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SCALING PHENOMENA IN RELATIVISTIC NUCLEUS - NUCLEUS COLLISIONS*

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ABSTRACT

We introduce new scaling variables in relativistic nucleus-nucleus collisions which allows a simple description of forward and backward angle reactions. They are the generalizations of the Feynman scaling variable to the case of lightly bound but heavy systems at finite energies.

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Recent experimental and theoretical studies of nucleus-nucleus collisions in the energy range of a few GeV per nucleon indicate that the use of the Feynman variable x_F should lead to scaling in the case of forward pion production.[1,2] Furthermore, when the cross section is represented by $(1-x_F)^g$, the index number g obeys the counting rule of Ref. 2. However, x_F scaling was not observed in proton and composite particle production,[3,4] nor in backward pion production[5,6] where the index number g is energy dependent and is much smaller than the index given by the counting rule. On the other hand, numerical evaluation[7] of the integral in the hard-scattering model of Ref. 2 gave good agreement to the pion production at moderate energies for both the forward and the backward angles. In the presence of these seemingly conflicting results it will be important to look for new scaling variables. Such variables will allow simple parametrizations of these reactions[6,8] and facilitate the search for models for the underlying mechanisms.

In the reaction $A+B \rightarrow C+X$, the two dominant processes of interest are illustrated in Fig. 1. From Fig. 1a, the direct projectile fragmentation (DF) inclusive cross section is proportional to the probability $G_{C/B}(x_D,\vec{C}_T)$ of finding a subsystem C in the projectile B with transverse momentum $\vec{C}_T$ and fractional momentum x_D defined by[9]

$$x_D = (C_0 + C_Z) \,/\, (B_0 + B_Z) \equiv x_F \, x_{max} \quad . \tag{1}$$

where the letter labels also denote the corresponding momentum four-vectors. In terms of $s=(A+B)^2$, x_{max} for <u>projectile</u> fragmentation is given by

$$x_{max} = \frac{s - D^2 + C^2 + \lambda\left(s, C^2, D^2\right)}{s - A^2 + B^2 + \lambda\left(s, A^2, B^2\right)} \simeq \frac{s - D^2}{s - A^2} \quad , \qquad (2)$$

where D is the mass of the "missing mass" X and λ is the usual λ-function as defined in Ref. 2. The direct fragmentation cross section is also proportional to an integral over the total reaction cross section for the collision of the complementary system $(B - C)$ with A and when the energy is sufficiently high, one finds

$$E_C \left(\frac{d\sigma}{d^3C}\right)_{DF} \simeq S(0^o, s)\; G_{C/B}\left(x_D, \vec{C}_T\right) \quad . \qquad (3)$$

In the near forward direction where S(s) is slowly varying. In the near backward direction one finds

$$E_C \left(\frac{d\sigma}{d^3C}\right)_{DF} \simeq S(180^o, s)\; G_{C/A}\left(x_D, \vec{C}_T\right)$$

where

$$x_D = \left(C_0 - C_Z\right) / \left(A_0 - A_Z\right) \equiv \left|x_F\right|\, x_{max} \quad , \qquad (5)$$

In this <u>target</u> fragmentation case, x_{max} is obtained from Eq. (2) by interchanging A and B.

As the direct fragmentation process is expected to dominate the proton production in the near forward and backward directions, the cross section there should be simply a function of x_D which can be used as a "direct fragmentation" scaling variable. Indeed, when we plot the invariant cross section[3] for forward proton production in the reaction of $\alpha + {}^{12}C$ for different values of proton transverse momentum p_T, we find that the data points appear to fall on the same curve (Fig. 2). Scaling with respect to x_D occurs for $x_D \gtrsim 0.2$ and $p_\alpha \gtrsim 1.74$ GeV/c/N. For $x_D < 0.2$, there are contributions from more complicated rescattering and inelastic processes, and one does not expect x_D scaling in this lower momentum regime. We report in passing that we have fitted the data represented by Fig. (2) with a combination of the direct fragmentation and hard-scattering processes.[10] The resultant structure function for $G_{p/\alpha}$ has an abrupt change of slope at $x_D \sim 0.3$ which may indicate[11] the presence of nuclear correlations in the ground states of ${}^4He$ and/or final state interactions.

The scaling variable x_D provides a natural way to link the forward projectile fragmentation data with target fragmentation data (involving the same nucleus) because the same structure function is involved. One finds that the 180° data[4] also approaches x_D scaling and can be used to extract the high momentum tail of the structure function.

