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THE EFFECTIVE HAMILTONIAN FOR NUCLEON DECAY*

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ABSTRACT

Renormalization effects for the $SU(3) \otimes SU(2) \otimes U(1)$ invariant baryon-number violating operators of lowest dimension are calculated. Linear relations involving these operators are presented and a minimal set is given for nucleon decay processes.

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1. Introduction

Renormalization group analyses[1] of SU(3), SU(2), and U(1) gauge theories have shown that unification of the strong, weak and electromagnetic interactions is possible at a mass scale of order 10^{15} GeV. New interactions resulting from this unification may violate baryon number conservation as in the SU(5) model of Georgi and Glashow.[2,3] If they do, nucleon decay will provide us with important information about these interactions and could help to determine how the SU(3), SU(2), and U(1) theories are unified. However, the parameters measured in a nucleon decay experiment refer to a mass scale of order the proton mass, m_p, whereas the mass scale relevant to grand unified models is 10^{15} GeV. In this paper, we calculate the SU(3), SU(2), and U(1) renormalization effects[4] which allow one to relate parameters at these two widely different mass scales.

If nucleon decay is governed by a mass of order 10^{15} GeV, then only those baryon number violating operators of lowest possible dimension will contribute at an observable rate. Such operators, consistent with SU(3) $\otimes$ SU(2) $\otimes$ U(1) and Lorentz symmetry have been enumerated by Weinberg and by Wilczek and Zee.[5] In the notation of Weinberg, the operators are:

$$O^{(1)}_{abcd} = \left(d_{\alpha aR}\, u_{\beta bR}\right)\left(q_{i\gamma cL}\, \ell_{jdL}\right) \varepsilon_{\alpha\beta\gamma}\, \varepsilon_{ij} \tag{1.1}$$

$$O^{(2)}_{abcd} = \left(q_{i\alpha aL}\, q_{j\beta bL}\right)\left(u_{\gamma cR}\, \ell_{dR}\right) \varepsilon_{\alpha\beta\gamma}\, \varepsilon_{ij} \tag{1.2}$$

$$O^{(3)}_{abcd} = \left(q_{i\alpha aL}\, q_{j\beta bL}\right)\left(q_{k\gamma cL}\, \ell_{\ell dL}\right) \varepsilon_{\alpha\beta\gamma}\, \varepsilon_{ij}\, \varepsilon_{k\ell} \tag{1.3}$$

$$O^{(4)}_{abcd} = \left(q_{i\alpha aL}\, q_{j\beta bL}\right)\left(q_{k\gamma cL}\, \ell_{\ell dL}\right) \varepsilon_{\alpha\beta\gamma}\, (\vec{\tau}\varepsilon)_{ij} \cdot (\vec{\tau}\varepsilon)_{k\ell} \tag{1.4}$$

$$O^{(5)}_{abcd} = \left(d_{\alpha aR}\, u_{\beta bR}\right)\left(u_{\gamma cR}\, \ell_{dR}\right)\varepsilon_{\alpha\beta\gamma} \tag{1.5}$$

where α,β,γ are SU(3) color indices; i,j,k,ℓ are SU(2) indices; a,b,c,d refer to generation numbers and L and R refer to left- and right-handed fields. We have used two-component spinor notation in Eqs. (1.1)-(1.5) with spinor indices contracted as in the appendix. The correspondence with four-component spinor notation is given in the appendix. The operator $O^{(6)}$ which appears in Ref. 5 can be expressed in terms of $O^{(5)}$ by the relation

$$O^{(6)}_{abcd} = \left[O^{(5)}_{cbad} - O^{(5)}_{cabd}\right] \tag{1.6}$$

and therefore need not be considered separately.

For renormalization group calculations it is useful to take into account any relations between the operators being considered. The operators $O^{(3)}$ and $O^{(4)}$ can be written as the symmetric and antisymmetric part (in the first two generation indices) of a single operator. We therefore find it most convenient to define an operator

$$\tilde{O}^{(4)}_{abcd} = \left(q_{\alpha iaL}\, q_{\beta jbL}\right)\left(q_{\gamma kcL}\, \ell_{\ell dL}\right)\varepsilon_{\alpha\beta\gamma}\,\varepsilon_{i\ell}\,\varepsilon_{jk} \tag{1.7}$$

and note that[6]

$$O^{(3)}_{abcd} = -\left(\tilde{O}^{(4)}_{abcd} + \tilde{O}^{(4)}_{bacd}\right) \tag{1.8}$$

and

$$O^{(4)}_{abcd} = -\left(\tilde{O}^{(4)}_{abcd} - \tilde{O}^{(4)}_{bacd}\right) \quad . \tag{1.9}$$

