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A POSSIBILITY OF DETECTING TRIPLE GLUON COUPLING AND ADLER-BELL-JACKIW ANOMALY IN POLARIZED DEEP INELASTIC SCATTERING*

C. S. Lam† and Bing An Li‡
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

We show that a way to detect experimentally the existence of triple gluon coupling and the Adler-Bell-Jackiw anomaly is to measure the Q^2-dependence of polarized deep inelastic scattering. These effects lead to a $\ln \ln Q^2$ term which we calculate by introducing a new gluon operator in the Wilson expansion.

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† Permanent address: Department of Physics, McGill University, Montreal, P.Q., Canada H3A 2T8.
‡ Permanent address: Institute of High Energy Physics, Academia Sinica, Beijing, China.

We have calculated the Q^2-dependence of the flavor-singlet part $I_S(Q^2)$ of $\int_0^1 \nu G_1(Q^2,x)\,dx = I_{NS}(Q^2) + \overline{e^2}\, I_S(Q^2)$ to be

$$I_S(Q^2) = \frac{1}{2}\Delta q \left[1 - \frac{\bar{\alpha}_s(Q^2)}{\pi} - \frac{2\alpha_s f}{\pi\beta_0}\ln\frac{\alpha_s}{\bar{\alpha}_s(Q^2)} + \frac{10f}{3\pi\beta_0}\left(\alpha_s - \bar{\alpha}_s(Q^2)\right)\right]$$

$$+ \Delta G \left[-\frac{\alpha_s f}{4\pi}\ln\frac{\alpha_s}{\bar{\alpha}_s(Q^2)} + \frac{5f}{12\pi}\alpha_s\right] \quad , \qquad (1)$$

where νG_1 is one of the structure functions in deep inelastic scattering defined in (3) and Q^2 is the squared momentum transfer. $\overline{e^2}$ is the average charge squared of all the quarks, $\bar{\alpha}_s(Q^2)$ is the running coupling constant in QCD and $\alpha_s = \bar{\alpha}_s(\mu^2)$ is its value at some renormalization point μ^2. f is the number of flavors of quarks and $\beta_0 = 11-2f/3$. $\Delta q/2$ and ΔG are parameters which may be interpreted to be the amount of helicities carried by the quarks and antiquarks, and respectively by the gluons, at $Q^2 = \mu^2$. Finally, we will let M and P to be the mass and momentum of the nucleon, q to be the momentum of the photon, $\nu = q\cdot P/M$, $Q^2 = -q^2$, and $x = Q^2/2M\nu$.

The $\ln\left(\alpha_s/\bar{\alpha}_s(Q^2)\right) \sim \ln\ln Q^2$ term comes from the triple gluon coupling (TGC) and the Adler-Bell-Jackiw [1] anomaly (ABJA). If either one of them were absent, this term will disappear. The detection of this term amounts to a confirmation of the existence of the triple gluon coupling and the existence of the highly theoretical object: the ABJ anomaly. It is therefore very interesting and important to find it. This term has an opposite sign than the usual parton term $\Delta q/2$ and it dominates over the latter at large Q^2. These signatures should make such a term relatively distinct. As a matter of fact, this term may be isolated by plotting $[I_S(Q^2) - I_S(Q_0^2)]/[\bar{\alpha}_s(Q^2) - \bar{\alpha}_s(Q_0^2)]$ versus $[\ell n\,\bar{\alpha}_s(Q^2) - \ell n\,\bar{\alpha}_s(Q_0^2)]/[\bar{\alpha}_s(Q^2) - \bar{\alpha}_s(Q_0^2)]$. This plot yields a straight line whose slope and intercept are related to Δq and ΔG. Finally, we present in Fig. 1 an illustration for the amount of Q^2-variation of the coefficients of the $\Delta q/2$ term (Fig. 1(a)) and of the ΔG terms (Fig. 1(b)).