The hard scattering cross section[2] is given by (see Fig. 1b)

$$E_C \frac{d\sigma}{d^3C} \sim \int G_{a/A}\left(x,\vec{a}_T\right) G_{b/B}\left(y,\vec{b}_T\right) E_C \frac{d\sigma}{d^3C} \left(ab \to Cd; x,\vec{a}_T,y,\vec{b}_T,\vec{C}\right), \quad (6)$$

where the integral is over the $x,\vec{a}_T$ and $y,\vec{b}_T$ variables of a and b, and the basic cross section vanishes outside the physical region of the internal process $a + b \to C + d$. We shall take this process to be nucleon-nucleon elastic scattering for proton production and nucleon-nucleon pion inclusive reaction for pion production. In the <u>projectile</u> fragmentation region the integrand is strongly peaked when a and b are on mass shell (since they are virtual, they cannot be exactly on-shell).

Expanding about the point $x = m_a/m_A$, $\vec{a}_T = \vec{b}_T = 0$, one finds that the cross section (6) is predominatly a function of $\vec{C}_T$ and x_H where x_H is the solution of y satisfying the energy conservation condition in the basic process in the case of proton production, and x_H is the lower limit of the y-integration in pion production. In either case, x_H is given by

$$x_H = \frac{\kappa^2 + \sqrt{\kappa^4 - b^2\left(u' + C_T^2\right)}}{u' + C_T^2} \left[\frac{m_a}{m_A} \frac{\left(A_0 + A_Z\right)}{\left(B_0 + B_Z\right)} - \frac{C_0 + C_Z}{B_0 + B_Z} \right] \tag{7}$$

where

$$\kappa^2 = \frac{1}{2}\left(d^2 - b^2 - u'\right) \quad , \tag{8}$$

$$u' = \left(m_a A / m_A - C\right)^2 \quad , \tag{9}$$

m_i is the mass of particle i, and d is the nucleon mass in proton production and is the minimum mass of d in pion production. In the target fragmentation region, the analogous variable x_H can be obtained by replacing $C_Z \to -C_Z$ and interchanging $a \leftrightarrow b$, $A_0 \leftrightarrow B_0$, $A_Z \leftrightarrow -B_Z$, $m_a \leftrightarrow m_b$ and $m_A \leftrightarrow m_B$. A simplified relation between x_H and x_F can be obtained in the case when $C^2 = m_C^2$ can be neglected and the expansion in power of $s - A^2 - B^2$ can be made

$$x_H \cong x_D + \frac{m_i\, m_I}{s - A^2 - B^2}\left(3 + \frac{C_T^2}{m_i^2}\right) \quad , \tag{10}$$

where for projectile fragmentation, x_D is given by Eq. (1), $i = a$, and $I = A$, and for target fragmentation, x_D is given by Eq. (5), $i = b$ and $I = B$.

Since a pion is not a normal constituent of the nucleus, we shall consider the hard-scattering process as the production mechanism. Accordingly, the cross section should depend on the variables x_H and $\vec{C}_T$. We plot in Fig. (3) the experimental data[5,12] for the backward pion production in the reaction of $p + Cu \rightarrow \pi^{\pm} + X$. As one observes, the experimentsal data scale quite well with x_H indicating its usefulness as a "hard-scattering" scaling variable. The experimental data[1] for the forward pion production in the reaction $\alpha + {}^{12}C \rightarrow \pi^- + X$ also scale well with respect to x_H ($\approx$ x_F in this regime).

The counting rules derived in Ref. (2) imply that the x_H behavior in the target region above the quasi-elastic peak should be of the form $(1 - x_H)^{g_A}$, where $g_A = 2T(N_A - N_a) + H$. Here, N_i is the number of nucleons in nucleus i, $H \sim 3(-1)$ for the pion (nucleon) inclusive case and T was near 3. The magnitude of the slope g should be even larger as one approaches the quasi-elastic peak at $x_H \sim N_a/N_A$. The discussion in Ref. 5 showed that if one tries to fit $(1 - x_F)^g$ to the data, one finds $g << g_A$, and that g depends strongly on the energy. It then was shown[6] that small clusters in the nucleus were required to adequately fit the data. However, the new scaling variable x_H considerably changes and simplifies this situation as Fig. (3) shows. The slope is somewhat larger than the prediction, as expected.[13] The 400 GeV data of Bayukov et al.,[5] at 160° is also plotted and the approach to scaling is clear.

In the case of proton production, the hard-scattering process becomes dominant at the quasi-elastic peak when $p_T >> 0.1$ GeV/c. It can be shown that $x_H = x_D$ for $p_T = 0$ and $x_H \approx x_D$ as p_T increases. This is why x_D scaling persists even for $p_T = 0.3$ GeV/c as we observed earlier in Fig. (2).

