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On the Magnetically Bound Monopole Pair, A Possible Structure for Fermions*

by

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Summary

Arguments are presented that two magnetically bound monopoles will fall into a ground state characterized by an orbital angular momentum quantum number $\ell = \frac{1}{2}$. That such a state forms a suitable structure for point-like fermions is proposed.

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Dirac, who was the first to speculate about the incorporation of magnetic charge into elementary particle theory,[1] obtained the quantization condition

(1) $$\frac{e_0 g_0}{\hbar c} = \frac{n}{2}$$

where e_0 and g_0 are respectively the basic electric and magnetic charge quanta, and n is an integer. Gaussian units are used here; $\hbar$ and c have their usual significance. Schwinger, who considered a particle carrying both electric and magnetic charge, which he named the dyon, has proposed "A Magnetic Model of Matter,"[2] in which the baryons are composed of three dyons (thus fundamentally differing from the approach here). He obtained the quantization condition

(2) $$e_1 g_2 - e_2 g_1 = n\hbar c \quad ,$$

where 1 and 2 denote two interacting dyons. By now there has been an enormous amount of theoretical work on magnetic monopoles, too voluminous for discussion here; the reader may consult a review article[3] for details and references.

Since this letter is concerned with a magnetically bound monopole pair which is expected to be magnetically neutral, one may write

(3) $$g_2 = -g_1 \equiv g_0 \quad .$$

Substituting eq. (3) into eq. (2) yields

(4) $$(e_1 + e_2)\, g_0 = n\hbar c \quad .$$

Now $(e_1 + e_2)$ is just the total electric charge which we attribute to the bound pair. Since it is proposed herein that the magnetically bound pair is a possible structure for fermions, one may set

(5) $$\left|e_1 + e_2\right| = e \text{ or } 0 \quad ,$$

where e is the charge of the positron [An analogous equation made for fractionally charged quarks would not change any of the conclusions of the following analysis (except the magnitude of g_0) and is hence omitted as being redundant.] Since there are two monopoles (or dyons, or vortons*) in the pair, one may make the phenomenological allocation:

(6) $$\left|e_1\right| = \left|e_2\right| = e_0 = e/2 \quad .$$

Consequently, one sees that for the structure under consideration here, eq. (1) and eq. (2) give the same result. Setting $n = 1$, one then obtains

(7) $$g_0 = \frac{\hbar c}{2e_0} = \frac{\hbar c}{e} = \frac{e}{\alpha} \quad \text{and} \quad \frac{g_0^2}{\hbar c} = \frac{1}{\alpha} \quad ,$$

where α is the fine structure constant.

Since the magnetic force is so much stronger than the electric force, there is no problem for a monopole pair to exist in an electrically charged, magnetically bound state. Such a structure, then, could furnish an answer to the old question[5] of the internal force ensuring the

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*The vorton is a recently proposed [4] semi-classical electromagnetic monopole configuration. It should be noted that the semi-classical value for the vorton charge, 25.8 e, would not satisfy either the Dirac or Schwinger quantization condition. But possible unaccounted quantum mechanical effects might appropriately modify the semi-classical result.

stability (against electrical self-repulsive forces) of the electron.* However, while solving this problem, the superstrong magnetic force leads also to what is in some sense the opposite problem. This is the problem of collapse, which we now take up.

It is well known that if the laws of classical mechanics prevailed, then atomic structures could not exist; atomic electrons, bound to the nucleus (of charge Ze) by the dimensionless coupling constant $Z\alpha(<1)$ would, through their centripetal acceleration, radiate away their energy and ultimately fall into the nucleus. Quantum mechanics explains why atomic structures are stable and this catastrophe does not occur.