In parton model, $I_S = \Delta q/2$. In the leading log approximation (LLA) of QCD, $I_S(Q^2) = \Delta\tilde{q}(Q^2)/2$, where $\Delta\tilde{q}(Q^2)$ ($\Delta\tilde{G}(Q^2)$) is the Q^2-dependent helicity carried by all the quarks and antiquarks (gluons). It turns out that $\Delta\tilde{q}(Q^2) = \Delta q$ in LLA. Thus, $I_S(Q^2)$ scales and it behaves very differently than the non-leading logarithm result (NLLA) of (1). But then, as we shall see, TGC and/or ABJA effects are absent in LLA. These effects are also absent in the integral of the non-singlet part of $\nu G_1(Q^2,x)$ and indeed the Q^2-dependence of this $I_{NS}(Q^2)$ differs from the parton result only by a small amount [2],

$$I_{NS}(Q^2) = \frac{1}{2}\,\Delta q_{NS}\left(1 - \frac{\bar{\alpha}_s(Q^2)}{\pi}\right) \quad . \qquad (2)$$

Again its Q^2-variation nowhere resembles that of (1). This shows that the $\ell n\,\ell n\,Q^2$ term is indeed a characteristic feature of TGC and ABJA.

The rest of this Letter contains an expose on the physical mechanism of how the $\ell n\,\ell n\,Q^2$ term arises from the presence of TGC and ABJA. It also contains a brief sketch of how these terms are calculated. In the course of the calculation we will find it necessary to introduce a new gluon operator in the Wilson expansion which carries these effects but is hitherto ignored.

First, a word on the definition of G_1 and how it is measured. It is defined via the relation

$$W_{\mu\nu} \equiv \frac{1}{4M\pi}\int e^{iq\cdot x}\, d^4x \langle P,S|[J_\mu^{em}(x)\ ,\ J_\nu^{em}(0)]|P,S\rangle$$

$$= -g_{\mu\nu}W_1 + \frac{P_\mu P_\nu}{M^2}W_2 + \frac{i}{M}\varepsilon_{\mu\nu\rho\sigma}q^\rho S^\sigma G_1$$

$$+ \frac{i}{M^3}\varepsilon_{\mu\nu\rho\sigma}q^\rho [S^\sigma(q\cdot P) - P^\sigma(q\cdot S)]G_2 \qquad , \qquad (3)$$

and it can be determined from the asymmetry in cross sections from longitudinally polarized electrons and nucleons. Here S is the nucleon polarization vector normalized to $S^2 = -1$, $S\cdot P = 0$. Terms proportional to q_μ or q_ν have been dropped.

We now turn to a physical explanation of the unusual Q^2-behavior in Eq. (1). The term $\ell n\, \ell n\, Q^2$ can be traced back to a term which says that the gluon helicity $\Delta\tilde{G}(Q^2)$ grows like $\ell n\, Q^2$ in LLA. This is a consequence of TGC. When a gluon with positive helicity decays into two gluons, the final gluons may carry helicities ++, +-, -+ and --. The cases +- and -+ do not contribute to the total helcity and may be ignored. The case -- actually does not occur because the TGC for this case is zero. We are thus left with ++ only. This implies that after N levels of such gluon decays, the total helicity is amplified by a factor 2^N, which grows with Q^2 like $(\ell n\, Q^2)^a$ because $N \propto \int^{\ell n Q^2} \alpha_s(t)dt \sim \ell n\, \ell n\, Q^2$ as can be seen for example from the Altarelli-Parisi equation [3]. By calculation it turns out that $a = 1$. This $\ell n\, Q^2$ dependence is rather remarkable because very few moments in QCD grow with $\ell n\, Q^2$. But unfortunately, it can be shown that chirality conservation (we take our quark masses to be zero) and charge conjugation invariance prevent any pair-created quarks from inheriting any of these growing helicities from the gluon, and hence this growing $\Delta\tilde{G}(Q^2)$ is unobservable in electron scattering (calculated to LLA). As far as the total quark helicity $\Delta\tilde{q}(Q^2)$ is concerned, it is Q^2-independent because chirality conservation preserves the quark helicity no matter how many gluons it emits.

