that contribute to the corresponding diagrams (only d and s quarks contribute to (a), and only $\bar{u}$ and $\bar{c}$ quarks contribute to (b) in the four quark model). If we take the discontinuities of (5) in x and use (1)-(3), then we get the parton formulas for the neutrino structure functions:

$$\begin{aligned}
2MW_1 &= q(x) \\
\nu W_2 &= xq(x) \\
\nu W_3 &= q'(x) \\
\nu G_1 &= \frac{1}{2}\,\Delta q(x)\left(\frac{m}{Mx}\right) = \frac{1}{2}\,\Delta q(x) \\
\frac{\nu^2}{M}\,G_2 &= 0 \\
\nu G_3 &= \frac{1}{2}\,\Delta q'(x)\left(\frac{m}{Mx}\right) = \frac{1}{2}\,\Delta q'(x) \\
\nu G_4 &= \frac{1}{2}\,x\Delta q'(x)\left(\frac{m}{Mx}\right) = \frac{1}{2}\,x\Delta q'(x) \\
\frac{\nu^2}{M}\,G_5 &= 0 \\
\nu G_6 &= \frac{1}{2}\,x\Delta q(x)\left(\frac{m}{Mx}\right) = \frac{1}{2}\,x\Delta q(x)
\end{aligned} \tag{7}$$

where

$$\begin{aligned}
q_\pm(x) &= 2\left(d_\pm(x) + s_\pm(x) + \bar{u}_\pm(x) + \bar{c}_\pm(x)\right) \\
q'_\pm(x) &= 2\left(d_\pm(x) + s_\pm(x) - \bar{u}_\pm(x) - \bar{c}_\pm(x)\right) \\
\Delta q(x) &= q_+(x) - q_-(x) \quad , \qquad q(x) = q_+(x) + q_-(x) \\
\Delta q'(x) &= q'_+(x) - q'_-(x) \quad , \qquad q'(x) = q'_+(x) + q'_-(x) \quad ,
\end{aligned} \tag{8}$$

$\nu = P\cdot q/M$ and $q_\pm^i$ is the positive/negative helicity distribution function of the ith quark. For antineutrino scattering, these formulas are still valid if interchange d and u, c and s, and their corresponding antiparticles. For electron scattering, Eq. (7) is again valid for

W_1, W_2, G_1 and G_2, provided we interpret the distribution functions to be

$$q_\pm(x) = \sum_{i=1}^{2f} e_i^2 \, q_\pm^i(x) \tag{9}$$

summed over f flavors of quarks and f flavors of antiquarks with charges e_i.

These parton formulas are of course well known. The point of all of these though is that the G_i's in Eq. (7) involve the quark mass m, which suggests that we will have trouble if we insist on putting $m = 0$ in our calculations from the outset. As has been pointed out by other authors,[6,7] this trouble has its origin in the difficulty of polarizing a zero mass quark in a direction perpendicular to its direction of motion. In fact, if we parametrize the momentum p of a particle with mass m by its rapidity y,

$$p^\mu = m(\mathrm{ch}y,\ 0,\ 0,\ \mathrm{sh}y) \quad , \tag{10}$$

then the polarization vector along the direction of motion is

$$s_\parallel^\mu = \pm(\mathrm{sh}y,\ 0,\ 0,\ \mathrm{ch}y) \tag{11}$$

and that in the perpendicular direction is

$$s_\perp^\mu = \pm(0,\ 1,\ 0,\ 0) \text{ or } \pm(0,\ 0,\ 1,\ 0) \tag{12}$$

When $y \to \infty$ and/or $m \to 0$, the components in (11) become infinitely larger than those in (12), thus making it virtually impossible to polarize it transversely.

It would be very inconvenient if we could not calculate any G_i by setting $m = 0$. After all, QCD calculations are usually carried out by ignoring higher twist effects and by setting $m = 0$. Actually, since we may still polarize a zero mass quark along its direction of motion, we expect that those structure functions G_i related to longitudinal polarizations may still be calculated directly by setting $m=0$. In contrast, the structure functions G_i related to transverse polarizations must be calculated with finite m. For simplicity, we shall call the former G_i the <u>mass-insensitive</u> structure functions and the latter the <u>mass-sensitive</u> structure functions.

To be more specific, let us remark that as long as $y \gg 1$, it would be a good approximation to set:

$$s_{\parallel}^{\mu} = \pm p^{\mu}/m + O(e^{-y}) \tag{13}$$

A similar relation would be true when the symbols in Eq. (13) refer to the nucleon rather than the quark:

$$S_{\parallel}^{\mu} = \pm P^{\mu}/M + O(e^{-y}) \tag{14}$$

Substituting (14) into (1), we obtain, for $S = S_{\parallel}$,

$$W_{\mu\nu} = -g_{\mu\nu}W_1' + \frac{1}{M^2} P_{\mu}P_{\nu}W_2' + \frac{i}{2M^2} \varepsilon_{\mu\nu\rho\sigma} q^{\rho}P^{\sigma}W_3' \tag{15}$$

with

$$\begin{aligned} W_1' &= W_1 \mp \frac{\nu}{M} G_3 \\ W_2' &= W_2 \mp 2G_4 \\ W_3' &= W_3 \mp 2G_1 \end{aligned} \tag{16}$$

By polarizing both parallel and antiparallel to P, we have sufficient information to obtain from (15) and (16) the structure functions W_1, W_2, W_3, G_1, G_3 and G_4. These are then the <u>mass-insensitive</u> structure functions. The <u>mass-sensitive</u> structure functions are those that disappear from $W_{\mu\nu}$ when (14) is used, and they are G_2, G_5 and G_6.

These mass-insensitive structure functions may be computed directly in the zero mass naive parton model. To do this, we note that in the zero mass limit, spin projection operator becomes chirality projection operator, where all references to masses disappear. In other words, when (13) is used,

$$(m+\gamma p)(1+\gamma_5\gamma s_{\parallel}) = (m+\gamma p)\left(1 \mp \frac{\gamma p}{m}\ \gamma_5\right) = (m+\gamma p)(1 \mp \gamma_5) \tag{17}$$

After substituting (17) in Eq. (5), we may now set m = 0 to obtain directly

$$\frac{1}{2}\,(T_a)_{\mu\nu} = \frac{1}{4}\,\mathrm{Tr}\left\{(1+\gamma_5)\gamma_\nu\gamma p(1\mp\gamma_5)(1+\gamma_5)\gamma_\mu\gamma(p+q)\right\}\Big/\left[-(q+p)^2 - i\epsilon\right]$$

$$= \begin{cases} 0 & \text{upper sign} \\ 4\langle\mu,p,\nu,q+p\rangle - 4i[\mu,\nu,q,p] & \text{lower sign} \end{cases} \tag{18}$$

Setting p = xP and comparing Eqs. (18) and (15), we see that we can indeed obtain the relations in Eq. (7) for the mass-insensitive structure functions. The results for electromagnetic interactions are identical.

It is convenient for subsequent QCD calculations to identify flavor-singlet and flavor-non-singlet parts of the structure functions. From Eqs. (7) and (8), we see that W_3, G_3, G_4 for neutrino or anti-neutrino scatterings are always flavor non-singlets; the rest will in general contain singlet and non-singlet components. The relevant flavor combinations are listed in Table I for $f = 3$ and $f = 4$, and for neutrino, antineutrino, and electron scatterings on proton (p) and neutron (n) targets. If $N = (p+n)/2$, then for the structure function

νG_1, for example, we have

$$\frac{1}{2}\left[\nu G_1^{\nu N} + \nu G_1^{\bar{\nu} N}\right] = \frac{1}{2}\sum_{i=1}^{2f} \Delta q^i \equiv \frac{1}{2}\Delta q_s \quad , \qquad (f=4) \quad ;$$

$$\left[9\nu G_1^{eN} - \frac{3}{2\cos^2\theta_c}\nu G_1^{\nu N}\right] = \frac{1}{2}\sum_{i=1}^{2f} \Delta q^i \equiv \frac{1}{2}\Delta q_s \quad , \qquad (f=3) \quad .$$