Monopole pairs, on the other hand, would be bound by the superstrong magnetic force characterized by the coupling constant $g^2/\hbar c = 1/\alpha >> 1$. Unfortunately, perturbation theory, the technique so successfully used in QED, will not work in this regime. Since, as yet no other technique has been developed, we resort to a simple qualitative argument† which

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*An excellent review of the history of physical models and analyses of the electron has been written by Rohrlich[(6)]

†Arguments along these lines are well known and found in the literature in varying degrees of complexity, e.g., the book by Bohm.[(7)] The question of stability of matter has been recently discussed in some detail[(8)] where it is shown that the use of the Heisenberg principle cannot be used for a general argument covering all conceivable wavefunctions. But the simplicity and directness of the argument, and its ease of extension to the relativistic domain make it suitable for the purposes of this letter.

shows that electrically bound atomic structures are stable against collapse (and which gives the correct atomic scale), while magnetically bound monopole pairs must collapse. To do this we employ quantum mechanics as embodied in the Heisenberg uncertainty principle:*

$$\Delta x \, \Delta p \geq \hbar \tag{8}$$

and minimize total energy

$$W = T + V \quad , \tag{9}$$

where T and V are kinetic and potential energies, respectively. Particle masses, which are assumed to be constant, are omitted from eq. (9).

In the case of atomic structures

$$T = \frac{p^2}{2m} \quad \text{and} \quad V = \frac{-Ze^2}{r} \quad . \tag{10}$$

Using eq. (8) to set $p = \hbar/r$ in eq. (10), one finds a minimum in W at

$$a = \frac{\hbar^2}{Ze^2 m} \quad , \tag{11}$$

where a has been used (for r) to represent the scale of the bound state. For $Z = 1$, a is just the Bohr raidus, the correct scale for atomic structures. With $Z\alpha << 1$, $T = \frac{1}{2}(Z\alpha)^2 mc^2$, and the motion is nonrelativistic.

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*One recognizes, of course, that this derivation is qualitative in nature, and eq. (8) could just as well be modified by a factor of order unity.

If one considers stronger couplings, i.e., as $Z\alpha \to 1$, the electron becomes relativistic (a statement independent of its mass) and a better approximation for the kinetic energy becomes $T = pc$. In this relativistic regime, then, the mass of the bound particles becomes irrelevant and the energy in which to find a minimum, in order to determine the appropriate scale of the bound state, is

$$W = \frac{\hbar c}{a} - Z\alpha \frac{\hbar c}{a} = \frac{\hbar c}{a}(1 - Z\alpha) \quad . \tag{12}$$

Extrapolating to $Z\alpha >> 1$, which is the regime equivalent to that of the magnetic force, one sees that the Heisenberg localization energy is easily furnished by the superstrong binding; eq. (12) shows no minimum in energy for any $a > 0$. One concludes, then, that the scale of the bound state $\to 0$, i.e., collapse ensues, and the magnetically bound monopole pair will tend to be point-like.* (The fact that $W < 0$ is a deficiency of the approximation which does not necessarily negate the conclusion that $Z\alpha >> 1$ implies collapse; see below.)

The extent of collapse of the magnetically bound monopole pair depends upon one's assumptions about the extent of the monopole. Some previous models have assumed that monopoles have an intrinsic dimension which would limit this collapse.[9-12] Here, on the other hand, we explore the assumptions that the monopole is a point-like particle interacting only locally with the photon and that the mass and spatial extent

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*It is also possible to show, using a relativistic version of the Sobolev inequality (more rigorous than the analysis here using the Heisenberg uncertainty principle) that a large coupling constant implies collapse; E. H. Lieb, private communication.

of the free monopole are consequently due to electromagnetic self-interactions. In this case the intrinsic nature of the free monopole would not furnish a scale for the magnetically bound monopole pair.

Using physical intuition to argue against singularities leads one to expect some new phenomenon to enter the picture and prevent the bound pair from becoming a true geometric point — hence the term "point-like" is used advisedly. While at present, what new phenomenon ultimately steps in to set the scale of the magnetically bound monopole pair is a matter of conjecture, it is relevant to note that Landau and his collaborators[(13)] have suggested that graviational phenomena taking place on the scale of the Planck length $\ell_P = (\hbar G/c^3)^{\frac{1}{2}} = 1.616 \times 10^{-33}$ cm might "save" QED. In looking at the QED self-mass problem along these lines, it has been suggested[(14)] that due to vacuum polarization, there may be a reversal of the electromagnetic force beyond the Landau singularity (at or near ℓ_P). This reversal would have the effect of a hard-core repulsive potential preventing the complete collapse of the magnetically bound monopole pair state beyond a scale on the order of ℓ_P. In any case, the issue of the precise scale of the bound state of the monopole pair is not crucial here since ℓ_P is many orders of magnitude beyond present experimental data on the point-like nature of fermions.

