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HEAVY QUARKS IN ELECTROPRODUCTION*

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ABSTRACT

The idea of an effective light-particle field theory is used to analyze the effects of heavy quarks in electroproduction. The b-quark distribution is predicted at large Q^2 and a systematic treatment of the changing number of quark flavors in a QCD analysis of F_2 is presented.

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The concept of an effective light-particle field theory has proven useful for calculations in grand unified theories[1] and in nonleptonic weak decays.[2] In this paper the effective field theory approach is applied to compute heavy particle effects in deep-inelastic scattering. This approach was first discussed by E. Witten[3] in a moment analysis of charm quark effects. Here we will discuss b- and t-quarks using an equivalent but more intuitive method which involves dealing directly with the structure functions themselves. We will derive, using the leading-logarithmic approximation, predictions for the b-quark momentum distribution at $Q^2 > m_b^2$ utilizing data from the region $m_c^2 < Q^2 < m_b^2$. These predictions are reliable only for $Q^2 \gg m_b^2$, however, such large Q^2 values should be attainable at electron-proton colliders.[4]

The basic idea of an effective field theory is that at momentum scales much smaller than some heavy particle mass Feynman diagrams not containing external heavy particle legs can be described by an effective field theory involving only light-particle fields. In electroproduction one is concerned with those Feynman diagrams which contribute to the forward Compton amplitude so in order to apply this formalism one must assume that there are no primordial heavy particle contributions to the wave function of the nucleon. This assumption is suggested by the small measured value of the s-quark distribution[5] and by the success of naive valence quark model predictions for the static properties of baryons.[6] Having made this assumption the effective field theory formalism allows for a systematic treatment of heavy quark distributions and of the changing number of flavors in deep-inelastic scattering.

At present there is experimental evidence for the existence of five quark flavors. QCD with five quark flavors may or may not be an effective "light" field theory depending on whether there exist quarks with a mass greater than that of the b-quark.[7] In the five quark theory the structure function for electron-proton scattering is given by

$$F_2^{(5)} = \frac{4}{9}\left(u+\bar{u}+c+\bar{c}\right) + \frac{1}{9}\left(d+\bar{d}+s+\bar{s}+b+\bar{b}\right) \tag{1}$$

where u,c,d,s,b and $\bar{u},\bar{c},\bar{d},\bar{s},\bar{b}$ are the appropriate quark and anti-quark momentum distributions for the proton. It is convenient for analysing the Q^2 dependence of $F_2^{(5)}$ to divide it into a sum of singlet and non-singlet pieces. Therefore we write

$$F_2^{(5)} = \frac{11}{45}\, F_S^{(5)} + \frac{1}{5}\, F_{NS}^{(5)} \tag{2}$$

where

$$F_S^{(5)} = u+\bar{u}+c+\bar{c}+d+\bar{d}+s+\bar{s}+b+\bar{b} \tag{3}$$

and

$$F_{NS}^{(5)} = u+\bar{u}+c+\bar{c} - \frac{2}{3}\left(d+\bar{d}+s+\bar{s}+b+\bar{b}\right) \tag{4}$$

are respectively the singlet and non-singlet pieces. $F_S^{(5)}$ and $F_{NS}^{(5)}$ evolve for Q^2 in the five quark region according to the Altarelli-Parisi[5] equations with five quark flavors. The singlet piece mixes with the gluon distribution during the evolution.

Much of the present experimental data on F_2 come from measurements at Q^2 in the four quark region. There QCD is described by an effective four quark theory where the relevant electroproduction structure function is

$$F_2^{(4)} = \frac{4}{9}\left(u+\bar{u}+c+\bar{c}\right)+\frac{1}{9}\left(d+\bar{d}+s+\bar{s}\right) \quad . \tag{5}$$

Once again it is convenient to divide this into singlet and non-singlet pieces but according to the SU(4) flavor group instead of the SU(5) flavor group which was used in Eq. (3). We find that

$$F_2^{(4)} = \frac{5}{18} F_S^{(4)} + \frac{3}{18} F_{NS}^{(4)} \tag{6}$$

where

$$F_S^{(4)} = u+\bar{u}+c+\bar{c}+d+\bar{d}+s+\bar{s} \tag{7}$$

and

$$F_{NS}^{(4)} = u+\bar{u}+c+\bar{c}-\left(d+\bar{d}+s+\bar{s}\right) \quad . \tag{8}$$

It is the singlet and non-singlet structure functions $F_S^{(4)}$ and $F_{NS}^{(4)}$ which are determined for example from fits to electroproduction data in the four quark region $m_c^2 < Q^2 < m_b^2$.