3. OPERATOR PRODUCT EXPANSION AND SUM RULES

We use Wilson expansion and the renormalization group method to calculate the moments of the structure functions in (1). To that end, we make the following expansion of $\widetilde{W}_{\mu\nu}$:

$$\begin{aligned}
\widetilde{W}^{\mu\nu} = \frac{1}{\pi}\sum_n^{\infty} q_{\mu_1}\cdots q_{\mu_n}\left(\frac{2}{Q^2}\right)^n \sum_{a=q,G,\ldots} \Big\{ & -g^{\mu\nu}\widetilde{C}^n_{1a}\frac{1}{4M}\langle O_a^{\mu_1\cdots\mu_n}\rangle \\
& + \widetilde{C}^{n+2}_{2a}\frac{1}{MQ^2}\langle O_a^{\mu\nu\mu_1\cdots\mu_n}\rangle - \frac{i}{4M}\,\varepsilon^{\mu\nu}_{\rho\sigma}\, g^{\rho\mu_1}\,\widetilde{C}^n_{3a}\,\langle O_a^{\sigma\mu_2\cdots\mu_n}\rangle \\
& + \frac{i}{M}\,\varepsilon^{\mu\nu}_{\rho\sigma}\, g^{\rho\mu_1}\,\frac{1}{4}\left[\frac{1}{n}\widetilde{C}^n_{4a}\,\langle R_{sa}^{\sigma\mu_2\cdots\mu_n}\rangle + \widetilde{C}^n_{5a}\,\langle R_{Aa}^{\sigma\mu_2\cdots\mu_n}\rangle\right] \\
& + \frac{1}{4nM}\,g^{\mu\nu}\widetilde{C}^n_{6a}\,\langle R_{sa}^{\mu_1\cdots\mu_n}\rangle - \frac{1}{MQ^2(n+2)}\,\widetilde{C}^{n+2}_{7a}\,\langle R_{sa}^{\mu\nu\mu_1\cdots\mu_n}\rangle \\
& - \frac{1}{4MQ^2}\,\widetilde{C}^{n+2}_{8a}\,\langle R_{Aa}^{\mu\mu_1\nu\mu_2\cdots\mu_n} + R_{Aa}^{\nu\mu_1\mu\mu_2\cdots\mu_n}\rangle \\
& - \frac{i}{4MQ^2}\,\varepsilon^{\mu\nu}_{\rho\sigma}\,\widetilde{C}^{n+2}_{9a}\,\langle R_{Aa}^{\rho\sigma\mu_1\cdots\mu_n}\rangle \Big\}
\end{aligned} \qquad (19)$$

Here $\widetilde{C}^n_{ia}(Q^2/\mu^2, g^2)$ are the Wilson coefficients, g^2 is the strong coupling constant determined at the renormalization point μ^2. Three kinds of operators are used in the expansion. The operators O_a are tensor operators symmetric and traceless in all their indices; the operators R_{sa} are pseudotensor operators symmetric and traceless in all their indices; and the operators R_{Aa} are pseudotensor operators antisymmetric in the first two indices and symmetric and traceless in the remaining indices. The subscript a runs over q (quark) and G gluon) in O_a and R_{sa}, and it runs also over other degenerate indices[6] for R_{Aa}. The angular brackets around the operators indicate diagonal nucleon matrix elements for the nucleon state $|P,S\rangle$. These symmetry properties for the operators imply that their nucleon matrix elements must be of the following forms:

$$\langle O_a^{\mu_1\cdots\mu_n}\rangle \equiv 2F_a^n\left(\prod_{i=1}^{n} P^{\mu_i} - \text{Traces}\right)$$

$$\langle R_{sa}^{\mu_1\cdots\mu_n}\rangle \equiv 4M\Delta F_a^n\left(\sum_{i=1}^{n} S^{\mu_i}\prod_{j\neq i} P^{\mu_j} - \text{Traces}\right) \tag{20}$$

$$\langle R_{Aa}^{\mu_1\mu_2\cdots\mu_n}\rangle \equiv 4M\delta F_a^n\left(S^{\mu_1}P^{\mu_2} - S^{\mu_2}P^{\mu_1}\right)\left(\prod_{i=3}^{n} P^{\mu_i} - \text{Traces}\right)$$

From (2), (19) and (20), we deduce that

$$2M\tilde{W}_1 = \frac{1}{\pi}\sum_{n=0}^{\infty}\left(\frac{1}{x}\right)^n \sum_a \tilde{C}_{1a}F_a^n$$

$$\nu\tilde{W}_2 = \frac{1}{\pi}\sum_{n=1}^{\infty}\left(\frac{1}{x}\right)^n \sum_a \tilde{C}_{2a}^{n+1}F_a^{n+1}$$

$$\nu\tilde{W}_3 = \frac{1}{\pi}\sum_{n=0}^{\infty}\left(\frac{1}{x}\right)^n \sum_a \tilde{C}_{3a}^{n}F_a^{n}$$

$$\nu\tilde{G}_1 = \frac{1}{\pi}\sum_{n=1}^{\infty}\left(\frac{1}{x}\right)^n \sum_a \tilde{C}_{4a}^{n}\Delta F_a^{n}$$

$$\frac{\nu^2}{M}\tilde{G}_2 = \frac{1}{\pi}\sum_{n=2}^{\infty}\left(\frac{1}{x}\right)^n \sum_a \left[\tilde{C}^n_{5a}\delta F^n_a - \frac{n-1}{n}\tilde{C}^n_{4a}\Delta F^n_a\right]$$

$$\nu\tilde{G}_3 = \frac{1}{\pi}\sum_{n=1}^{\infty}\left(\frac{1}{x}\right)^n \sum_a \tilde{C}^n_{6a}\Delta F^n_a$$

$$\nu\tilde{G}_4 = \frac{1}{\pi}\sum_{n=1}^{\infty}\left(\frac{1}{x}\right)^n \sum_a \tilde{C}^{n+1}_{7a}\Delta F^{n+1}_a \qquad (21)$$

$$\frac{\nu^2}{M}\tilde{G}_5 = \frac{1}{\pi}\sum_{n=2}^{\infty}\left(\frac{1}{x}\right)^n \sum_a \left[\tilde{C}^{n+1}_{8a}\delta F^{n+1}_a - \frac{2n}{n+2}\tilde{C}^{n+1}_{7a}\Delta F^{n+1}_a\right]$$

$$\nu\tilde{G}_6 = \frac{1}{\pi}\sum_{n=1}^{\infty}\left(\frac{1}{x}\right)^n \sum_a \tilde{C}^{n+1}_{9a}\delta F^{n+1}_a$$

where terms down by $O(M^2/Q^2)$ are ignored. For definiteness, we shall always normalize the operators so that their <u>lowest order matrix elements</u> are

$$F^n_a = \Delta F^n_G = 1$$

$$\Delta F^n_q = \frac{1}{2} \qquad (22)$$

Here, F^n_a etc., are defined in (20) with the states in $\langle\ldots\rangle$ being taken to be the corresponding parton states. For zero mass partons, MS^μ should be interpreted as $\pm P^\mu$. See Eq. (14).

We can read out the moments of the untilde structure functions from Eq. (21). Specifically, if $\tilde{H}(Q^2,x)$ is an even/odd function of x, and if

$$\tilde{H}(Q^2,x) = \frac{1}{\pi}\sum_n h_n\left(\frac{1}{x}\right)^n \qquad (23)$$

Then $H(Q^2,x) \equiv -i \text{ Disc } \tilde{H}(Q^2,x)$ is odd/even in x and

$$\int_0^1 H(Q^2,x)\, x^{n-1}\, dx = h_n \quad , \quad n = \text{even/odd} \qquad (24)$$

Applying this to the electron or the $(\nu \pm \bar{\nu})$ structure functions, and using (5), we can read off directly from (21) the appropriate nth (even or odd, whichever is appropriate) moment of the untilde structure functions. In particular, for $\nu^2 G_2$ and $\nu^2 G_5$, we have

$$0 = \int_0^1 \nu^2 G_2(Q^2,x)\, dx = \int_0^1 \nu^2 G_5(Q^2,x)\, dx \qquad (25)$$

The sum rules in Eq. (25) had previously been obtained in Ref. 8. Our treatments here differ from those in Ref. 8 by having an extra structure function G_6 in Eq. (1), and also in having three terms containing R_{Aa} in the Wilson expansion rather than just one term as in Ref. 8. However, as we saw, the sum rules of (25) are not affected.

More relations can be derived in the leading log approximation (LLA). In the parton level, F_a^n, ΔF_a^n and δF_a^n are given by Eq. (22). $\tilde{C}_{ia}^n$ are Q^2-independent constants that can be obtained by comparing (21) with the parton result (4) and (5) after we expand $\left(m^2-(q+p)^2-i\epsilon\right)^{-1} \simeq [Q^2(1-\xi/x)]^{-1}$ in powers of $1/x$. In LLA, $\tilde{C}_{ia}^n$ becomes Q^2-dependent in a way determined by the renormalization group equation and its anomalous dimension. Since the latter depends only on the operator type involved, there are all together three types of Q^2-dependences: those involving $\tilde{C}_{ia}^n$ for $i = 1,2,3$; those for $i = 4,6,7$; and those for $i = 5,8,9$. Thus the parton relations [see (7)]

$$\begin{aligned} x(2MW_1) &= \nu W_2 \\ x(\nu G_3) &= \nu G_4 \end{aligned} \qquad (26)$$

remain valid in LAA because their Q^2-dependences are the same. On the other hand, the parton relations

$$x(\nu G_1) = \nu G_6$$
$$\nu^2 G_2 = 0 \qquad (27)$$
$$\nu^2 G_5 = 0$$

are no longer valid in LLA.

4. Q^2-DEPENDENCES OF $\int_0^1 \nu G_i(Q^2,x)\,dx$ FOR $i=1,3,4$

We shall now concentrate on the mass-insensitive polarized structure functions νG_1, νG_3 and νG_4 and shall carry out all subsequent calculations by putting the quark masses equal to zero. Moreover, we shall concentrate on their $n=1$ moments for these lead to interesting and unusual results. The parton model expressions for them are given in (7), and we see that νG_3 and νG_4 contain only flavor non-singlet parts whereas νG_1 in general contains both singlet and non-singlet parts. See Table I and the discussion at the end of Section 2.