While the above line of reasoning concludes that the magnetically bound monopole pair will tend to be point-like, the S state, which is the ground state of binding by small coupling constants, needs special consideration because of the effects of (superstrongly coupled) vacuum fluctuations. Vacuum fluctuations are known to decrease the binding energy (raise the enrgy level) of S state electrons in atoms, leading to what is known as the Lamb shift. The reduction in binding in hydrogen[15] is given by

$$\Delta E_{n\ell m} = \frac{4}{3}\,\hbar c\alpha^2 \lambda\!\!\!^{-}{}_e^2 L \left|\psi_{n\ell m}(0)\right|^2 \tag{13}$$

where $L \equiv \ln(m_e c^2 / \langle W \rangle)$ and $\lambda\!\!\!^{-}_e$ is the (reduced) electron Compton wavelength. In hydrogen the average excitation energy $\langle W \rangle$ (for the 2S level) has been calculated[15] to be 17.8 Ry, yielding $L \sim 7.6$. For small r, the wavefunction

$$\psi_{n\ell m} \sim r^\ell Y_{\ell m} \tag{14}$$

where $Y_{\ell m}$ is the usual spherical harmonic. For S states the substitution

$$|\psi(0)|^2 = \frac{1}{\pi (na)^3} \tag{15}$$

where (na) is the scale of the state, may be used. Using eqs. (14) and (15) in eq. (13) shows that

$$\Delta E_{n\ell m} = \frac{4}{3}\,\hbar c\alpha^2 \lambda\!\!\!^{-}{}_e^2 L \,\frac{1}{\pi (na)^3} \quad \text{for } \ell = 0, \tag{16}$$

while

$$\Delta E_{n\ell m} \to 0 \quad \text{for } \ell > 0 \quad . \tag{17}$$

One may use the substitutions $\alpha \to Z\alpha$, $\lambda\!\!\!^{-}_e \to \lambda\!\!\!^{-}_M$, the Compton wavelength of the monopole, and $n = 1$ in eq. (16) to convert it to an appropriate (estimated) magnetic monopole Lamb shift potential to add to eq. (9). Including this potential and minimizing total energy, the scale of the S state bound monopole pair is then estimated to be

$$a_S \cong 2\left(\frac{Z\alpha L}{\pi}\right)^{\frac{1}{2}} \lambda\!\!\!^{-}_M \sim 35\lambda_M \quad , \tag{18}$$

where $Z\alpha = 137$ has been used, and L, since it is a logarithmic function (which is insensitive to large variations in its argument), was assumed to be the same as the value employed in eq. (13). It is important to note that since eq. (18) yields $a_S >> \lambdabar_M$, one can argue that at a scale characterized by a_S we are not yet in a fully relativistic regime, i.e., the non-relativistic Lamb shift potential approximated by eq. (16) would still be valid. Thus vacuum fluctuations which cause a 10^{-6} effect in hydrogen, an effect measured with considerable difficulty, assume a dominant role in the superstrongly bound monopole pair due to the a^{-3} dependence in eq. (16).

