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# WEAK AND ELECTROMAGNETIC CONTRIBUTIONS TO QUARK MASSES*

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ABSTRACT

We study the lowest order electromagnetic and weak contributions to quark masses in quantum chromodynamics. We review the arguments that these quantities are ultraviolet divergent in perturbation theory. A suggestion due to Brodsky, Schmidt, and de Téramond, based on a study of the Dyson equation for the quark self-energy, that electroweak corrections are controlled by low-energy physics if the number of quark flavors is greater than 10, is investigated. The meaning of the electromagnetic mass shift defined by these authors is clarified. In models possessing natural relations among quark masses and spontaneous symmetry breakdown the prescription of these authors is shown to lead to the same results as those of conventional analyses. In particular, corrections to these relations remain sensitive to the symmetry violating scale for any number of quark flavors.

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## I. Introduction

The problem of calculating electromagnetic and weak contributions to hadronic masses and mass differences is an old one.[1] Attached to this problem is an old question: Are electroweak corrections to these masses ultraviolet finite? Since we now have, in quantum chromodynamics (QCD), a promising candidate for a theory of strong interactions, it should be possible to address these issues. Of course, hadrons have yet to emerge from any QCD calculation, but, since the theory is asymptotically free, it should be possible to study the question of ultraviolet convergence.

In QCD, this problem reduces to the question: Are electroweak corrections to quark masses (or appropriate mass differences) finite? In the standard version of the Weinberg-Salam model[2], the mass of each quark arises from a separate Yukawa coupling to Higgs particles, and thus all quark masses are free parameters. As a result, even if electromagnetic and weak corrections to quark masses were finite, no relations among quark masses could be computed. Counterterms (albeit finite) could still be introduced to adjust each quark mass to any desired value. The issue becomes more meaningful, however, in models which possess "natural" mass relations. Natural relations are those imposed by the symmetries of the theory, and cannot be violated by counterterms. If the symmetry is spontaneously broken, one expects that at low energies (momentum scales well below the symmetry violating scale) corrections to these relations well be sensitive (logarithmically) to the symmetry violating scale. If corrections to quark masses were individually ultraviolet finite, one would expect this sensitivity to high energy scales to disappear.

## II. Electromagnetic Contributions to Quark Masses in Perturbation Theory

In a recent paper[3], Brodsky, Schmidt, and de Téramond (hereafter referred to as BST) addressed the problem of computing electromagnetic corrections to quark masses, and arrived at some interesting conclusions. In particular, they argued that in QCD, if the number of quark flavors, $n_f$, is greater than 10 (but less than 16, so as not to spoil asymptotic freedom) then the first order electromagnetic contributions to quark masses are in some sense calculable. Their analysis began with the Dyson equation for the running quark mass (Figs. 1-3):[4]

$$m(p^2) = m(\Lambda^2) + \int_{p^2}^{\Lambda^2} \frac{dq^2}{q^2}\left\{\frac{3C_F}{4\pi} m(q^2)\alpha_s(q^2) + \frac{3\alpha}{4\pi} m(q^2)\right\} + O(\alpha^2,\alpha_s^2) \ . \qquad (1)$$

In this equation, $\Lambda$ is an ultraviolet cutoff, p is a large, Euclidean momentum, $C_F$ is the quadratic Casimir operator of the fermion representation (3/4 for SU(3)), and $\alpha_s(q^2)$ is the running coupling constant of QCD:

$$\alpha_s(q^2) = \frac{4\pi}{\beta_o \ell n(q^2/\Lambda^2_{QCD})} \qquad (2)$$

where

$$\beta_o = 11 - \frac{2}{3}n_f \qquad (3)$$

and $\Lambda_{QCD}$ is the scale parameter of QCD. If we set $\alpha$ to zero, this equation has the solution

$$m(p^2) = m(\Lambda^2)\left(\frac{\ln(\Lambda^2/\Lambda^2_{QCD})}{\ln(p^2/\Lambda^2_{QCD})}\right)^{\gamma} \tag{4}$$