Divide the integral of νG_i into non-singlet and singlet parts:

$$\int_0^1 \nu G_i(Q^2,x)\,dx = I^i_{NS}(Q^2) + a_i I_S(Q^2) \qquad (28)$$

Then $a_3 = a_4 = 0$ and we will choose

$$a_1 = \overline{e^2} = \frac{1}{2f}\sum_{i=1}^{2f} e_i^2 \qquad (29)$$

for electromagnetic interaction. Thus in the parton model,

$$I_s = \frac{1}{2}\,\Delta q = \frac{1}{2}\sum_{i=1}^{2f} \Delta q^i$$

represents the helicity carried by all the quarks.

Much is known about $I_{NS}(Q^2)$. Its Q^2-variation, up to the first non-leading logarithm order (NLLA), is given by[6]

$$I_{NS}(Q^2) = \text{Const}\left(1 - \bar{\alpha}_s(Q^2)/\pi\right) \tag{30}$$

The constant in (30) is even known for a particular non-singlet part. This is the Bjorken sum rule,

$$\int_0^1 \left[\nu G_1^p(Q^2,x) - \nu G_1^n(Q^2,x)\right] dx = \frac{1}{6}\left(G_A/G_V\right)\left(1 - \bar{\alpha}_s(Q^2)/\pi\right), \tag{31}$$

where G_A/G_V is the weak interaction axial vector to vector coupling constant ratio for the nucleon.

We now concentrate on the singlet part, normalized according to (29). According to (21) and (24) it is of the form

$$I_s(Q^2) = \Delta F \cdot \tilde{C} \tag{32}$$

where

$$\Delta F = \left[\Delta F_q, F_G\right]$$

$$\tilde{C} = \begin{bmatrix} \tilde{C}_q\left(Q^2/\mu^2, g^2\right) \\ \tilde{C}_G\left(Q^2/\mu^2, g^2\right) \end{bmatrix}$$

All subscripts 4 and superscripts $n=1$ in (21) have been dropped for convenience.

The Q^2-dependence of the $C_a(Q^2/\mu^2, g^2)$ satisfy the renormalization group equations. Consequently

$$\tilde{C} = WC(\bar{g}^2) \tag{33}$$

where

$$C(\bar{g}^2) \equiv \begin{bmatrix} \tilde{C}_q\left(1, \bar{g}^2(Q^2)\right) \\ \tilde{C}_G\left(1, \bar{g}^2(Q^2)\right) \end{bmatrix} \tag{34}$$

and W is given by the g'-ordered exponential matrix

$$W = \left(\exp \int_{\bar{g}(Q^2)}^{g} dg' \, \frac{\gamma(g')}{\beta(g')} \right)_+ = 1 + \sum_{k=1}^{\infty} W_k \quad , \tag{35}$$

$$W_k = \int_{\bar{g}}^{g} dg_k \, \frac{\gamma(g_k)}{\beta(g_k)} \int_{\bar{g}}^{g_{k-1}} dg_{k-1} \, \frac{\gamma(g_{k-1})}{\beta(g_{k-1})} \int \cdots \int_{\bar{g}}^{g_2} dg_1 \, \frac{\gamma(g_1)}{\beta(g_1)}, \tag{36}$$

in terms of the coupling constant β-function and the anomalous dimension matrix $\gamma(g)$ for the operators R^{μ}_{sa}.

Using (33), we can rewrite (32) as

$$I_S(Q^2) = \Delta\tilde{F} \cdot C(\bar{g}^2) = \Delta F \cdot W \cdot C(\bar{g}^2) \tag{37}$$

where

$$\Delta\tilde{F} = \Delta F \cdot W = \left[\Delta\tilde{F}_q, \Delta\tilde{F}_G \right] \tag{38}$$

The evaluation of W and $\Delta\tilde{F}$ can be done by expanding γ, β, and C in power series of g:

$$\begin{aligned} \gamma(g) &= \frac{g^2}{16\pi^2} \gamma^{(0)} + \left(\frac{g^2}{16\pi^2} \right)^2 \gamma^{(1)} + \cdots \\ \beta(g) &= -\frac{g^3}{16\pi^2} \beta_0 - \frac{g^5}{(16\pi^2)^2} \beta_1 - \cdots \\ C(\bar{g}^2) &= C^{(0)} + \frac{\bar{g}^2}{16\pi^2} C^{(1)} + \cdots \end{aligned} \tag{39}$$

The coefficients β_0 and β_1 are known,[1]

$$\beta_0 = 11 - 2f/3 \quad , \qquad \beta_1 = 102 - 38f/3 \quad , \tag{40}$$

but the anomalous dimensions and the Wilson coefficients have to be calculated.

The running coupling constants in LLA and NLLA are[1]

$$\frac{\bar{\alpha}_s(Q^2)}{4\pi} = \frac{\bar{g}^2(Q^2)}{16\pi^2} = \frac{1}{\beta_0 \ln \frac{Q^2}{\Lambda^2}}$$

$$\Lambda^2 = \mu^2 \exp\left[-\frac{16\pi^2}{\beta_0 g^2}\right] \qquad \text{(LLA)} \qquad (41)$$

$$g^2 = \bar{g}^2(\mu^2)$$

and

$$\frac{\bar{\alpha}_s(Q^2)}{4\pi} = \frac{1}{\beta_0 \ln \frac{Q^2}{\Lambda^2}} - \frac{\beta_1 \ln \ln \frac{Q^2}{\Lambda^2}}{\beta_0^3 \ln^2 \frac{Q^2}{\Lambda^2}}$$

$$\Lambda^2 = \mu^2 \exp\left[-\frac{16\pi^2}{\beta_0 g^2} + \frac{\beta_1}{\beta_0^2} \ln\left(\beta_o g^2\right)\right] \qquad \text{(NLLA)} \qquad (42)$$

$$g^2 = \bar{g}^2(\mu^2)$$

To proceed further, we need the specific forms of the singlet operator R_{sa}. This is given in Ref. 8 to be

$$R_{sq}^{\mu_1\cdots\mu_n} = i^{n-1}\left(\bar{q}\gamma^{\mu_1}\gamma_5 D^{\mu_2}\cdots D^{\mu_n} q + \text{Permutations} - \text{Traces}\right)$$

$$R_{sG}^{\mu_1\cdots\mu_n} = i^{n-1}\,\epsilon^{\mu_1\alpha\beta\gamma}\left(G^a_{\alpha\beta} D^{\mu_2}\cdots D^{\mu_{n-1}} G^{a\mu_n}_{\gamma} + \text{Permutations} - \text{Traces}\right) \qquad (43)$$

with g,G being the quark and gluon fields, D the covariant derivative, and a the color index. Note that the gluon operator in (43) is not defined for n = 1. To do this, we note that only diagonal matrix elements are required in inclusive reactions, and for the diagonal matrix elements for $n=1$, we can take the gluon operator to be

$$\tilde{a}_\mu(x) = -\varepsilon_{\mu\alpha\beta\gamma}\left[G_a^{\alpha\beta}(x)B_a^\gamma(x) - \frac{g}{3}C_{abc}B_a^\alpha(x)B_b^\beta(x)B_c^\gamma(x)\right] \tag{44}$$

with

$$G_a^{\alpha\beta} = \partial^\alpha B_a^\beta - \partial^\beta B_a^\alpha + gC_{abc}B_b^\alpha B_c^\beta \tag{45}$$

It follows from (44) and (45) that the divergence of a_μ is given simply by

$$\partial^\mu a_\mu(x) = -\frac{1}{2}\varepsilon_{\alpha\beta\gamma\delta}G_a^{\alpha\beta}(x)G_a^{\gamma\delta}(x) \tag{46}$$

Note that the operator $a_\mu(x)$ is the familiar one used in the discussion of topological solutions of the Yang-Mills fields.[5] Note also that its diagonal matrix elements are gauge invariant.

The quark operator for $n=1$ is just the familiar singlet axial vector current operator

$$A^\mu(x) \equiv R^\mu_{sq}(x) = \bar{q}(x)\gamma^\mu\gamma_5 q(x) \tag{47}$$

whose divergence is given by the Adler-Bell-Jackiw (ABJ) anomaly relation to be (See Appendix A)

$$\partial_\mu A^\mu(x) = \frac{\alpha}{4\pi}\left(-2T(R)\right)\ \partial_\mu a^\mu(x) \tag{48}$$

Here $\alpha = g^2/4\pi$ and $T(R) = f/2$. This relation is also reflected in the result of the calculation of the triangle diagrams of Fig. 2. These diagrams are calculated in Appendix A, and the results are indicated in Figs. 2 and 3. We see that these results are consistent with Eq. (48).