If chirality were strictly conserved, the quarks will never inherit any gluon helicity, no matter how high a perturbational order we go to. The dramatic growth of $\Delta\tilde{G}(Q^2)$ and the TGC would then be unobservable and wasted. Fortunately, because of ABJA, chirality conservation is violated in $O(\alpha_s)$. This enables quarks to inherit gluon helicity in NLLA. We would therefore expect a correction term of $O\left(\tilde{\alpha}_s(Q^2)\right)\cdot\Delta\tilde{G}(Q^2) \sim O(1)$ to $I_S(Q^2)$. Actually a mathematical accident turns this term into a $\ell n\, \ell n\, Q^2$ term. We see therefore how the presence of such a term relies crucially on the existence of both TGC and ABJA.

Now we will sketch the actual calculations leading to Eq. (1). For that, we perform a Wilson expansion on the time ordered product matrix element [4]

$$\begin{aligned}
\tilde{W}_{\mu\nu} &\equiv \frac{i}{4M\pi}\int e^{iq\cdot x}\, d^4x\, \langle P,S|T\left(J_\mu^{em}(x)\;\; J_\nu^{em}(0)\right)|P,S\rangle \\
&= \frac{i}{M}\,\varepsilon_{\mu\nu\rho\sigma}\, q^\rho\, S^\sigma\, \tilde{G}_1 + \ldots \\
&= \frac{1}{\pi}\frac{i}{M}\,\varepsilon_{\mu\nu\rho\sigma}\, q^\rho\, \frac{1}{2Q^2}\left[\tilde{C}_q\left(\frac{Q^2}{\mu^2},\alpha_s\right)\langle P,S|A^\sigma(0)|P,S\rangle\right. \\
&\quad \left. + \tilde{C}_G\left(\frac{Q^2}{\mu^2},\alpha_s\right)\langle P,S|a^\sigma(0)|P,S\rangle\right] + \ldots \qquad , \qquad (4)
\end{aligned}$$

in which, we have ignored terms that do not contribute to $I_S(Q^2)$. $\tilde{C}_q$ and $\tilde{C}_G$ are the Wilson coefficients and A^σ, a^σ are the corresponding quark and gluon axial vector operators. If we introduce ΔF_q and ΔF_G

through the matrix elements

$$\langle P,S|A^{\sigma}(0)|P,S\rangle = 4M\,\Delta F_q\, S^{\sigma}$$
$$\langle P,S|a^{\sigma}(0)|P,S\rangle = 4M\,\Delta F_G\, S^{\sigma} \qquad (5)$$

then Eq. (4) implies the relation

$$I_S(Q^2) = \tilde{C}_q\,\Delta F_q + \tilde{C}_G\,\Delta F_G = [\Delta F_q\,,\,\Delta F_G]\cdot W\cdot \begin{bmatrix} C_q \\ C_G \end{bmatrix} . \qquad (6)$$

Renormalization group arguments [4] have been used to obtain the second expression in Eq. (6), where $C_i \equiv \tilde{C}_i\left(1,\bar{\alpha}_s(Q^2)\right)$ and W is the g'-ordered exponential matrix ($g^2 \equiv 4\pi\alpha_s$)

$$W = \left(\exp\int_{\bar{g}(Q^2)}^{g} \frac{\gamma(g')}{\beta(g')}\,dg'\right)_{+} \qquad (7)$$

involving the anomalous dimension matrix $\gamma(g)$ and the coupling constant β-function $\beta(g)$. The evaluation of W and $I_S(Q^2)$ will be carried out perturbatively by expanding γ, β, C in power series of α_s: $\gamma(g) = \gamma^{(0)}(\alpha_s/4\pi) + \gamma^{(1)}(\alpha_s/4\pi)^2 + \ldots$; $\beta(g)/g = -\beta_0(\alpha_s/4\pi) - \beta_1(\alpha_s/4\pi)^2$; $C_i = C_i^{(0)} + (\alpha_s/4\pi)C_i^{(1)} + \ldots$