In the projectile fragmentation region, x_D and x_H approach x_F when $s \gg D^2 - A^2$ and in the target fragmentation region where $s \gg D^2 - B^2$. Thus, as the bombarding energy is increased, x_D and x_H approach x_F earlier in the projectile fragmentation region than in the target fragmentation region, when a projectile collides with a heavier target.

It is clear that the scaling variables introduced here can be applied not only to proton and pion production but also to the production of other elementary or composite particles.

In conclusion, we have obtained new scaling variables which are the generalization of the Feynman scaling variable for cases where the rest masses are not small compared to s. Its introduction clarifies the underlying physics leading to scaling phenomenon in nucleus-nucleus collisions, and shows that the same model of the structure functions may be applicable to forward and backward collisions.

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9. Here, we have used the Lorentz-invariant definition of $x_F = \left(C_0 + C_Z\right) / \left(C_0 + C_Z\right)_{max}$. In terms of the rapidity variables y_C and y_B for the detected particle C and projectile B, $x_D = \left(\left(m_C^2 + C_T^2\right) / \left(m_B^2 + B_T^2\right)\right) \exp\left(y_C - y_B\right)$.

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13. If we fit the data in Fig. (3) by $(1-x_H)^n$, then the index n has the value of 511 ($T\sim 4$) which is greater than $g=371$ ($T\sim 3$) from the counting rule of Ref. (2). Incidentally, the fact that $n\sim 2TA$ and hence gets very large for large A may be disturbing to some. However, note that in the frame in which the projectile (target) is at rest, this distribution for projectile (target) fragmentation becomes exp $\left\{-\alpha\left(C_0+C_Z\right)\right\}$ where $\alpha\left(=2T/m_p\right)$ is independent of energy and A. The effective temperature (see Ref. 4) is then $\alpha(1\pm r\cos\theta)^{-1}$, where $r=C_0/\left|C_Z\right|$. In fitting data, one finds that T = 4 results in the observed proton and pion temperatures as well as their angular variation.

FIGURE CAPTIONS

Fig. 1. —The two dominant processes contributing to the reactions of interest.

Fig. 2. Experimental invariant cross section for the reaction $\alpha + {}^{12}C \rightarrow p + X$ plotted as a function of the scaling variable x_D. Data are from Ref. (3).

Fig. 3. Experimental invariant cross section for $p + Cu \rightarrow \pi^{\pm} + X$ plotted as a function of the scaling variable x_H.

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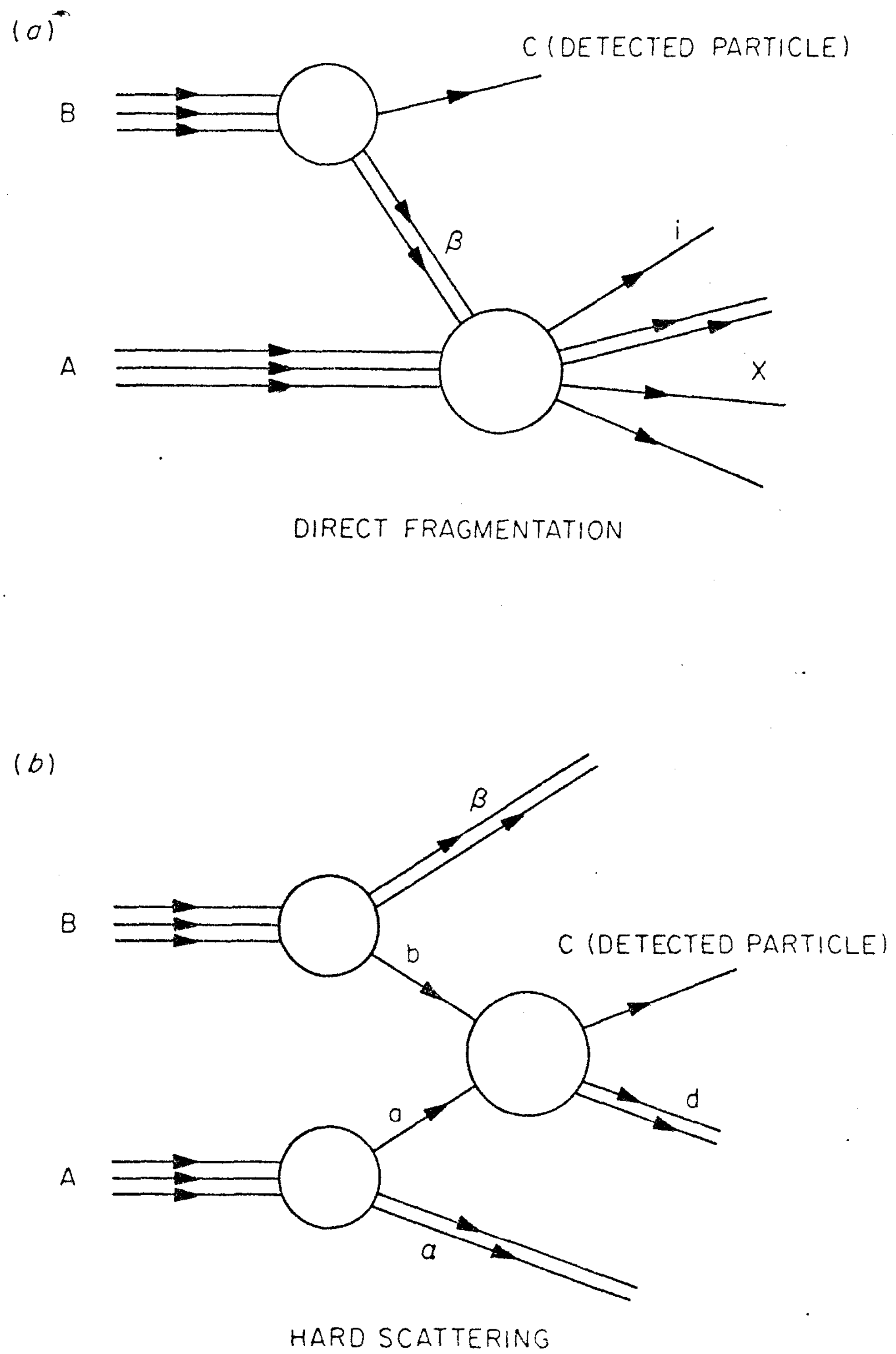