The operators A_ρ and $\mathcal{a}_\rho$ are respectively the quark and the gluon helicity operators in the sense that their lowest order matrix elements, for massless free quarks and free gluons with momentum p and helicity ±1, are

$$\begin{aligned} \langle p\pm a|A_\rho(0)|p\pm b\rangle &= \delta_{ab}\bar{u}_\pm(p)\gamma_\rho\gamma_5 u_\pm(p) = \left(\pm\frac{1}{2}\right)4p_\rho\delta_{ab} \\ \langle p\pm a|\mathcal{a}_\rho(0)|p\pm b\rangle &= \delta_{ab}(\pm 1)4p_\rho \end{aligned} \tag{49}$$

The relation (48) therefore states that gluon helicities may be converted into quark helicities whenever the ABJ anomaly exist. Moreover, because of (13), this also shows that the normalization conditions (22) are satisfied. Combining this with (29), we may now interpret

$$\Delta F = \left[\frac{1}{2}\,\Delta q, \Delta G\right]$$

and

$$\Delta\tilde{F} = \left[\frac{1}{2}\,\Delta\tilde{q}, \Delta\tilde{G}\right] \tag{50}$$

as the helicity carried by the quarks and the gluons.

Now we turn to the computation of the anomalous dimensions. In LLA, the anomalous dimensions are calculated from the $O(g^2)$ matrix elements of the operators A_ρ and $\mathcal{a}_\rho$. Because of Ward's identity for A_ρ, $\gamma^{(0)}_{qq} = 0$. The gluon matrix element of A_ρ is shown in Fig. 2(a) and it is finite. Hence also $\gamma^{(0)}_{Gq} = 0$. Next, Fig. 4 yields a divergent term given by ($\alpha_s = g^2/4\pi$, $\Lambda' =$ cutoff)

$$\frac{\alpha_s}{4\pi}\left(12C_2(R)\right)\,\ln\frac{\Lambda'}{\mu}\,\tilde{A}_\rho = \left(Z^{(0)}_{qG} - 1\right)\tilde{A}_\rho \tag{51}$$

so that

$$\gamma^{(0)}_{qG} = \frac{4\pi}{\alpha_s}\,\mu\,\frac{\partial}{\partial\mu}\,Z^{(0)}_{qG} = -12C_2(R) = -16 \tag{52}$$

Finally, $\gamma_{GG}^{(0)}$ is given by Fig. 5. Fig. 5(a) yields the divergent terms $(\alpha_s/4\pi)\left(10C_2(G)\right)\ \ell n(\Lambda'/\mu)\ \tilde{a}'_\rho$, where $\tilde{a}'_\rho$ is defined in Fig. 3(b). Fig. 5(b) yields the divergent term $(\alpha_s/4\pi)\ \left(-6C_2(G)\right)\ \ell n(\Lambda'/\mu)\ \tilde{a}'_\rho$, and Fig. 5(c) yields $(\alpha_s/4\pi)\left(10C_2(G)/3 - 8T(R)/3\right)\ \ell n(\Lambda'/\mu)\ \tilde{a}'_\rho$. Adding up these contributions, we get

$$\frac{\alpha_s}{4\pi}\ (-2\beta_0)\ \ell n\,\frac{\Lambda'}{\mu}\ \tilde{a}'_\rho = \left(Z_{GG}^{(0)} - 1\right)\tilde{a}'_\rho \tag{53}$$

thereby yielding

$$\gamma_{GG}^{(0)} = -2\beta_0 \tag{54}$$

To summarize, the anomalous dimension matrix in LLA is

$$\gamma^{(0)} = \begin{bmatrix} 0 & -16 \\ 0 & -2\beta_0 \end{bmatrix} . \tag{55}$$

The property

$$\gamma_{iq}^{(0)} = 0, \qquad (i = q,G) \tag{56}$$

is very useful in the evaluation of ordered exponential W in (36) because neither matrix index of any γ in the middle of (36) may be a quark index. In terms of the LLA running coupling constant, we can express the result as

$$W^{(0)} = \exp\left[\int_{\bar{g}(Q^2)}^{g} \frac{dg'}{g'} \left(\frac{-\gamma^{(0)}}{\beta_0}\right)\right]_+ = \begin{bmatrix} 1 & \frac{8}{\beta_0}\left(\frac{\alpha_s}{\bar{\alpha}_s(Q^2)} - 1\right) \\ 0 & \frac{\alpha_s}{\bar{\alpha}_s(Q^2)} \end{bmatrix} \tag{57}$$

Substituting (57) into (38), and using (41) and (50), we conclude that in LLA, the total quark and gluon helicities are given by

$$\frac{1}{2}\,\Delta\tilde{q}(Q^2) = \frac{1}{2}\Delta\, q$$

$$\Delta\tilde{G}(Q^2) = -\frac{8}{\beta_0}\,\frac{1}{2}\,\Delta q + \left(\frac{8}{\beta_0}\,\frac{1}{2}\,\Delta q + \Delta G\right)\frac{\alpha_s \beta_0}{4\pi}\,\ln\frac{Q^2}{\Lambda^2} \tag{58}$$

Thus the total quark helicity remains Q^2-independent but the total gluon helicity grows linearly with $\ln Q^2/\Lambda^2$. This result agrees with the result of Altarelli and Parisi[12] obtained by a completely different method, thus confirming the correctness and the relevance of the operator a_ρ defined in (44).

The conclusion of Eq. (58) is very strange. Quarks can be pair created from gluons, yet quarks do not seem to inherit any of the growing helicity of $\Delta\tilde{G}(Q^2)$. This non-coupling is expressed mathematically as $\gamma_{Gq}^{(0)} = 0$. Physically, this is because of a combination of chirality conservation and charge conjugation invariance. The former asserts that the quark helicity must be opposite to the antiquark helicity, and the latter assures us that there must be a charge conjugated configuration where the quark also carry this opposite helicity. Thus the total quark helicity is zero whatever the gluon helicity is.

Since quarks cannot inherit helicity from gluons, they must get it from quarks. But chirality conservation again asserts that the quark helicity is unchanged no matter how many gluons it emits, or, no matter what Q^2 is. Hence $\Delta\tilde{q}(Q^2) = \Delta q$.

Finally, gluon helicity grows with Q^2 because of the triple gluon coupling. This can be seen in the following way. There are three terms in the triple gluon vertex, each with a metric tensor $g_{\alpha\beta}$ between the space time indices α,β of a pair of gluons. For $A \to B + C$, these three terms then contain respectively $e_B^* \cdot e_A$, $e_C^* \cdot e_A$, $e_B^* \cdot e_C$, where e_A is the spin wave function of gluon A, etc. All the three terms are zero if A has positive helicity but B anc C both have negative helicities. Moreover, as far as total gluon helicity is concerned, the two cases when the B, C helicities are one positive and the other one negative do not count, because they both give zero total helicity. The only case that counts then is when both B and C also carry positive helicity. Thus if the total helicity (that of A) is +1 before the three gluon interaction, the total helicity after the interaction is 2 (that of B+C). After N such triple gluon splits, the helicity factor is amplified by a 2^N. Thus $\Delta\tilde{G}(Q^2)$ grows with Q^2. The rate of growth with Q^2 can also be estimated. N should be proportional to the strong coupling constant $\bar{\alpha}_s$, and should increase with increasing Q^2. From the Altarelli-Parisi equation,[12] for example, one sees that

$$N \,\alpha \int^{\ell n Q^2} \bar{\alpha}_s(t)\, dt \sim \ell n\, \ell n\, Q^2$$

Thus 2^N grows like $(\ell n\, Q^2)^a$ for some a. We saw from (58) that calculation shows $a = 1$.

The phenomena of the growing helicity is remarkable because very few moments in QCD grows with $\ell n\, Q^2$. Moreover, the origin of the growth is the triple gluon coupling. Thus we can hope to have a good way of detecting experimentally the existence of such a coupling so vital for correctness of non-abelian theories. But alas, gluon helicities cannot be seen directly by the photon, and the physical argument given before seems to show that if gluons convert themselves into quarks so as to interact with the photon, the quarks will inherit none of this growing helicity. It would therefore seem that this remarkable growth of gluon helicity cannot be seen in lepton scattering experiments after all. Fortunately, this is not the case. Quarks do not inherit gluon helicities only when chirality is conserved, which is the case in LLA. When $O(\alpha_s)$ effects are taken into account in NLLA, chirality is no longer conserved because of the existence of the Adler-Bell-Jackiw (ABJ) anomaly.[4] Quarks can therefore inherit gluon helicities of amount of order of $\alpha_s \cdot \Delta\tilde{G}(Q^2) \sim O(1)$. Calculation below shows that this is indeed the case, and actually because the coupling constant runs, a more careful argument shows that this nominally $O(1)$ term actually behaves like $\int (d\bar{\alpha}_s/\bar{\alpha}_s)\ \bar{\alpha}_s \cdot \Delta G(Q^2) \sim \ell n\bar{\alpha}_s \sim \ell n\, \ell n\, Q^2$.