With the relations (1.8) and (1.9) the effective Hamiltonian for nucleon decay can be expressed in terms of only four types of operators:

$$O^{(1)}_{abcd} \;,\; O^{(2)}_{abcd} \;,\; \tilde{O}^{(4)}_{abcd} \;\text{ and }\; O^{(5)}_{abcd} \quad .$$

To avoid confusion we will retain the original numbering in Eqs. (1.1)-(1.5). Thus, no $O^{(3)}_{abcd}$ will occur in our analysis. Note that there are still some relations between these remaining operators, namely:[6]

$$O^{(2)}_{abcd} = O^{(2)}_{bacd} \quad , \tag{1.10}$$

and

$$\tilde{O}^{(4)}_{abcd} + \tilde{O}^{(4)}_{bacd} - \tilde{O}^{(4)}_{cabd} - \tilde{O}^{(4)}_{cbad} = 0 \quad . \tag{1.11}$$

In terms of the original operators $O^{(3)}$ and $O^{(4)}$ Eq. (1.11) becomes

$$O^{(3)}_{abcd} = \frac{1}{2}\left(O^{(3)}_{cabd} + O^{(4)}_{cabd} + O^{(3)}_{cbad} + O^{(4)}_{cbad}\right) \quad . \tag{1.12}$$

In the following section, we derive the one loop renormalization factors for the operators $O^{(1)}_{abcd}$, $O^{(2)}_{abcd}$, $\tilde{O}^{(4)}_{abcd}$ and $O^{(5)}_{abcd}$ from SU(3), SU(2) and U(1) interactions, and then apply our results to nucleon decay into non-strange and strange final states. We will ignore the extremely small effects from light Higgs renormalization of the operators. Our results allow one to include SU(3), SU(2) and U(1) renormalization effects in calculations of nucleon decay rates and branching ratios in grand unified models.[7] For example, operators of type one and two can occur from vector boson exchange while operators of type four and five originate from Higgs boson exchange.[5]

2. Results

To determine renormalization effects in a renormalization group approach one needs to know the anomalous dimension matrix for the operators of interest.[8] This is determined from the renormalization

Z factors which relate the bare and renormalized operators. In our calculation of these Z factors, we have used dimensional regularization in $n = 4-2\varepsilon$ dimensions and minimal subtraction. The calculation was performed both in the Landau gauge and in the Feynman gauge (where external wave function renormalization must be taken into account). Our results are:

$$O^{(1)^o}_{abcd} = \left[1 + \frac{\alpha_s}{4\pi\varepsilon}\,(2) + \frac{\alpha_2}{4\pi\varepsilon}\left(\frac{9}{4}\right) + \frac{\alpha_1}{4\pi\varepsilon}\left(\frac{11}{12}\right)\right] O^{(1)}_{abcd} \quad , \tag{2.1}$$

$$O^{(2)^o}_{abcd} = \left[1 + \frac{\alpha_s}{4\pi\varepsilon}\,(2) + \frac{\alpha_2}{4\pi\varepsilon}\left(\frac{9}{4}\right) + \frac{\alpha_1}{4\pi\varepsilon}\left(\frac{23}{12}\right)\right] O^{(2)}_{abcd} \quad , \tag{2.2}$$

$$\tilde{O}^{(4)^o}_{abcd} = \left[1 + \frac{\alpha_s}{4\pi\varepsilon}\,(2) + \frac{\alpha_2}{4\pi\varepsilon}\left(\frac{3}{2}\right) + \frac{\alpha_1}{4\pi\varepsilon}\left(\frac{1}{6}\right)\right] \tilde{O}^{(4)}_{abcd} + \frac{\alpha_2}{4\pi\alpha}\,(2)\left(\tilde{O}^{(4)}_{bacd} + \tilde{O}^{(4)}_{cbad} + \tilde{O}^{(4)}_{acbd}\right) \quad , \tag{2.3}$$

$$O^{(5)^o}_{abcd} = \left[1 + \frac{\alpha_s}{4\pi\varepsilon}\,(2) + \frac{\alpha_1}{4\pi\varepsilon}\,(1)\right] O^{(5)}_{abcd} + \frac{\alpha_1}{4\pi\varepsilon}\left(\frac{10}{3}\right) O^{(5)}_{acbd} \quad . \tag{2.4}$$