where

$$\gamma = \frac{3C_F}{\beta_o} . \tag{5}$$

This is just the standard result one obtains for the running mass in QCD using renormalization group arguments.[5] BST noted that for $n_f \geqslant 11$, $\gamma > 1$. Under such circumstances, the integral equation, Eq. (1) continues to make sense if we set $\Lambda$ to $\infty$ and $m(\Lambda^2)$ to zero. They argued that in this case one could then use the Dyson equation to identify strong and electromagnetic components of (renormalized) quark masses,

$$m(p^2) = m_s(p^2) + \delta m(p^2) \tag{6}$$

where $m_s(p^2)$ and $\delta m(p^2)$ satisfy the equations

$$m_s(p^2) = \frac{3C_F}{4\pi} \int_{p^2}^{\infty} \frac{dq^2}{q^2}\, \alpha_s(q^2) m_s(q^2) \tag{7}$$

$$\delta m(p^2) = \frac{3\alpha}{4\pi} \int_{p^2}^{\infty} \frac{dq^2}{q^2}\, m_s(q^2) + \frac{3C_F}{4\pi} \int_{p^2}^{\infty} \frac{dq^2}{q^2}\, \alpha_s(q^2)\, \delta m(q^2) . \tag{8}$$

These equations have the solutions

$$m(p^2) = m_s(p_o^2)\left(\frac{\ln(p_o^2/\Lambda^2_{QCD})}{\ln(p^2/\Lambda^2_{QCD})}\right)^{\gamma} \tag{9}$$

$$\delta m(p^2) = -\frac{3\alpha}{4\pi} \ln\left(\frac{p^2}{\Lambda^2_{QCD}}\right) m_s(p^2) + A m_s(p^2) . \tag{10}$$

BST argued that A has no dependence on $\alpha$, and thus should be absorbed into $m_s(p^2)$. Alternatively one might argue that since one can set $\Lambda$ to infinity, $\Lambda_{QCD}$ is the natural scale for the problem. From this result, they concluded that electromagnetic corrections to quark masses are determined by physics at momentum scales less than or of order the threshold for the eleventh quark flavor.

In order to decide whether or not this separation is appropriate, we must first point out (as did BST) that the electromagnetic corrections to the quark masses are not ultraviolet finite.[6,7,8] This fact is readily established using the Cottingham formula[1] and the operator product expansion (OPE).[9] The Cottingham formula gives for the electromagnetic contribution to the mass

$$\Delta m(p^2) = \frac{-i}{32\pi^4}\int \frac{dq^2}{q^2}\, T_\mu{}^\mu(q,p) \tag{11}$$

where $T_{\mu\nu}(q,p)$ is the virtual forward Compton amplitude for scattering of a virtual photon of momentum q on a target of momentum p. For the question of interest, only the integration region

$$q^2 >> p^2, p\cdot q \tag{12}$$

is relevant. In this region, one may study $T_\mu{}^\mu$ using the OPE. The leading operators in the expansion are $m_i\bar{\psi}_i\psi_i$ and $g^2F^a_{\mu\nu}\, F^{a\,\mu\nu}$, where $m_i$ and $\psi_i$ are quark masses and fields, respectively, and $F^a_{\mu\nu}$ is the Yang-Mills field strength. Since both of these contributions are renormalization group invariant, $T_\mu{}^\mu$ behaves as a constant at large $q^2$, and the integral diverges.[6,7,8]

In order to understand the BST result in light of this divergence, a graphical analysis is instructive. We can reorganize the perturbation series so that $\Delta m$ is given by a sum of terms, each of which appears to be ultraviolet finite, and each of which appears to receive its dominant contribution (for $n_f > 10$) from low energy phenomena. We first sum all QCD contributions to the quark mass, obtaining $m_s(p^2)$ as in Eq. (4). Consider then the diagram of Fig. 2. This diagram gives (from now on we choose units in which $\Lambda_{QCD} = 1$)

$$\Delta m^{(0)}(p) = \frac{3\alpha}{4\pi} \int_{p^2}^{\Lambda^2} \frac{dq^2}{q^2}\, m_s(q^2) = \frac{3\alpha}{4\pi}\,\frac{1}{\gamma-1}\left\{m_s(p^2)\ln p^2 - m_s(\Lambda^2)\ln\Lambda^2\right\} . \quad (13)$$