To proceed further, we need the explicit forms for A_σ and a_σ. The quark operator A_σ is known [5] and it is just the familiar singlet axial vector current operator

$$A_\sigma(x) = \bar{q}(x)\,\gamma_\sigma\,\gamma_5\,q(x) \qquad . \qquad (8)$$

On the other hand, the gluon operator in Ref. [5] does not exist for $n=1$ and is then taken to be zero in Ref. [6]. This is where we differ from the previous treatments. A gluon axial vector operator actually exists. We only need the diagonal matrix element of the gluon axial vector operator $\langle P,S|a^{\sigma}|P,S\rangle$, and for this purpose, it can be taken to be the following operator well known in discussions of topological solutions [7] of the Yang-Mill fields:

$$a_{\sigma}(x) = -\varepsilon_{\sigma\alpha\beta\gamma}\left[G_{a}^{\alpha\beta}(x)B_{a}^{\gamma}(x) - \frac{g}{3}\,C_{abc}B_{a}^{\alpha}(x)B_{b}^{\beta}(x)B_{c}^{\gamma}(x)\right]$$

$$G_{\alpha\beta}^{a}(x) = \partial_{\alpha}B_{\beta}^{a}(x) - \partial_{\beta}B_{\alpha}^{a}(x) + gC_{abc}B_{\alpha}^{b}(x)B_{\beta}^{c}(x) \quad . \tag{9}$$

The subscript a is the color index and the diagonal matrix element of a_{σ} is gauge invariant. The operators A_{σ} and a_{σ} are the helicity operators for free quarks and gluons respectively, i.e., $\Delta F_q = 1/2$ and $\Delta F_G = 1$ in Eq. (5) for free particle states. Moreover, their divergences are related:

$$\partial^{\sigma}A_{\sigma}(x) = \frac{\alpha_s}{4\pi}\left(-2T(R)\right)\left(-\frac{1}{2}\varepsilon_{\alpha\beta\gamma\delta}G_{a}^{\alpha\beta}(x)G_{a}^{\gamma\delta}(x)\right) = \frac{\alpha_s}{4\pi}\left(-2T(R)\right)\partial^{\sigma}a_{\sigma}(x) \; . \tag{10}$$

Here T(R) = f/2 and the non-Abelian generalization of ABJA relation [1] has been used. A corollary of (10) which can be verified by direct computation states that the triangle graphs in Fig. 3 are proportional to the lowest order vertices for a_{σ} (Fig. 2) in the manner shown in Fig. 3. The minus sign in Fig. 3 is eventually responsible for the minus sign in the $(-\ln\ln Q^2)$ term in Eq. (1).

With these operators, the anomalous dimensions can be calculated in the usual way [4]. $\gamma_{qG}^{(0)}$ and $\gamma_{GG}^{(0)}$ are calculated from Figs. 4 and 5 respectively. The result is $\gamma_{qG}^{(0)} = -16$ and $\gamma_{GG}^{(0)} = -2\beta_0$. Moreover, $\gamma_{qq}^{(0)} = 0$ because of the axial vector current Ward's identity, and $\gamma_{Gq}^{(0)} = 0$ becasue Fig. 3(a) is finite. These results agree with those in Ref. [3]

obtained by a completely different method.* The minus sign in front of $\gamma_{GG}^{(0)}$ is responsible for the growth of $\Delta\tilde{G}(Q^2)$ with Q^2; $\gamma_{Gq}^{(0)} = 0$ asserts the impossibility of quarks inheriting helicities from the gluons; and $\gamma_{qq}^{(0)} = 0$ leads to scaling results for $\Delta\tilde{q}(Q^2)$. The property $\gamma_{iq}^{(0)} = 0$ $(i = q,G)$ is useful mathematically in that it greatly simplifies the evaluation of W in (7). If we keep $I_S(Q^2)$ to $O(\alpha_s)$ and $O\left(\bar{\alpha}_s(Q^2)\right)$, then this evaluation of W will lead to

