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ξ-SCALING ? *

J. F. Gunion
University of California at Davis
Physics Department, Davis, California 95616

and

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

Abstract: A class of purely kinematical corrections to ξ-scaling is exposed. These corrections are inevitably present in any realistic hadron model with spin and gauge invariance and lead to phenomenologically important M^2_{hadron}/Q^2 corrections to Nachtmann moments.

Résumé: Nous présentons une classe de corrections purement cinématiques au scaling en ξ. Ces corrections sont inévitables dans tout modèle hadronique réaliste qui possède une invariance de jauge et de spin, et elles entraînent des corrections d'ordre M^2_{hadron}/Q^2 aux moments de Nachtmann qui sont phénoménologiquement importantes.

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It has been claimed[1] that Nachtmann moments of deep inelastic structure functions absorb all non-dynamical M^2_{hadron}/Q^2 corrections. For instance for scalar photons, the Nachtmann moment is defined by

$$M_n \equiv \int_0^1 \frac{dx}{x^2} \xi^{n+1} \frac{1}{\pi} \operatorname{Im} T \tag{1}$$

where T is the deep inelastic amplitude, $x = Q^2/2p\cdot q$ with p = target momentum, q = photon momentum ($p^2 = M^2$, $q^2 = -Q^2$), and

$$\xi = \frac{2x}{1 + \sqrt{1 + \frac{4M^2x^2}{Q^2}}} \quad . \tag{2}$$

The statement of ξ-scaling is that

$$M_n = f_0^n(Q^2)\, A_0^n(M^2) + f_1^n(Q^2)\frac{M^2}{Q^2} A_1^n(M^2) + \ldots \tag{3}$$

with $A^n_{i\geq 1} = 0$ so that the Q^2 dependence, predicted by QCD, and the model-dependent hadron structure information, dependent on M^2, factorize. The derivations of Nachtmann scaling[1] all rely on forcing the quark (struck by the deep inelastic probe) to be on-shell. We will attack the question of ξ-scaling using diagrammatic techniques[2] in which the off-shell nature of the struck quark is explicitly accounted for.

In our approach[3] we calculate T in the Euclidean region as an expansion of the form

$$T = \sum_{n=0}^{\infty} M_n \left(\frac{p}{q}\right)^n C_n(\hat{p}\cdot\hat{q}) \tag{4}$$

where $C_n \equiv C_n^1$ are Gegenbauer polynomials. The M_n, when continued to the physical region, are the Nachtmann moments, (1). To illustrate, we first consider a simple scalar photon example in which ξ-scaling works. We calculate T from Fig. 1, assuming all particles are spinless. There K represents the hadron bound state structure and H is the deep inelastic quark amplitude. In particular consider the simplest "Born" term for which (in the Euclidean domain)

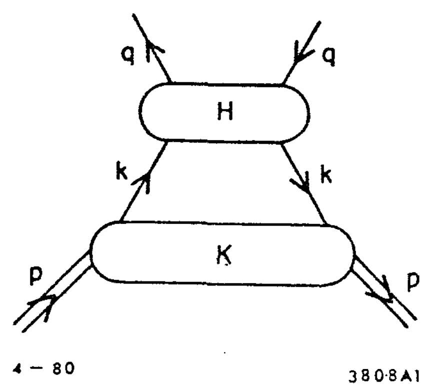

Fig. 1. General diagram for deep inelastic scattering on a bound state. K is the two particle irreducible bound state wave function kernel. H is the amplitude for deep inelastic scattering on a quark.

