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# MAGNETIC MONOPOLES IN GRAND UNIFIED THEORIES*

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## ABSTRACT

We suggest that the magnetic monopoles predicted by grand unified theories would not be produced in significant numbers if electromagnetic gauge invariance is spontaneously broken when the temperature T is greater than $T_c \gtrsim 1$ TeV. A model possessing this behavior is displayed and the cosmological implications are discussed.

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There has been considerable discussion[1-4] recently that if Grand Unified Theories[5] (GUT's) are correct then an unacceptably large number of superheavy ($m_m \gtrsim 10^{16}$ GeV) magnetic monopoles[6] may have been produced immediately after the big bang. Magnetic monopoles of the 't Hooft-Polyakov type[6] can exist if a semi-simple gauge group G is spontaneously broken down to a subgroup H which contains a $U_1$ factor. The monopole mass $m_m$ is of order $M_X/\alpha_g$, where $M_X$ is a typical mass of a gauge boson associated with a broken generator, g is the gauge coupling, and $\alpha_g = g^2/4\pi$. In the Georgi-Glashow[5] $SU_5$ model $M_X \simeq 10^{14} - 10^{15}$ GeV and $m_m \simeq 10^{16}$ GeV.

It is likely that G was unbroken immediately after the big bang when the temperature T was large compared to $M_X$. As the universe cooled it presumably underwent one or more phase transitions, finally entering the phase in which G is broken down to H (containing the $U_1$ factor) at some temperature $T_i$. Preskill has argued[2] that the ratio $r(T) \equiv n_m(T)/n_\gamma(T)$ of monopole to photon density must have been less than $10^{-19}$ initially (i.e., when $T \lesssim T_i$). However, Preskill[2] and Einhorn, Stein, and Toussaint[3] have estimated that if the phase transition to the H phase is second order or weakly first order then $r(T_i) \simeq 10^{-6}$, thirteen orders of magnitude too large, unless unacceptably large values for the Higgs self coupling are assumed. One attractive solution to this problem, suggested by Preskill,[2] Einhorn et al.,[3] and Guth and Tye,[4] is that the phase transition at which the $U_1$ factor occurs is strongly first order, in which case it may be possible to have $r < 10^{-19}$.

In this paper we propose an alternative scenario for the suppression of monopoles, in which the universe undergoes two or more phase transitions

(which can be second order)

$$G \underset{T_1}{\rightarrow} H_1 \underset{T_2}{\rightarrow} \cdots \underset{T_n}{\rightarrow} H_n \underset{T_c}{\rightarrow} SU_3^c \times U_1^{EM} \quad , \qquad (1)$$

where $U_1^{EM}$ is not a subgroup of $H_n$. The critical temperature $T_c$ at which $U_1^{EM}$ appears is $T_c \gtrsim 1$ TeV. Since $T_c \lll m_m \simeq 10^{16}$ GeV, no monopoles will be produced. For example, $SU_5$ could break down to $SU_3^c$ at $T_1 \lesssim M_X$ and undergo a second phase transition to the higher symmetry group $SU_3^c \times U_1^{EM}$ at $T_c \gtrsim 1$ TeV.

We consider a model which at $T = 0$ is the standard Georgi-Glashow $SU_5$ model,[5] with $SU_5$ broken to $SU_3^c \times SU_2 \times SU_1$, by an adjoint Higgs representation and then to $SU_3^c \times U_1^{EM}$ by one or more five-dimensional Higgs representations. We assume that a hierarchy exists, i.e., that $M_{W\pm,Z} \lll M_{X,Y}$ and that the color triplet components of the Higgs fields have masses $\lesssim M_X$. For $0 \leq T \ll M_X$ we need only consider the $SU_2 \times U_1$ part of the model (we assume that $SU_3^c$ is never broken).