Now, using eqs. (17) and (18), one can make the argument that states with $\ell > 0$, which do not have a ("first order") Lamb shift $[\psi_{\ell>0}(0) \approx 0]$, will collapse to a smaller radius than will the S state. Thus, the S state will be prevented from being the ground state by the Lamb shift!*

Since the radial function near the origin goes like r^{ℓ}, the $\ell = \frac{1}{2}$ state will be one which minimizes the 1/r potential, the kinetic, and the Lamb shift energies. As the monopole pair is (radiatively) cascading

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*This estimate ignores the vacuum polarization graph which in hydrogen is the same order (α^2) as the first order Lamb shift, but of opposite sign. For vacuum polarization loops consisting of leptons (electrically coupled), this graph will be down by a factor of α^2. It is further assumed that vacuum polarization loops of monopoles are never able to form because the superstrong coupling effectively prevents a spatial separation of the monopole-antimonopole pair.

down through integral ℓ angular momentum states, the final transition, say, could be from an $\ell = 1$ state to an $\ell = \frac{1}{2}$ state by the emission of an antifermion. Since it is proposed that the original monopole pair will become a fermion, then the final result would be a fermion-antifermion pair, conserving angular momentum (as well as other quantum numbers). Hence we have an argument that the $\ell = \frac{1}{2}$ state is, in fact, the ground state of the magnetically bound monopole pair, and could become a spin $\frac{1}{2}$ entity, a point-like fermion. Since it is a pair of monopoles under consideration here, the spin of a single monopole does not enter these arguments in any crucial way. However, since this letter proposes a composite structure for fermions, consistency and simplicity would imply an integral spin for the monopole, which itself would then be elementary. (One would not suppose that there are two basic structures for fermions.)

Now in proposing this composite structure, two difficulties must be acknowledged. The first is that the above Lamb shift argument is based upon a non-relativistic perturbation analysis which is of questionable applicability to the superstrong binding of magnetic monopoles. However, while a proper solution to the strong binding problem, using the Bethe-Salpeter equation, say, has never been done, we note that a straightforward physically based argument[16] gives a result (for hydrogen) that is in qualitative accord with more sophisticated perturbation calculations. Thus, we argue that in the strong coupling regime we can rely upon the same physical arguments by which one would expect the local magnetic fields associated with vacuum fluctuations to act to separate oppositely charged particles, militating against states with a finite $\psi(0)$, i.e., against S states, favoring states for which $\psi(0) \simeq 0$, i.e., states of $\ell > 0$.

The second is that arguments have been advanced as far back as[17] 1939 to show that these half odd integral orbital states would be precluded in the physical world. In a review article on this question Whippman[18] counters all arguments opposing the state with $\ell = \frac{1}{2}$, save the one attributed to Nordsieck[19] and one published by Schwinger[20] while Whippman's paper was in proof. More recently the pros and cons of half-integral orbital angular momentum have been discussed by van Winter.[21] These papers cite what is by now an extensive literature on this subject. A perusal of this literature reveals that none of the arguments against $\ell = \frac{1}{2}$ appear, in the final analysis, to be conclusive. That is, $\ell = \frac{1}{2}$ does not violate any basic laws of quantum mechanics.[18] In fact, there are at least two published elementary particle models[10,22] (both differing from this one in essential ways) which incorporate the possibility of $\ell = \frac{1}{2}$.

It was argued above that the strong magnetic binding force would cause the (magnetically neutral) monopole pair to collapse to a point-like state. Since it has been assumed that the free monopole mass is due to magnetic self-interactions (neglecting the electric components of the charge as being negligible relative to the magnetic components), it follows that the binding energy causing the collapse will be furnished by and limited to the mass (energy) of the original unbound, or free, monopoles. This being the case, one may argue that in the point-like limit, the binding energy will be identically equal to the sum of the masses of the free monopoles (a statement independent of the monopole mass!);* it will not become negative as implied by (the simple approximation given in) eq. (12).

From a qualitative point of view, the point-like bound monopole pair would be a massless entity because the magnetic charges are essentially in full spatial overlap,† the magnetic charges effectively cancelling each other, precluding their coupling to the vacuum and consequently precluding the generation of any individual or pair mass due to (magnetic) self-interactions. By the self-mass assumption, objects which are electromagnetically (completely) neutral cannot couple to the vacuum and will hence be massless.

Any residual electric charge on the pair could, in fact, go on to generate a self-mass for this composite entity as a pair (electron or muon, say) as described in further detail in ref. 14. In this latter process, as in other fermion interactions, the magnetic substructure would remain hidden.