Now we shall carry out the NLLA calculation to obtain this effect quantitatively. The first task is to calculate the ordered exponential of (35) and (36), making the expansion (39) and keeping only terms in $I_s(Q^2)$ that give at least an $O(\alpha_s)$ or $O\left(\bar{\alpha}_s(Q^2)\right)$ contribution. Once again because of (56), the evaluation of W is greatly simplified. This we shall carry out in detail in Appendix B. The result is

$$W_{qq} = 1 + \frac{1}{8\pi\beta_0}\left(\frac{8}{\beta_0}\gamma_{Gq}^{(1)} - \gamma_{qq}^{(1)}\right)\left(\alpha_s - \bar{\alpha}_s(Q^2)\right) - \frac{1}{\pi\beta_0^2}\gamma_{Gq}^{(1)}\alpha_s \ell n \frac{\alpha_s}{\bar{\alpha}_s(Q^2)}$$

$$W_{qG} = \frac{8}{\beta_0}\left(\frac{\alpha_s}{\bar{\alpha}_s(Q^2)} - 1\right)$$

$$W_{Gq} = -\frac{1}{8\pi\beta_0}\gamma_{Gq}^{(1)}\alpha_s \ell n \frac{\alpha_s}{\bar{\alpha}_s(Q^2)}$$

$$W_{GG} = \frac{\alpha_s}{\bar{\alpha}_s(Q^2)} \qquad (59)$$

When we substitute (50) and (59) into (37), and keep only terms at least as large as $O(\alpha_s)$ and $O\left(\bar{\alpha}_s(Q^2)\right)$, we get

$$I_s(Q^2) = \frac{1}{2}\Delta q\left\{\left[1+\frac{1}{8\pi\beta_0}\left(\frac{8}{\beta_0}\gamma_{Gq}^{(1)}-\gamma_{qq}^{(1)}\right)\left(\alpha_s-\bar{\alpha}_s(Q^2)\right) - \frac{\alpha_s}{\pi\beta_0^2}\gamma_{Gq}^{(1)}\ln\frac{\alpha_s}{\bar{\alpha}_s(Q^2)}\right]\right.$$
$$\left.\times\, C_q^{(0)} + \frac{\bar{\alpha}_s(Q^2)}{4\pi}\,C_q^{(1)} + \frac{2}{\pi\beta_0}\left(\alpha_s-\bar{\alpha}_s(Q^2)\right)\,C_G^{(1)}\right\}$$
$$+\,\Delta G\left\{-\frac{\alpha_s}{8\pi\beta_0}\,\gamma_{Gq}^{(1)}\ln\frac{\alpha_s}{\bar{\alpha}_s(Q^2)}\;C_q^{(0)} + \frac{\alpha_s}{4\pi}\;C_G^{(1)}\right\}. \qquad (60)$$

The $(-\ln\ln Q^2)$ term discussed above now appears as the $-\ln\left(\alpha_s/\bar{\alpha}_s(Q^2)\right)$ term. To obtain the numerical details, we must now compute the first order anomalous dimensions and Wilson coefficients.

Let us first consider $\gamma_{qq}^{(1)}$. Again because of axial vector current Ward's identity, only diagrams involving the anomalous triangle diagrams survive. This of course agrees with the physical argument discussed above that any new effect must go through the ABJ anomaly. The relevant diagram is shown in Fig. 6. The divergent term of Fig. 6 can be read out from Figs. 2 and 4 and Eq. (51) to be

$$\left(Z_{qq}^{(1)}-1\right)\tilde{A}_\rho = \frac{\alpha_s}{4\pi}\left(-2T(R)\right)\left(Z_{qG}^{(0)}-1\right)\tilde{A}_\rho$$

Thus

$$\gamma_{qq}^{(1)} = \left(\frac{4\pi}{\alpha_s}\right)^2\mu\frac{\partial}{\partial\mu}\,Z_{qq}^{(1)} = \left(-2T(R)\right)\gamma_{qG}^{(0)} = 16f \qquad (61)$$

Next consider $\gamma_{Gq}^{(1)}$. Again we need to go through the triangle anomaly diagrams of Fig. 2 to obtain any contribution. This means only diagrams in Fig. 4 contribute (c.f., Appendix A). The result can similarly be read out from Figs. 2 and 5, and Eq. (53) to be

$$Z_{Gq}^{(1)} - 1 = \frac{\alpha_s}{4\pi} \left(-2T(R)\right) \left(Z_{GG}^{(0)} - 1\right)$$

so that

$$\gamma_{Gq}^{(1)} = \left(\frac{4\pi}{\alpha_s}\right)^2 \mu\frac{\partial}{\partial\mu} Z_{Gq}^{(1)} = \left(-2T(R)\right) \gamma_{GG}^{(0)} = 2f\beta_o \quad . \qquad (62)$$

Now we turn to the Wilson coefficients. If we apply (37) to a flavor singlet quark target and note the normalizations in (29) and (22), we see that $C_q^{(0)} = 1$. Similarly applying (37) to a gluon target shows $C_G^{(0)} = 0$. The next order Wilson coefficient $C_q^{(1)}$ has been calculated[6,9] and $C_q^{(1)} = -4$. The remaining Wilson coefficient that we must calculate is $C_G^{(1)}$. For this we consider photon-gluon scattering with gluon carrying helicity ±1. The scattering amplitude is depicted by the six diagrams of Fig. 8. We shall compute only the part of the amplitude $T_{\mu\nu}$ antisymmetric in μ and ν and independent of q·P, and shall use a number of dots to indicate the left out parts. Then

$$T_{\mu\nu}(a+c) = \frac{\alpha_s}{2\pi} T(R) \frac{\overline{e^2}}{Q^2} \left(e_\pm^\alpha\right)\left(e_\pm^\beta\right)^* \left\{\varepsilon_{\mu\nu\lambda\rho} p^\lambda \varepsilon_{\alpha\beta\sigma}^{\rho} q^\sigma - \frac{1}{3} \varepsilon_{\mu\nu\lambda\rho} q^\lambda \varepsilon_{\alpha\beta\sigma}^{\rho} p^\sigma\right\}$$
$$+ \ . \ . \ . \qquad (63)$$

$$T_{\mu\nu}(b) = \frac{\alpha_s}{2\pi} T(R) \frac{\overline{e^2}}{Q^2} \left(e_\pm^\alpha\right)\left(e_\pm^\beta\right)^* \left\{-\varepsilon_{\mu\nu\lambda\rho} p^\lambda \varepsilon_{\alpha\beta\sigma}^{\rho} q^\sigma + 3\varepsilon_{\mu\nu\lambda\rho} q^\lambda \varepsilon_{\alpha\beta\sigma}^{\rho} p^\sigma\right\}$$
$$+ \ . \ . \ . \qquad (64)$$

Adding up all six diagrams, we have

$$T_{\mu\nu} = 2\left[T_{\mu\nu}(a+c) + T_{\mu\nu}(b)\right] = \pm i \ \overline{e^2} \frac{\alpha_s}{4\pi} T(R) \frac{1}{Q^2} \frac{32}{3} \varepsilon_{\mu\nu\lambda\rho} q^\lambda p^\rho$$
$$+ \ . \ . \ . \qquad (65)$$

This is to be compared with (See Eq. (15))

$$T_{\mu\nu} = (4m\pi)\ \widetilde{W}_{\mu\nu} = \pm 4\pi i\ \overline{e^2}\ \varepsilon_{\mu\nu\rho\sigma} q^\rho p^\sigma \frac{2\nu x}{Q^2} \widetilde{G}_1 + \ . \ . \ . \qquad (66)$$

We thus obtain

$$\nu\widetilde{G}_1 = \frac{\overline{e^2}}{\pi} \frac{\alpha_s}{4\pi} T(R) \frac{4}{3} \frac{1}{x} + \ . \ . \ . \qquad (67)$$

On comparing this with (28), (29) and (37), we get

$$\frac{\alpha_s}{4\pi} T(R) \frac{4}{3} = \left(\Delta F_q^{(1)} C_q^{(0)} + \Delta F_G^{(0)} C_G^{(1)} \right) \frac{\alpha_s}{4\pi}$$

where

$$\Delta F_a = \Delta F_a^{(0)} + \frac{\alpha_s}{4\pi} \Delta F_a^{(1)} + \ . \ . \ .$$

But $C_q^{(0)} = 1$, and from Fig. 1, $\Delta F_q^{(1)} = -2T(R)$. Moreover, $\Delta F_G^{(0)} = 1$ from (22). Thus

$$C_G^{(1)} = \frac{10}{3} T(R) = 5\,f/3 \qquad (69)$$

To summarize, we have

$$\gamma_{qq}^{(1)} = 16\,f$$

$$\gamma_{Gq}^{(1)} = 2\,f\beta_0$$

$$C_q^{(0)} = 1$$

$$C_q^{(1)} = -4$$

$$C_G^{(1)} = 5\,f/3 \qquad (70)$$

Substituting these quantities into (60), we obtain

$$I_S(Q^2) = \frac{1}{2}\,\Delta q\left\{1 - \frac{\bar{\alpha}_s(Q^2)}{\pi} - \frac{2\alpha_s f}{\pi\beta_0}\,\ln\frac{\alpha_s}{\bar{\alpha}_s(Q^2)} + \frac{10f}{3\pi\beta_0}\left(\alpha_s - \bar{\alpha}_s(Q^2)\right)\right\}$$

$$+ \Delta G\left\{-\frac{\alpha_s f}{4\pi}\,\ln\frac{\alpha_s}{\bar{\alpha}_s(Q^2)} + \frac{5f}{12\pi}\,\alpha_s\right\} \tag{71}$$

Substituting (71) and (30) into (28), we obtain finally the Q^2-variation of the integral of νG_1 to NLLA.

For a numerical estimate of the contribution of (71), See Ref. 3.