Here a superscript o refers to a bare operator, α_s is the SU(3) coupling constant, which at some large mass M is given by

$$\alpha_s(M) = \frac{4\pi}{\beta_0^{(3)} \log\,(M^2/\Lambda^2)} \quad , \tag{2.5}$$

and α_1 and α_2 are the SU(2) and U(1) couplings related to the electromagnetic coupling α_{EM} at the W-boson mass, M_W, by

$$\alpha_1(M_W) = \alpha_{EM}(M_W)/\cos^2\theta_W \tag{2.6}$$

and

$$\alpha_2(M_W) = \alpha_{EM}(M_W)/\sin^2\theta_W \quad . \tag{2.7}$$

Note that under renormalization $\tilde{O}^{(4)}_{abcd}$ mixes with $\tilde{O}^{(4)}_{bacd}$, $\tilde{O}^{(4)}_{cbad}$, and $\tilde{O}^{(4)}_{acbd}$ and $O^{(5)}_{abcd}$ mixes with $O^{(5)}_{acbd}$.

We apply these results to two relevant cases. First consider $a=b=c=d=1$ so that we have operators relevant (apart from Cabibbo suppressed modes) to nucleon decay into non-strange final states.[9] In this case there are only four linearly independent operators which we denote by

$$Q_1 = O^{(1)}_{1111} = \left(d_{\alpha R}u_{\beta R}\right)\left(u_{\gamma L}e_L - d_{\gamma L}\nu^e_L\right)\varepsilon_{\alpha\beta\gamma} \quad , \tag{2.8}$$

$$Q_2 = -\frac{1}{2}O^{(2)}_{1111} = \left(d_{\alpha L}u_{\beta L}\right)\left(u_{\gamma R}e_R\right)\varepsilon_{\alpha\beta\gamma} \quad , \tag{2.9}$$

$$Q_3 = \tilde{O}^{(4)}_{1111} = \left(d_{\alpha L}u_{\beta L}\right)\left(u_{\gamma L}e_L - d_{\gamma L}\nu^e_L\right)\varepsilon_{\alpha\beta\gamma} \quad , \tag{2.10}$$

and

$$Q_4 = O^{(5)}_{1111} = \left(d_{\alpha R}u_{\beta R}\right)\left(u_{\gamma R}e_R\right)\varepsilon_{\alpha\beta\gamma} \quad . \tag{2.11}$$

These do not mix under renormalization. If $A_1^{GUM},\ldots,A_4^{GUM}$ are the tree level coefficients of $Q_1,\ldots,Q_4$ in a grand unified model, then the corresponding coefficients at a mass scale of order the proton mass, $\mu \simeq m_p$, are given by

$$A_1(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_{GUM}}\right]^{2/\beta_0^{(3)}}\left[\frac{\alpha_2(M_W)}{\alpha_{GUM}}\right]^{9/4\beta_0^{(2)}}\left[\frac{C^2\alpha_1(M_W)}{\alpha_{GUM}}\right]^{11/12C^2\beta_0^{(1)}} A_1^{GUM} \quad , \tag{2.12}$$

$$A_2(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_{GUM}}\right]^{2/\beta_0^{(3)}}\left[\frac{\alpha_2(M_W)}{\alpha_{GUM}}\right]^{9/4\beta_0^{(2)}}\left[\frac{C^2\alpha_1(M_W)}{\alpha_{GUM}}\right]^{23/12C^2\beta_0^{(1)}} A_2^{GUM} \quad , \tag{2.13}$$

$$A_3(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_{GUM}}\right]^{2/\beta_0^{(3)}}\left[\frac{\alpha_2(M_W)}{\alpha_{GUM}}\right]^{15/2\beta_0^{(2)}}\left[\frac{C^2\alpha_1(M_W)}{\alpha_{GUM}}\right]^{1/6C^2\beta_0^{(1)}} A_3^{GUM} \quad , \tag{2.14}$$

and

$$A_4(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_{GUM}}\right]^{2/\beta_0^{(3)}} \left[\frac{C^2\alpha_1(M_W)}{\alpha_{GUM}}\right]^{13/3C^2\beta_0^{(1)}} A_4^{GUM} \quad . \qquad (2.15)$$