One can also look at this question from the point of view of Feynman diagrams. The lowest order self-mass diagram for the monopole is shown in fig. 1, and the lowest order binding diagram in fig. 2. As the two magnetic monopoles enter into the point-like state, these evolve into

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*We see, then, that the assumption that the monopole has only the electromagnetic interaction furnishes in a natural way a reason that the binding energy can exactly cancel the mass of the monopoles.

†One expects the monopole wavefunctions to expand to the size of the bound state just as the electronic wavefunction in hydrogen expands to $\sim 1\text{Å}$.

diagrams such as are shown in fig. 3. The (sum of the) integrands associated with these four diagrams will cancel for scales greater than that of the point-like state. The other possible time orderings as well as the vacuum bubbles will similarly cancel. The contributions to the integrals for photon momenta $k \gtrsim \hbar/a$ will depend upon the physics which determines a. But if there is a cutoff as suggested by Landau, then this ultraviolet "end effect" would be expected to be much less than the monopole mass because of the restricted range in which the sum of the integrands would be nonzero. And since potential energy tends to a minimum, it is not unreasonable to expect this magnetic self-energy for the point-like pair to be null or nearly null.* This argument easily extends to all orders because every photon line terminating on the pair can have its vertex either on a $+g_0$ or a $-g_0$ and all other factors are equal.

In conclusion, arguments have been presented that the ground state of two magnetically bound monopoles may be characterized by an orbital angular quantum number $\ell = \frac{1}{2}$. As a consequence, that two oppositely charged monopoles in a tightly bound state form a suitable structure for point-like fermions is proposed.

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*It might be more appropriate to describe the bound, point-like state as "massless-like" which, like the term "point-like", would allow for the possibility of a small discrepancy from mathematical perfection.

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Figure Captions

Fig. 1. The lowest order self-mass diagram of a magnetic monopole.

Fig. 2. The lowest order binding diagram of a monopole pair.

Fig. 3. The four possible lowest order time ordered (x earlier than y) Feynman diagrams for the point-like bound monopole pair. The (ultraviolet) photons binding the pair are omitted. The upper two diagrams derive from self-mass diagrams for the individual monopoles, the lower two from the binding diagram. Because the pair is in a point-like state, tightly bound in both space and time, the bottom two diagrams are distinct (below the ultraviolet cutoff).

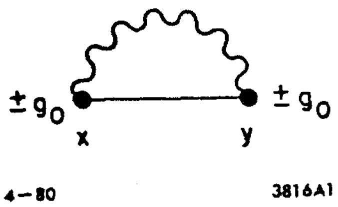

Fig. 1

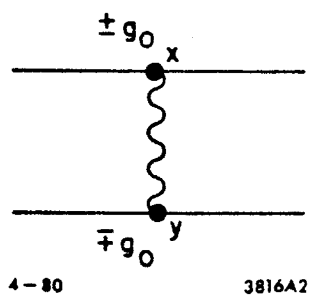

Fig. 2

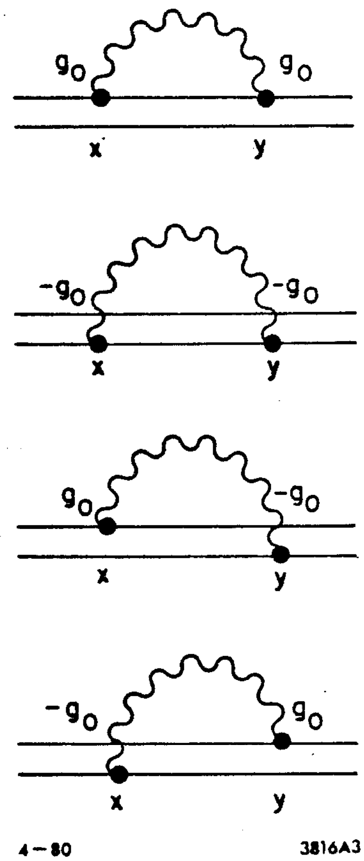

Fig. 3