$$I_S(Q^2) = \Delta F_q \left\{ \left[1 + \frac{1}{8\pi\beta_0}\left(\frac{8}{\beta_0}\gamma_{Gq}^{(1)} - \gamma_{qq}^{(1)}\right)\left(\alpha_s - \bar{\alpha}_s(Q^2)\right) - \frac{\alpha_s}{\pi\beta_0^2}\gamma_{Gq}^{(1)} \ln \frac{\alpha_s}{\bar{\alpha}_s(Q^2)} \right] C_q^{(0)} + \frac{\bar{\alpha}_s(Q^2)}{4\pi} C_q^{(1)} + \frac{2\bar{\alpha}_s(Q^2)}{\pi\beta_0}\left(\frac{\alpha_s}{\bar{\alpha}_s(Q^2)} - 1\right) C_G^{(1)} \right\}$$
$$+ \Delta F_G \left\{ -\frac{\alpha_s}{8\pi\beta_0}\gamma_{Gq}^{(1)} \ln \frac{\alpha_s}{\bar{\alpha}_s(Q^2)} C_q^{(0)} + \frac{\alpha_s}{4\pi} C_G^{(1)} \right\} \quad . \qquad (11)$$

The anomalous dimension $\gamma_{qq}^{(1)}$ in Eq. (11) receives non-zero contribution (because of the Ward's identity again) only from Fig. 6, and $\gamma_{Gq}^{(1)}$ receives non-zero contribution also only from the anomaly-related diagrams of Fig. 7. Using the results for Figs. 3-5, we can easily read off the answer $\gamma_{qq}^{(1)} = (-2T(R))(-16) = 16f$ and $\gamma_{Gq}^{(1)} = (-2T(R))(-2\beta_0) = 2f\beta_0$. As to the Wilson coefficients, it follows from the parton result that $C_q^{(0)} = 1$ and $C_G^{(0)} = 0$. Also, $C_q^{(1)} = -4$ as is already given in Refs. [2] and [6]. Finally, $C_G^{(1)}$ can be obtained by comparing the

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* This incidentally confirms the existence and relevence of a_ρ. Our $\gamma_{qG}^{(0)}$ is actually twice that in Ref. [3] because of different normalizations. For free particles, we have $\Delta F_q = 1/2$ but $\Delta F_G = 1$ in Eq. (5).

amplitude T for the six graphs in Fig. 8 with Eq. (6). We obtain**
$T = (\alpha_s/4\pi)T(R)(-32/3Q^2) \cdot \varepsilon_{\mu\nu\lambda\rho}\, q^\lambda\, \varepsilon^\rho_{\alpha\beta\gamma}\, p^\gamma$, which yields $C_G^{(1)} = 5f/3$. Substituting all these anomalous dimensions and Wilson coefficients into Eq. (11), we obtain the final result of Eq. (1).

A more detailed discussion and other related results will be published elsewhere.

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** This differs from the result of Ref. [6] by a factor 2/3. However, the qualitative dependence of Eqs. (1) and (11) on Q^2 is of course insensitive to the exact coefficients involved.

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FIGURE CAPTIONS

Fig. 1. The dependence of $I_S(Q^2)$ on Q^2 or $\bar{\alpha}_s(Q^2)$ from Eq. (1), $\mu^2 = 10(\mathrm{GeV}/c)^2$, $f = 4$ and $\Lambda = .5(\mathrm{GeV}/c)$ are taken. In (a), $\Delta G = 0$ and $\frac{1}{2}\Delta q = 1$; in (b), $\Delta q = 0$ and $\Delta G = 1$.

Fig. 2. Feynman rules for the lowest order vertices of the axial vector operators A_ρ (diagram a) and a_ρ (diagram b and c). The total momentum of the three gluons in (c) is zero.

Fig. 3. Triangle diagrams and their relation with the gluon vertices of Fig. 2. The total momentum of the three gluons in (b) is zero.