Fig. 1

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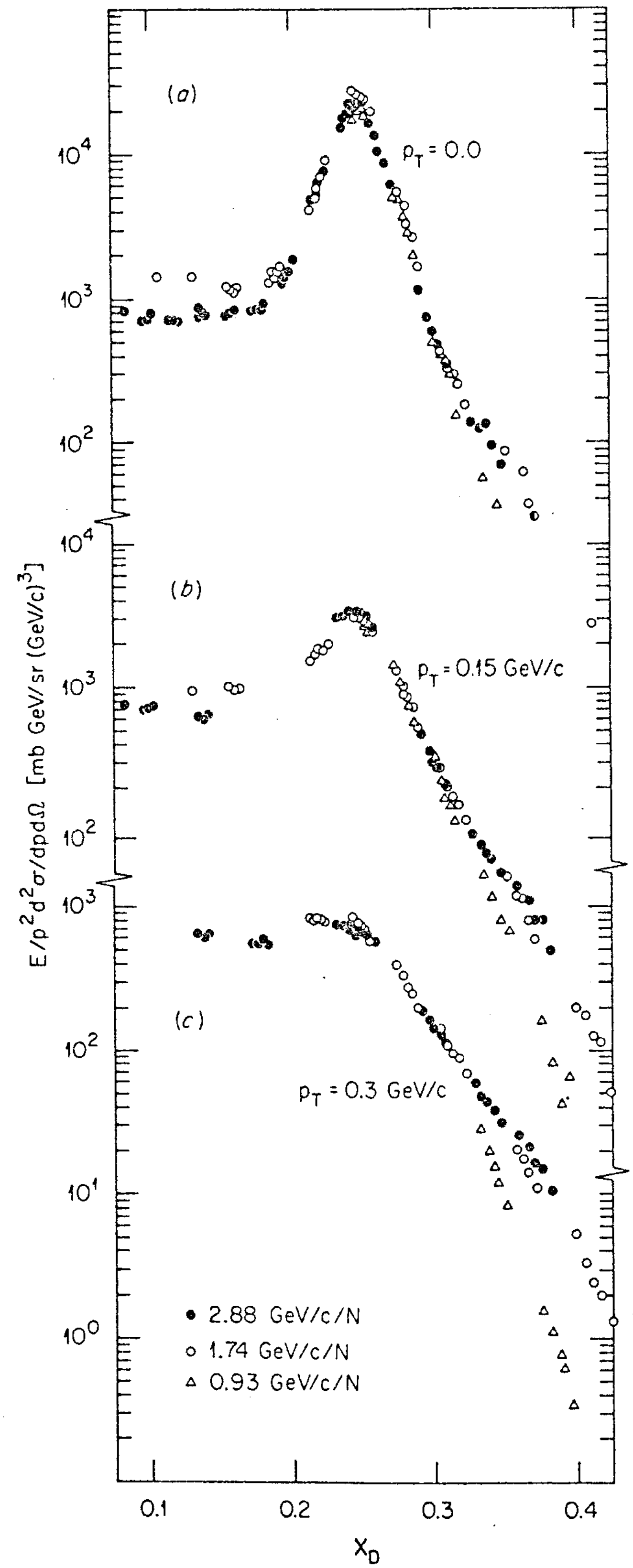