From our knowledge of $F_S^{(4)}$ and $F_{NS}^{(4)}$ at some $Q_0^2 < m_b^2$, we can however make predictions for $F_2^{(5)}$ in the five quark region. This is done by using the Altarelli-Parisi equations with four quark flavors to evolve $F_S^{(4)}$ and $F_{NS}^{(4)}$ from Q_0^2 to some Q_{th}^2, which is approximately equal to m_b^2, and matching them at this threshold to $F_S^{(5)}$ and $F_{NS}^{(5)}$. The matching formulas are easily derived by noting that in the leading-logarithmic approximation the transition between the four and five quark theories can be treated as if it were a sharp step. Comparing Eqs. (3), (4), (7) and (8) with $b=\bar{b}=0$ gives the matching conditions

$$F_S^{(5)} = F_S^{(4)} \tag{9}$$

and

$$F_{NS}^{(5)} = \frac{1}{6} F_S^{(4)} + \frac{5}{6} F_{NS}^{(4)} \tag{10}$$

at $Q^2 = Q^2_{th}$. The $F_S^{(5)}$ and $F_{NS}^{(5)}$, determined from Eqs. (9) and (10), are then evolved from Q^2_{th} to some larger value of Q^2 using the Altarelli-Parisi equations with five quark flavors. This procedure thus takes into account the change from four to five quark flavors which occurs when one applies QCD over the Q^2 range from $Q^2 \simeq m_c^2$ to $Q^2 >> m_b^2$.

A similar procedure can be used to predict the b-quark momentum distribution at very large Q^2 on the basis of electroproduction data in the four quark region, $m_c^2 < Q^2 < m_b^2$. The sum of b and $\bar{b}$ distributions can be written in terms of the $F_S^{(5)}$ of Eq. (3) and $F'^{(5)}_{NS}$, a new flavor non-singlet distribution, in the following manner

$$b + \bar{b} = \frac{1}{5}\left(F_S^{(5)} - F'^{(5)}_{NS}\right) \tag{11}$$

where

$$F'^{(5)}_{NS} = u + \bar{u} + c + \bar{c} + d + \bar{d} + s + \bar{s} - 4(b + \bar{b}) \quad . \tag{12}$$

The $F_S^{(5)}$ and $F'^{(5)}_{NS}$ at large Q^2 are again predicted by using the Altarelli-Parisi equations with four quark flavors to evolve $F_S^{(4)}$ and $F_{NS}^{(4)}$ from some Q_0^2 up to Q^2_{th}, then matching up with the five quark theory via the matching formulas

$$F_S^{(5)} = F_S^{(4)} \tag{13}$$

and

$$F'^{(5)}_{NS} = F_S^{(4)} \tag{14}$$

and finally evolving $F_S^{(5)}$ and $F'^{(5)}_{NS}$ up to the required Q^2 using the

Altarelli-Parisi equations with five quark flavors. The b-quark distribution alone can be extracted from Eq. (11) by noting that due to charge conjugation invariance of the strong interactions the difference between the b and anti-b distributions, $b-\bar{b}$, does not mix with the gluon distribution and hence is zero to the order which we are working.

In Figs. 1 and 2 we present predictions for the b-quark momentum distribution and F_2 at $Q^2 = 500$ GeV^2 and 5000 GeV^2. These are based on a fit[9] to SLAC-MIT electroproduction data performed in Ref. 10. We have used such large Q^2 values so that the leading-logarithmic approximation can be used with some confidence. For the transition point between the four and five quark theories we have used $Q^2_{th} = m^2_b = 20.25$ GeV^2. To get an idea of the uncertainties involved in our calculation Fig. 3 contains predictions for the b-quark distribution at $Q^2 = 500$ GeV^2 and 5000 GeV^2 when a transition point $Q^2_{th} = 4m^2_b = 80.1$ GeV^2 is used. It is important to realize that one can improve the treatment of the threshold by going beyond the leading logarithmic approximation[11] using the two-loop anomalous dimensions for the singlet, non-singlet and gluon operators and calculating the one-loop corrections to the matching conditions in Eqs. (9), (10), (13) and (14). Such a treatment would still neglect "higher-twist" effects of order m^2_b/Q^2 and Λ^2/m^2_b (where Λ is a typical light hadronic mass of order 1 GeV) however it is possible that some of the higher-twist effects could be included by using a modified scaling variable.[12] Finally, at very low x our perturbative predictions become unreliable due to the singular nature of the gluon evolution.