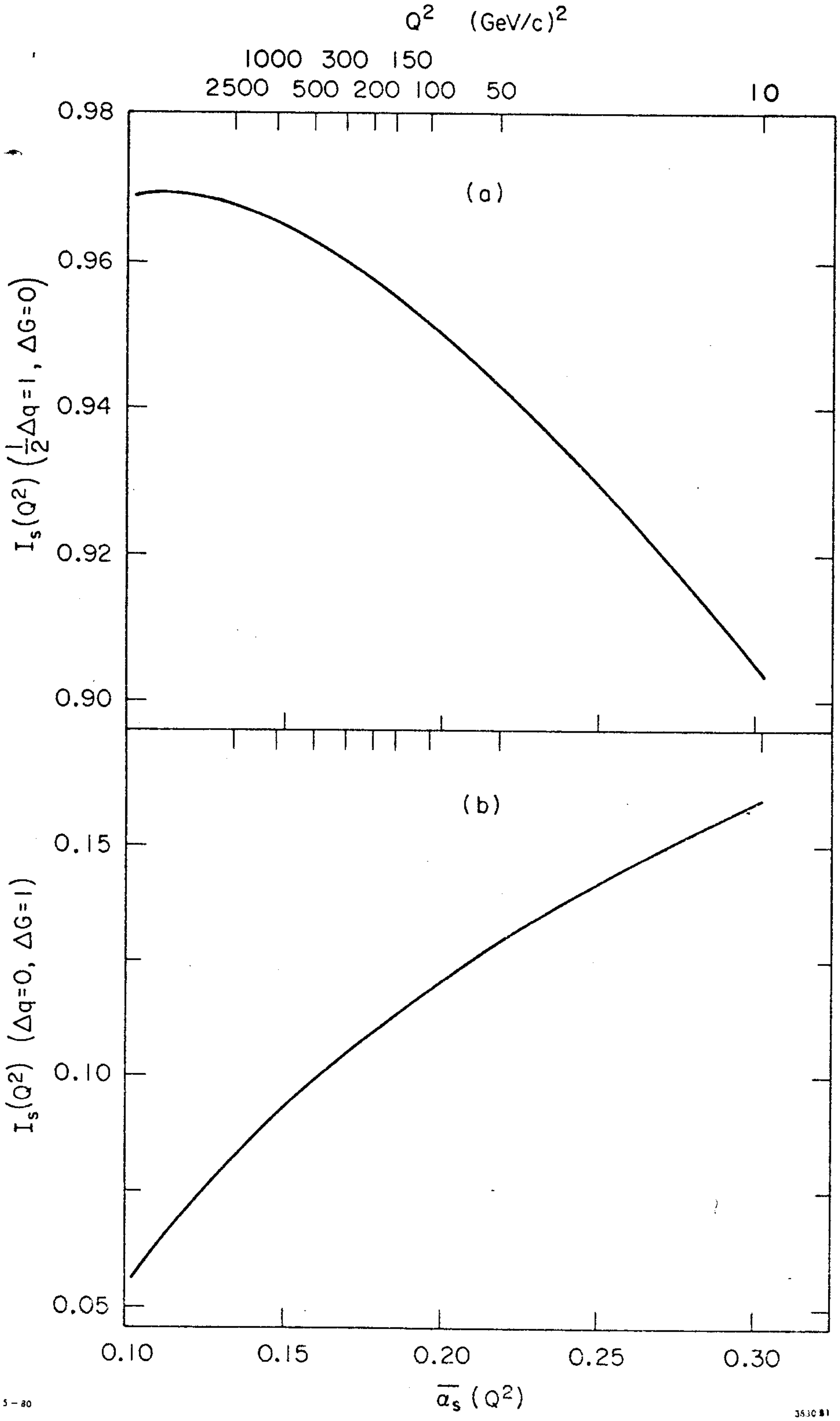
Fig. 4. Graph for the calculation of $\gamma_{qG}^{(0)}$.

Fig. 5. Graphs for the calculation of $\gamma_{GG}^{(0)}$.

Fig. 6. Graph for the calculation of $\gamma_{qq}^{(1)}$.