If $n_f \geqslant 11$, this diagram has a finite limit as $\Lambda^2 \to \infty$. Similarly, diagrams of the type shown in Fig. 3 all separately have finite limits. For the diagram with n gluons outside the photon, we obtain

$$\Delta m^{(n)}(p) \xrightarrow[\Lambda^2 \to \infty]{} \frac{3\alpha}{4\pi}\, m_s(p^2)\, \frac{\ln p^2}{\gamma-1}\left(\frac{\gamma}{\gamma-1}\right)^n . \quad (14)$$

The sum

$$\Delta m(p) \xrightarrow[\Lambda^2 \to \infty]{} \sum_{n=0}^{\infty} \Delta m^{(n)}(p) = \frac{3\alpha}{4\pi}\, m_s(p^2)\, \frac{\ln p^2}{\gamma-1} \sum_{n=0}^{\infty}\left(\frac{\gamma}{\gamma-1}\right)^n . \quad (15)$$

is divergent. (Note that we would obtain the same result if we attempted to solve Eqs. (7-8) (with $\Lambda \to \infty$) by iteration.) If our expression for $\Delta m^{(n)}(p)$ was correct for $|\frac{\gamma}{\gamma-1}| < 1$, then we could sum the series by analytic continuation, and obtain the BST result, Eq. (10). This is not the case, however, since the integrals diverge for such values of $\gamma$. Thus it is necessary to keep the cutoff in these integrals. One can easily demonstrate by induction that

$$\Delta m^{(n)}(p) = \frac{3\alpha}{4\pi} m_s(p^2) \frac{\ell n p^2}{\gamma-1}\left(\frac{\gamma}{\gamma-1}\right)^n - \frac{3\alpha}{4\pi} m_s(\Lambda^2) \frac{\ell n \Lambda^2}{\gamma-1} \sum_{m=0}^{n} \left(\frac{\gamma}{\gamma-1}\right)^m \frac{1}{(n-m)!} \left\{\gamma \ell n\left(\frac{\ell n\Lambda^2}{\ell n p^2}\right)\right\}^{n-m} . \tag{16}$$

This expression remains valid for $\left|\frac{\gamma}{\gamma-1}\right| < 1$. For such values of $\gamma$, we can sum the series, interchanging summation orders with impunity.

$$\begin{aligned}
\Delta m(p^2) &= \sum_{n=0}^{\infty} \Delta m^{(n)}(p) \\
&= \frac{3\alpha}{4\pi} m_s(p^2) \frac{\ell n p^2}{\gamma-1} \sum_{n=0}^{\infty} \left(\frac{\gamma}{\gamma-1}\right)^n - \frac{3\alpha}{4\pi} m_s(\Lambda^2) \frac{\ell n\Lambda^2}{\gamma-1} \sum_{n=0}^{\infty} \sum_{m=0}^{n} \left(\frac{\gamma}{\gamma-1}\right)^m \\
&\quad \times \frac{1}{(n-m)!} \left\{\gamma \ell n\left(\frac{\ell n\Lambda^2}{\ell n p^2}\right)\right\}^{n-m} \\
&= -\frac{3\alpha}{4\pi} m_s(p^2)\, \ell n p^2 - \frac{3\alpha}{4\pi} m_s(\Lambda^2) \frac{\ell n\Lambda^2}{\gamma-1} \sum_{m=0}^{\infty} \sum_{n=m}^{\infty} \left(\frac{\gamma}{\gamma-1}\right)^m \\
&\quad \times \frac{1}{(n-m)!} \left\{\gamma \ell n\left(\frac{\ell n\Lambda^2}{\ell n p^2}\right)\right\}^{n-m} \\
&= \frac{3\alpha}{4\pi} m_s(p^2) \ell n(\Lambda^2/p^2) .
\end{aligned} \tag{17}$$