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APPENDIX A

We discuss the Adler-Bell-Jackiw anomaly for QCD in this Appendix. We shall perform a Pauli-Villars regularization by introducing a heavy fermion with mass M. The regulated amplitude is obtained by subtracting this amplitude with mass M from the original zero mass amplitude. Eventually we will let $M \to \infty$.

We shall use naive Ward's identity for the regularized amplitude, on the grounds that a shift in the internal momentum space is allowed if the integral is less than linearly divergent. If there are internal gluons attached to the sides of the triangle, as in Fig. 9(c), then we shall first hold all the gluon momenta fixed, do the fermion loop integration and apply the Ward's identity whenever this is allowed by the degree of divergence of the fermion loop. In this way, diagram 9(b) is logarithmically divergent before applying the 'differential operator' $iq_\rho = ik_\rho - i(k-q)_\rho$, but will become linearly divergent after doing so. But then the Pauli-Villars regularization reduces the divergence once again to logarithmic and we may therefore use the naive Ward's identity on the regulated amplitude. This argument also shows that we may apply naive Ward's identity to any diagram with more than three gluons attached to the fermion loop, for then we have more fermion propagators and the fermion loop integration becomes more convergent.

The only suspect, as far as the application of the naive Ward's identity on the regulated amplitude is concerned, is Fig. 9(a). Here, by power counting, the regulated and differentiated amplitude still seems to diverge linearly. But then the top vertex is an axial vector and the whole amplitude must contain a factor

$$\varepsilon_{\alpha\beta\rho\lambda} q^{\rho} p^{\lambda} \tag{A.1}$$

The remaining Feynman integral is then at most logarithmically divergent and naive Ward's identity may still be used.

Because the original diagram contains zero mass quarks, the right hand side of the naive Ward's identity contains only diagrams involving the heavy quark. Thus,

$$\partial_{\rho} \langle A^{\rho} \rangle = -2iM \langle j_5 \rangle_M \tag{A.2}$$

where $\langle A^{\rho} \rangle$ refers to any regulated diagram with an axial vector vertex $\gamma^{\rho}\gamma_5$ and $\langle j_5 \rangle_M$ refers to the heavy quark component of the same diagram with the vertex $\gamma^{\rho}\gamma_5$ replaced by γ_5. The minus sign on the right hand side comes in because the heavy quark diagrams have a minus sign in front.

We shall now calculate the anomaly by calculating the right hand side of (A.2) and by letting $M \to \infty$. If $\langle j_5 \rangle_M = O(1/M)$, then anomaly exists. If $\langle j_5 \rangle_M = o(1/M)$, then anomaly is absent. Remembering that the factors in (A.1) are always present, we can use power counting to show that any fermion loop with four or more gluon lines attached to it is $o(1/M)$ and thus anomaly free. Thus only Figs. 9(a) and 9(b), and their permuted diagrams, may contain an anomaly.

The $\langle j_5 \rangle_M$ contribution of Fig. 9(a) is $\big($up to $O(1/M)\big)$

$$\langle j_5 \rangle_M = \frac{g^2}{16\pi^2} (-1)\, T(R)\, \delta_{ab} \cdot 4iM[\alpha\beta pq] \cdot \frac{1}{2M^2} \tag{A.3}$$

Because of the existence of a charge conjugated diagram, the total anomaly of diagrams of the type Fig. 9(a) (2 diagrams) is

$$\partial_\rho \langle A^\rho \rangle = iq_\rho \langle A^\rho \rangle = 2(-2iM)\,\frac{g^2}{16\pi^2}\left(-T(R)\delta_{ab}\right)\frac{2i}{M}\,[\alpha\beta pq]$$

$$= \frac{g^2}{16\pi^2}\left(-2T(R)\right)\,4[\alpha\beta pq]\,\delta_{ab} \qquad (A.4)$$

$$= iq^\rho\,\frac{g^2}{16\pi^2}\left(-2T(R)\right)\,\tilde{\alpha}'_\rho$$

where

$$\tilde{\alpha}_\rho{}' = 4i\,\varepsilon_{\rho\alpha\beta\lambda}p^\lambda\delta_{ab} \qquad (A.5)$$

is the elementary vertex of Fig. 3(b).

The first diagram in Fig. 2(a) is identical to Fig. 9(a), except with $q=0$. Since the top vertex is $\gamma_\rho\gamma_5$, This amplitude must be proportional to $\varepsilon_{\rho\alpha\beta\lambda}p^\lambda$. The coefficient can be read out from (A.4) and (A.5) and the result is indicated in Fig. 2(a).

We now come to Fig. 9(b). A straightforward calculation shows that (up to $O(1/M)$)

$$-2iM\langle j_5\rangle_M = \frac{1}{4\pi^2}\,g^3\,\varepsilon_{\alpha\gamma\beta\lambda}\,\left(p_1+P_3\right)^\lambda\,\mathrm{Tr}\left(t_a t_c t_b\right). \qquad (A.6)$$

Now we have six distinct diagrams of the type 9(b) obtained by permuting the gluon lines. In particular, add up (A.6) and the one with the p_1,p_3 gluon lines interchanged, we get

$$\frac{1}{4\pi^2}\,g^3\,\varepsilon_{\alpha\gamma\beta\lambda}\,\left(p_1+p_3\right)^\lambda\,\mathrm{Tr}\left(t_a t_c t_b - t_c t_a t_b\right) = \frac{i}{4\pi^2}\,g^3\,\varepsilon_{\alpha\gamma\beta\lambda}\,\left(p_1+p_3\right)^\lambda\,C_{acb}T(R) \qquad (A.7)$$

Adding in the other two sets of diagrams, we obtain the total anomaly for the (six) diagrams of type 9(b) to be

$$\partial^\rho \langle A_\rho \rangle = iq^\rho \langle \tilde{A}_\rho \rangle = -\frac{i}{2\pi^2} g^3 \varepsilon_{\alpha\gamma\beta\rho} q^\rho C_{acb} T(R)$$
$$= iq^\rho \frac{g^2}{16\pi^2} \left(-2T(R)\right) \tilde{a}''_\rho \qquad (A.8)$$

where

$$\tilde{a}''_\rho = -4g C_{abc} \varepsilon_{\rho\alpha\beta\gamma} \qquad (A.9)$$

is the elementary vertex in Fig. 3(c). Finally, Eqs. (A.8) and (A.9) also lead to the rules depicted in Fig. 2(b).

APPENDIX B

This Appendix contains a derivation of Eq. (59). The matrix W is given in Eqs. (35) and (36), and W will be calculated only to an order so that $I_S(Q^2)$ contains nothing smaller than $O(\alpha_s)$ and $O(\bar{\alpha}_s(Q^2))$. From Eq. (37) and the fact that $C_G(\bar{g}^2) = O(\bar{g}^2)$, the W_{iG} matrix elements $(i = q,G)$ may be calculated to $O(1)$ although the W_{iq} matrix elements must be calculated to $O(\alpha_s)$ and $O(\bar{\alpha}_s)$.

Employing Eq. (39), we can expand $\gamma(g)/\beta(g)$ in powers of g:

$$\frac{\gamma(g)}{\beta(g)} = -\frac{1}{g}\left[\frac{\gamma^{(0)}}{\beta_0} + \frac{g^2}{16\pi^2}\left(\frac{\gamma^{(1)}}{\beta_0} - \frac{\gamma^{(0)}\beta_1}{\beta_0^2}\right)\right] + \ldots$$

$$\equiv -\frac{1}{g}\left[d_0 + g^2 d_1\right] + \ldots \tag{B.1}$$

In calculating W_{iq} using (36), we should keep terms linear in d_1, although in the calculation for W_{iG}, we may keep only terms without d_1. This means that $W_{iG} = W_{iG}^{(0)}$ with $W^{(0)}$ given by (57).

Because of the special properties of (56), we have

$$(W_n)_{iq} = (-1)^n (d_0)_{iG} (d_0)_{GG}^{n-2} (d_i)_{Gq}\, J_n \qquad (n \geq 2)$$

$$(W_1)_{iq} = -(d_1)_{iq}\, J_1 \quad , \tag{B.2}$$

where

$$J_n = \int_{\bar{g}}^{g} \frac{dg_n}{g_n} \int_{\bar{g}}^{g_n} \frac{dg_{n-1}}{g_{n-1}} \int \cdots \int_{\bar{g}}^{g_2} \frac{dg_1}{g_1}\, g_1^2$$

$$= \int_{\bar{g}}^{g} \frac{dg_1}{g_1}\, g_1^2 \int_{g_1}^{g} \frac{dg_2}{g_2} \int_{g_2}^{g} \frac{dg_3}{g_3} \int \cdots \int_{g_{n-1}}^{g} \frac{dg_n}{g_n}$$

$$= \frac{1}{(n-1)!} \int_{\bar{g}}^{g} g_1\, dg_1\, \ln^{n-1}\left(\frac{g}{g_1}\right) \tag{B.3}$$