Fig. 2

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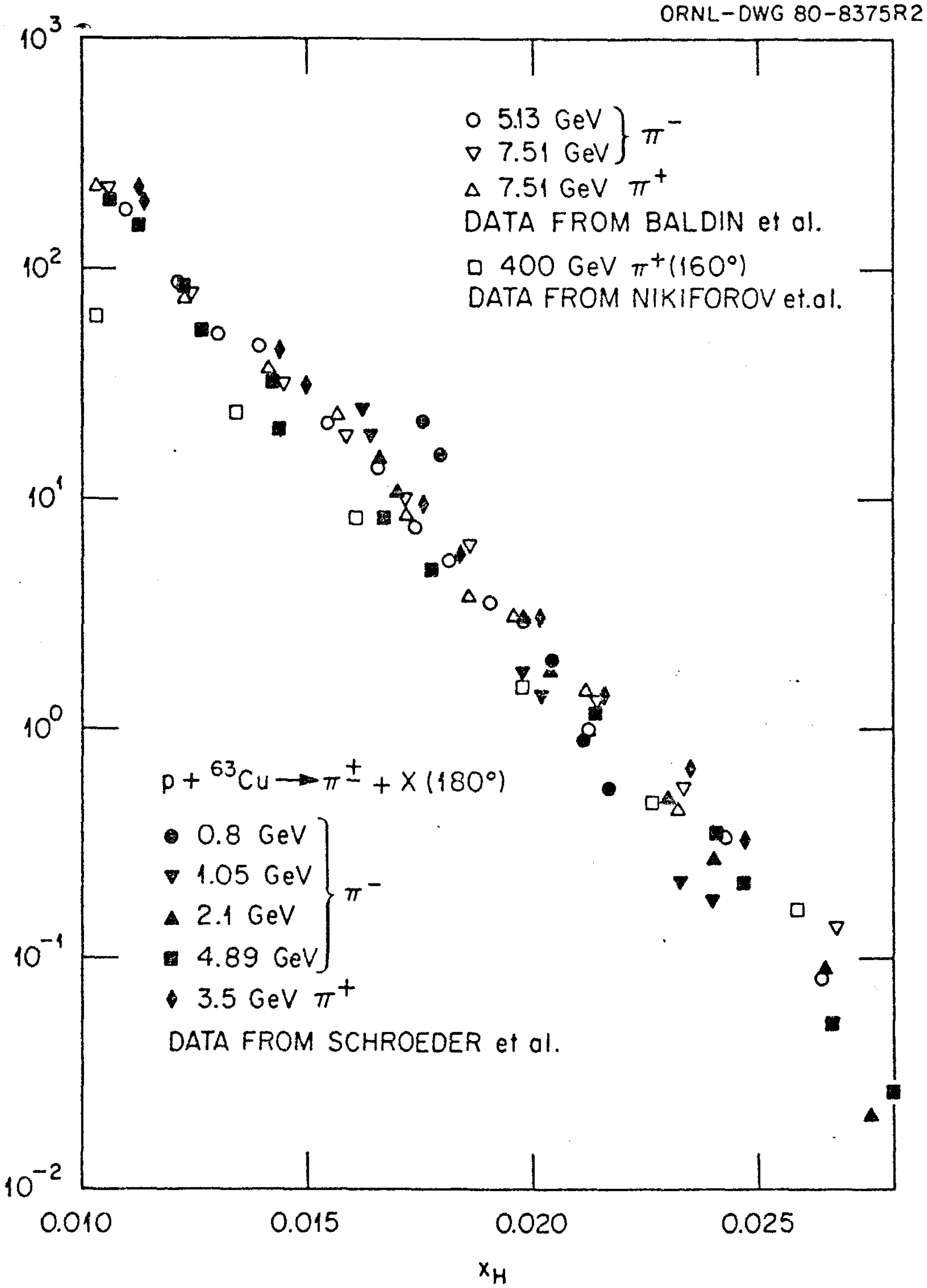

Fig. 3