Now we can continue $\gamma$ back to its correct value, yielding the divergent (cutoff dependent) result. A similar analysis can be performed for the diagrams such as those of Fig. 4, which were neglected in the

BST analysis. These diverge as $\ell n \ell n \Lambda^2$. Their contribution to running masses is suppressed compared to those of Fig. 1 by a power of a logarithm.[10]

This result is also readily obtained if we keep the cutoff, $\Lambda$, in the Dyson equation, Eq. (1). Then we obtain

$$m(p^2) = \left(m_s(\Lambda^2) + \delta m(\Lambda^2)\right)\left(\frac{\ell n \Lambda^2}{\ell n p^2}\right)^{\gamma}\left(1 + \frac{3\alpha}{4\pi}\,\ell n\left(\frac{\Lambda^2}{p^2}\right)\right) \tag{18}$$

where we have taken

$$m(\Lambda^2) \equiv m_s(\Lambda^2) + \delta m(\Lambda^2) \ . \tag{19}$$

The BST prescription corresponds to identifying

$$m_s(p^2) = m_s(\Lambda^2)\left(\frac{\ell n \Lambda^2}{\ell n p^2}\right)^{\gamma} \tag{20}$$

$$\delta m(\Lambda^2) = -\frac{3\alpha}{4\pi}\,\ell n\left(\frac{\Lambda^2}{\Lambda^2_{QCD}}\right) m_s(\Lambda^2) \ . \tag{21}$$

In the standard Weinberg-Salam model, such a separation is without observable significance, since $m_s(p^2)$ can be adjusted to any desired value.

Still, BST argued that this breakup was in some sense appropriate and that low energy physics would determine this $\delta m(p^2)$. They clearly had in mind cases where $m(\Lambda^2)$, the bare masses, were constrained in some way (or equivalently that the normalization of the $m_s(p^2)$ was fixed at some scale). In the following section, we consider their prescription in models which possess "natural" relations among quark masses.

## III. A Simple Model with a Natural Mass Relation

These questions become more meaningful if we consider a model with a "natural" mass relation.[11] In this section, we study such a model, using both the Cottingham-OPE (COPE) type analysis and the prescription of Brodsky, Schmidt and de Téramond. Since the theory makes a definite, observable, prediction for relations among quark masses, if the two procedures were to make different predictions, this would have definite physical consequences. At least one would certainly be wrong. Moreover, we might expect such a difference, based on our discussion in the introduction, for the BST analysis suggests that corrections to quark masses are determined (for $n_f \geqslant 11$) by low energy physics, in contrast to the COPE analysis, which suggests scales up to the symmetry breaking scale should be important.

The model we consider is based on the gauge group $SU(3)_c \times O(3)$. The color group is just the usual one, and will remain unbroken. Quarks will transform as triplets under this group. Under O(3), we take the quarks to lie in triplets (the O(3) coupling is taken vector-like),

$$\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix} . \qquad (22)$$

We also introduce a Higgs triplet, with potential arranged so that

$$< \phi >_o = \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix} . \qquad (23)$$

Two of the vector mesons, which we denote by $W^{\pm}$, will gain mass $M_W^2 = \bar{e}^2 v^2$, where e is the O(3) gauge coupling. A third, the photon of this model, remains massless. We also arrange the Higgs potential so that the surviving scalar $\phi^\circ$ has mass $m_\phi^2 \sim e^2 v^2$. The symmetry of the model, as well as the restriction to renormalizable couplings, limits the possible mass terms for the fermions to two:

$$\mathscr{L}_m = m\bar{\psi}\psi + G\sum_a \bar{\psi}\, T_a \psi \phi_a \tag{24}$$

where $T_a$ are the O(3) generators, and a sum over colors is implied. At tree level, then, we have