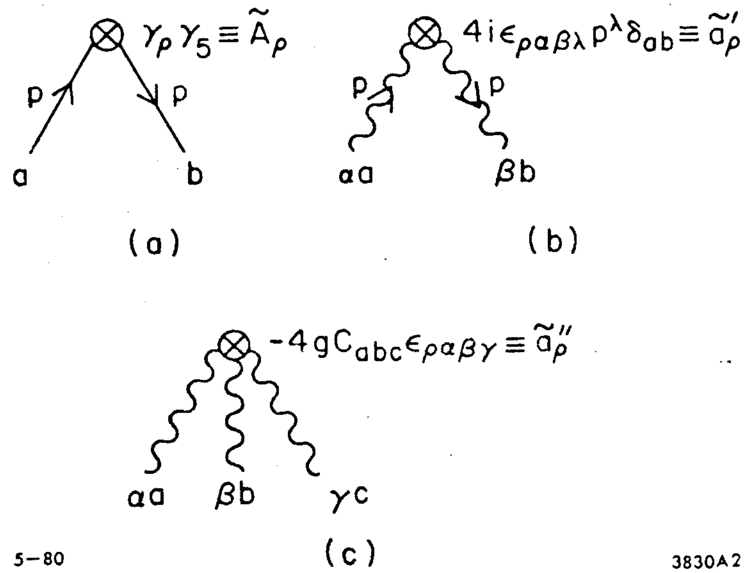
Fig. 7. Graphs for the calculation of $\gamma_{Gq}^{(1)}$.

Fig. 8. Gluon-photon scattering diagrams. The dotted and wiggler lines are photons and gluons respectively. There are six distinct diagrams of this type obtained by permutating the photon and gluon lines in the diagram shown.



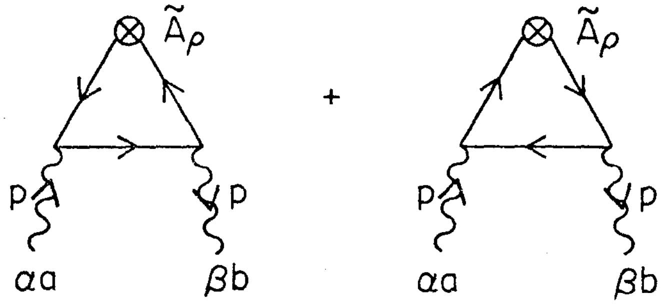
Q^2 (GeV/c)^2
1000 300 150
2500 500 200 100 50
10
0.98
0.96
0.94
0.92
0.90
(a)
$I_s(Q^2)$ $(\frac{1}{2}\Delta q=1, \Delta G=0)$
(b)
0.15
0.10
0.05
$I_s(Q^2)$ $(\Delta q=0, \Delta G=1)$
0.10
0.15
0.20
0.25
0.30
$\overline{\alpha_s}$ (Q^2)
5-80
3530B1

Fig. 1



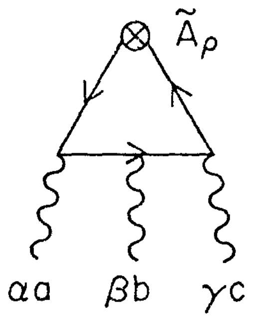
$\gamma_\rho \gamma_5 \equiv \tilde{A}_\rho$
p
p
a
b
(a)
$4i\epsilon_{\rho\alpha\beta\lambda} p^\lambda \delta_{ab} \equiv \tilde{a}'_\rho$
p
p
αa
βb
(b)
$-4gC_{abc}\epsilon_{\rho\alpha\beta\gamma} \equiv \tilde{a}''_\rho$
αa
βb
γc
(c)
5-80
3830A2

Fig. 2



$$= \frac{\alpha_s}{4\pi}(-2T(R))\,\tilde{a}'_\rho$$

(a)



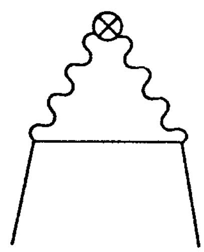
$$+ \text{ 5 other permuted diagram } = \frac{\alpha_s}{4\pi}(-2T(R))\,\tilde{a}''_\rho$$

5 – 80

(b)