Therefore,

$$W_{iq} = 1 + \sum_{n=1}^{\infty} (W_n)_{iq}$$

$$= 1 - \frac{1}{2} (d_1)_{iq}(g^2 - \bar{g}^2) + \frac{(d_0)_{iG}}{(d_0)_{GG}} (d_1)_{Gq} \frac{1}{2} (g^2 - \bar{g}^2)$$

$$- \int_{\bar{g}}^{g} g_1 dg_1 \frac{(d_0)_{iG}}{(d_0)_{GG}} (d_1)_{Gq} \left(\frac{g}{g_1}\right)^{-(d_o)_{GG}}$$

$$= 1 + \frac{1}{2} \left[\frac{(d_0)_{iG}}{(d_0)_{GC}} (d_1)_{Gq} - (d_1)_{iq} \right] (g^2 - \bar{g}^2)$$

$$- \frac{(d_0)_{iG} (d_1)_{Gq} g^2}{(d_0)_{GG} [(d_0)_{GG} + 2]} \left[1 - \left(\frac{\bar{g}}{g}\right)^{(d_0)_{GG} + 2} \right] \qquad (B.4)$$

Now from (B.1) and (55), $(d_0)_{GG} = -2$, $(d_0)_{qG} = -16/\beta_0$. Moreover, $(d_1)_{iq} = \gamma_{iq}^{(1)}/16\pi^2\beta_0$. Thus

$$W_{qq} = 1 + \frac{1}{32\pi^2\beta_0} \left[\frac{8}{\beta_0} \gamma_{Gq}^{(1)} - \gamma_{qq}^{(1)}\right] (g^2 - \bar{g}^2) - \frac{\gamma_{Gq}^{(1)}}{2\pi^2\beta_0^2} \ln \frac{g}{\bar{g}}$$

$$W_{Gq} = - \frac{\gamma_{Gq}^{(1)} g^2}{16\pi^2\beta_0} \ln \frac{g}{\bar{g}} \qquad (B.5)$$

and this result is reproduced in Eq. (59).

TABLE I

Flavor combinations of neutrino (ν), antineutrino ($\bar{\nu}$), and electron (e) scatterings on proton (p) and neutron (n) targets for three and four flavors. These flavor contents are valid for W_1, W_2, G_1 and G_6. Here u, etc., stand for u(x), xu(x), Δu(x) or xΔu(x), etc.; whatever the situation dictates.

	f = 4	f = 3
ep	$\frac{4}{9}(u+\bar{u}+c+\bar{c}) + \frac{1}{9}(d+\bar{d}+s+\bar{s})$	$\frac{4}{9}(u+\bar{u}) + \frac{1}{9}(d+\bar{d}+s+\bar{s})$
en	$\frac{4}{9}(d+\bar{d}+c+\bar{c}) + \frac{1}{9}(u+\bar{u}+s+\bar{s})$	$\frac{4}{9}(d+\bar{d}) + \frac{1}{9}(u+\bar{u}+s+\bar{s})$
νp	$2(d+s+\bar{u}+\bar{c})$	$2(d+\bar{u})\cos^2\theta_c$
νn	$2(u+s+\bar{d}+\bar{c})$	$2(u+\bar{d})\cos^2\theta_c$
$\bar{\nu}$p	$2(u+c+\bar{d}+\bar{s})$	$2(u+\bar{d})\cos^2\theta_c$
$\bar{\nu}$n	$2(d+c+\bar{u}+\bar{s})$	$2(d+\bar{u})\cos^2\theta_c$

FIGURE CAPTIONS

Fig. 1. Parton model diagrams for neutrino scattering.

Fig. 2. The gluon matrix elements of A_ρ.

Fig. 3. Elementary vertices of A_ρ and a_ρ.

Fig. 4. Diagram for $\gamma_{qG}^{(0)}$.

Fig. 5. Diagram for $\gamma_{GG}^{(0)}$.

Fig. 6. Diagram for $\gamma_{qq}^{(1)}$.

Fig. 7. Diagram for $\gamma_{Gq}^{(1)}$.

Fig. 8. Second order photon (dashed line) gluon (wiggle line) scattering diagrams.

Fig. 9. (a) and (b), diagrams containing anomaly; (c), a diagram with no anomaly.

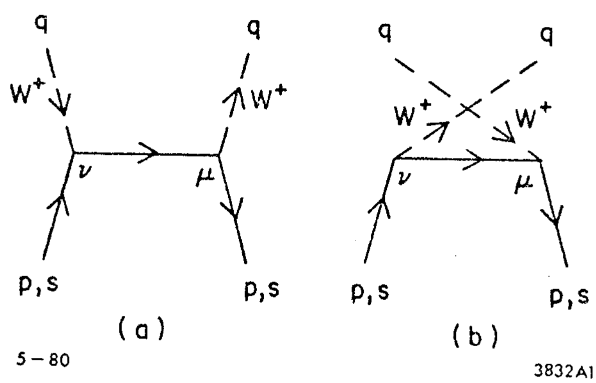
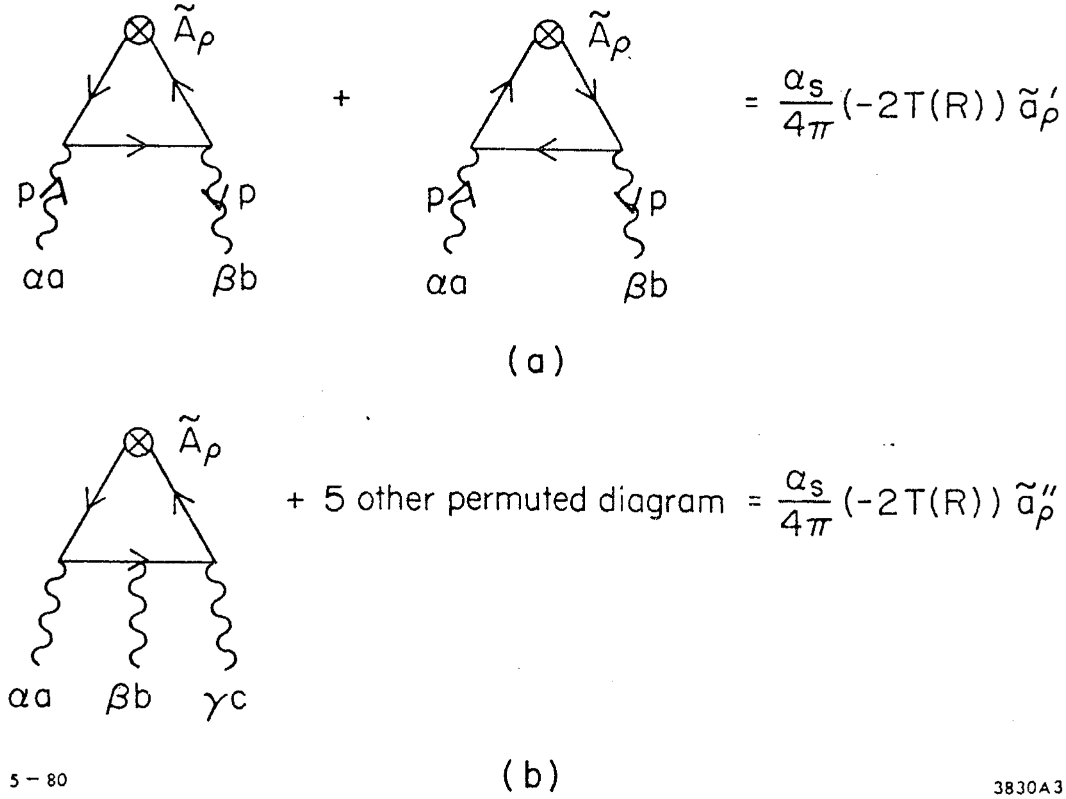
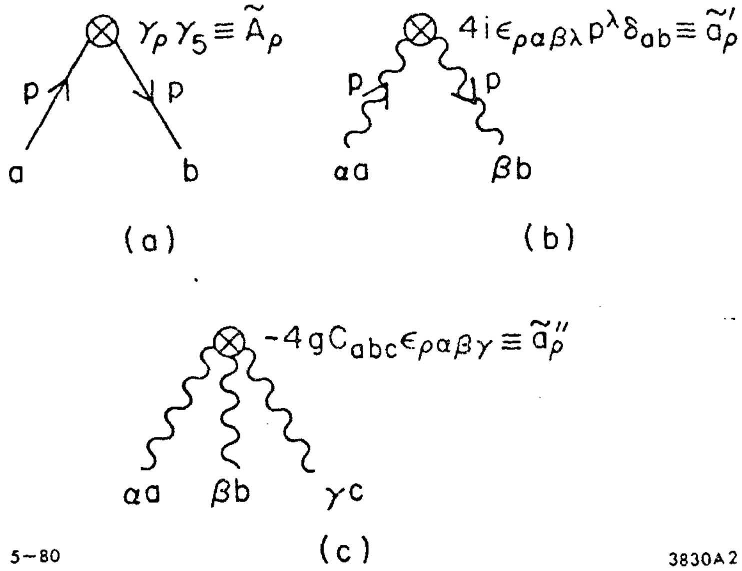