$$H(q,k) = \frac{1}{(q-k)^2} \quad , \quad K(k,p) = \frac{\rho(k^2)}{(p-k)^2 + \sigma} \quad ; \tag{5}$$

here for simplicity a single mass, $\sqrt{\sigma}$, has been assumed for the spectator system of K. The basic rational behind Nachtmann moments becomes apparent when one notes that Gegenbauers are the natural expansion polynomials for 4-dimensional propagators:

$$\begin{aligned} \frac{1}{(q-k)^2} &= \frac{1}{q^2} \sum_n \left(\frac{k}{q}\right)^n C_n(\hat{k}\cdot\hat{q}) \quad , \\ \frac{1}{(p-k)^2+\sigma} &= \frac{1}{k^2} \sum_j \left(\frac{p}{k}\right)^j C_j(\hat{p}\cdot\hat{k})(Z)^{j+1} \end{aligned} \tag{6}$$

where

$$Z = \frac{1}{2p^2}\left(p^2+\sigma+k^2 - \sqrt{(p^2+\sigma+k^2)^2 - 4p^2k^2}\right) \quad . \tag{7}$$

We calculate

$$T = \int \frac{k^2\, d^4k}{(2\pi)^4} \, \frac{1}{k^4} \, H(q,k)\, K(k,p) \tag{8}$$

using the natural orthogonality

$$\int \frac{d\Omega_k}{2\pi^2} \, C_n(\hat{k}\cdot\hat{q})\, C_j(\hat{k}\cdot\hat{p}) = \frac{\delta nj}{n+1} \, C_n(\hat{p}\cdot\hat{q}) \tag{9}$$

to obtain (4) with

$$\begin{aligned} M_n &= \left[\frac{1}{16\pi^2(n+1)q^2}\right]\left[\int \frac{dk^2}{k^2} \left\{Z(k^2,p^2,\sigma)\right\}^{n+1} \rho(k^2)\right] \\ &\equiv f_0^n(q^2) \quad A_0^n(p^2,\sigma) \quad . \end{aligned} \tag{10}$$

The q^2 and p^2 dependence have factorized naturally despite the expected presence of an integral, $\int dk^2$, over off-shell quark momenta. Equation (10) is easily extended to a complete leading log calculation (H and, hence, $f_0^n(q^2)$ develops a power of $\log q^2$) without destroying this factorization.

In a realistic situation with spin (for photons, target, etc.) the above no longer works. A model independent example is provided[3)] by a photon target. In leading log (i.e., extracting the leading $\int^{q^2} dk^2/k^2 \sim \log q^2$ factors) the "Born" terms of Fig. 2 yield, for the O(4) spin-0 Nachtmann moment,

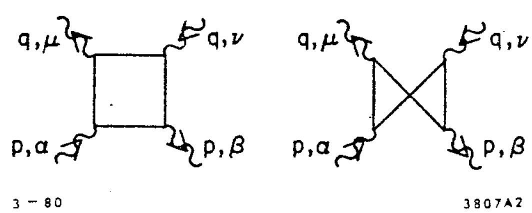

Fig. 2. "Born" diagrams for deep inelastic scattering on a photon target.

$$M_0^n \equiv \int_0^1 \frac{dx}{x^2}\,\xi^{n+1}\,\frac{1}{\pi}\,\mathrm{Im}\left(T^{\mu\nu\alpha\beta}\,g_{\mu\nu}\,g_{\alpha\beta}\right) \tag{11}$$

$$\underset{-q^2\to\infty}{\sim} -\frac{1}{8\pi^2}\log q^2\left\{\left(\frac{1}{n}+\frac{2}{n+2}-\frac{2}{n+1}\right)+\frac{p^2}{q^2}\left(\frac{6}{n+2}-\frac{2}{n+4}-\frac{2}{n+1}\right)\right.$$
$$\left.+\sum_{k=2}^{\infty}\left(\frac{p^2}{q^2}\right)^k(-)^k\left(\frac{2}{n+2k-2}+\frac{2}{n+2k+2}-\frac{4}{n+2k}\right)\right\} \quad . \tag{12}$$

The box or "handbag" diagram of Fig. 2a contributes to the $(p^2/q^2)^0$ and $(p^2/q^2)^1$ terms and the cross-box diagram of Fig. 2b to all terms. The emergence of p^2/q^2 correction factors to the Nachtmann moment is due both to the presence of numerator spin traces, which destroy the diagonalization of the spinless example, and to the necessity of including a crossed graph related to gauge invariance. The propagator information in the crossed-box graph is very different from that in the box graph and inevitably leads to an infinite series of p^2/q^2 powers.