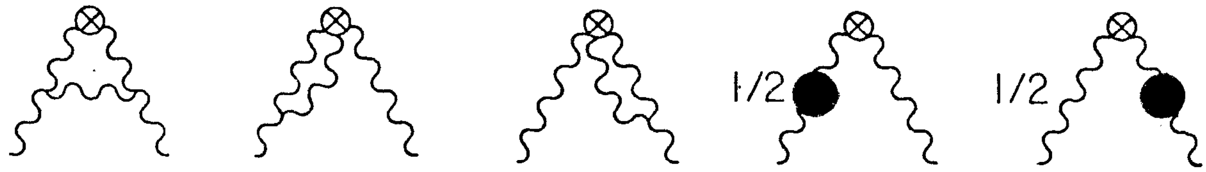
3830A3

Fig. 3



5–80 3830A4

Fig. 4



5 — 80

3830A5

Fig. 5

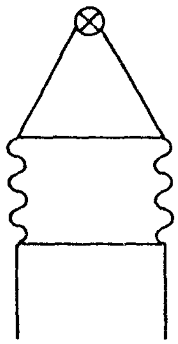
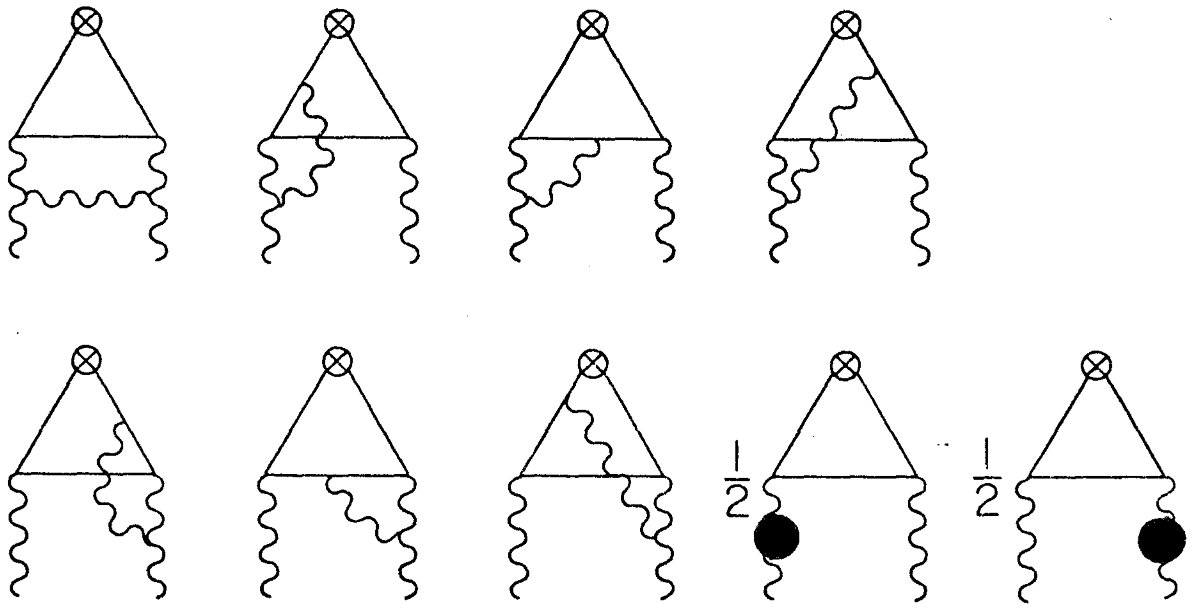


Fig. 6



5 – 80

Fig. 7

3830A7

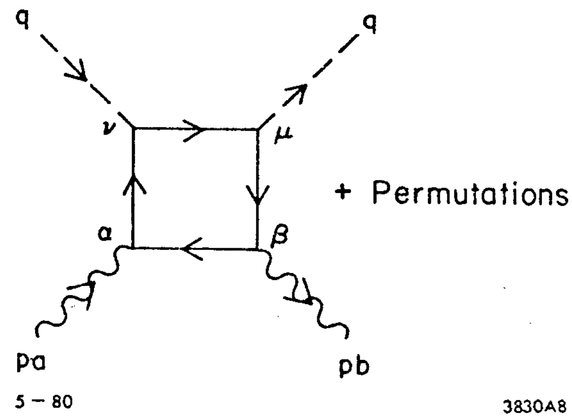


Fig. 8