Fig. 1



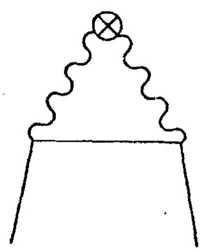
$\tilde{A}_\rho$
+
$= \frac{\alpha_s}{4\pi}(-2T(R))\,\tilde{a}'_\rho$
p
p
αa
βb
(a)
+ 5 other permuted diagram $= \frac{\alpha_s}{4\pi}(-2T(R))\,\tilde{a}''_\rho$
αa
βb
γc
5 – 80
(b)
3830A3

Fig. 2



$\gamma_\rho \gamma_5 \equiv \tilde{A}_\rho$
p
p
a
b
(a)
$4i\epsilon_{\rho\alpha\beta\lambda} p^\lambda \delta_{ab} \equiv \tilde{a}'_\rho$
p
p
αa
βb
(b)
$-4gC_{abc}\epsilon_{\rho\alpha\beta\gamma} \equiv \tilde{a}''_\rho$
αa
βb
γc
(c)
5–80
3830A2

Fig. 3



5–80 3830A4

Fig. 4

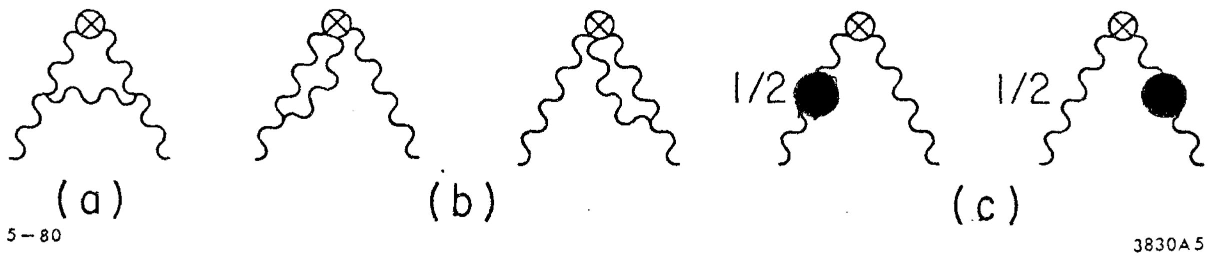


Fig. 5

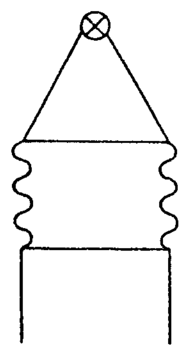


Fig. 6

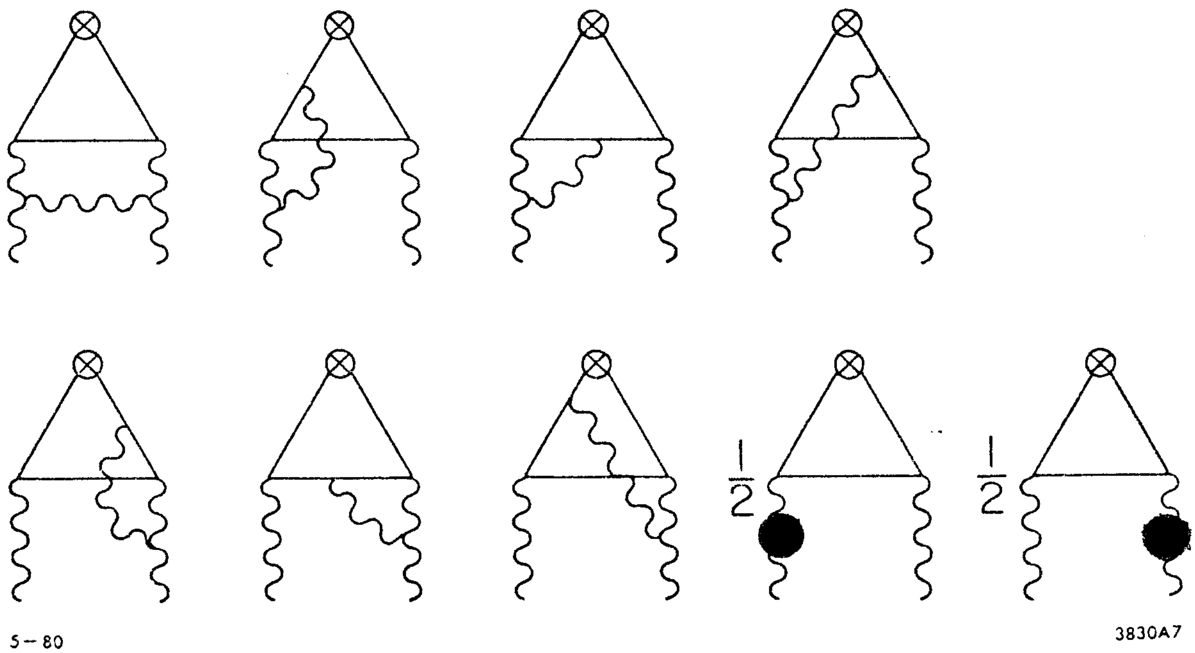


Fig. 7

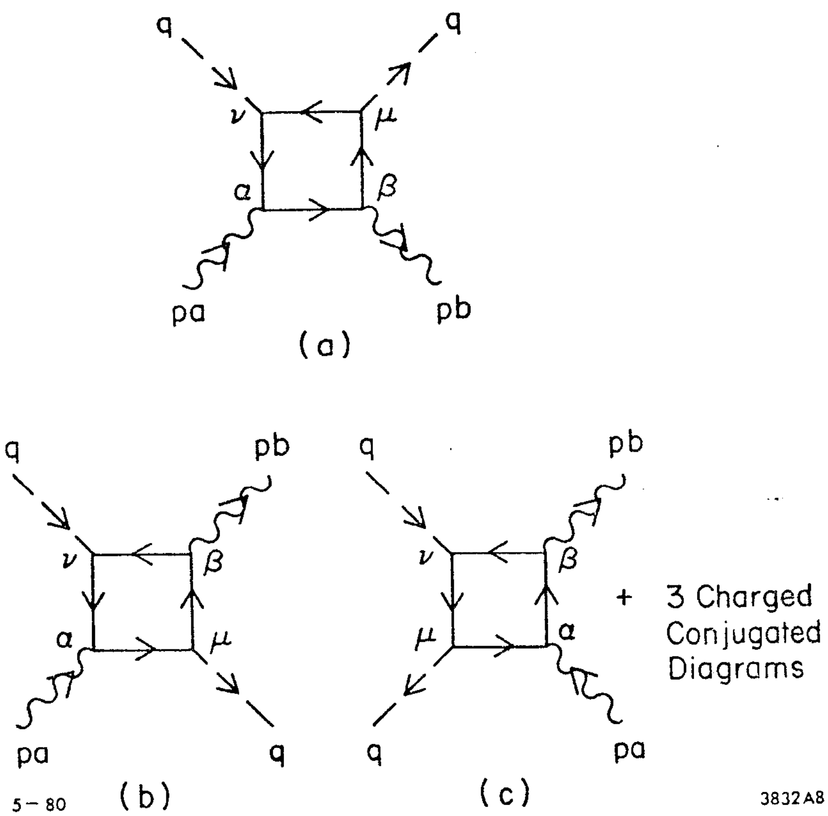

q
q
ν
μ
α
β
pa
pb
(a)
q
pb
ν
β
α
μ
pa
q
q
pb
ν
β
μ
α
q
pa
+ 3 Charged Conjugated Diagrams
5 – 80
(b)
(c)
3832A8

Fig. 8

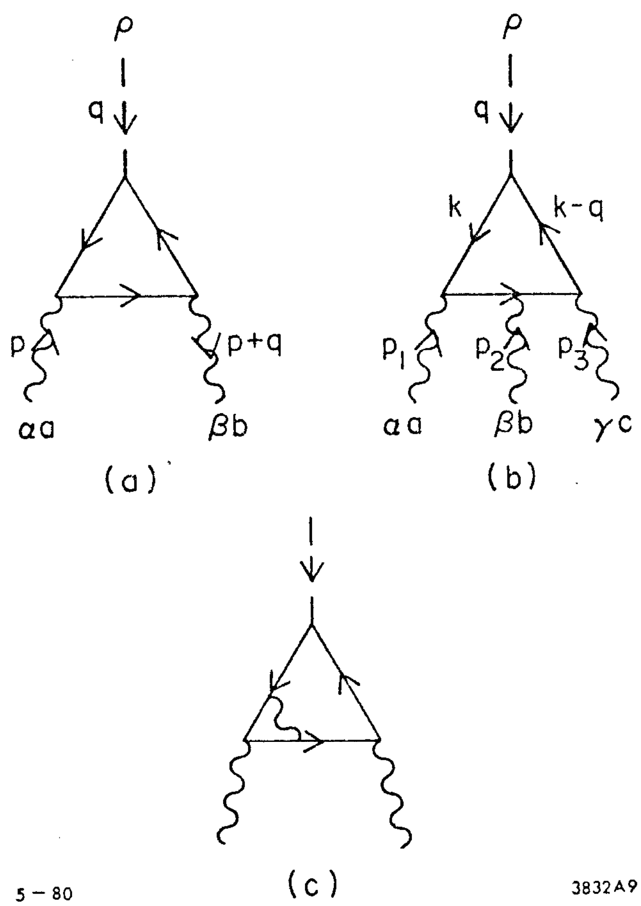


Fig. 9