$$\begin{aligned} m_u &= m + Gv \\ m_d &= m \\ m_s &= m - Gv \end{aligned} \tag{25}$$

or

$$\frac{m_u - m_d}{m_d - m_s} = 1 \ . \tag{26}$$

Any counterterms must respect this relation. Thus, in perturbation theory

$$\frac{m_u - m_d}{m_d - m_s} = 1 + f \tag{27}$$

where f is a calculable function of the parameters of the theory. We will renormalize the quark masses at scale $p^2$,

$$\Lambda_{QCD}^2 << p^2 << M_W^2, m_\phi^2 \ . \tag{28}$$

For the analysis, it is convenient to choose a renormalizable gauge for the $W^{\pm}$, since all diagrams individually have good high energy behavior, and power counting arguments are straightforward. If, for the moment, we turn off the color interactions, a simple calculation yields

$$\frac{m_u(p)-m_d(p)}{m_d(p)-m_s(p)} = 1 + \frac{6\alpha}{4\pi}\left(\ell n\left(\frac{M_W^2}{p^2}\right) + O(1)\right) . \qquad (29).$$

In particular, all infinities cancel, as expected. This result, Eq. (29), is easily understood physically. At momentum scales below the symmetry breaking scale, the primary corrections to the mass relation come from emission and absorption of virtual photons by the u and s quarks. The corresponding integrals are divergent, but in computing the ratio these integrals are cut off at scales $M_W$, yielding Eq. (29).

Now consider what happens in the presence of strong interactions. We can organize the calculation along the lines of the COPE analysis, for photon, $W^{\pm}$ and $\phi^{\pm}$ exchanges. Then the leading contributions to the quark masses are the same as those of the bare theory, except that the quark masses now include all strong corrections, evaluated at scale $p^2$. Thus Eq. (29) remains unchanged.

What of the BST prescription? The problem is most simply discussed in the language of the Dyson equation. The analysis of photon contributions is as before. For the $W^{\pm}$ and $\phi$ contributions, we can write similar Dyson equations with photon lines replaced by W and $\phi$ lines. Then, for the W contribution to the u-quark mass, for example (using 't Hooft-Feynman gauge),

$$\delta m_W^u(p^2) = \frac{3\alpha}{4\pi} \int_{p^2}^{\infty} \frac{dq^2}{q^2-m_W^2}\, m_d^s(q^2) + \frac{3C_F}{4\pi} \int_{p^2}^{\infty} \frac{dq^2}{q^2}\, \alpha_s(q^2)\, \delta m_W^u(q^2) \tag{30}$$

where we have followed BST in taking the cutoff to infinity. The solution to this equation which these authors instruct us to take is

$$\delta m_W^u(p^2) = -\frac{3\alpha}{4\pi}\, m_d^s(p^2)\, \ln\left(\frac{M_W^2}{\Lambda_{QCD}^2}\right)\left(1 + 0\left(\frac{1}{\ln M_W^2}\right)\right) . \tag{31}$$

A similar analysis can be performed for the other contributions to the ratio, Eq. (27). When we sum these contributions we again obtain the result, Eq. (29), provided that the quantities $m_s^i(p^2)$ for the various quarks obey the natural relation. Noting that (Eq. (19))

$$\begin{aligned} m_s^i(\Lambda^2) &= m^i(\Lambda^2) - \delta m_W^i(\Lambda^2) - \delta m_\gamma^i(\Lambda^2) - \delta m_\phi^i(\Lambda^2) \\ &\equiv m^i(\Lambda^2) - \delta m(\Lambda^2) \end{aligned} \tag{32}$$

and that the $m^i(\Lambda^2)$ necessarily satisfy Eq. (26), one can readily verify that the quantities $m_s^i(p^2)$ do in fact satisfy the natural relations. This is simply a consequence of the renormalizability of the theory: The divergent counterterms must have the same structure as the original Lagrangian.