Comparing F_2 with the b-quark distribution reveals, for example, that at $Q^2 = 5000\ \mathrm{GeV}^2$ and $x = .2$ about 1/2% of the electroproduction events result from a photon striking a b- or $\bar{b}$-quark.[13] One should keep in mind, however, that b-quarks are also produced from events where the photon strikes a light quark and a $b\bar{b}$ pair is produced through a virtual gluon.

Finally we note that the methods used here can easily be extended[14] to include neutrino scattering and to treat the multiple thresholds which will occur if a t-quark exists. For example, the t-quark distribution in electroproduction can be predicted at $Q^2 \gg m_t^2$ by noting that in the six quark region

$$t + \bar{t} = \frac{1}{6}\left(F_S^{(6)} - F_{NS}^{(6)}\right) \tag{15}$$

where

$$F_S^{(6)} = u + \bar{u} + c + \bar{c} + t + \bar{t} + d + \bar{d} + s + \bar{s} + b + \bar{b} \tag{16}$$

and

$$F_{NS}^{(6)} = u + \bar{u} + c + \bar{c} + d + \bar{d} + s + \bar{s} + b + \bar{b} - 5(t + \bar{t}) \quad . \tag{17}$$

$F_S^{(6)}$ and $F_{NS}^{(6)}$ are matched at some $Q_{th}^{\prime 2}$, which is approximately equal to m_t^2, to the structure function $F_S^{(5)}$, defined in Eq. (3), via the matching relations

$$F_S^{(6)} = F_S^{(5)} \tag{18}$$

and

$$F_{NS}^{(6)} = F_S^{(5)} \quad . \tag{19}$$

$F_S^{(5)}$ is then determined as was previously discussed. Evolution of $F_S^{(6)}$ and $F_{NS}^{(6)}$ for $Q^2 > Q_{th}^{\prime 2}$ is governed, of course, by the Altarelli-Parisi equations with six quark flavors.

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9. The values we have used correspond to a fit to this data setting all higher-twist effects to zero. The presence of higher-twist effects can significantly alter the parameters of this fit. Also, the gluon distribution was not determined in the fit of Ref. 10. We have tried gluon distributions of the form $(1-x)^3$, $(1-x)^5$ and

$(1-x)^7$ with an overall normalization determined by the momentum sum rule. The results of Figs. 1-3 (which were obtained using $(1-x)^5$) are not greatly changed when $(1-x)^3$ or $(1-x)^7$ gluon distributions are used.

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11. Such corrections to our predictions for the b-quark distribution are expected to be significant when $\left(\alpha_s(Q^2)/\alpha_s(Q^2_{th})\right) \approx 1$. Therefore the results presented in Fig. 1b should be interpreted as qualitative. These corrections will indicate that a better choice of expansion variable would have been $xW^2 = (1-x)Q^2$ instead of simply Q^2. We thank S. J. Brodsky for pointing this out.

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14. One can make predictions for the charm quark distribution using these methods, however, this requires treating the charm quark as heavy compared with typical hadronic masses and therefore the results would be very qualitative.

FIGURE CAPTIONS

Fig. 1. Predictions for F_2^{ep} and the b-quark momentum distribution at $Q^2 = 500\ \mathrm{GeV}^2$ with $Q_{th}^2 = m_b^2$.

Fig. 2. Predictions for F_2^{ep} and the b-quark momentum distribution at $Q^2 = 5000\ \mathrm{GeV}^2$ with $Q_{th}^2 = m_b^2$.

Fig. 3. Predictions for the b-quark momentum distribution at $Q^2 = 500\ \mathrm{GeV}^2$ and $Q^2 = 5000\ \mathrm{GeV}^2$ with $Q_{th}^2 = 4m_b^2$.