$\alpha_1(M_W)$ and $\alpha_2(M_W)$ are given by Eqs. (2.6) and (2.7), $\alpha_s(\mu)$ is the SU(3) running coupling constant evaluated at the renormalization point $\mu \simeq m_p$ and α_{GUM} is the grand unified coupling. C is the normalization factor between the U(1) of SU(2) $\otimes$ U(1) and the U(1) subgroup of the grand unified gauge group. In SU(5), $C^2 = 5/3$.[10] Renormalization effects due to the electromagnetic interactions have been neglected between the W-boson mass and the renormalization point mass since $\alpha_{EM}\, \ell n\, (M_W^2/\mu^2)$ is a small number. The β-functions are given by:[11]

$$\beta_0^{(3)} = 11 - \frac{2}{3}N_f \quad , \qquad (2.16)$$

$$\beta_0^{(2)} = \frac{22}{3} - \frac{2}{3}N_f - \frac{1}{6} \quad , \qquad (2.17)$$

and

$$\beta_0^{(1)} = -\frac{2}{3}N_f - \frac{1}{10} \quad , \qquad (2.18)$$

when one light Higgs doublet is included. The last terms in Eqs. (2.17) and (2.18) are the contributions of the light Higgs doublet. N_f is the number of quark flavors.

Now consider the case of nucleon decay into strange final states.[9] The linearly independent set of operators relevant to this case are

$$Q_1' = O_{2111}^{(1)} \quad , \qquad (2.19)$$

$$Q_2' = O_{1121}^{(1)} \quad , \qquad (2.20)$$

$$Q_3' = O_{2111}^{(2)} \quad , \tag{2.21}$$

$$Q_4' = \left(2\tilde{O}_{2111}^{(4)} + \tilde{O}_{1211}^{(4)}\right) \quad , \tag{2.22}$$

$$Q_5' = \left(\tilde{O}_{2111}^{(4)} - \tilde{O}_{1211}^{(4)}\right) \quad , \tag{2.23}$$

and

$$Q_6' = O_{2111}^{(5)} \quad . \tag{2.24}$$

$Q_1',\ldots,Q_6'$ have been defined in such a way that they do not mix under renormalization. Denoting the tree level coefficients of these operators in a grand unified model (i.e., the values of the coefficients at the superheavy mass scale) by $A_1'^{GUM},\ldots,A_6'^{GUM}$ it follows from Eqs. (2.1)-(2.5) that the coefficients determined at $\mu \simeq m_p$ are

$$A_1'(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_{GUM}}\right]^{2/\beta_0^{(3)}} \left[\frac{\alpha_2(M_W)}{\alpha_{GUM}}\right]^{9/4\beta_0^{(2)}} \left[\frac{C^2\alpha_1(M_W)}{\alpha_{GUM}}\right]^{11/12C^2\beta_0^{(1)}} A_1'^{GUM} \quad , \tag{2.25}$$

$$A_2'(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_{GUM}}\right]^{2/\beta_0^{(3)}} \left[\frac{\alpha_2(M_W)}{\alpha_{GUM}}\right]^{9/4\beta_0^{(2)}} \left[\frac{C^2\alpha_1(M_W)}{\alpha_{GUM}}\right]^{11/12C^2\beta_0^{(1)}} A_2'^{GUM} \quad , \tag{2.26}$$

$$A_3'(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_{GUM}}\right]^{2/\beta_0^{(3)}} \left[\frac{\alpha_2(M_W)}{\alpha_{GUM}}\right]^{9/4\beta_0^{(2)}} \left[\frac{C^2\alpha_1(M_W)}{\alpha_{GUM}}\right]^{23/12C^2\beta_0^{(1)}} A_3'^{GUM} \quad , \tag{2.27}$$

$$A_4'(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_{GUM}}\right]^{2/\beta_0^{(3)}} \left[\frac{\alpha_2(M_W)}{\alpha_{GUM}}\right]^{15/2\beta_0^{(2)}} \left[\frac{C^2\alpha_1(M_W)}{\alpha_{GUM}}\right]^{1/6C^2\beta_0^{(1)}} A_4'^{GUM} \quad , \tag{2.28}$$

$$A_5'(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_{GUM}}\right]^{2/\beta_0^{(3)}} \left[\frac{\alpha_2(M_W)}{\alpha_{GUM}}\right]^{3/2\beta_0^{(2)}} \left[\frac{C^2\alpha_1(M_W)}{\alpha_{GUM}}\right]^{1/6C^2\beta_0^{(1)}} A_5'^{GUM} \quad , \tag{2.29}$$

and

$$A_6'(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_{GUM}}\right]^{2/\beta_0^{(3)}} \left[\frac{C^2\alpha_1(M_W)}{\alpha_{GUM}}\right]^{13/3C^2\beta_0^{(1)}} A_6'^{GUM} \quad . \tag{2.30}$$