The lesson to be drawn from this analysis is that the BST separation of fermion masses into "strong", "weak", and "electromagnetic" components (and other components in models with additional interactions) will in general respect natural relations among fermion masses. If the strong interactions by themselves respect the symmetries represented by the natural relation, the quantities $m_s^i(p^2)$ defined by BST must as well, and the finite corrections obtained from the $\delta m^i(p^2)$ will agree with the results obtained with standard methods of analysis.

In cases where the strong interactions do not respect the natural relations, the quantities $m_s^i(p^2)$ will not obey them, and the BST analysis must be extended in order to find relations among these "strong" masses. Grand unified models possessing natural relations are examples of such theories. To extend the BST analysis to such theories one merely needs to start with their solutions to the Dyson equation for momenta well above the grand unification scale. Here the theory is symmetric, and masses will respect the natural relations. One can then evolve their equations down to lower $p^2$. Since the masses defined by BST satisfy the same scaling laws as those obtained using the renormalization group, the corrections to natural relations must necessarily be the same as those obtained with conventional analyses. For example, in the standard SU(5) grand unified model[12] with the simplest Higgs assignment, there are certain natural relations, such as $m_b = m_\tau$, at tree level. The analysis of BST again (to leading order) reproduces the result of conventional analysis[13] for corrections to this relation, but only if physics near the grand unification point is included.

We conclude by emphasizing that in these models, even if we use the BST prescription, it is not correct to say that the corrections to quark masses come only from physics at low energies (i.e. scales of the order of the 11th quark mass). As we have seen, both quark masses and natural relations among them receive important corrections from physics at much higher energy scales. In grand unified models, in particular, physics at the grand unification scale will give important contributions to relations among quark masses.

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## FIGURE CAPTIONS

Fig. 1 Dyson equation for quark masses, including first order electromagnetic effects. Label g denotes gluons, label $\gamma$ denotes photons.

Fig. 2 A contribution to the electromagnetic mass shift of a quark.

Fig. 3 Another class of diagrams which contribute to the electromagnetic mass shift of a quark. With an appropriate choice of gauge, these, along with the diagram of Fig. 2, are the leading contributions.

Fig. 4 A non-leading (but ultraviolet divergent) contribution to quark masses.

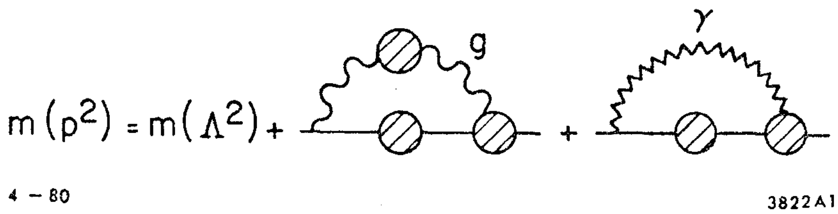


Fig. 1

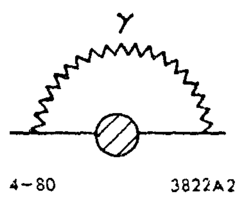


Fig. 2

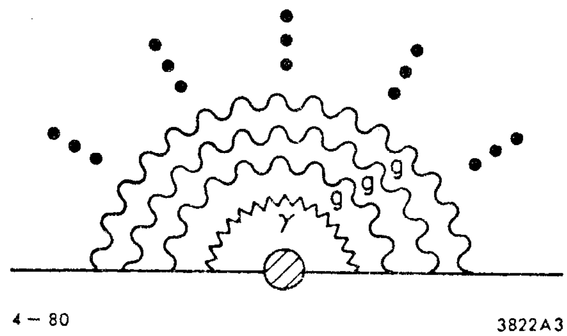


Fig. 3

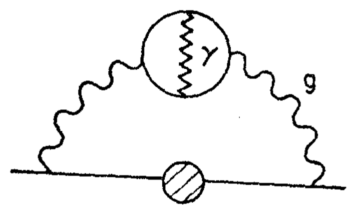


Fig. 4