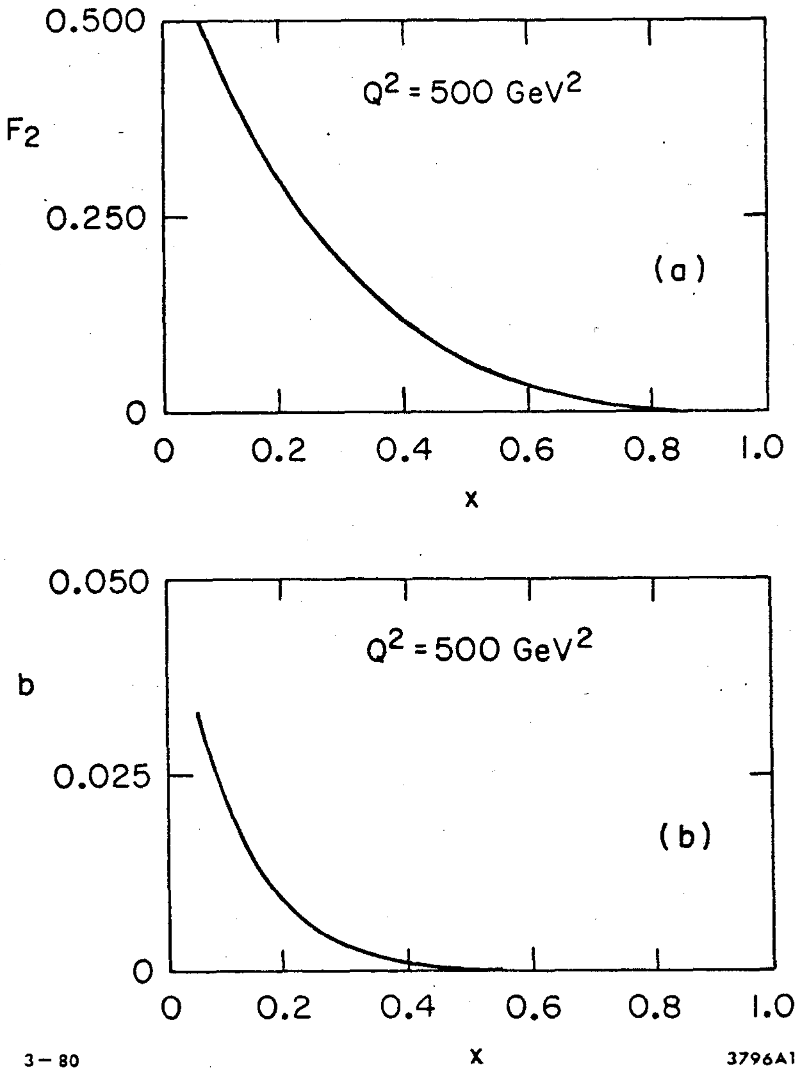
0.500
$Q^2 = 500\ GeV^2$
F_2
0.250
(a)
0
0
0.2
0.4
0.6
0.8
1.0
x
0.050
$Q^2 = 500\ GeV^2$
b
0.025
(b)
0
0
0.2
0.4
0.6
0.8
1.0
3-80
x
3796A1

Fig. 1

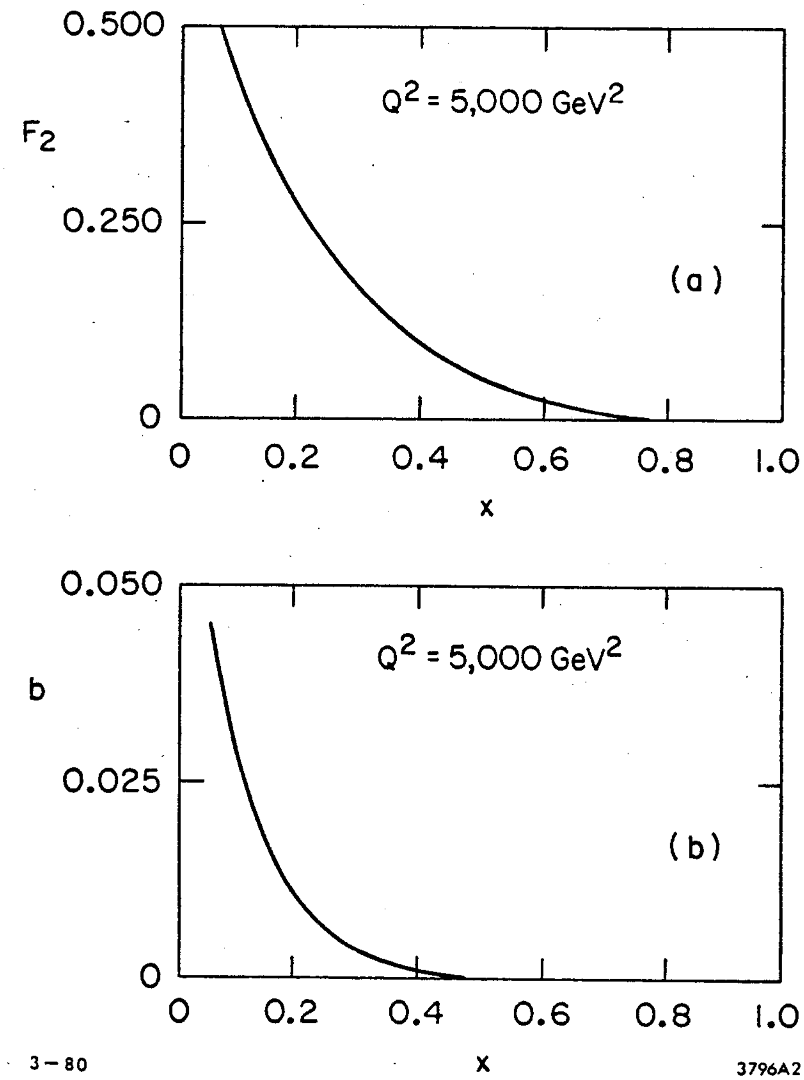
0.500
F2
0.250
0
Q2 = 5,000 GeV2
(a)
0 0.2 0.4 0.6 0.8 1.0
x
0.050
b
0.025
0
Q2 = 5,000 GeV2
(b)
0 0.2 0.4 0.6 0.8 1.0
x
3-80
3796A2