Therefore, consider an $SU_2 \times U_1$ model with n complex Higgs doublets $\phi_i$. It will turn out that at least three doublets (or two doublets and a singlet) are required, so we will taken $n = 3$. The Higgs potential (at $T = 0$) is

$$\begin{aligned} V = \sum_{i=1}^{3} & \left[-\mu_i^2 \phi_i^\dagger \phi_i + \lambda_i\left(\phi_i^\dagger \phi_i\right)^2\right] \\ + \sum_{i<j} & \left[\sigma_{ij} \phi_i^\dagger \phi_i \phi_j^\dagger \phi_j + \rho_{ij} \phi_i^\dagger \phi_j \phi_j^\dagger \phi_i \right. \\ & \left. + \eta_{ij}\left(\phi_i^\dagger \phi_j\right)^2 + \eta_{ij}^*\left(\phi_j^\dagger \phi_i\right)^2\right] \quad , \qquad (2) \end{aligned}$$

where we have imposed discrete symmetries under $\phi_i \rightarrow -\phi_i$ for simplicity.

If the minimum of V occurs when only one doublet (e.g., $\phi_1$) has a nonzero VEV$\langle\phi_i(0)\rangle$, then $SU_2 \times U_1$ is broken down to $U_1^{EM}$ and we can take $\langle\phi_1(0)\rangle = (0\ v_1)^T/\sqrt{2}$. If two doublets $\phi_1$ and $\phi_2$ both have nonzero VEV's then either $\langle\phi_2(0)\rangle = (v_2\ 0)^T/\sqrt{2}$ or $\langle\phi_2(0)\rangle = (0\ v_2)^T/\sqrt{2}$ which occur for $\rho_{12}$ greater or less than $2|\eta_{12}|$, respectively. $SU_2 \times U_1$ is either completely broken or broken to $U_1^{EM}$ for these two cases, respectively. We want $U_1^{EM}$ to be unbroken at $T=0$ but broken for $T > T_c$. Therefore, we take $\rho_{ij} > 2|\eta_{ij}|$, so that the VEV's want to be orthogonal, but we will arrange the other parameters so that $\langle\phi_2(0)\rangle = \langle\phi_3(0)\rangle = 0$. This occurs for $\mu_1^2 > 0$, $\mu_{2,3}^2 < 0$, and

$$|\mu_i|^2 + \frac{\sigma_{1i}\,\mu_1^2}{2\lambda_1} > 0 \quad , \quad i=2,3 \qquad . \tag{3}$$

Then $v_1^2$ is given by $\mu_1^2/\lambda_1 = (\sqrt{2}\,G_F)^{-1}$. We also require

$$\lambda_i > 0$$

$$\sigma_{ij} > -\sqrt{\lambda_i\,\lambda_j} \qquad , \tag{4}$$

which are sufficient conditions for V to be bounded from below.

For $T > 0$, the VEV's $\langle\phi_i(0)\rangle$ must be replaced by ensemble averages[7-9] $\langle\phi_i(T)\rangle$. It has been shown that the $\langle\phi_i(T)\rangle$ can be obtained at least for sufficiently large T, by minimizing the effective potential

$$V(T) \equiv V + \sum_{i=1}^{3} \frac{T^2}{2} F_i\,\phi_i^\dagger\,\phi_i \qquad , \tag{5}$$

where

$$F_i = \frac{3g^2 + g'^2}{8} + \lambda_i + \sum_{j\neq i}\left[\frac{\sigma_{ij}}{3} + \frac{\rho_{ij}}{6}\right] + \text{Yukawa terms} \ . \tag{6}$$

For small fermion masses the Yukawa terms in (6) will generally be negligible. If the $F_i$ are positive then for $T^2 \gtrsim 2\mu_1^2/F_1$ the coefficient of $\phi_1^\dagger\phi_1$ in V(T) will be positive and the system will undergo a transition to phase in which $SU_2 \times U_1$ is unbroken ($\langle\phi_i(T)\rangle = 0$). However, Weinberg[8] and more recently Mohapatra and Senjanovic[10] and Zee[11] have emphasized in analogous models that some of the $F_i$ can be negative; in this case the symmetry need not be restored at high T.