3. Conclusion

In the previous section we enumerated the linearly independent $SU(3) \otimes SU(2) \otimes U(1)$ invariant operators which enter the effective Hamiltonian for nucleon decay into strange and nonstrange final states and calculated the relationship between their coefficients at the grand unified and proton mass scales. A given grand unified theory predicts most directly (i.e., from tree level) values of these coefficients at the grand unified mass scale. However quark model-type estimates for the matrix elements[7] of the operators can be expected to be valid at the proton mass scale. Thus one must make use of Eqs. (2.12)-(2.15) and (2.25)-(2.30) in order to make predictions concerning proton decay from a grand unified theory or to extract from future experiments information on the physics occuring at the grand unified mass scale.

Finally, it is worth noting that our analysis is only valid in the simplest possible scenerio where there exist only two relevant mass scales. It is possible that new physics exists at intermediate mass scales. The grand unified group G could break down to $SU(3) \otimes SU(2) \otimes U(1)$ in a series of steps $G \supset G' \supset \ldots \supset SU(3) \otimes SU(2) \otimes U(1)$ in which case one must also calculate, for example, the renormalization of the operators due to a G' gauge theory in order to relate the coefficients of operators at the grand unified and proton mass scales.

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APPENDIX

Two component spinors come in two varieties which for fields that annihilate particles we refer to as left- and right-handed. These two transform under SL(2C) according to representations which are complex conjugates of each other. To distinguish these representations we denote left-handed fields with an undotted index and right-handed fields with a dotted index. Under complex conjugation of the fields undotted indices become dotted and vice versa. Either dotted or undotted indices may be lowered, raised, or contracted by the antisymmetric ε-tensors $\varepsilon_{\alpha\beta} = \varepsilon^{\alpha\beta} = \varepsilon_{\dot{\alpha}\dot{\beta}} = \varepsilon^{\dot{\alpha}\dot{\beta}}$.

The relationship between the four-component spinor notation and the two-component form is as follows. If we write

$$\gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \tag{A.1}$$

and

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \tag{A.2}$$

with

$$\sigma^\mu = (1,\vec{\sigma}) = (\sigma^\mu)_{\alpha\dot{\alpha}} \tag{A.3}$$

$$\bar{\sigma}^\mu = (1,-\vec{\sigma}) = (\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} \tag{A.4}$$

then a four-component spinor can be written as

$$\psi = \begin{pmatrix} L_\alpha \\ R^{\dot{\alpha}} \end{pmatrix} \tag{A.5}$$

$$\bar{\psi} = \left(\bar{R}^\alpha \; \bar{L}_{\dot{\alpha}} \right) \tag{A.6}$$

where a bar on a two-component spinor indicates complex conjugation. Familiar bilinears are

$$\bar{\psi}\psi = \bar{R}L + \bar{L}R \equiv \bar{R}^{\alpha}L_{\alpha} + \bar{L}_{\dot{\alpha}}R^{\dot{\alpha}} \tag{A.7}$$

and

$$\bar{\psi}\gamma_{\mu}\psi = \bar{R}\sigma_{\mu}R + \bar{L}\bar{\sigma}_{\mu}L \equiv \bar{R}^{\alpha}(\sigma_{\mu})_{\alpha\dot{\alpha}}R^{\dot{\alpha}} + \bar{L}_{\dot{\alpha}}(\bar{\sigma}_{\mu})^{\dot{\alpha}\alpha}L_{\alpha} \quad . \tag{A.8}$$

The charge conjugate 4 field is

$$\psi^{c} = \begin{pmatrix} \bar{R}_{\alpha} \\ \bar{L}^{\dot{\alpha}} \end{pmatrix} , \qquad \overline{\psi^{c}} = \begin{pmatrix} L^{\alpha}R_{\dot{\alpha}} \end{pmatrix} \tag{A.9}$$

where

$$\bar{R}_{\alpha} = \bar{R}^{\beta}\varepsilon_{\beta\alpha}, \ \bar{L}^{\dot{\alpha}} = \varepsilon^{\dot{\alpha}\dot{\beta}}\bar{L}_{\dot{\beta}}, \ L^{\alpha} = \varepsilon^{\alpha\beta}L_{\beta}, \ R_{\dot{\alpha}} = R^{\dot{\beta}}\varepsilon_{\dot{\beta}\dot{\alpha}} \tag{A.10}$$