One could still hope that the above breakdown of ξ-scaling is related to the point-like nature of the photon target which allows large values of the off-shell quark momentum, k^2. We have examined[3)] this question for a number of simple bound state models. The simplest is that illustrated in Fig. 3 where we construct a spin-½ "proton" from a chargeless, spin-0 particle of mass $\sqrt{\sigma}$ and a massless charge spin-½ quark. In this case we choose to calculate the moment of the neutrino structure function $F_3 = \nu/\pi\,\mathrm{Im}\,T_3$,

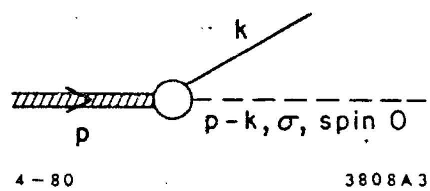

Fig. 3. A simple model for a spin-½ proton.

$$M_3^n = \frac{1}{n+2}\int\frac{dx}{x^2}\,F_3\xi^{n+1}\left[1+(n+1)\sqrt{1+\frac{4M^2x^2}{Q^2}}\;\right] \quad . \tag{13}$$

For a bound state wave function with power law behavior of the type predicted by QCD[4)]

$$\rho(k^2) \sim \left(\frac{\lambda}{k^2+\lambda}\right)^{\alpha} \tag{14}$$

the handbag type diagram, with the single quark propagator "Born" approximation to H(k,q) (see Fig. 1), combined with a gauge-invariance related vertex graph yields

$$M_3^n \underset{n\to\infty}{\sim} \frac{1}{32\pi}\left(\frac{\lambda}{\sigma}\right)^{\alpha}\frac{(\alpha-1)}{n^{\alpha+1}}\left[\left(2+\alpha+\alpha\frac{M^2}{\sigma}\right)+\frac{2n^2M^2}{\alpha Q^2}+\mathcal{O}\left(n\frac{M^2}{Q^2}\right)\right] \tag{15}$$

plus an infinite series of terms with higher M^2/Q^2 powers. If we recall that large n behavior probes $x \to 1$ the above form, (15), is equivalent to

$$F_3(x) \sim \left(2 + \alpha + \frac{M^2}{\sigma}\alpha\right)(1-x)^{\alpha} + \frac{2M^2}{Q^2}(\alpha-1)(1-x)^{\alpha-2} \quad . \qquad (16)$$

The M^2/Q^2 correction term has weaker x dependence than the leading term. This is the type of phenomenological "higher twist" term which Abbott and Barnett[5)] showed to be capable of explaining all observed scale-breaking; with such terms the QCD evolution does not need to be included for a consistent description of the Q^2 dependence of deep inelastic structure functions.

In summary we have shown, using diagrammatic techniques, that a class of phenomenologically important "higher twist" corrections to Nachtmann moments of deep inelastic structure functions inevitably arise for purely kinematical reasons when proper off-shell behavior for the probed quark is incorporated. These are not "dynamical" higher twist terms of the type that would arise from substructure (e.g., diquarks) inside the target. In addition, they are not small simply because the bound state scale for k^2 (λ in Eq. (14)) is small. Even though we have presented calculations only for the "Born" term contribution to the quark amplitude H, it is easily demonstrated that these M^2/Q^2 corrections survive the QCD ladderization leading log development for H. Alternatively, it is interesting to note that in the limit

$$p^2, q^2 \to \infty \quad , \qquad q^2/p^2 = r \text{ fixed} \quad , \qquad (17)$$

where corrections of the form p^2/q^2 are explicitly more important than $\alpha_s(q^2)$ corrections from non-leading logarithms, the leading log series for H collapses to the "Born" term and the higher twist terms dominate scale-breaking. It is not impossible that the low Q^2 region of proton target data is closer to such a limit than to the standard QCD leading log approximation.

Acknowledgements

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