It is even possible to have a transition to a state of lower symmetry.[8] We will choose parameters so that $F_{1,2} < 0$. This turns out to require $F_3 > 0$ so that for sufficiently large T we may have a transition to the phase with $SU_2 \times U_1$ completely broken. As an existence proof that all of these conditions can be satisfied, choose

$$\begin{aligned}
\lambda_1 = \lambda_2 \equiv \lambda &\gg g^4, |\rho_{ij}| \\
\sigma_{12} &> -\lambda \\
-\sigma_{13} = -\sigma_{23} \equiv \sigma &> 3\lambda + \sigma_{12} + 3X \\
\lambda_3 &> \sigma^2/\lambda \quad , \qquad (7)
\end{aligned}$$

where $X = (3g^2 + g'^2)/8 \simeq 0.16$. The condition $\lambda \gg g^4$ allows us to neglect radiative corrections to V. For a typical set of numbers, choose $\lambda \simeq -\sigma_{12} \simeq g^2 \simeq 0.4$, $\sigma \gtrsim 1.3$, $\lambda_3 \gtrsim 4.1$. The only purpose of introducing $\phi_3$ was to lower the energy of $\phi_1$ and $\phi_2$ at high temperatures because of their coupling to $\phi_3$. We see that there is a range of parameters which satisfy the above conditions, but a rather large value for $\lambda_3$ is required. This value is not so large as to violate tree level unitarity, which would occur[12] for $\lambda_3 \gtrsim 8\pi/3$, but it may lead to serious difficulties with the renormalization group equations[13] for running quartic couplings.[14]

For large T, the effective mass quantities $M_1^2(T)$ and $M_2^2(T)$ defined by $M_i^2(T) \equiv \mu_i^2 - F_i T^2/2$ will be positive. V(T) will have an extremum, $\langle\phi_1(T)\rangle = (0 \; v_1(T))^T/\sqrt{2}$ and $\langle\phi_2(T)\rangle = (v_2(T) \; 0)^T/\sqrt{2}$, with

$$\begin{pmatrix} v_1(T)^2 \\ v_2(T)^2 \end{pmatrix} = \frac{\begin{pmatrix} \lambda_2 & -\sigma_{12}/2 \\ -\sigma_{12}/2 & \lambda_1 \end{pmatrix}\begin{pmatrix} M_1^2(T) \\ M_2^2(T) \end{pmatrix}}{\lambda_1\lambda_2 - \sigma_{12}^2/4} \quad , \tag{8}$$

and $\langle\phi_3(T)\rangle = 0$. This will be (at least) a local minimum if

$$v_{1,2}^2 > 0$$

$$2 > \frac{\sigma_{12}}{\sqrt{\lambda_1\lambda_2}} > -1$$

$$|M_3^2| + \frac{\sigma_{13}v_1^2}{2} + \frac{\sigma_{23}v_2^2}{2} > 0 \quad . \tag{9}$$

The second order transition between these phases occurs at $T_c$ such that $v_2(T_c) = 0$. For the special case (6) these conditions are fulfilled if $2 > \sigma_{12}/\lambda > -1$ and $|\mu_2^2| > \mu_1^2$. In this case $|F_1| = |F_2| \leq O(\lambda \simeq g^2)$, $F_3 \simeq \lambda_3$ and the phase transition occurs for

$$T_c = A\mu_1/\sqrt{\lambda} = (246 \text{ GeV})A \quad , \tag{10}$$

where

$$A = \left[\frac{\lambda|\mu_2|^2/\mu_1^2 + \sigma_{12}/2}{\frac{1}{2}|F_2|(1-\sigma_{12}/2\lambda)}\right]^{\frac{1}{2}} \quad . \tag{11}$$

A is typically of order unity, but can be made much larger or smaller by adjusting parameters. We will assume $T_c \gtrsim 1$ TeV.

We have therefore demonstrated the existence of a model for which $SU_3^c \times SU_2 \times U_1$ is broken to $SU_3^c$ for $T > T_c$.