so that for example

$$\overline{\psi^{c}}\psi = L^{2} + R^{2} \equiv L^{\alpha}L_{\alpha} + R_{\dot{\alpha}}R^{\dot{\alpha}} \tag{A.11}$$

and

$$\overline{\psi^{c}}\gamma^{5}\psi = -L^{2} + R^{2} \equiv -L^{\alpha}L_{\alpha} + R_{\dot{\alpha}}R^{\dot{\alpha}} \ . \tag{A.12}$$

These formulas can be used to express the operators defined in Eqs. (1.1)-(1.6) in four-component form.

In order to prove some of the identities involving the operators (1.1)-(1.5) and to calculate the renormalization effects one needs the Fierz transformation rules:

$$\left(A_R B_R\right)\left(C_L D_L\right) = -\tfrac{1}{2}\left(A_R \bar{\sigma}_{\mu} D_L\right)\left(C_L \sigma^{\mu} B_R\right) \tag{A.13}$$

$$\left(A_R B_R\right)\left(C_R D_R\right) = -\tfrac{1}{2}\left\{\left(A_R D_R\right)\left(C_R B_R\right) + \tfrac{1}{4}\left(A_R \bar{\sigma}_{\mu\nu} D_R\right) \times \left(C_R \bar{\sigma}^{\mu\nu} B_R\right)\right\} \tag{A.14}$$

$$\left(A_L B_L\right)\left(C_L D_L\right) = -\frac{1}{2}\Big\{\left(A_L D_L\right)\left(C_L B_L\right) + \frac{1}{4}\left(A_L \sigma_{\mu\nu} D_L\right)\left(C_L \sigma^{\mu\nu} B_L\right)\Big\} \tag{A.15}$$

where

$$\sigma_{\mu\nu} = \frac{i}{2}\left(\sigma_\mu \bar{\sigma}_\nu - \sigma_\nu \bar{\sigma}_\mu\right) \tag{A.16}$$

and

$$\bar{\sigma}_{\mu\nu} = \frac{i}{2}\left(\bar{\sigma}_\mu \sigma_\nu - \bar{\sigma}_\nu \sigma_\mu\right) \quad . \tag{A.17}$$

It is also useful to note that

$$\left(A_R \bar{\sigma}_{\mu\nu} B_R\right) = -\left(B_R \bar{\sigma}_{\mu\nu} A_R\right) \quad , \tag{A.18}$$

$$\left(A_L \sigma_{\mu\nu} B_L\right) = -\left(B_L \sigma_{\mu\nu} A_L\right) \quad , \tag{A.19}$$

$$\left(A_L \sigma^\mu B_R\right) = -\left(B_R \bar{\sigma}^\mu A_L\right) \quad , \tag{A.20}$$

and

$$\left(A_R \bar{\sigma}_{\mu\nu} B_R\right)\left(C_L \sigma^{\mu\nu} D_L\right) = 0 \quad . \tag{A.21}$$

Eqs. (A.13)-(A.21) can be used to write all $SU(3) \otimes SU(2) \otimes U(1)$ invariant baryon number violating nucleon decay operators (of lowest possible dimension) as linear combinations of those in Eqs. (1.1)-(1.5).

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6. These relations come from using the identities in the appendix and from the identity

$$\varepsilon_{ij}\varepsilon_{k\ell} - \varepsilon_{kj}\varepsilon_{i\ell} + \varepsilon_{ik}\varepsilon_{\ell j} = 0 \qquad .$$

This can be proved by noting that the L.H.S. is completely anti-symmetric on its four indices i,j,k,ℓ and thus is zero since each indice only takes on two values. We thank B. Ovrut for pointing out this simple proof.

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9. We have written the operators for a positron or electron anti-neutrino in the final state. Our results hold equally well for $d = 2$ in which case there is a anti-muon or muon anti-neutrino in the final state. Note also that the quark fields d_L and s_L are weak eigenstates and related to the mass eigenstates d_L', s_L' by the rotation:

$$\begin{pmatrix} d_L \\ s_L \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d_L' \\ s_L' \end{pmatrix} \qquad .$$

10. It is possible that such a C factor could enter into the relations of the SU(3) or SU(2) couplings as well. These could be accomodated in a similar manner.

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