Fig. 2

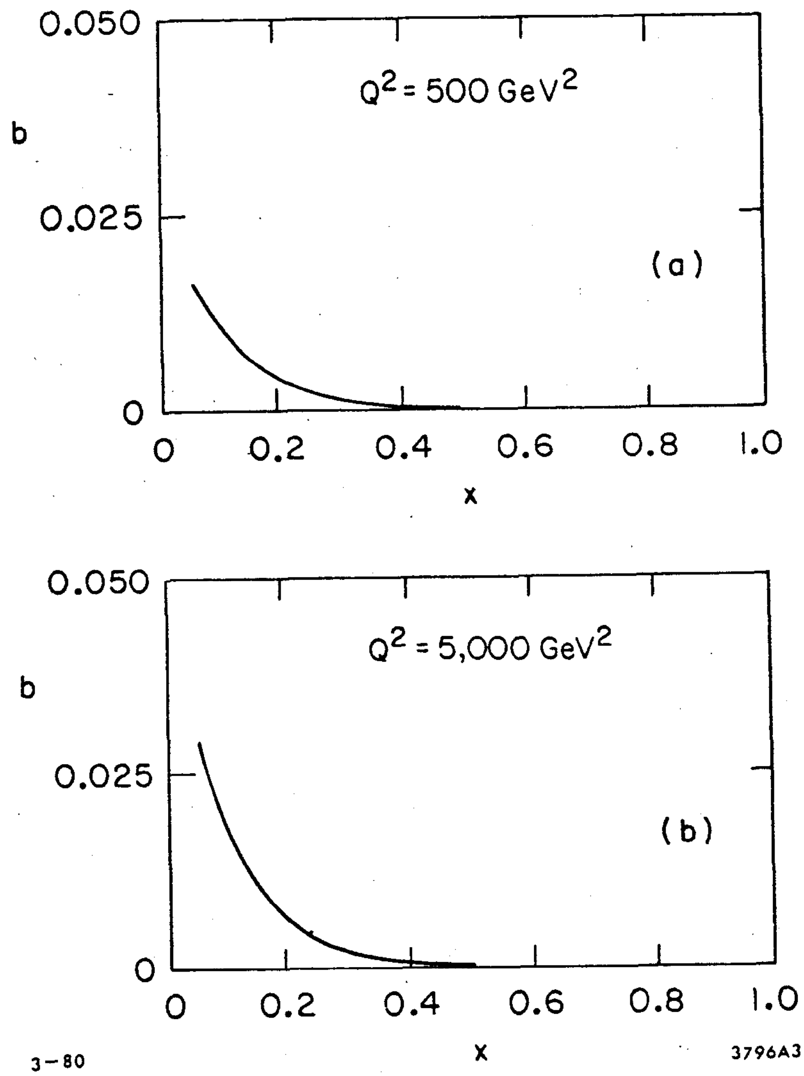
0.050
0.025
0
b
Q² = 500 GeV²
(a)
0
0.2
0.4
0.6
0.8
1.0
x
0.050
0.025
0
b
Q² = 5,000 GeV²
(b)
0
0.2
0.4
0.6
0.8
1.0
x
3-80
3796A3

Fig. 3