For $M_X >> T >> T_c$, we have

$$v_1(T) \simeq v_2(T) \sim \frac{\sqrt{-F_i T}}{\sqrt{\lambda}} \lesssim T$$

$$m_1, m_2 \simeq \sqrt{-F_1}\, T < T$$

$$m_3 \simeq \sqrt{F_3}\, T \gtrsim T \qquad , \qquad (12)$$

where the four massive gauge bosons (mixtures of $W^{\pm}$, Z, $\gamma$) have masses $\simeq gv_i \leq gT$. $m_{1,2}$ are the masses of the Higgs particle eigenstates which are mixtures of $\phi_1$ and $\phi_2$, and $m_3$ are the masses of the bosons in $\phi_3$ (which do not mix with $\phi_{1,2}$). Fermion masses are of order

$$m_F(T) \sim \frac{m_F(0)}{v_i(0)} v_i(T) \sim m_F(0)\, G_F^{\frac{1}{2}} T << T \qquad . \qquad (13)$$

For $T \lesssim M_X$ the superheavy scalar and vector particles can no longer be neglected and additional terms will be added to (6). A phase transition to an unbroken $SU_5$ phase is probable. There may also be intermediate phases (e.g., with $SU_3^c \times SU_2 \times U_1$ unbroken) for $T \lesssim M_X$, either due to the onset of superheavy thresholds or possibly from the effects of T dependent effective coupling constants.[10]

There should be essentially no magnetic monopoles in our model. Any monopoles produced during intermediate phases at $T \lesssim M_X$ will become unstable once the $SU_3^c$ phase is entered. They would presumably either decay or be confined in pairs which could subsequently annihilate. Stable monopoles of mass $m_m \simeq 10^{16}$ GeV could, in principle, exist for

$T < T_c$, but the number $r \simeq \exp(-m_m/T_c)$ expected from thermal fluctuations when $T \simeq T_c$ is utterly negligible.

Fermion masses are always << T, so the usual scenarios for producing a baryon asymmetry will be unchanged. Also, for $T < T_c$, $U_1^{EM}$ is restored so nucleosynthesis at $T \simeq 1$ MeV is not affected.

The most interesting feature is that electric charge is violated and the gauge bosons (including the photon) and the fermions and Higgs particles are massive for $T_c < T \lesssim M_X$. In fact, the gauge boson masses $M \simeq gT$ are negligible compared to the electron plasma frequency

$$\omega_p(T) \sim \left[\frac{4\pi n_e(T)e^2}{m_e(T)}\right]^{\frac{1}{2}} \simeq 400T \qquad , \qquad (14)$$

and can therefore be ignored. The fermion and (hopefully) the Higgs masses are small enough not to be problematic.

The reaction rate for charge violating reactions is[15]

$$\begin{aligned} \Gamma(T) &= \langle\sigma v\rangle_T \, n(T) \\ &\simeq 10^{23} \, g\alpha^2 \, T(\text{GeV})/\text{sec} \qquad , \qquad (15) \end{aligned}$$

where we have assumed

$$\langle\sigma v\rangle_T \simeq c\langle\sigma\rangle_T \approx \frac{\alpha^2 v_2(T)^2}{T^4} c \approx \frac{\alpha^2 c}{T^2} \qquad (16)$$

and a number density $n(T) \sim g n_\gamma(T)/2$, with[15] $g = g_B + 7g_F/8 \gtrsim 100$. $g_{B,F}$ are the number of boson and fermion light degrees of freedom at T. This is large compared to $t^{-1}$, where $t(\text{sec}) = 2.4 \times 10^{-6} g^{-\frac{1}{2}} T^{-2}(\text{GeV})$ is the age of the universe ($\Gamma(T)t(T) \sim 10^{14}/T$ (GeV)), so the charge violating reactions are in equilibrium for $T \geq T_c$. There will be a small net charge density $n_Q$ in the present universe left over from fluctuations from equilibrium at $T > T_c$. Only charge fluctuations on the scale of the observable universe are distinguishable from the standard scenario, so we will assume a total charge $N_Q \leq \sqrt{N_\gamma}$ in the observable universe (actually, the net charge will probably be much smaller because charged Higgs bosons become massless for $T \sim T_c$. They could be produced prolifically out of the vacuum to neutralize any excess charge produced earlier[16]). With $N_\gamma \simeq 10^{86}$, this implies $n_Q < 10^{-43} n_\gamma \sim 10^{-34} n_B$ in the present universe, where $n_B$ is the baryon density. This is far smaller than the observational limit[17,18] $n_Q/n_B \simeq 10^{-18}$ from galaxies and cosmology.

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